The Distributional Effects of Uneven Regional Growth*

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Abstract

Economic growth has varied tremendously across regions of the United States over the past several decades. In this paper, I develop a dynamic quantitative spatial model to study the distributional implications of this uneven growth. The model incorporates two key mechanisms that link local economic conditions to individual welfare: migration costs and homeownership. Using the model, I show that uneven regional growth has important distributional consequences. On average, a 1% shock to local productivity raises residents' welfare by 0.43%. The pass-through from local productivity shocks to welfare varies substantially across the age and wealth distribution. I find that homeownership plays a central role in spatial redistribution: in an otherwise similar model without homeownership, the average welfare effect of the same shock is just 0.16%. Finally, I analyze the effects of two housing policy counterfactuals: relaxing land-use regulations and eliminating the mortgage interest deduction. Both policies mitigate spatial redistribution. However, their effects are quantitatively small, so the link between local economic conditions and welfare remains strong.

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1 Introduction

Economic growth has varied tremendously across regions of the United States over the past several decades. While some areas have experienced extraordinary growth, others have stagnated or even declined. This disparity has led to heightened concerns that uneven regional growth may have important distributional consequences. Such worries are magnified by falling migration rates and rapidly increasing cost of living in high-growth areas. There is a growing sense that the benefits of rapid growth in select areas of the country are available only to a subset of incumbent residents, while being inaccessible to others.[1]

While there are good reasons to believe that uneven regional growth may have substantial distributional effects, its quantitative importance is not clear *a priori*. It depends on the strength of migration frictions and the importance of assets whose values are tied to the local economy, such as owner-occupied housing. Without migration costs or local assets, the welfare effect of a productivity shock to one location would not vary across space because utility would equalize across locations. Even without migration frictions, the welfare effects of such a shock would vary across space if some people own housing where they live. In this case, the effects of the shock would differ between local homeowners and others due to capital gains/losses on owner-occupied housing. Migration frictions further increase spatial redistribution. By making it costly to migrate in response to changing economic conditions, such frictions concentrate the effects of the shock on local residents.

Ultimately, the degree to which uneven regional growth causes redistribution at the household level is a quantitative question. Existing quantitative spatial economic models contain many of the ingredients needed to answer this question.[2] Specifically, they account for realistic geography, migration costs, and local congestion and agglomeration forces. However, virtually all abstract from homeownership, which is of first-order importance in this context.[3] Most households own a home, and the typical household invests the majority of its wealth in it: in the 2019 Survey of Consumer Finances, the homeownership rate was 64.9% and the median homeowner held 77.2% of its wealth in owner-occupied housing. The ownership of housing fundamentally shapes the distributional effects of local shocks. A productivity shock which raises wages in a given location will also raise local house prices in equilibrium. If housing is rented, housing costs move in the same direction as wages, thus mitigating the distributional effects of the shock. If instead housing is owned, residents receive capital gains and so house price changes can *exacerbate* the distributional effects of the shock.

In this paper, I develop a dynamic quantitative spatial model that is suitable for studying the distributional effects of uneven regional growth. The model incorporates incomplete markets, lifecycle

[1] For evidence of regional divergence and a discussion of its distributional consequences, see Moretti (2012). Molloy et al. (2011) and Kaplan and Schulhofer-Wohl (2017) document a long-term decline in internal migration in the United States. Ganong and Shoag (2017) show that high-income cities have experienced a disproportionate increase in house prices during the past thirty years, and argue that this has especially affected low-income households.

[2] See Redding and Rossi-Hansberg (2017) for a review of the quantitative spatial economics literature.

[3] A notable exception is Giannone et al. (2020). In contemporaneous work, they develop a spatial model with incomplete markets and homeownership to study the role of credit constraints and other frictions in determining migration patterns.

dynamics, and realistic homeownership into a quantitative spatial economic framework. It accounts for critical mechanisms that shape the relationship between local growth and welfare. Households face uninsurable income risk and can either rent or own housing. There are transaction costs associated with buying or selling housing, as well as a collateral constraint which limits borrowing as a function of housing wealth. These frictions endogenously generate observed lifecycle patterns of housing investment, and allow the model to capture the welfare effects of house price changes on various types of people. Households can live in one of numerous discrete locations, which correspond to cities in the data. They can move between locations subject to a migration cost, which limits their ability to respond to shocks via migration. The model also incorporates empirically estimated local house price elasticities. These ingredients allow it to match the endogenous responses of populations, wages, and house prices to local shocks.

While the model is well-suited for quantifying the distributional effects of uneven local growth, solving it presents a computational challenge. The combination of incomplete markets, lifecycle dynamics, housing frictions, and geography leads to a large state space. Realistic housing frictions also imply that the household's problem is nonconvex, which precludes the use of efficient discrete-time numerical algorithms.[4] It is imperative that the model be solved efficiently, because estimating a quantitative spatial model requires choosing a large number of parameters (specifically, location-specific parameters such as amenities). Moreover, analyzing the effects of local shocks when households make dynamic housing and migration decisions requires computing transition dynamics. I overcome these challenges by setting the model in continuous time and taking advantage of numerical techniques recently developed by Achdou et al. (2022). These are much more efficient at solving incomplete market models than discrete-time methods, and can handle nonconvexities without difficulty. An additional contribution of this paper is to show how continuous-time computational methods can be applied to quantitative spatial models. Doing so expands the types of heterogeneity and frictions that can be considered in such models, making it possible to ask new questions and incorporate model features that would otherwise be infeasible.

I calibrate the model to match important features of the data on housing investment and economic geography. Using the calibrated model, I quantitatively analyze the distributional effects of uneven regional growth. My analysis is based on the economy's dynamic response to spatially heterogeneous local productivity shocks. The particular shock I consider is based on U.S. cities' heterogeneous exposures to industries with varying growth rates during the past two decades. My first main result is that uneven regional growth has important distributional consequences. On average, a 1% increase in a city's productivity is estimated to raise welfare of local residents by 0.43%. The pass-through from local productivity shocks to welfare varies substantially across households depending on their age and housing tenure. Local productivity shocks affect homeowners more than renters, and middle-aged households more than the young or old. Whereas a productivity increase leads to welfare gains for most households, it *decreases* welfare for old renters. These households do

[4] In particular, the endogenous gridpoint method (Carroll, 2006) cannot be used because the first-order conditions may not be sufficient.

not not benefit from increased wages because they do not participate in the labor market, and do not receive any capital gains from higher house prices.

In the next set of results, I examine the role of homeownership in shaping the welfare effects of uneven local growth. To do this, I repeat the baseline estimation and quantitative analysis in a special case of the model where homeownership is prohibited. Without homeownership, the average welfare effect of a unit local productivity shock is just 0.16. This suggests that accounting for homeownership is essential when analyzing the distributional effects of shocks that have heterogenous impacts across space. Models that do not include homeownership are likely to underestimate such effects.

Motivated by these findings, I next analyze two housing policy counterfactuals that have the potential to mitigate the distributional effects of uneven local growth. The first is to reduce land-use regulations. Such regulations increase capital gains for incumbent homeowners in areas that receive positive shocks, and vice versa. On the other hand, they mitigate welfare effects for renters by raising rental costs in places that receive positive shocks and lowering them in places that receive negative shocks. On balance, I find that even a large reduction in land-use regulations would have only a quantitatively small effect on spatial redistribution. Capping the level of regulations at that of the least-regulated city in my dataset would only reduce the average welfare effect of a unit local productivity shock from 0.43 to 0.42. The second counterfactual I consider is to eliminate the mortgage interest deduction for homeowners. This policy reduces both homeownership and housing investment conditional on owning. The average welfare effect of a unit local productivity shock without the mortgage interest deduction is 0.38. These experiments suggest that there is some scope for policy to mitigate the distributional effects of uneven local growth. However, even if these policies were instituted, uneven regional growth would likely still cause substantial redistribution.

Related Literature This paper contributes to a growing literature that uses dynamic spatial models to analyze questions for which space and forward-looking decisions are salient. Important papers in this literature include Desmet and Rossi-Hansberg (2014), Giannone (2017), Desmet et al. (2018), Caliendo et al. (2019), Eckert and Kleineberg (2019), Lyon and Waugh (2019), Giannone et al. (2020), Bilal and Rossi-Hansberg (2021), Kleinman et al. (2021), and Lagakos et al. (2021). My primary contribution to this literature is to incorporate a realistic model of homeownership into a quantitative spatial framework. Solving such a model is challenging because accounting for lifecycle and illiquid housing (which are needed to match housing investment behavior) requires dealing with nonconvexities and a large state space. I overcome these challenges using continuous-time computational methods recently developed by Achdou et al. (2022).

My work is also closely related to a literature that explores the distributional effects of uneven local growth. Important papers in this literature include Moretti (2013), Yagan (2014), Diamond (2016), Eckert (2019), Ganong and Shoag (2017), Fogli and Guerrieri (2019), and Notowidigdo (2020). I contribute to this literature by estimating the distributional effects of local shocks in a framework that explicitly accounts for incomplete markets, lifecycle, and homeownership. Since housing is one of the most important channels through which welfare is tied to local economic conditions, and the effects of a local shock vary depending on age and wealth, this enables me to obtain more accurate and disaggregated estimates.

Another set of related papers use incomplete market models with housing to study the welfare effects of house price changes. Examples of papers that study the effects of housing wealth on consumption include Fernandez-Villaverde and Krueger (2011), Berger et al. (2018), Kaplan et al. (2020b), and Jones et al. (2022). Also related is Kiyotaki et al. (2011), who study how the effects of an aggregate house price shock differ across the population. I contribute to this literature by accounting for economic geography. This allows me to quantify the importance of housing wealth in transmitting spatially heterogenous shocks to the welfare of different types of households.

Finally, this paper connects to a literature that examines the macroeconomic effects of land-use regulation. Herkenhoff et al. (2018) and Hsieh and Moretti (2019) find that these regulations are an important source of spatial misallocation and have substantially reduced growth in the United States during the past several decades. Ganong and Shoag (2017) argue that they have also contributed to the slowdown in U.S. internal migration and regional income convergence. I contribute to this literature by quantifying the *distributional* effects of land-use regulations. For this purpose, it is crucial to account for dynamics and homeownership in order to capture the capital gains/losses that accrue to incumbent homeowners in areas that experience different rates of economic growth.

The remainder of the paper is organized as follows. In Section 2, I develop the theoretical framework. Section 3 describes how the model is estimated. In Section 4, I use the model to quantitatively analyze the distributional effects of uneven regional growth. Section 5 assesses the impact of policies that have the potential to mitigate such distributional effects. Finally, in Section 6 I conclude.

2 Model

In this section, I describe the theoretical framework. The model contains two primary components. The first is geography. There are a large number of locations in which households live and work. Locations differ in their wages, house prices, and amenities. Households can migrate across locations, but must pay a moving cost to do so. This component is based on standard quantitative spatial models, reviewed by Redding and Rossi-Hansberg (2017).

The second component incorporates homeownership. Most households own a home, and owner-occupied housing accounts for the majority of the typical household's wealth. Local shocks lead to house price changes that affect homeowner's wealth, so it is essential to account for homeownership when estimating the distributional effects of uneven regional growth. In order to realistically model homeownership, I use a lifecycle model with incomplete markets and idiosyncratic risk. Housing services can be obtained either by renting or investing in illiquid owner-occupied housing. These ingredients are necessary to capture important features of housing investment, such as the fact that homeownership rates and the fraction of wealth invested in housing vary substantially over the lifecycle. They are also needed to accurately estimate the welfare effects of shocks that cause house prices to change. This component of the model is based on standard incomplete market models with homeownership, reviewed by Piazzesi and Schneider (2016).

2.1 Environment

Time t is continuous. The economy is populated by a unit mass of finitely-lived households. A household's age is indexed by $j \in [0, J]$, where J is the maximum possible age. They can live in one of I locations, indexed by $i \in \{1, ..., I\}$. Households demand two types of goods: a consumption good c and housing services $\mathbf{h}$. The consumption good is freely traded and serves as the numeraire. Housing is non-tradeable across space, and so its price differs by location. All markets are competitive and there is no aggregate uncertainty.

2.2 Households

Preferences Households derive utility from location-specific amenities A_i, consumption c, and housing services $\mathbf{h}$ according to the flow utility function[5]

$$u(A, c, \mathbf{h}) = \ln(Ac^{1-\eta}\mathbf{h}^{\eta})$$

They discount the future at rate ρ and do not have any bequest motive.

Migration Migration between locations is only allowed at discrete time intervals of length Δ. I denote the set of ages at which migration is allowed by $\mathcal{S} \equiv \{0, \Delta, 2\Delta, ...J - \Delta\}$.[6] When migration is allowed, households draw a vector of idiosyncratic location preferences $\varepsilon = (\varepsilon_1, ..., \varepsilon_I)$. The elements of ε are drawn independently from a Type-1 Extreme Value (Gumbel) distribution with scale parameter ν[7]

$$F(\varepsilon) = \exp(-\exp(-\varepsilon/\nu - \overline{\gamma}))$$

After observing ε, they choose their new location. The variance of ε is increasing in ν, so higher values imply that idiosyncratic location preference draws are more important determinants of migration decisions. As is common in the literature, I refer to ν as the "inverse migration elasticity." The higher is ν, the less responsive are migration flows to changes in local prices and amenities, and vice versa. There is also a moving cost $\kappa_{ii'}(j)$ required to move from i to $i' \neq i$, which may vary by origin, destination, and age. Both idiosyncratic location preferences and moving costs are measured

[5] Diamond and Moretti (2021) find that local price indices are well-approximated using only data on local housing costs. Accordingly, I do not include non-tradeables other than housing in the model.

[6] I assume that J is divisible by Δ.

[7] $\overline{\gamma} \equiv \int_{-\infty}^{\infty} x \exp(-x - \exp(-x))dx$ is the Euler-Mascheroni constant, which is added so that $\mathrm{E}(\varepsilon) = 0$.

in terms of utility and are enjoyed/paid continuously until the next migration opportunity occurs. As such, the present discounted value of idiosyncratic location preference ε and moving cost κ when migration occurs is

$$\int_0^{\Delta} e^{-\rho s}(\varepsilon - \kappa)ds = \tilde{\rho}(\varepsilon - \kappa)$$

where $\tilde{\rho} \equiv (1 - e^{-\rho\Delta})/\rho$.

This way of modeling migration is common in dynamic quantitative spatial models. It is a tractable way of generating important features of the migration data, such as the fact that gross flows are larger than net flows and that population allocations respond gradually to shocks. In discrete time, it is natural to only allow migration at the frequency with which time is discretized. In continuous time, the frequency with which migration should be allowed is a less straightforward choice. With Gumbel-distributed idiosyncratic location preferences, allowing migration at all times would lead to households continuously migrating, which is obviously counterfactual. I instead make explicit the assumption that migration is only allowed with finite frequency, which is the implicit assumption made in similar discrete-time models.

Earning Households work from birth until the retirement age J_R. They receive a stochastic stream of labor endowments l, which they supply inelastically at wage w_{it} in their location. Labor endowments consist of a deterministic lifecycle component $\bar{l}(j)$ and an idiosyncratic component ℓ:

$$\ln l = \bar{l}(j) + \ell$$

The idiosyncratic component follows an AR(1) process with innovations drawn at ages $j \in \mathcal{S}$:

$$\ell_j = \theta \ell_{j-\Delta} + \zeta_j$$
$$\zeta_j \sim N(0, \sigma^2)$$

Labor endowment shocks are observed prior to making migration decisions. From now on, I denote the probability density function of ζ by f_ζ. After retirement, households receive a pension ϖ_t.

Housing Households can obtain housing services either by renting or owning. The quantity of housing a household rents is denoted by h^r and the quantity it owns by h. Renting and owning simultaneously is prohibited, and households can only rent or own in the location where they live. Rental prices are denoted by p^r_{it} and house prices by p_{it}.

Owner-occupied housing can be bought or sold at any time, but there are proportional transaction costs required to do so. Whenever a household changes its location or quantity of owner-occupied housing, it must pay fraction f_s of the value of its old house and fraction f_b of the value of its new house. Housing depreciates at rate δ^h.

Rented housing translates one-for-one into housing services, while each unit of owner-occupied housing provides χ units of services. The amount of housing services enjoyed by a household with h^r units of rental housing and h units of owner-occupied housing is

$$\mathbf{h} = \begin{cases} h^r & \text{if } h = 0 \\ \chi h & \text{if } h^r = 0 \end{cases}$$

Borrowing and Saving Households can borrow and save in a liquid, risk-free asset a at net interest rate r. They face a collateral constraint which restricts borrowing to no more than a fraction ϕ of their housing wealth:

$$a \geq -\phi p_{it} h$$

Taxes Households pay an income tax of rate τ on all income (earnings, pensions, and capital income). Homeowners are allowed to deduct mortgage interest payments from their taxable income.

Demographics Households survive with probability $\psi(j)$ upon reaching age $j \in \mathcal{S}$. I denote the unconditional probability of surviving to age j by $\Psi(j)$. When a household dies, it is replaced by a new one. Newborn households do not have any wealth, draw their initial labor endowment from the invariant distribution, and their initial location from a population distribution $N_t^0(i)$.

Household Value Function It is convenient to work with the variable

$$x = a + \phi p_{it} h$$

which is liquid wealth held in excess of the borrowing constraint. From now on, I refer to x as "voluntary wealth." Using this notation, the collateral constraint is $x \geq 0$. The household's state variables are x, owner-occupied housing h, location i, idiosyncratic labor endowment ℓ, age j, idiosyncratic location preference ε, and moving cost κ.

In between housing and location changes, x evolves according to the budget constraint

$$\dot{x} = (1-\tau)\left(ra + \underbrace{\begin{cases} w_{it}\exp(\bar{l}(j) + \ell) & \text{if } j < J_r \\ \varpi_t & \text{if } j \geq J_r \end{cases}}_{\text{earnings and pension}} \right) - c - p_{it}^r h^r - \underbrace{[\delta^h p_{it} - (1+\phi)\dot{p}_{it}]h}_{\text{user cost of } h}$$

The first term of this expression is after-tax income. The final term captures the effects of owner-occupied housing on voluntary wealth. This includes housing depreciation, as well as capital gains and changes in borrowing capacity due to house price changes.

Voluntary wealth after adjusting owner-occupied housing to h' and/or moving to location to i' is

$$x'(h',i') = \begin{cases} x & \text{if } i' = i \text{ and } h' = h \\ x + (1 - f_s - \phi) p_{it} h - (1 + f_b - \phi) p_{i't} h' & \text{otherwise} \end{cases}$$

At ages $j \notin \mathcal{S}$ between when idiosyncratic shocks occur, the household's value function satisfies the Hamilton-Jacobi-Bellman (HJB) equation

$$\rho V_t(x,h,i,\ell,j,\varepsilon,\kappa) = \max_{c,h^r} u(A_i, c, \mathbf{h}) + \varepsilon - \kappa + \partial_x V_t(x,h,i,\ell,j,\varepsilon,\kappa)\dot{x} + \dot{V}_t(x,h,i,\ell,j,\varepsilon,\kappa) \quad (1)$$

$$\begin{aligned} \text{subject to} \quad & h^r = 0 \text{ if } h > 0 \\ & \dot{x} \geq 0 \text{ if } x = 0 \\ & V_t(x,h,i,\ell,j,\varepsilon,\kappa) \geq \max_{h':x'(h',i)\geq 0} V_t(x'(h',i),h',i,\ell,j,\varepsilon,\kappa) \end{aligned}$$

The first constraint prevents owning and renting housing simultaneously. The second ensures that the collateral limit is obeyed. The final constraint reflects the fact that households always choose owner-occupied housing h optimally (subject to the collateral constraint). In Appendix B.1, I characterize the household's policy functions. I show there that the policy functions are all independent of ε and κ, conditional on location i. This is due to the fact that ε and κ affect utility additively, and is key to the model's tractability.

For what follows, it is convenient to work with the value function net of idiosyncratic location preferences and moving costs, which I denote by

$$\overline{V}_t(x,h,i,\ell,j) \equiv V_t(x,h,i,\ell,j,0,0)$$

Note that

$$V_t(x,h,i,\ell,j,\varepsilon,\kappa) = \overline{V}_t(x,h,i,\ell,j) + \int_0^{\Delta(j)} e^{-\rho s}(\varepsilon - \kappa) ds$$

where $\Delta(j) \equiv \min\{s \in \mathcal{S} \cup J | s > j\} - j$ is the length of time until the next realization of idiosyncratic shocks (if $j < J - \Delta$) or death (if $j \geq J - \Delta$). Denote the value of $\overline{V}$ at ages immediately after idiosyncratic shocks are realized by

$$\overline{V}_t^+(x,h,i,\ell,j) \equiv \lim_{\iota \downarrow 0} \overline{V}_{t+\iota}(x,h,i,\ell,j+\iota), \ j \in \mathcal{S}$$

Optimal owner-occupied housing conditional on choosing location i' when migration is allowed is

$$h^m(i') = \operatorname*{argmax}_{h':x'(h',i')\geq 0} \overline{V}_t^+(x'(h',i'),h',i',\ell,j)$$

and the associated value of x is $x^m(i') \equiv x'(h^m(i'),i')$. Indirect utility net of idiosyncratic location preferences and moving costs conditional on choosing i' when migration is allowed is

$$V_t^m(x,h,i,\ell,j,i') \equiv \overline{V}_t^+(x^m(i'),h^m(i'),i',\ell,j)$$

In Appendix B.3, I show that the value function at ages $j \in \mathcal{S}$ is

$$V_t(x,h,i,\ell,j) = \psi(j) \int \left[\tilde{\nu} \ln \sum_{i'} \exp(V_t^m(x,h,i,\theta\ell+\zeta,j,i') - \tilde{\kappa}_{ii'}(j))^{1/\tilde{\nu}} \right] f_\zeta(\zeta) d\zeta \tag{2}$$

where $\tilde{\nu} \equiv \tilde{\rho}\nu$ and $\tilde{\kappa}_{ii'}(j) \equiv \tilde{\rho}\kappa_{ii'}(j)$. Note that since idiosyncratic location preferences and migration costs are only enjoyed/paid for Δ units of time after being incurred (and ε is i.i.d.), ε and κ are not state variables at ages $j \in \mathcal{S}$. Equation (2) is expected indirect utility after realizing the mortality, location preference, and labor endowment shocks. The fact that ε has a Gumbel distribution makes it possible to write this expression in closed form.

Since there is no bequest motive, the value function at the maximum age is $V_t(x,h,i,\ell,J) = 0$. Together with this boundary condition, equations (1) and (2) characterize the household value function and optimal behavior throughout the lifecycle.

Density of State Variables Denote the density of state variables by $g_t(x,h,i,\ell,j)$.[8] Ignoring owner-occupied housing adjustments, at ages $j \notin \mathcal{S}$ the density of state variables evolves according to the Kolmogorov Forward (KF) equation[9]

$$\dot{g}_t(x,h,i,\ell,j) = -\partial_x[\dot{x} g_t(x,h,i,\ell,j)] \tag{3}$$

In Appendix B.2, I show that the fraction of households who choose location i' when migration is allowed is

$$m(i') = \frac{\exp(V_t^m(x,h,i,\ell,j,i') - \tilde{\kappa}_{ii'}(j))^{1/\tilde{\nu}}}{\sum_{i''} \exp(V_t^m(x,h,i,\ell,j,i'') - \tilde{\kappa}_{ii''}(j))^{1/\tilde{\nu}}}$$

Denote the density of state variables at ages immediately before idiosyncratic shocks are realized by

$$g_t^-(x,h,i,\ell,j) \equiv \lim_{\iota \downarrow 0} g_{t-\iota}(x,h,i,\ell,j-\iota),\ j \in \mathcal{S}$$

The density of state variables at ages $j \in \mathcal{S}$ when idiosyncratic shocks occur is

$$g_t(x,h,i,\ell,j) = \psi(j) \int\int\int \left[\sum_{\tilde{i}=1}^{I} \mathbb{1}(x^m(i) = x, h^m(i) = h) m(i) g_t^-(\tilde{x},\tilde{h},\tilde{i},\tilde{\ell},j) \right] f_\zeta(\ell - \theta\tilde{\ell}) d\tilde{x} d\tilde{h} d\tilde{\ell} \tag{4}$$

[8]As shown below, it is not necessary to keep track of the state variables ε and κ when computing equilibria or the welfare effects of shocks, which are the only things I use the density of state variables for.

[9]As discussed by Kaplan et al. (2020a), it is not straightforward to mathematically characterize the Kolmogorov Forward equation of optimal stopping time problems. For expositional clarity, I show the KF equation here without housing adjustments. It is, however, straightforward to describe how housing adjustments are dealt with in the numerical algorithm, which I do in Appendix D.

This expression integrates over the subset of the state space in which households who choose location i will have wealth and owner-occupied housing (x, h) after doing so, weighted by the probability of choosing i. It accounts for both mortality risk and the likelihood that the idiosyncratic component of labor supply will be ℓ after the labor endowment shock is realized.

Together with the distribution of initial conditions described above, equations (3) and (4) determine the density of state variables.

2.3 Production

In each location, a representative firm produces the numeraire good using the Cobb-Douglas technology

$$Y_{it} = \overline{Z}_{it} L_{it}^{\alpha} K_{it}^{1-\alpha} \tag{5}$$

where $\overline{Z}$ is total factor productivity, L is labor, and K is physical capital. I assume that factor markets are competitive and that capital is freely mobile across locations. Under these assumptions, wages can be written as

$$w_{it} = Z_{it} \equiv \left[\overline{Z}_{it} \alpha \left(\frac{1-\alpha}{\alpha R_t} \right)^{1-\alpha} \right]^{1/\alpha} \tag{6}$$

where R_t is the gross return to capital. Understanding the welfare implications of shocks to local labor productivities Z_{it} is the focus of my quantitative analysis in Section 4.

In the baseline model, I abstract from both congestion and agglomeration forces in production. I examine the effects of incorporating such forces in Appendix C.1.

2.4 Housing and Capital Markets

Rental housing is owned by absentee landlords, who supply housing inelastically at rental price

$$p_{it}^{r} = \overline{p}_{it}^{r} N_{it}^{\xi_i} \tag{7}$$

where $\overline{p}_{it}^{r}$ is an exogenous price shifter, ξ_i are location-specific house price elasticities, and

$$N_{it} = \int \int \int \int g_t(x, h, i, \ell, j) dx dh d\ell dj$$

are local populations. If $\xi_i > 0$ (the empirically relevant case), rental costs are increasing in local population. This could be due, for example, to a scarce factor of production such as land or regulations that limit population growth. I choose this specification to be consistent with Saiz (2010)'s house price elasticity estimates, which I use later in the quantitative analysis.

I assume that house prices are the discounted value of future rents:

$$p_{it} = \int_{t}^{\infty} e^{-(r+\delta^h)(s-t)} p_{is}^{r} ds \tag{8}$$

Equation (8) ensures that the return to rental housing and the risk-free asset are the same. Note that since there is no uncertainty regarding rental prices, there is no reason that future rents should be discounted at a different rate than the opportunity cost of investing in the risk-free asset. In addition, in a stationary equilibrium in which prices are time-invariant, rents and house prices are related by $p_i^r = (r + \delta^h)p_i$.

Finally, I assume that both the interest rate on the risk-free asset and the return to physical capital are set exogenously. This would be the case, for example, in a small open economy.

2.5 Government

There is a government which receives revenue from taxing income and accidental bequests. Accidental bequests are fully expropriated, so there are no inheritances. The government uses its revenue to fund the pension system and other government spending G_t, which does not affect households' marginal propensities to consume. I assume that the government always runs a balanced budget.

2.6 Definition of Equilibrium

An equilibrium is household value and policy functions $V_t(x, h, i, \ell, j, \varepsilon, \kappa)$, $c_t(x, h, i, \ell, j)$, $h_t^r(x, h, i, \ell, j)$, and $h_t(x, h, i, \ell, j)$; density of state variables $g_t(x, h, i, \ell, j)$; prices w_{it}, p_{it}^r, and p_{it}; and income tax rate τ, such that, given an interest rate r_t, pension benefit ϖ_t, and government spending G_t:

1. Households optimize: the value and policy functions satisfy (1) and (2)
2. The density of state variables is consistent with the policy functions: (3) and (4) are satisfied
3. Wages are set competitively: (6) holds
4. Rental and house prices are set according to (7) and (8)
5. The government's budget balances

A stationary equilibrium is one in which all equilibrium objects are time-invariant. The algorithm used to numerically compute equilibria is described in Appendix D.

3 Estimation

In this section, I describe how the model is estimated. My overall goal is to choose parameters so that the model matches important features of the spatial data and data on household saving and homeownership. I begin by describing the data in subsection 3.1. In subsection 3.2, I discuss my procedure for bringing the model to this data. Lastly, in subsection 3.3, I examine the model's ability to match untargeted moments of interest.

The unit of time used for estimation is one year. The locations are the 50 most populous Metropolitan Statistical Areas (MSAs) in the United States as of the year 2000. At that time, these cities accounted for 54.3% of the total U.S. population and 69.7% of the population that lived in an MSA.

3.1 Data

In this subsection, I describe the data sources and procedures used to construct the variables used for estimation. Statistics for each city are displayed in Appendix Table 9.

3.1.1 Data Sources

Decennial Census My primary source of geographical data is the 5% public use sample of the 2000 Decennial Census, obtained from IPUMS (Ruggles et al., 2021). In order to be consistent with the model calibration, I keep only individuals between the ages of 20 and 99. I also exclude individuals who are living in group quarters. Results obtained using this dataset are computed using person weights.

American Community Survey I take migration data from the public use sample of the 2005 American Community Survey (ACS). This is the earliest year in which annual migration data at the MSA level is available. I also use data from the public use sample of the 2019 ACS to construct the productivity shock on which my quantitative analysis is based. These data were obtained from IPUMS. As with the Decennial Census data, I exclude individuals outside the ages of 20 and 99 or who live in group quarters. Results obtained using these datasets are computed using person weights.

Survey of Consumer Finances The main source of data for household saving and homeownership decisions is the 2001 Survey of Consumer Finances (SCF). The SCF is available on a triennial basis, so these data are based on observations from 1999 to 2001. As with the Census data, I keep only people between the ages of 20 and 99. To be consistent with the model, I also keep only observations with non-negative wealth. Results obtained using this dataset are computed using household weights.

3.1.2 Data Construction

Population and Employment Shares I compute population and employment shares for each MSA using the 2000 Decennial Census. Population shares are computed with the full sample, while employment shares are computed with the sample used to compute wages (described below).

Wages I compute wages in each MSA using the 2000 Decennial Census. The sample of workers consists of individuals between the ages of 20 and 64 who are employed and worked at least 35 hours per week for 40 weeks during the past year. I exclude people who are self-employed, have farm or business income, whose income is imputed, or have missing data on income, education, or demographics.

An individual's raw wage is their wage and salary income during the past year divided by weeks worked in the last year times usual hours worked per week. In order to control for selection on worker characteristics that my model abstracts from, wages are the residual of a regression that controls for observables that are important determinants of earnings. In particular, I estimate the regression

$$\ln w_n = \text{c} + \mathbf{\Gamma X}_n + \epsilon_n$$

where w_n is the raw wage of person n, c is a constant, $\mathbf{X}_n$ is a vector of observable characteristics, and ϵ_n is the residual. $\mathbf{X}$ contains age, age squared, a dummy for gender, a series of race dummies, and a series of education dummies. The race dummies are black and white (with all other categories the omitted group) and the education dummies are high school, some college, college, and post-graduate (with less than high school the omitted group). The wage in a location is the average residual wage $\exp(\epsilon_n)$ among workers in that location. Without loss of generality, I normalize the population-weighted average wage across locations to 1.

House Prices I compute house prices using the 2000 Decennial Census. The house price in each MSA is the median house value reported by homeowners. I exclude values that are top-coded or imputed. As with wages, I normalize the population-weighted average house price to 1.

Migration Rates Migration rates are calculated using the 2005 ACS. The migration rate is the number of individuals who changed MSAs in the last year divided by the number who lived in an MSA both now and one year ago.

Household Portfolios I compute the median wealth/income ratio, homeownership rate, and median housing wealth share of homeowners using the 2001 SCF. I define wealth as net worth (value of assets minus liabilities), and housing wealth as the value of owner-occupied housing.

3.2 Estimation Procedure

In this subsection, I describe my estimation procedure. As is standard in quantitative spatial models, I choose location-specific amenities, productivities, and house price shifters to exactly match population and price data for each location. The parameters related to geographic mobility are selected to match important moments of the migration data. Finally, the parameters related to borrowing/saving and housing are chosen to match key features of household wealth and portfolio data.

3.2.1 Externally Calibrated Parameters

I take many of the model's parameters either directly from data or existing literature. In this subsection, I describe how these parameter values are chosen. Table 1 provides a summary of the externally calibrated parameters.

Demographics Households live for at most $J = 80$ years, from age 20 to 100. They retire at age 65 ($J_R = 45$). Idiosyncratic shocks occur at annual frequency: $\Delta = 1$. I take survival rates $\psi(j)$ from Bell and Miller (2005). When new households are born, they draw their initial location from the current population distribution.

Earning I set the persistence and volatility parameters of the idiosyncratic earning process equal to the values estimated in Floden and Lindé (2001). These are $\theta = 0.9136$ and $\sigma^2 = 0.0426$, respectively. The deterministic lifecycle component of earnings $\bar{l}(j)$ is taken from Hansen (1993). The pension benefit is set equal to 50% of average worker earnings in the initial steady state.

Housing I set the housing transaction costs equal to the median costs estimated by Gruber and Martin (2005). The selling cost is $f_s = 7\%$ of the house's value, and the buying cost is $f_b = 2.5\%$. I set the housing depreciation rate to $\delta^h = 2\%$ and the collateral constraint to $\phi = 0.8$, both of which are within the range of commonly used values discussed by Piazzesi and Schneider (2016).

Other I set the interest rate to $r = 4\%$, which is the standard value mentioned by Davis and Van Nieuwerburgh (2015). As discussed above, the location-specific house price elasticities ξ_i are taken from Saiz (2010).

3.2.2 Internally Estimated Parameters

The remaining parameters of the model are internally estimated. These are the discount factor ρ, homeownership preference χ, housing preference weight η, migration costs $\tilde{\kappa}_{ii'}(j)$, inverse migration elasticity $\tilde{\nu}$, tax rate τ, local amenities A_i, local productivities Z_i, and local house price shifters $\overline{p}_i^r$. All except $\tilde{\nu}$ are chosen so that the initial stationary equilibrium of the model matches cross-sectional data from (or as close as possible to) the year 2000. $\tilde{\nu}$ is chosen to match an empirical estimate of the long-run labor supply response to an MSA-level productivity shock. I use data from

Table 1: Externally Calibrated Parameters

Parameter	Description	Value	Source
Demographics			
J	Maximum age	80	Max lifespan 20-100
J_R	Retirement age	45	Retire at age 65
$\psi(j)$	Survival rates		Bell and Miller (2005)
Earning			
θ	Persistence	0.9136	Floden and Lindé (2001)
σ^2	Volatility	0.0426	Floden and Lindé (2001)
$\bar{l}(j)$	Lifecycle		Hansen (1993)
ϖ	Pension		Replacement ratio = 50%
Housing			
f_s	Transaction cost: sell	7%	Gruber and Martin (2005)
f_b	Transaction cost: buy	2.5%	Gruber and Martin (2005)
δ^h	Depreciation rate	2%	Piazzesi and Schneider (2016)
ϕ	Collateral constraint	80%	Piazzesi and Schneider (2016)
Other			
r	Interest rate	4%	
ξ_i	House price elasticities		Saiz (2010)

Notes: This table displays the parameter values which are either taken directly from data, come from existing literature, or are set to standard values. The unit of time used throughout my estimation procedure is one year.

2000 to estimate the initial stationary equilibrium because it is the earliest date at which all variables are available on a consistent basis. Table 2 provides a summary of the internally estimated parameters.

Preferences The discount factor ρ, homeownership preference χ, and housing preference weight η are chosen to match the median wealth to income ratio (2.35), homeownership rate (71.7%), and median housing wealth share of owners (78.5%) from the 2001 SCF, respectively.

Migration Cost I parameterize the migration cost as a fixed (across locations) cost that increases linearly with age:

$$\tilde{\kappa}_{ii'}(j) = \tilde{\kappa}_0 + \tilde{\kappa}_1 j$$

$\tilde{\kappa}_0$ and $\tilde{\kappa}_1$ are chosen to match the overall migration rate (3.01%) and the difference in migration rates between working-age and retired individuals (1.99%: 3.33% for working-age and 1.34% for retired) from the 2005 ACS. Allowing the migration cost to increase with age is necessary to generate the observed decline in migration rates over the lifecycle. When I impose $\tilde{\kappa}_1 = 0$, the model predicts too small of a decrease in migration rates over the lifecycle. The model does include some features that are likely important reasons why mobility decreases with age. Specifically, younger households have longer to enjoy the benefits of moving to a more desirable location, and older households are more likely to own a home, which raises the cost of moving due to transaction costs. However, there are other important factors which are not included in the model, such as changing family sizes and local social networks that deepen over time. The parameter $\tilde{\kappa}_1$ is a reduced-form way to capture such factors.

Inverse Migration Elasticity I estimate the inverse migration elasticity $\tilde{\nu}$ using indirect inference. In particular, I choose $\tilde{\nu}$ to match the long-run effect of local productivity shocks on MSA-level employment estimated by Hornbeck and Moretti (2022, henceforth HM). HM estimate the instrumental variable regression

$$\Delta \ln L_i = \pi \Delta \ln \hat{A}_i + \alpha_r + \epsilon_i \tag{9}$$

where $\Delta \ln L_i$ is the change in log employment of city i between 1980 and 2010, $\Delta \ln \hat{A}_i$ is the predicted change in log productivity[10] of city i between 1980 and 1990, α_r is a Census region fixed effect, and ϵ_i is the residual. They obtain the estimate $\hat{\pi} = 4.03$. I use this "identified moment" (Nakamura and Steinsson, 2018) to estimate $\tilde{\nu}$.

In order to replicate HM's experiment in my model, I require a set of local productivity shocks. In principle, any shock could be used for this purpose. I focus on a shock that is caused by heterogeneous local exposures to industry-specific growth rates. An advantage of this choice is that it is based on an empirically important source of local variation in economic growth. The productivity shock I use is based on an industry shift-share variable constructed using the methodology of Bartik (1993). I first compute 3-digit industry employment shares in each location using the 2000 Decennial Census. The sample is the same as that used to compute wages. I then compute average residual wages in 2000 and 2019 for each industry using the same procedure used to compute residual wages by location. The data source for 2000 industry wages is the 2000 Decennial Census, and the data source for 2019 industry wages is the 2019 ACS. I use the time period 2000-2019 to limit the extent to which my results are driven by short-run fluctuations or extraordinary disruptions associated with the mid-2000s housing boom-bust episode and Covid-19 pandemic. Let f_{is} denote location i's industry-s employment share in 2000 and w_{st} the industry-s, year t wage. The unscaled productivity shock in location i is

$$\Delta \ln Z_i = \ln \left(\sum_s f_{is} w_{s2019} \right) - \ln \left(\sum_s f_{is} w_{s2000} \right)$$

In order to focus on distributional (versus aggregate) effects, I scale productivity shocks so that the population-weighted average shock is 0. As such, these should be thought of as relative productivity shocks.

Given the local productivity shock, I estimate $\tilde{\nu}$ as follows. First, I compute the initial stationary equilibrium conditional on a guess for $\tilde{\nu}$ (and the other internally estimated parameters). I then compute the new steady state that would obtain after local log productivities $\ln Z_i$ change by $\Delta \ln Z_i$.

[10] More specifically, the manufacturing revenue total factor productivity.

I use this model-generated data to estimate the regression

$$\Delta \ln L_i = c + \pi^{\text{model}} \Delta \ln Z_i + \epsilon_i$$

using weighted least squares, where c is a constant, $\Delta \ln L_i$ is the log change in location i's labor supply between the initial and post-shock stationary equilibria, and ϵ_i is the residual. As in HM's estimation, I weight observations by initial population share. I iterate on $\tilde{\nu}$ until the estimated coefficient $\hat{\pi}^{\text{model}}$ matches the auxiliary statistic of 4.03.

Income Tax The income tax rate τ is chosen so that the ratio of government spending to GDP in the initial steady state is 18%. I assume that both the tax rate and pension benefit remain fixed over time, so that government spending besides pensions payments G_t adjusts to balance the government's budget after shocks.[11]

Local Parameters The remaining internally estimated parameters are the location-specific amenities A_i, productivities Z_i, and rental price shifters $\overline{p}_i^r$. I choose these so that the initial stationary equilibrium matches local population shares, wages, and house prices. Without loss of generality, I normalize the average amenity value to 1. The location-specific parameters for each city are displayed in Appendix Table 10.

As is common in quantitative spatial models, estimating local productivities Z_i and rental price shifters $\overline{p}_i^r$ poses no computational difficulty. Given population and labor supply allocations in the initial steady state, these can be immediately backed out from the data using the closed-form equations (6)-(8).[12] In contrast to standard quantitative spatial models, it is *not* trivial to estimate local amenities. There is no closed-form relationship between amenities and population allocations, so these cannot be backed out from data. Furthermore, neither the hat algebra (Dekle et al., 2008) or dynamic hat algebra (Caliendo et al., 2019) techniques can be applied, so the *level* of amenities must be estimated. The only way to do this is to iterate on guesses for each of the I amenity values until a vector of amenities is chosen that matches the population allocation observed in the data. This is one of the reasons why it is imperative that the model be solved efficiently.

3.3 Untargeted Moments

In this subsection, I evaluate how the calibrated model compares to the data with regard to various non-targeted moments of interest. Given their importance for my analysis, I focus on statistics re-

[11] I verify *ex post* that G_t is always strictly positive, that is, government revenue is sufficient to cover the costs of the social security system.

[12] In fact, there is no need to recover the level of these parameters for estimation or counterfactual analysis. Households do not need to know them to make decisions in the initial stationary equilibrium – all they care about are prices, which exactly match data. And given a guess for population and labor supply allocations on the transition path after a shock, wages and house prices can be computed using the hat algebra techniques of Dekle et al. (2008), which requires only knowledge of how Z_i and $\overline{p}_i^r$ *change* over time.

Table 2: Internally Calibrated Parameters

Parameter	Description	Value	Target
ρ	Discount factor	0.019	Median wealth / income = 2.35
χ	Homeownership preference	1.07	Homeownership rate = 71.7%
η	Housing preference weight	0.21	Median house wealth share = 78.5%
$\tilde{\kappa}_0$	Migration cost: intercept	16.16	% Migrate = 3.03%
$\tilde{\kappa}_1$	Migration cost: slope	0.07	% Migrate, working-age - retired = 2.02%
$\tilde{\nu}$	Inverse migration elasticity	2.53	See text
τ	Income tax rate	16.2%	Government spending / GDP = 18%
A_i	Local amenities	See Table 10	Population shares
Z_i	Local productivities	See Table 10	Wages
$\overline{p}_i^r$	Local house price shifters	See Table 10	House Prices

Notes: This table displays the parameter values which are internally calibrated. All parameters except the inverse migration elasticity $\tilde{\nu}$ are chosen so that the initial steady state of the model matches cross-sectional data moments in (or as near as data allows to) the year 2000. $\tilde{\nu}$ is chosen to match the long-run employment effect of local productivity shocks estimated by Hornbeck and Moretti (2022). The unit of time is one year.

lated to homeownership, housing investment, migration, the wealth distribution, and the dynamic responses of the model economy to local productivity shocks.

Homeownership Figure 1a compares homeownership rates over the lifecycle in the model versus data.[13] Recall that the estimation procedure ensures that the model matches the aggregate homeownership rate. Overall, the model does a fairly good job of matching the steady increase in homeownership over the lifecycle. It does predict too low of a homeownership rate at the beginning of the working life (17.7% for 20-29 year-olds in the model versus 36.2% in the data) and correspondingly too high of a homeownership rate at the end of the working life (96.3% for 50-64 year-olds in the model versus 82.8% in the data). However, as shown in the top panel of Table 3, the model does a good job of separately matching the homeownership rates among working-age households (69.1% in the model versus 69.2% in the data) and retired households (78.8% in the model versus 80.3% in the data).

Figure 3a compares the homeownership rate across deciles of the wealth distribution in the model versus data. Qualitatively, the model generates the observed monotonically increasing relationship between homeownership rate and wealth. The model predicts somewhat too low of a homeownership rate below the median of the wealth distribution and correspondingly too high of a homeownership rate above the median. This is likely due to the fact that there are idiosyncratic reasons for some poor people to own and some rich people to rent that are not included in the model.

Housing Investment Figure 1b displays the median house wealth share among homeowners for various age groups in the model and data. Recall that the median house wealth share is one of the

[13]The values from Figures 1-3 are shown in Appendix Tables 11-13.

Table 3: Untargeted Moments by Age and Tenure

	Model		Data	
By Employment	Working-Age	Retired	Working-Age	Retired
Homeownership rate	69.1%	78.8%	69.2%	80.3%
Median house wealth share	74.9%	88.3%	84.3%	65.7%
Migration rate	3.6%	1.5%	3.4%	1.3%
Median wealth / income	1.88	4.92	1.73	7.30
By Tenure	Renters	Owners	Renters	Owners
Migration rate	4.7%	2.4%	6.0%	1.8%
Median wealth / income	0.00	4.10	0.34	3.63

Notes: This table compares untargeted moments by age group and homeownership status in the calibrated model versus data. "Working-age" refers to households between the ages of 20 and 64, while "retired" refers to households age 65 and over. The data sources used for this table are the 2001 Survey of Consumer Finances (for the homeownership rate, median house wealth share, and median wealth to income ratio) and 2005 American Community Survey (for the migration rate). See Section 3.1 for more details.

estimation targets. The model over-predicts housing wealth shares at the beginning and end of life, while under-predicting it for middle-age households. The discrepancy is largest for the youngest and oldest households. For 20-29 year-olds, the model predicts a median housing wealth share of 221.4%, whereas in the data the median share is 171.0%. For retired households, the model predicts a housing wealth share of 88.3%, whereas in the data the median share is 65.7%.

Figure 2a compares selected percentiles of the house wealth share distribution in the model versus data. The model does a fairly good job of matching the house wealth share distribution until about the 60th percentile, while predicting too much leverage at the top of the distribution. The discrepancy is largest at the top: the 90th percentile is 323.5% in the model versus 231.1% in the data.

Finally, Figure 3b compares the median housing wealth share across deciles of the wealth distribution in the model versus the data.[14] Qualitatively, the model matches the fact that the housing wealth share is decreasing with wealth. This may seem surprising since the utility function is homothetic. The negative relationship between housing wealth share and wealth is caused by the illiquidity of housing. Since it is expensive to adjust owner-occupied housing, young (wealth-poor) households invest more in housing than they would in a frictionless world because they expect to desire a larger house in the future. The opposite is true of old (wealth-rich) households. Quantitatively, the model predicts too high of a housing wealth share at the bottom of the wealth distribution (500% in the 2nd wealth decile versus 312% in the data). The discrepancy between the model and data shrinks with wealth, and the fit is quite close for the 7th-10th percentiles.

Migration Figure 1c compares the migration rate in the model versus data over the lifecycle. Recall that both the overall migration rate and difference in migration rates between working-age and

[14] I exclude the first decile of the wealth distribution in Figure 3b for two reasons. First, the homeownership rate for this subgroup is 0 in the model. Second, in the data the homeownership rate is quite low (4.6%) and the median housing wealth share is quite high (3,199%). See Table 13 for more details.

Figure 1: Statistics over the Lifecycle

Notes: This figure compares statistics in the calibrated model versus data for various age groups. The data sources used for this figure are the 2001 Survey of Consumer Finances (homeownership rate, median house wealth share, and median wealth to income ratio) and 2005 American Community Survey (migration rate). See Section 3.1 for more details.

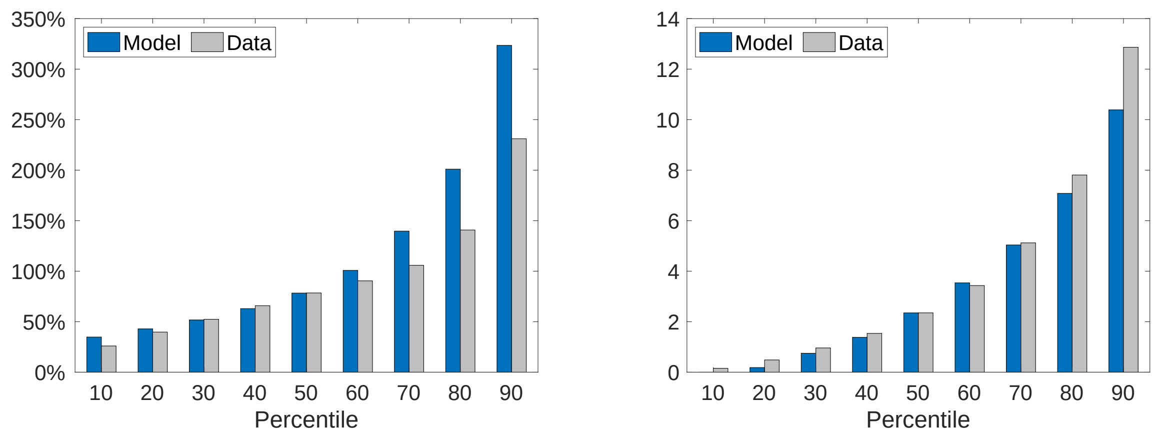
retired households are estimation targets. As such, it is not surprising that the model does a fairly good job of matching the steady decline in migration rates observed over the lifecycle. The bottom panel of Table 3 compares the migration rates of renters and owners in the model versus the data. The model under-predicts the migration rate of renters (4.7% in the model versus 6.0% in the data) and correspondingly over-predicts the migration rate of owners (2.4% in the model versus 1.8% in the data). However, qualitatively the model does generate the large observed gap in migration rates between renters and owners.

Wealth Distribution Figure 1d displays the median wealth to income ratio over the lifecycle in the model and data. Recall that the model is calibrated to match the median wealth to income ratio. Qualitatively, the model generates the increasing wealth to income ratio over the working

Figure 2: Distributional Statistics

(a) House Wealth Share

(b) Wealth / Income



Notes: This figure compares deciles of the house wealth share and wealth to income ratio in the calibrated model versus the data. The data source used for this figure is the 2001 Survey of Consumer Finances. See Section 3.1 for more details.

life observed in the data. The largest discrepancy between the model and data is at the end of the lifecycle. In the data, the median wealth to income ratio for retired households is 4.92, while in the data it is 7.30. Conversely, the model overpredicts the wealth to income ratio at the end of the working life: the model predicts a ratio of 5.03 for 50-64 year-olds, which is higher than the ratio of 3.48 observed in the data. As shown in Figure 2b, the model does a good job of matching the wealth/income distribution up until the 80th percentile. The model under-predicts the top deciles of the wealth/income distribution: the 90th percentile is 10.39 in the model versus 12.87 in the data. This is to be expected given that the model does not contain features that are important for explaining the upper tail of the wealth distribution.[15]

Dynamic Effects of Productivity Shock The final set of untargeted moments pertain to the dynamic effects of the local productivity shock $\Delta \ln Z_i$ described in Section 3.2. In Section 4, I use this shock to quantify the distributional effects of uneven local growth. Throughout the remainder of the paper, I assume that the economy is initially in the calibrated stationary equilibrium at time $t = 0$ when the productivity shock occurs. The shock is entirely unexpected and happens all at once. Households have rational expectations, and so immediately know all future prices as soon as the shock occurs.[16]

I summarize the average cumulative effect of the shock on a variable y after t periods by estimating the regression

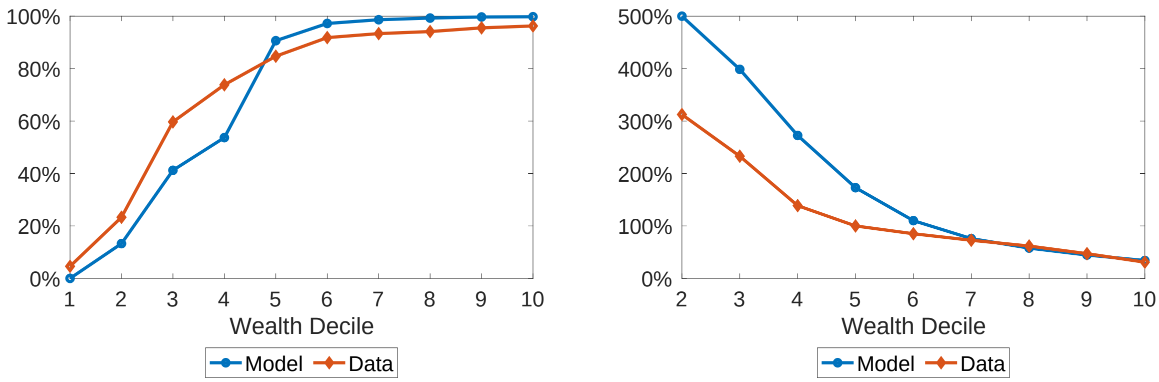
$$\ln y_{it} - \ln y_i^{ss} = c_t + \gamma_t \Delta \ln Z_i + \epsilon_{it} \tag{10}$$

[15]See De Nardi and Fella (2017) for a discussion of the mechanisms that are necessary for matching various features of the wealth distribution.

[16]The numerical algorithm used to compute transition dynamics after the shock is described in Appendix D.

Figure 3: Statistics by Wealth Decile

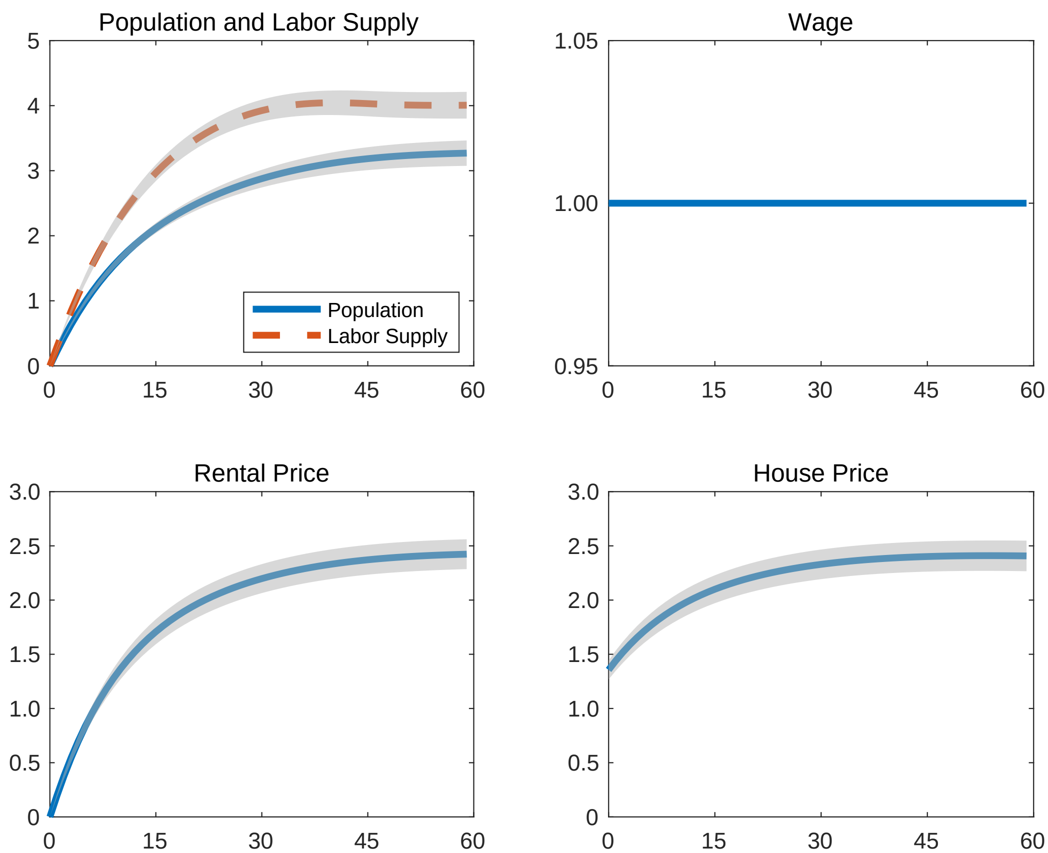
(a) Homeownership Rate   (b) Housing Wealth Share (Owners)



Notes: This figure compares deciles of the house wealth share and wealth to income ratio in the calibrated model versus the data. The data source used for this figure is the 2001 Survey of Consumer Finances. See Section 3.1 for more details.

where y_i^{ss} is the value of y in location i in the initial stationary equilibrium, c_t is a time fixed effect, and ϵ_{it} is a residual. I estimate (10) using weighted least squares, with weights equal to population shares in the initial steady state. Figure 4 displays the estimated coefficients $\hat{\gamma}_t$ for various outcomes of interest. In Table 4, I compare the long-run effects of the shock in the model versus the empirical estimates obtained by Hornbeck and Moretti (2022) using the estimating equation (9).

Figure 4: Dynamic Effects of Productivity Shock



Notes: This figure displays the estimated coefficients γ_t^y from the regression (10) for various outcomes. The x axis shows periods since the shock t. The shaded areas represent 95% confidence intervals. All coefficients are estimated using weighted least squares, with weights equal to local population shares in the initial steady state.

The upper left panel of Figure 4 displays the dynamic effects of the shock on local populations and labor supplies. Recall that the model is calibrated to match the long-run labor supply response of 4.03 estimated by Hornbeck and Moretti (2022). The response of local populations is smaller (long-run effect 3.27). This is not surprising – the higher wages caused by a productivity shock are attractive only to working-age households, and are irrelevant for retirees.

The upper right panel of Figure 4 shows the evolution of wages after the productivity shock. Since there are no agglomeration or congestion forces in the baseline production function, local wages move one-for-one with productivity. In Appendix C.1, I extend the model to include agglomeration and congestion in production. The long-run wage response is smaller in the model than HM (1.00 in the model versus 1.46 in HM), but well within their 95% confidence interval of $[0.48, 2.44]$.

The bottom two panels of Figure 4 display the dynamic responses of rents and house prices to the productivity shock. Note that rent prices do not change on impact. The way that migration costs

Table 4: Long-Run Effects of Productivity Shock

	Model	Hornbeck and Moretti (2022)
Wage	1.00	1.46
	(0.00)	(0.50)
Rental Price	2.41	1.09
	(0.07)	(0.48)
House Price	2.41	3.05
	(0.07)	(0.98)

Notes: This table compares the average long-run effects of the productivity shock in the model versus the empirical estimates from Hornbeck and Moretti (2022). The model estimates are obtained using regression equation (10). The estimates from Hornbeck and Moretti (2022) are displayed in their Table 2. Standard errors are in parenthesis.

and idiosyncratic location preferences are modeled imply that population allocations change only gradually. Given the rental price specification (7), it follows that rents adjust only gradually as well. In contrast, house prices jump instantaneously in response to the shock. This is because future rent increases are immediately capitalized into house prices. There is no way for homeowners to avoid the capital gains/losses that coincide with productivity shocks – as soon as the shock occurs, it causes an immediate change in homeowners' wealth. The no-arbitrage condition (8) that links rents and house prices implies that their long-run responses to a shock must be equal. The long run effect in the model (2.41) is higher than HM's estimate for rental prices (1.09) and lower than their estimate for house prices (3.05). It is outside the 95% confidence interval for rental prices, but within the 95% confidence interval for house prices.

4 Quantitative Analysis

In this section, I use the theoretical framework described above to quantify the distributional effects of uneven regional growth. My analysis is focused on how the productivity shock described in Section 3 affects the welfare of different types of households. The effects of the shock vary depending on location, age, homeownership status, wealth, and income. The model accounts for all of these dimensions of heterogeneity, which makes it possible to analyze the disaggregate effects of uneven local growth.

4.1 Measuring Welfare

I measure the welfare effect of a shock as the percentage change in the utility aggregator $Ac^{1-\eta}\mathbf{h}^{\eta}$, applied for the remainder of a household's life in the initial equilibrium, that would make it indifferent between remaining in the initial equilibrium or experiencing the shock. Let $V'(x, h, i, \ell, j, \varepsilon, \kappa)$ denote the value function immediately after the shock occurs. Formally, the welfare effect of the shock for a household with state variables $(x, h, i, \ell, j, \varepsilon, \kappa)$ is the value ω that satisfies

$$\mathrm{E} \sum_{s \in j \cup \{z \in \mathcal{S} | z > j\}} \frac{\Psi(s)}{\Psi(j)} \int_{s}^{s+\Delta(s)} e^{-\rho(\varsigma - j)} [\ln([1+\omega] A_\varsigma c_\varsigma^{1-\eta} \mathbf{h}_\varsigma^{\eta}) + \varepsilon_\varsigma - \kappa_\varsigma] d\varsigma = V'(x, h, i, \ell, j, \varepsilon, \kappa)$$

The left hand side of this expression computes expected lifetime utility, accounting for survival probabilities and idiosyncratic shocks at each of the remaining ages at which individual shocks occur. The expectation is taken with respect to all future draws of idiosyncratic labor endowment (conditional on current ℓ) and idiosyncratic location preferences.

In Appendix B.4, I show that

$$\omega(x, h, i, \ell, j) = \exp \left[\frac{\overline{V}'(x, h, i, \ell, j) - \overline{V}(x, h, i, \ell, j)}{\sum_{s \in j \cup \{z \in \mathcal{S} | z > j\}} \frac{\Psi(s)}{\Psi(j)} e^{-\rho(s-j)} \tilde{\rho}(s)} \right] - 1 \qquad (11)$$

where $\tilde{\rho}(j) \equiv (1 - e^{-\rho \Delta(j)})/\rho$. As noted previously, welfare effects are independent of ε and κ because these affect utility additively (and ε is i.i.d.). The welfare effects of the shock do depend on all other state variables. The assumptions that ε has a Gumbel distribution and households have log preferences make it possible to obtain a closed-form expression for $\omega(x, h, i, \ell, j)$.

4.2 Quantifying the Distributional Effects of Uneven Local Growth

I now turn to the main question of this paper: does uneven regional growth have unequal welfare effects across different types of households? I quantify the distributional effects of uneven growth by estimating the average effect of a productivity shock on local residents' welfare. Specifically, I use model-generated data to estimate the regression

$$\omega(x, h, i, \ell, j) = \mathrm{c} + \beta \Delta \ln Z_i + \epsilon(x, h, i, \ell, j) \qquad (12)$$

where c is a constant and ϵ is a residual. I estimate (12) using weighted least squares, with weights equal to the density of state variables in the initial stationary equilibrium. The parameter β is a measure of the extent to which a productivity shock passes through to local residents' welfare. A high value of β suggests that there is a close link between local economic conditions and individual welfare, while a low value of β suggests that the welfare effects of local productivity shocks are evenly distributed across space.

The first entry of Table 5 shows the estimated coefficient from regression (12), which is $\hat{\beta} = 0.43$. On average, after controlling for the economy-wide average welfare effect, a 1% increase in local productivity increases residents' welfare by 0.43%. This is the first main result of the paper: there is high pass-through from local productivity shocks to welfare. This suggests that shocks which have heterogeneous effects across space have important distributional consequences.

Table 5: Welfare Regression (12) Estimates

Tenure / Age	All Ages	20-29	30-39	40-49	50-64	65+
All Tenures	0.43	0.39	0.52	0.55	0.44	0.32
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Renters	0.27	0.34	0.39	0.40	0.26	-0.09
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Owners	0.50	0.61	0.59	0.57	0.45	0.43
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Notes: This table displays the estimated coefficient $\hat{\beta}$ from the welfare regression (12) for various subgroups. Observations are weighted by initial population share. Standard errors are shown in parentheses.

This first result is an average effect across all households, and obscures potentially important variation across people with different characteristics. In order to understand how the relationship between local growth and welfare varies throughout the population, in the remaining entries of Table 5 I report estimates of regression (12) by housing tenure and age group. These estimates are obtained in the same way as for the full population, but with the sample confined to households within a particular category.

The first column presents the average welfare effect separately for renters and owners. The average welfare effect of a local productivity shock is higher for owners (0.50) than renters (0.27). Estimates by age group are shown along the rows of Table 5. These entries show that β is higher for owners than renters even within age groups. At the beginning of the lifecycle, the average welfare effect of a local productivity shock is increasing with age. This is not surprising given that owners are more sensitive to productivity shocks than renters, and (as shown in Figure 1a) the homeownership rate is initially increasing with age. At the end of the lifecycle, the average welfare effect of a local productivity shock is decreasing with age. Overall, a positive productivity shock benefits all groups except retired renters. These households are actually *harmed* by higher productivity ($\hat{\beta} = -0.09$).

4.3 Intuition

Before proceeding, it is instructive to analyze several special cases of the model that provide context and intuition behind the results shown in Table 5. I consider three special cases. The first two abstract from borrowing/saving, lifecycle, and owner-occupied housing. When migration costs are infinite (no mobility) or zero (free mobility), it is possible to obtain analytical results in these cases. These simple models illustrate the role of moving costs in determining the distributional effects of local productivity shocks. The third special case is identical to the baseline model except that there is only one location. This special case is useful for illustrating the disaggregate welfare effects of changes in wages and house prices.

Special Case 1: No Wealth or Migration The first special case I consider is one in which households live hand-to-mouth, do not own housing, and cannot migrate.[17] For simplicity, here I also assume that households are infinitely lived and do not face any earning risk. In this case, lifetime utility in the initial steady state is the same for all households in a given location and is given by

$$V(i) = \ln[A_i w_i (p_i^r)^{-\eta}]/\rho$$

Since there is no migration, population and labor supply allocations are not affected by productivity shocks. As a result, the shock affects wages one-for-one and has no effect on rental prices. Lifetime utility after the productivity shock is thus

$$V^{'}(i) = \ln[A_i \exp(\Delta \ln Z_i) w_i (p_i^r)^{-\eta}]/\rho$$

It follows that the welfare effect of the productivity shock for location-i residents is

$$\omega(i) = \exp(\Delta \ln Z_i) - 1$$

Taking logs of both sides and using the approximation $\ln(1 + \omega(i)) \approx \omega(i)$ yields[18]

$$\omega(i) \approx \Delta \ln Z_i$$

In this special case, local productivity shocks pass through approximately one-to-one to welfare: $\beta \approx 1$.

Special Case 2: No Wealth, Free Mobility The second special case I consider is identical to the first, except that now I assume there are no migration costs. For simplicity, here I also assume that there are no idiosyncratic location preferences (or, equivalently, $\tilde{\nu} = 0$ so that the migration elasticity is infinite). Since households are freely mobile, in this case lifetime utility is equalized in every location both before and after the productivity shock. Note that lifetime utility may change after the shock, but the change will be the same for all households regardless of their initial location. Hence in this special case, there is no pass-through from local productivity shocks to welfare: $\beta = 0$.

Special Case 3: One Location The final special case of the model I consider is one in which there is only one location. This environment is useful for illustrating how changes in wages and house prices affect different types of households within a location.

I examine the effects of two partial equilibrium shocks in this setting: a permanent, 1% increase in wages, and a permanent, 1% increase in rental prices. Note from equation (8) that the rental price

[17] This would happen endogenously if $r - \rho$ and χ were sufficiently low and migration costs were sufficiently high.

[18] This is a good approximation for $\Delta \ln Z_i$ near 0. In my dataset, the maximum absolute value of $\Delta \ln Z_i$ is 0.0225. See Appendix Table 9 for the entire distribution of productivity shocks.

shock also implies a permanent, 1% increase in house prices. In both cases, I assume that the shock is completely unexpected and occurs all at once. The initial stationary equilibrium of this model is calibrated in the same way as Section 3 (except for the migration parameters, which are irrelevant when there is only one location).

Figure 5a shows the welfare effect of the wage shock by age group and housing tenure.[19] Not surprisingly, the benefits of this shock are decreasing with age. This is because earnings make up a larger fraction of lifetime income for young households (who have little financial or housing wealth and will not receive their pension for many years) than older households. This also explains why renters (who are wealth-poor compared to owners) benefit more from the shock than owners. All age groups prior to the retirement age are made strictly better off by the increase in wages. Since pension payments are not affected by wage changes, this shock has no welfare effect on retired households.

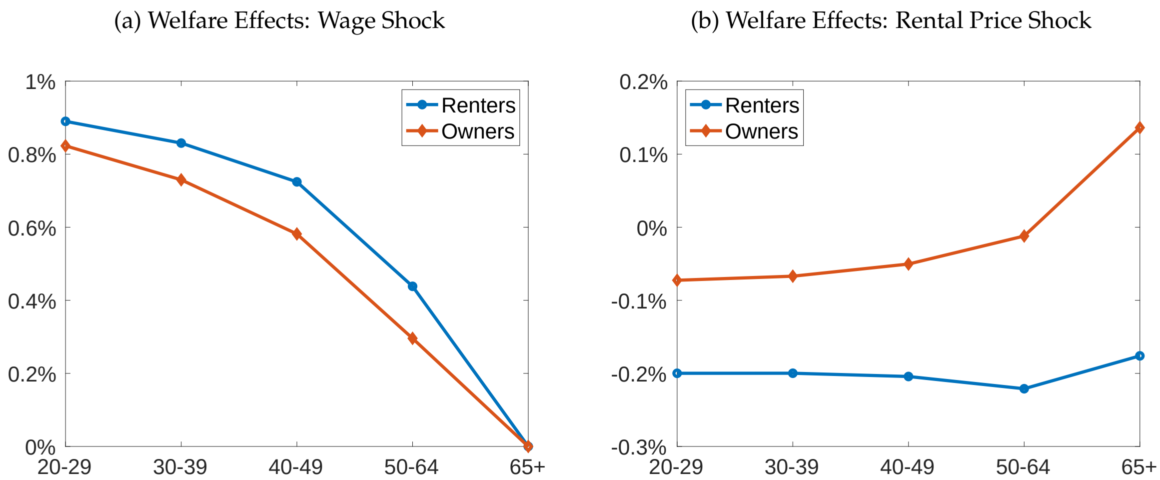
Figure 5b displays the welfare effect of the rental price shock by age group and housing tenure. Not surprisingly, renters of all ages are made worse off. This shock causes housing costs to increase, and since house prices immediately jump to their new long-run level, there is no opportunity for renters to capture capital gains by purchasing housing after the shock occurs. Owners of all ages are hurt less by the shock than renters, and retired homeowners actually benefit. There are two reasons why higher house prices can hurt owners. The first is that higher prices increase maintenance and transaction costs. Second, owners who are "short" in housing (expected to be net buyers of housing in the future) face higher future housing costs. On the other hand, owners experience capital gains on the housing they currently own when house prices increase. Since there is no bequest motive, owners will, in expectation, eventually realize this capital gain when they downsize at the end of their life.[20] The net welfare effect on owners depends on the present discounted value of higher costs versus the present discounted value of capital gains. Younger households tend to be short in housing and do not expect to realize capital gains for many years, while the opposite is true of older households. This explains why older owners benefit from higher house prices more than younger owners.

The difference in the welfare effects of house price changes for renters versus owners is critical for understanding the main results of this paper. As shown in Figure 4, a local productivity shock endogenously raises wages and house prices. Both renters and owners benefit from higher wages, and most are hurt by higher house prices. The fact that owners receive capital gains (losses) when house prices increase (decrease) implies that the house price changes affect them less than renters. As a result, the net welfare effect of a local productivity shock is larger for owners than renters. This explains why $\hat{\beta}$ is higher for owners than renters in the welfare regression (12). It also suggests that policies which reduce owner-occupied housing investment would mitigate the distributional effects of uneven regional growth. I examine this idea in more detail in Section 5.

[19] The values from Figure 5 are shown in Appendix Table 14.

[20] If there were a bequest motive, owners would still "realize" capital gains when they bequeathed their wealth.

Figure 5: One-Location Model



Notes: The left panel shows the welfare effect of a permanent, 1% increase in wages by age group and housing tenure in the one-location model. The right panel shows the welfare effect of a permanent, 1% increase in rental and house prices. The parameters of the one-location model are calibrated in the same way as the full quantitative model.

Summary As shown in Figure 4, an increase in local productivity raises wages, rental prices, and house prices. If migration costs are high, there will be high pass-through from local productivity changes to welfare via the wage channel, as in Special Case 1. In contrast, if migration is not costly, the welfare effects of the wage channel will not differ much across space, as in Special Case 2. As illustrated in Special Case 3, the effects of the wage channel are decreasing with age. Furthermore, the rental price channel mitigates the effects of wage changes for renters. The same is true, to a lesser extent, for homeowners. On average, higher house prices hurt older homeowners less than young homeowners, and can even benefit the oldest owners.

4.4 Role of Homeownership

The results of the previous two subsections suggest that homeownership plays a key role in determining the distributional effects of uneven regional growth. As mentioned previously, quantitative spatial models typically do not account for homeownership, or if they do, they incorporate it in a stylized manner. In this section, I explore how omitting homeownership affects conclusions about spatial redistribution.

For this purpose, I consider two variations of the baseline model in which there is no ownership. In the first, which I refer to as the "no ownership" model, households are prohibited from owning. In the second, which I refer to as the "local ownership model," owning is also prohibited, but rental payments in each location are redistributed lump-sum to current residents (instead of accruing to absentee landlords, as in the baseline model). As discussed in Redding and Rossi-Hansberg

Table 6: Comparison to Models without Homeownership

	All Ages	20-29	30-39	40-49	50-64	65+
Baseline	0.43	0.39	0.52	0.55	0.44	0.32
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
No ownership	0.16	0.33	0.34	0.29	0.10	-0.12
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Local Ownership	0.16	0.28	0.29	0.25	0.11	-0.05
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Notes: This table compares the estimated coefficient $\hat{\beta}$ from the welfare regression (12) by age group in the baseline model versus the models without homeownership. The "no ownership" model is the same as the baseline except that owning is prohibited. Ownership is also prohibited in the "local ownership" model, but local rental payments are also redistributed lump-sum to current residents. Both models are calibrated in the same way as the baseline model (except that χ is set to 0). Observations are weighted by population share. Standard errors are shown in parentheses.

(2017), this is a common way to capture the welfare effects of changing house prices for local owners without explicitly modeling homeownership. I estimate both models using the same procedure employed for the baseline model,[21] and compute transition dynamics after the same productivity shock. I then estimate the welfare regression (12) as above.

Table 6 compares estimates from the baseline model and the models without homeownership by age group. The estimated welfare effect of a local productivity shock is more than twice as large in the baseline model than the models without ownership (0.45 in the baseline versus 0.16 in both models without ownership). Consistent with the intuition discussed in Section 4.3, the difference between the models is smallest at the beginning of the lifecycle. This is when the homeownership rate is lowest, as well as when house price increases hurt owners the most. In the baseline model the estimated coefficient is positive for all age groups, but in both models without ownership it decreases with age until becoming negative for retired households. The estimated welfare effects in the local ownership model are smaller in magnitude than (but of the same sign as) the no ownership model. Quantitatively, results are similar for both models without ownership.

The disparity between estimated welfare effects in the baseline model versus the models without ownership provides further evidence that homeownership is a crucial channel through which local shocks affect individual welfare. The results of this section suggest that models which do not incorporate realistic homeownership are likely to underestimate the distributional effects of shocks that have heterogeneous impacts across space.

[21]The results of these estimations are shown in Appendix Table 15.

5 Policy Counterfactuals

In this section, I analyze policies that have potential to mitigate the distributional effects of uneven regional growth. The counterfactuals I consider are motivated by the importance of homeownership for spatial redistribution documented in Section 4. The first policy experiment is to reduce land-use regulations. This policy would lower house price elasticities, thus reducing the capital gains (losses) that accrue to incumbent homeowners in areas that receive positive (negative) shocks. The second experiment is to eliminate the mortgage interest deduction. This policy reduces incentives to invest in owner-occupied housing, thus diminishing the importance of the house price channel of regional redistribution.

To be clear, it is not the goal of this section to determine whether the policies being considered should or should not be implemented. There may be good reasons for both land-use regulation and the mortgage interest deduction that are not considered in the model. My objective is simply to quantify how these policies affect the link between local economic conditions and individual welfare.

5.1 Land-Use Deregulation

My first policy experiment explores how lessening land-use regulations would impact the distributional effects of uneven local growth. A large literature argues that such regulations are an important source of house price growth and spatial misallocation. In recent years, they have also gained increased attention as a source of inequality.[22]

Recall from Section 3 that the local house price elasticity parameters ξ_i used in the baseline model are taken from Saiz (2010). Saiz estimates house price elasticities as a function of observable geographic features and land-use regulations, as measured by the Wharton Residential Land Use Regulatory Index (WRLURI, Gyourko et al. 2008). Using his estimates, it is possible to predict the house price elasticities that would obtain under counterfactual WRLURI values.[23] I consider a counterfactual in which observed regulation levels are replaced with those of the least restrictive city in my dataset (New Orleans). This is a rather extreme scenario, and can be thought of as representing near the upper bound of the effects of plausible (uniform) deregulation. House price elasticities in the calibrated model and counterfactual are displayed in Appendix Table 16. For this counterfactual, I re-estimate the house price shifters $\overline{p}_i^r$ so that house prices still match the data in the initial stationary equilibrium after the change in ξ_i. All other parameters are held fixed at the values from the baseline calibration.

The first two rows of Table 7 compare the welfare coefficient from regression (12) in the calibrated model versus the deregulation counterfactual. The average welfare effect of a local productivity

[22] See for example Furman (2015) and Ganong and Shoag (2017).

[23] Hsieh and Moretti (2019) use this strategy to estimate the effects of land-use regulation on spatial misallocation and aggregate growth.

Table 7: Results of Policy Counterfactuals

	Welfare Coefficient
Baseline	0.43
	(0.000)
Land-use deregulation	0.42
	(0.000)
House price elasticity = 0	0.30
	(0.000)
Eliminate mortgage deduction	0.38
	(0.000)

Notes: This table compares the estimated coefficient $\hat{\beta}$ of the welfare regression (12) in the baseline model versus various counterfactual scenarios. In all cases, observations are weighted by initial population share. Standard errors are shown in parentheses. "Land-use deregulation" refers to the case when land-use regulations, as measured by the Wharton Residential Land Use Regulatory Index, are capped at the level of the least-regulated city in the dataset (New Orleans). "House price elasticity = 0" refers to the case in which ξ_i is exogenously set to 0, so that rental and house prices remain fixed at their initial values. The final row shows estimates when the mortgage interest deduction is eliminated.

shock is only slightly smaller with reduced land-use regulations than in the baseline (0.42 with reduced regulations versus 0.43 in the baseline). Given the modest effect of such a large deregulation, it is natural to wonder whether inelastic housing supply is an important determinant of spatial redistribution at all. To test this, I estimate the welfare coefficient when house price elasticities are exogenously set to 0 (so that house prices remain fixed at their initial values after the shock). The result of this exercise is shown in the third column of Table 7. In this counterfactual, the estimated welfare coefficient falls substantially relative to the baseline, from 0.43 to 0.30. This suggests that the limited impact of land-use deregulation is not due to house price elasticities being unimportant for redistribution, but instead because land-use regulation has a relatively minor effect on them. This is consistent with the conclusions of Saiz (2010), who argues that geography, moreso than land-use regulation, is the key factor in determining the evolution of urban house prices in the United States.[24]

5.2 Eliminating the Mortgage Interest Deduction

In this subsection, I analyze the effects of eliminating the mortgage interest deduction. This deduction constitutes a subsidy to owner-occupied housing, and empirical evidence suggests that it has a considerable effect on housing investment.[25] Given the finding from Section 4 that much of the distributional effects of uneven regional growth are driven by owner-occupied housing, it therefore

[24]This is illustrated in the last two rows of Appendix Table 16, which show the population-weighted average and median house price elasticities in the calibrated model and deregulation counterfactual. While deregulation does decrease house price elasticities, the effect is quantitatively small.

[25]See for example Gruber et al. (2017). There is little evidence, however, that the mortgage interest deduction has a meaningful effect on the homeownership rate.

seems likely that eliminating the mortgage interest deduction could mitigate such redistribution. To quantify the distributional effects of this policy counterfactual, I re-compute the initial stationary distribution and transition dynamics after the calibrated productivity shock in a model that is in every way the same as the calibrated baseline model (including parameter values) except that homeowners are not able to deduct mortgage payments. I then estimate the welfare regression (12) using the model-generated data.

The final row of Table 7 displays the results of this exercise. Eliminating the mortgage interest deduction reduces the estimated welfare coefficient (0.38 without the deduction versus 0.43 in the baseline). As expected, abolishing the mortgage interest deduction reduces owner-occupied housing investment. This is shown in Table 8, which compares statistics from the initial steady state in the baseline model versus the one with no mortgage deduction. Eliminating the mortgage interest deduction lowers both the homeownership rate (71.6% in the baseline versus 60.2% in the counterfactual) and the fraction of wealth homeowners invest in housing (median of 78.3% in the baseline versus 65.2% in the counterfactual). Other statistics, such as the aggregate housing expenditure share, migration rate, and median wealth to income ratio, are nearly unchanged.[26] Since the welfare effects of local productivity shocks are higher for owners than renters (as documented in Section 4.2), an outcome of this policy is to reduce the distributional effects of uneven local growth.

Table 8: Initial Steady State Comparison: Baseline vs. No Mortgage Deduction

	Baseline	No Mortgage Deduction
Homeownership Rate	71.6%	60.2%
Median House Wealth Share	78.3%	65.2%
Housing Expenditure Share	20.8%	20.6%
Migration Rate	3.0%	3.1%
Median Wealth/Income	2.35	2.48

Notes: This table compares statistics from the initial steady state in the baseline calibration versus when the mortgage interest deduction is eliminated.

6 Conclusion

Economic growth tends to be highly uneven across regions within a country. There are numerous reasons why individual welfare is tied to local economic conditions. Two of the most important are migration frictions, which make it difficult to move across labor markets, and owner-occupied housing investments, which link financial wealth to local house prices. Accounting for both of these channels is necessary to measure the distributional effects of uneven regional growth and to quantify the importance of the mechanisms that determine them.

[26]While housing expenditure does fall for owners, the housing expenditure share is higher for renters than owners, and the counterfactual causes the homeownership rate to drop. These two effects approximately cancel, leaving the aggregate housing expenditure share nearly unchanged.

In this paper, I develop a dynamic quantitative spatial model and use it to quantify the disaggregate effects of uneven regional growth. The model incorporates realistic homeownership into an otherwise standard quantitative spatial model. This increases computational burden considerably, but this challenge can be overcome using continuous-time computational techniques. The computational challenge is worth confronting because homeownership fundamentally changes the relationship between local shocks and individual welfare. Using my calibrated model, I estimate that the average welfare effect of a unit local productivity shock is 0.43. Without homeownership, the effect is only 0.16. An additional benefit of accounting for realistic household heterogeneity is that it makes it possible to quantify how the welfare effects of local shocks vary across different types of households. I find that both the magnitude and even, in some cases, the sign of such effects vary across households of different age groups and for renters versus owners. Finally, I analyze the effects of two policies that seem likely to mitigate the distributional effects of uneven regional growth: land-use deregulation and eliminating the mortgage interest deduction. While both policies reduce the distributional effects of uneven growth, the link between local growth and welfare remains strong.

These results suggest that the ongoing regional divergence observed in the United States and other countries has important distributional consequences. Developing a more complete understanding of these effects and the mechanisms through which they occur remains a challenging but important task.

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A Additional Tables and Figures

Table 9: MSA Statistics

City	Population	Wage	House Price	$\Delta \ln Z$
Atlanta-Sandy Springs-Roswell, GA	0.0262	0.9980	0.6850	-0.0003
Austin-Round Rock, TX	0.0088	0.9229	0.6850	0.0066
Baltimore-Columbia-Towson, MD	0.0171	0.9826	0.6850	-0.0005
Birmingham-Hoover, AL	0.0069	0.9035	0.4110	0.0025
Boston-Cambridge-Newton, MA-NH	0.0320	1.0507	1.0047	0.0016
Buffalo-Cheektowaga-Niagara Falls, NY	0.0080	0.8623	0.4110	-0.0097
Charlotte-Concord-Gastonia, NC-SC	0.0102	0.9349	0.5937	-0.0048
Chicago-Naperville-Elgin, IL-IN-WI	0.0554	1.0375	0.8220	-0.0020
Cincinnati, OH-KY-IN	0.0130	0.9432	0.5024	-0.0042
Cleveland-Elyria, OH	0.0153	0.9312	0.5937	-0.0045
Columbus, OH	0.0107	0.9332	0.5937	-0.0021
Dallas-Fort Worth-Arlington, TX	0.0343	0.9835	0.5937	0.0010
Denver-Aurora-Lakewood, CO	0.0145	0.9843	0.8220	0.0018
Detroit-Warren-Dearborn, MI	0.0256	1.0624	0.5937	-0.0225
Hartford-West Hartford-East Hartford, CT	0.0086	1.0502	0.6850	0.0034
Houston-The Woodlands-Sugar Land, TX	0.0290	0.9772	0.5024	0.0099
Indianapolis-Carmel-Anderson, IN	0.0094	0.9229	0.5024	-0.0016
Jacksonville, FL	0.0080	0.8880	0.5024	0.0043
Kansas City, MO-KS	0.0103	0.9245	0.5024	-0.0042
Knoxville, TN	0.0062	0.7904	0.4110	-0.0028
Las Vegas-Henderson-Paradise, NV	0.0098	0.9592	0.6850	-0.0020
Los Angeles-Long Beach-Anaheim, CA	0.0873	1.0294	2.2834	0.0008
Louisville/Jefferson County, KY-IN	0.0085	0.8739	0.5024	-0.0102
Memphis, TN-MS-AR	0.0076	0.9198	0.5024	-0.0053
Miami-Fort Lauderdale-West Palm Beach, FL	0.0361	0.9123	0.5937	0.0026
Milwaukee-Waukesha-West Allis, WI	0.0072	0.9441	0.6850	-0.0074
Minneapolis-St. Paul-Bloomington, MN-WI	0.0147	0.9903	0.5937	0.0001
Nashville-Davidson–Murfreesboro–Franklin, TN	0.0090	0.8998	0.5937	-0.0097
New Orleans-Metairie, LA	0.0088	0.8527	0.5024	0.0093
New York-Newark-Jersey City, NY-NJ-PA	0.1291	1.1407	1.6441	0.0021
Oklahoma City, OK	0.0069	0.8123	0.4110	-0.0016
Orlando-Kissimmee-Sanford, FL	0.0120	0.8589	0.5024	-0.0026
Philadelphia-Camden-Wilmington, PA-NJ-DE-MD	0.0354	1.0244	0.5937	0.0021
Phoenix-Mesa-Scottsdale, AZ	0.0230	0.9311	0.5937	0.0036
Pittsburgh, PA	0.0168	0.8562	0.4110	0.0030
Portland-Vancouver-Hillsboro, OR-WA	0.0130	0.9455	0.8220	0.0003
Providence-Warwick, RI-MA	0.0115	0.9176	0.6850	-0.0013
Raleigh, NC	0.0063	0.9472	0.6850	0.0076
Richmond, VA	0.0069	0.9246	0.5937	-0.0042
Riverside-San Bernardino-Ontario, CA	0.0200	0.9727	0.5937	-0.0079
Rochester, NY	0.0078	0.8860	0.4110	-0.0110
St. Louis, MO-IL	0.0157	0.9223	0.5024	-0.0023
San Antonio-New Braunfels, TX	0.0120	0.8093	0.4110	-0.0002
San Diego-Carlsbad, CA	0.0193	0.9521	1.2787	0.0036
San Francisco-Oakland-Hayward, CA	0.0308	1.1807	2.2834	0.0035
San Jose-Sunnyvale-Santa Clara, CA	0.0123	1.2723	2.2834	0.0108
Seattle-Tacoma-Bellevue, WA	0.0207	1.0111	1.0047	0.0017
Tampa-St. Petersburg-Clearwater, FL	0.0181	0.8670	0.4110	0.0020
Virginia Beach-Norfolk-Newport News, VA-NC	0.0099	0.8105	0.5937	-0.0024
Washington-Arlington-Alexandria, DC-VA-MD-WV	0.0341	1.0671	1.0047	0.0002

Notes: This table displays statistics for each city in the dataset. Total population and population-weighted average wage and house price are normalized to 1. $\Delta \ln Z$ is the change in log productivity from the productivity shock. Productivity shocks are scaled so that the population-weighted average productivity shock is 0.

Table 10: Estimated Location-Specific Parameters

City	Amenity	Productivity	Rental Price Shifter
Atlanta-Sandy Springs-Roswell, GA	1.0200	0.9980	0.1711
Austin-Round Rock, TX	0.9825	0.9229	0.1984
Baltimore-Columbia-Towson, MD	0.9974	0.9826	1.1062
Birmingham-Hoover, AL	0.8711	0.9035	0.2511
Boston-Cambridge-Newton, MA-NH	1.0777	1.0507	3.3270
Buffalo-Cheektowaga-Niagara Falls, NY	0.9146	0.8623	0.3470
Charlotte-Concord-Gastonia, NC-SC	0.9579	0.9349	0.1568
Chicago-Naperville-Elgin, IL-IN-WI	1.0845	1.0375	1.7440
Cincinnati, OH-KY-IN	0.9398	0.9432	0.1762
Cleveland-Elyria, OH	0.9985	0.9312	2.1002
Columbus, OH	0.9632	0.9332	0.1900
Dallas-Fort Worth-Arlington, TX	1.0216	0.9835	0.1680
Denver-Aurora-Lakewood, CO	1.0201	0.9843	0.7869
Detroit-Warren-Dearborn, MI	0.9380	1.0624	0.6834
Hartford-West Hartford-East Hartford, CT	0.8852	1.0502	0.9895
Houston-The Woodlands-Sugar Land, TX	0.9787	0.9772	0.1402
Indianapolis-Carmel-Anderson, IN	0.9253	0.9229	0.0968
Jacksonville, FL	0.9349	0.8880	2.8896
Kansas City, MO-KS	0.9330	0.9245	0.1264
Knoxville, TN	0.9381	0.7904	0.8796
Las Vegas-Henderson-Paradise, NV	0.9643	0.9592	1.1603
Los Angeles-Long Beach-Anaheim, CA	1.3992	1.0294	6.7211
Louisville/Jefferson County, KY-IN	0.9518	0.8739	0.2316
Memphis, TN-MS-AR	0.9065	0.9198	0.4801
Miami-Fort Lauderdale-West Palm Beach, FL	1.0878	0.9123	9.4508
Milwaukee-Waukesha-West Allis, WI	0.9454	0.9441	4.9577
Minneapolis-St. Paul-Bloomington, MN-WI	0.9483	0.9903	0.6579
Nashville-Davidson–Murfreesboro–Franklin, TN	0.9726	0.8998	0.2923
New Orleans-Metairie, LA	0.9716	0.8527	10.5593
New York-Newark-Jersey City, NY-NJ-PA	1.2268	1.1407	1.4655
Oklahoma City, OK	0.9350	0.8123	0.1115
Orlando-Kissimmee-Sanford, FL	0.9987	0.8589	1.5658
Philadelphia-Camden-Wilmington, PA-NJ-DE-MD	0.9907	1.0244	0.2717
Phoenix-Mesa-Scottsdale, AZ	1.0342	0.9311	0.3687
Pittsburgh, PA	0.9911	0.8562	0.7315
Portland-Vancouver-Hillsboro, OR-WA	1.0423	0.9455	2.8866
Providence-Warwick, RI-MA	1.0135	0.9176	0.6627
Raleigh, NC	0.9290	0.9472	0.4534
Richmond, VA	0.9264	0.9246	0.2415
Riverside-San Bernardino-Ontario, CA	0.9879	0.9727	2.2536
Rochester, NY	0.8955	0.8860	0.7828
St. Louis, MO-IL	0.9729	0.9223	0.1757
San Antonio-New Braunfels, TX	0.9967	0.8093	0.1087
San Diego-Carlsbad, CA	1.1806	0.9521	27.0543
San Francisco-Oakland-Hayward, CA	1.1600	1.1807	26.3355
San Jose-Sunnyvale-Santa Clara, CA	1.0020	1.2723	44.3107
Seattle-Tacoma-Bellevue, WA	1.0746	1.0111	4.9036
Tampa-St. Petersburg-Clearwater, FL	0.9885	0.8670	1.3762
Virginia Beach-Norfolk-Newport News, VA-NC	1.0551	0.8105	10.1660
Washington-Arlington-Alexandria, DC-VA-MD-WV	1.0689	1.0671	0.4941

Notes: This table displays the estimated amenity A_i, productivity Z_i, and rental price shifter $\overline{p}_i^r$ for each city. These are chosen to match population shares, wages, and house prices in the initial stationary equilibrium. The population-weighted average amenity, wage, and house price are all normalized to 1.

Table 11: Values from Figure 1

	Homeownership Rate		House Wealth Share		Migration Rate		Wealth/Income	
Age Group	Model	Data	Model	Data	Model	Data	Model	Data
20-29	17.7%	36.2%	221.4%	171.0%	5.5%	6.6%	0.01	0.47
30-39	65.9%	63.1%	126.7%	117.7%	3.9%	3.7%	1.15	1.12
40-49	87.5%	76.3%	74.7%	81.5%	3.0%	2.1%	2.95	2.06
50-64	96.3%	82.8%	54.2%	68.0%	2.3%	1.7%	5.03	3.48
65+	78.8%	80.3%	88.3%	65.7%	1.5%	1.3%	4.92	7.30

Notes: This table displays the values shown in Figure 1. It compares statistics in the calibrated model versus data over the lifecycle. The data sources are the 2001 Survey of Consumer Finances (for the homeownership rate, median house wealth share, and median wealth to income ratio) and 2005 American Community Survey (for the migration rate). See Section 3.1 for more details.

Table 12: Values from Figure 2

Percentile	House Wealth Share		Wealth/Income	
	Model	Data	Model	Data
10	34.8%	26.0%	0.00	0.16
20	42.9%	39.7%	0.18	0.49
30	51.7%	52.4%	0.75	0.96
40	62.9%	65.9%	1.38	1.54
50	78.3%	78.5%	2.35	2.35
60	100.8%	90.5%	3.54	3.43
70	139.7%	105.9%	5.04	5.12
80	201.0%	140.8%	7.08	7.81
90	323.5%	231.1%	10.39	12.87

Notes: This table displays the values shown in Figure 2. It compares selected percentiles of the house wealth share and wealth to income ratio distributions in the calibrated model versus the data. The data source is the 2001 Survey of Consumer Finances. See Section 3.1 for more details.

Table 13: Values from Figure 3

Percentile	Homeownership Rate		House Wealth Share	
	Model	Data	Model	Data
1	0.0%	4.6%		3198.7%
2	13.3%	23.3%	500.0%	312.5%
3	41.2%	59.7%	398.6%	233.0%
4	53.7%	73.8%	272.5%	138.7%
5	90.6%	84.7%	172.9%	100.0%
6	97.2%	91.8%	110.1%	85.1%
7	98.6%	93.3%	75.9%	72.7%
8	99.3%	94.1%	57.7%	61.7%
9	99.7%	95.5%	44.5%	47.1%
10	99.8%	96.3%	34.1%	30.9%

Notes: This table displays the values shown in Figure 3. It compares the homeownership rate and median house wealth share (among owners) within deciles of the wealth distribution in the calibrated model versus the data. The data source is the 2001 Survey of Consumer Finances. See Section 3.1 for more details.

Table 14: Values from Figure 5

	Wage Shock		Rental Price Shock	
Age Group	Renters	Owners	Renters	Owners
20-29	0.89%	0.82%	-0.20%	-0.07%
30-39	0.83%	0.73%	-0.20%	-0.07%
40-49	0.72%	0.58%	-0.20%	-0.05%
50-64	0.44%	0.30%	-0.22%	-0.01%
65+	0.00%	0.00%	-0.18%	0.14%

Notes: This table displays the values from in Figure 5. It shows the welfare effects of a 1% increase in wages ("wage shock") and a 1% increase in rental and house prices ("rental price shock") in the one-location model. Both shocks are unexpected and permanent. The parameters of the one-location model are calibrated in the same way as in the full quantitative model (where applicable).

Table 15: Calibration Results: Baseline versus Models without Ownership

Parameter	Description	Baseline	No Ownership	Local Ownership
ρ	Discount factor	0.019	0.017	0.017
χ	Homeownership preference	1.07	0.00	0.00
η	Housing preference weight	0.21	0.21	0.21
$\tilde{\kappa}_0$	Migration cost: intercept	16.16	16.71	15.53
$\tilde{\kappa}_1$	Migration cost: slope	0.07	0.07	0.07
$\tilde{\nu}$	Inverse migration elasticity	2.53	2.59	2.41
τ	Income tax rate	16.2%	14.3%	10.9%

Notes: This table compares the internally calibrated parameters in the baseline model with those from the models without homeownership. The "no ownership" model is the same as the baseline except that owning is prohibited. Ownership is also prohibited in the "local ownership" model, and local rental payments are also redistributed lump-sum to current residents. Both models are calibrated in the same way as the baseline model (except that χ is set to 0).

Table 16: MSA House Price Elasticities

City	Baseline	Minimum Regulation
Atlanta-Sandy Springs-Roswell, GA	0.3916	0.2392
Austin-Round Rock, TX	0.3330	0.2116
Baltimore-Columbia-Towson, MD	0.8098	0.5408
Birmingham-Hoover, AL	0.4663	0.3399
Boston-Cambridge-Newton, MA-NH	1.1654	0.8903
Buffalo-Cheektowaga-Niagara Falls, NY	0.5473	0.4206
Charlotte-Concord-Gastonia, NC-SC	0.3234	0.2287
Chicago-Naperville-Elgin, IL-IN-WI	1.2325	1.0815
Cincinnati, OH-KY-IN	0.4063	0.3170
Cleveland-Elyria, OH	0.9757	0.8419
Columbus, OH	0.3686	0.1961
Dallas-Fort Worth-Arlington, TX	0.4597	0.3327
Denver-Aurora-Lakewood, CO	0.6540	0.4359
Detroit-Warren-Dearborn, MI	0.8057	0.6515
Hartford-West Hartford-East Hartford, CT	0.6684	0.4772
Houston-The Woodlands-Sugar Land, TX	0.4344	0.3250
Indianapolis-Carmel-Anderson, IN	0.2498	0.1805
Jacksonville, FL	0.9441	0.7970
Kansas City, MO-KS	0.3134	0.2501
Knoxville, TN	0.7042	0.5921
Las Vegas-Henderson-Paradise, NV	0.7218	0.6460
Los Angeles-Long Beach-Anaheim, CA	1.5959	1.4040
Louisville/Jefferson County, KY-IN	0.4273	0.3253
Memphis, TN-MS-AR	0.5667	0.3246
Miami-Fort Lauderdale-West Palm Beach, FL	1.6799	1.4541
Milwaukee-Waukesha-West Allis, WI	0.9722	0.7833
Minneapolis-St. Paul-Bloomington, MN-WI	0.6909	0.5085
Nashville-Davidson–Murfreesboro–Franklin, TN	0.4470	0.3385
New Orleans-Metairie, LA	1.2371	1.2371
New York-Newark-Jersey City, NY-NJ-PA	1.3178	1.1134
Oklahoma City, OK	0.3036	0.1914
Orlando-Kissimmee-Sanford, FL	0.8927	0.7155
Philadelphia-Camden-Wilmington, PA-NJ-DE-MD	0.6079	0.3694
Phoenix-Mesa-Scottsdale, AZ	0.6197	0.4187
Pittsburgh, PA	0.8299	0.6717
Portland-Vancouver-Hillsboro, OR-WA	0.9372	0.7641
Providence-Warwick, RI-MA	0.6228	0.3367
Raleigh, NC	0.4741	0.2709
Richmond, VA	0.3847	0.2732
Riverside-San Bernardino-Ontario, CA	1.0602	0.8658
Rochester, NY	0.7124	0.5686
St. Louis, MO-IL	0.4245	0.3532
San Antonio-New Braunfels, TX	0.3353	0.2068
San Diego-Carlsbad, CA	1.4864	1.2970
San Francisco-Oakland-Hayward, CA	1.5115	1.3017
San Jose-Sunnyvale-Santa Clara, CA	1.3152	1.1472
Seattle-Tacoma-Bellevue, WA	1.1340	0.9096
Tampa-St. Petersburg-Clearwater, FL	1.0021	0.8742
Virginia Beach-Norfolk-Newport News, VA-NC	1.2259	1.0657
Washington-Arlington-Alexandria, DC-VA-MD-WV	0.6227	0.4460
Average	0.9579	0.7833
Median	0.9722	0.7970

Notes: This table displays house price elasticities in the calibrated model and counterfactual land-use regulation scenario. "Minimum regulation" refers to the case where regulations, as measured by the Wharton Residential Land Use Regulatory Index (Gyourko et al., 2008), are capped at that of the least-regulated city in my dataset (New Orleans). The effect of regulation on house price elasticities is taken from Saiz (2010).

B Mathematical Derivations

B.1 Policy Functions

The necessary first-order conditions of (1) are (the second only applies for renters)

$$\begin{aligned} \frac{1-\eta}{c} &\geq \partial_x V_t(x,h,i,\ell,j,\varepsilon,\kappa) \\ \frac{\eta}{h^r} &\geq \partial_x V_t(x,h,i,\ell,j,\varepsilon,\kappa) p^r_{it} \end{aligned}$$

These hold with equality if $x > 0$. As in Section 2, define $\overline{V}_t(x,h,i,\ell,j) \equiv V_t(x,h,i,\ell,j,0,0)$. Since $\partial_x V_t(x,h,i,\ell,j,\varepsilon,\kappa) = \partial_x \overline{V}_t(x,h,i,\ell,j)$, the first-order conditions can be written as

$$\begin{aligned} c &\leq \frac{1-\eta}{\partial_x \overline{V}_t(x,h,i,\ell,j)} \\ h^r &\leq \frac{\eta}{\partial_x \overline{V}_t(x,h,i,\ell,j) p^r_{it}} \end{aligned}$$

For notational convenience, denote after-tax income by

$$y = (1-\tau)\left(ra + \begin{cases} w_{it} \exp(\bar{l}(j) + \ell) & \text{if } j < J_r \\ \varpi_t & \text{if } j \geq J_r \end{cases} \right)$$

For renters ($h = 0$), the consumption and rental policy functions are

$$\begin{aligned} c &= \begin{cases} \frac{1-\eta}{\partial_x \overline{V}_t(x,h,i,\ell,j)} & \text{if } x > 0 \\ \min\left\{ \frac{1-\eta}{\partial_x \overline{V}_t(x,h,i,\ell,j)},\ (1-\eta)y \right\} & \text{if } x = 0 \end{cases} \\ h^r &= \begin{cases} \frac{\eta}{\partial_x \overline{V}_t(x,h,i,\ell,j) p^r_{it}} & \text{if } x > 0 \\ \min\left\{ \frac{\eta}{\partial_x \overline{V}_t(x,h,i,\ell,j) p^r_{it}},\ \frac{\eta y}{p^r_{it}} \right\} & \text{if } x = 0 \end{cases} \end{aligned}$$

and for owners ($h > 0$), they are

$$\begin{aligned} c &= \begin{cases} \frac{1-\eta}{\partial_x \overline{V}_t(x,h,i,\ell,j)} & \text{if } x > 0 \\ \min\left\{ \frac{1-\eta}{\partial_x \overline{V}_t(x,h,i,\ell,j)},\ y - [\delta^h p_{it} - (1+\phi)\dot{p}_{it}]h \right\} & \text{if } x = 0 \end{cases} \\ h^r &= 0 \end{aligned}$$

Finally, the optimal choice of owner-occupied housing, denoted h', is

$$\begin{aligned} h' &= \underset{\tilde{h}}{\text{argmax}}\; V_t(x'(\tilde{h},i),\tilde{h},i,\ell,j,\varepsilon,\kappa) \\ &= \underset{\tilde{h}}{\text{argmax}}\; \overline{V}_t(x'(\tilde{h},i),\tilde{h},i,\ell,j) \end{aligned}$$

As mentioned in Section 2, neither ε nor κ appear in any of the policy functions. This is due to the assumption that they affect utility additively, and is key to the model's tractability. Note that this does *not* imply that idiosyncratic location preferences and moving costs are irrelevant for household decisions. They do affect the optimal location choice i, and location in turn affects consumption and housing decisions. However, the policy functions are independent of ε and κ *conditional* on a location choice.

B.2 Migration Flows

Denote the difference (net of idiosyncratic location preferences) in the value of choosing location i' and i'' when migration is allowed by

$$\bar{\varepsilon}_{i'i''} \equiv V_t^m(x,h,i,\ell,j,i') - \tilde{\kappa}_{ii'}(j) - [V_t^m(x,h,i,\ell,j,i'') - \tilde{\kappa}_{ii''}(j)]$$

The probability that a household chooses i' when migration is allowed is

$$m(i') = \int_{-\infty}^{\infty} f(\tilde{\varepsilon}_{i'}) \prod_{i''\neq i'} F(\bar{\varepsilon}_{i'i''} + \tilde{\varepsilon}_{i'}) d\tilde{\varepsilon}_{i'}$$

where $\tilde{\varepsilon} \equiv \tilde{\rho}\varepsilon$. Substitute

$$\begin{aligned} F(\tilde{\varepsilon}) &= \exp(-\exp(-\tilde{\varepsilon}/\tilde{\nu} - \overline{\gamma})) \\ f(\tilde{\varepsilon}) &= (1/\tilde{\nu})\exp(-\tilde{\varepsilon}/\tilde{\nu} - \overline{\gamma})\exp(-\exp(-\tilde{\varepsilon}/\tilde{\nu} - \overline{\gamma})) \end{aligned}$$

to obtain

$$\begin{aligned} m(i') &= \int_{-\infty}^{\infty} (1/\tilde{\nu}) \exp\left(-\tilde{\varepsilon}_{i'}/\tilde{\nu} - \overline{\gamma} - \exp(-\tilde{\varepsilon}_{i'}/\tilde{\nu} - \overline{\gamma}) - \sum_{i''\neq i'} \exp(-[\bar{\varepsilon}_{i'i''} + \tilde{\varepsilon}_{i'}]/\tilde{\nu} - \overline{\gamma}) \right) d\tilde{\varepsilon}_{i'} \\ &= \int_{-\infty}^{\infty} (1/\tilde{\nu}) \exp\left(-\tilde{\varepsilon}_{i'}/\tilde{\nu} - \overline{\gamma} - \sum_{i''} \exp(-[\bar{\varepsilon}_{i'i''} + \tilde{\varepsilon}_{i'}]/\tilde{\nu} - \overline{\gamma}) \right) d\tilde{\varepsilon}_{i'} \end{aligned}$$

where I have used $\bar{\varepsilon}_{i'i'} = 0$. Define $\zeta_{i'} \equiv \tilde{\varepsilon}_{i'}/\tilde{\nu} + \overline{\gamma}$. Then

$$\begin{aligned} m(i') &= \int_{-\infty}^{\infty} (1/\tilde{\nu}) \exp\left(-\zeta_{i'} - \sum_{i''} \exp(-\bar{\varepsilon}_{i'i''}/\tilde{\nu} - \zeta_{i'})\right) d\tilde{\varepsilon}_{i'} \\ &= \int_{-\infty}^{\infty} (1/\tilde{\nu}) \exp\left(-\zeta_{i'} - \exp(-\zeta_{i'}) \sum_{i''} \exp(-\bar{\varepsilon}_{i'i''}/\tilde{\nu})\right) d\tilde{\varepsilon}_{i'} \end{aligned}$$

Define $\lambda_{i'} \equiv \ln \sum_{i''} \exp(-\bar{\varepsilon}_{i'i''}/\tilde{\nu})$. Then

$$\begin{aligned} m(i') &= \int_{-\infty}^{\infty} \exp(-\zeta_{i'} - \exp(-(\zeta_{i'} - \lambda_{i'}))) d\zeta_{i'} \\ &= \exp(-\lambda_{i'}) \int_{-\infty}^{\infty} \exp(-(\zeta_{i'} - \lambda_{i'}) - \exp(-(\zeta_{i'} - \lambda_{i'}))) d\zeta_{i'} \end{aligned}$$

where I have used $d\zeta_{i'} = (1/\tilde{\nu}) d\tilde{\varepsilon}_{i'}$. Define $y_{i'} \equiv \zeta_{i'} - \lambda_{i'}$. Then

$$m(i') = \exp(-\lambda_{i'}) \int_{-\infty}^{\infty} \exp(-y_{i'} - \exp(-y_{i'})) dy_{i'}$$

Since $\exp(-y_{i'} - \exp(-y_{i'}))$ is a probability density function (specifically, the Gumbel distribution with location parameter 0 and scale parameter 1), we have

$$\begin{aligned} m(i') &= \exp(-\lambda_{i'}) \\ &= \sum_{i''} \exp\left(\frac{V_t^m(x,h,i,\ell,j,i') - \tilde{\kappa}_{ii'}(j) - V_t^m(x,h,i,\ell,j,i'') + \tilde{\kappa}_{ii''}(j)}{\tilde{\nu}}\right) \\ &= \frac{\exp(V_t^m(x,h,i,\ell,j,i') - \tilde{\kappa}_{ii'}(j))^{1/\tilde{\nu}}}{\sum_{i''} \exp(V_t^m(x,h,i,\ell,j,i'') - \tilde{\kappa}_{ii''}(j))^{1/\tilde{\nu}}} \end{aligned}$$

B.3 Expected Value of Migration and Equation (2) Derivation

The expected value of the optimal migration choice is

$$\mathrm{E}_{\varepsilon} \max_{i'} [V_t^m(x,h,i,\ell,j,i') - \tilde{\kappa}_{ii'}(j) + \tilde{\varepsilon}_{i'}] = \sum_{i'} \int_{-\infty}^{\infty} [V_t^m(x,h,i,\ell,j,i') - \tilde{\kappa}_{ii'}(j) + \tilde{\varepsilon}_{i'}] f(\tilde{\varepsilon}_{i'}) \prod_{i'' \neq i'} F(\bar{\varepsilon}_{i'i''} + \tilde{\varepsilon}_{i'}) d\tilde{\varepsilon}_{i'}$$

Follow the steps of Section B.2 to write the right-hand side as

$$\begin{aligned}
&\sum_{i'}\int_{-\infty}^{\infty}[V_t^m(x,h,i,\ell,j,i')-\tilde{\kappa}_{ii'}(j)+\tilde{\varepsilon}_{i'}](1/\tilde{v})\exp\left(-\tilde{\varepsilon}_{i'}/\tilde{v}-\overline{\gamma}-\sum_{i''}\exp(-[\overline{\varepsilon}_{i'i''}+\tilde{\varepsilon}_{i'}]/\tilde{v}-\overline{\gamma})\right)d\tilde{\varepsilon}_{i'}\\
=&\sum_{i'}\exp(-\lambda_{i'})\int_{-\infty}^{\infty}[V_t^m(x,h,i,\ell,j,i')-\tilde{\kappa}_{ii'}(j)+\tilde{\varepsilon}_{i'}]\exp(-y_{i'}-\exp(-y_{i'}))dy_{i'}\\
=&\sum_{i'}\exp(-\lambda_{i'})\int_{-\infty}^{\infty}[V_t^m(x,h,i,\ell,j,i')-\tilde{\kappa}_{ii'}(j)+(y_{i'}-\overline{\gamma}+\lambda_{i'})\tilde{v}]\exp(-y_{i'}-\exp(-y_{i'}))dy_{i'}\\
=&\sum_{i'}\exp(-\lambda_{i'})\left[V_t^m(x,h,i,\ell,j,i')-\tilde{\kappa}_{ii'}(j)+(\lambda_{i'}-\overline{\gamma})\tilde{v}+\tilde{v}\int_{-\infty}^{\infty}y_{i'}\exp(-y_{i'}-\exp(-y_{i'}))dy_{i'}\right]\\
=&\sum_{i'}\exp(-\lambda_{i'})\left[V_t^m(x,h,i,\ell,j,i')-\tilde{\kappa}_{ii'}(j)+\lambda_{i'}\tilde{v}\right]\\
=&\sum_{i'}m(i')[V_t^m(x,h,i,\ell,j,i')-\tilde{\kappa}_{ii'}(j)-\tilde{v}\ln m(i')]
\end{aligned}$$

where I have used the definition of $\overline{\gamma}$ in the second to last step. Since

$$-\tilde{v}\ln m(i')=-V_t^m(x,h,i,\ell,j,i')+\tilde{\kappa}_{ii'}(j)+\tilde{v}\ln\sum_{i''}\exp(V_t^m(x,h,i,\ell,j,i'')-\tilde{\kappa}_{ii''}(j))^{1/\tilde{v}}$$

This simplifies to

$$\begin{aligned}
&\sum_{i'}m(i')\tilde{v}\ln\sum_{i''}\exp(V_t^m(x,h,i,\ell,j,i'')-\tilde{\kappa}_{ii''}(j))^{1/\tilde{v}}\\
=&\tilde{v}\ln\sum_{i'}\exp(V_t^m(x,h,i,\ell,j,i')-\tilde{\kappa}_{ii'}(j))^{1/\tilde{v}}
\end{aligned}$$

Multiplying this by survival probability and integrating over the probability density of labor endowment shocks yields equation (2).

B.4 Derivation of Welfare Measure (11)

For notational convenience, throughout this section I suppress state variables besides i, j, ε, and κ. Note that the expression for the value function at ages $j\in\mathcal{S}$ (2) can be written as

$$\begin{aligned}
\mathrm{E}V(i,j)&=\psi(j)\mathrm{E}\left[\tilde{v}\ln\sum_{i'}\exp(V^m(i,j,i')-\tilde{\kappa}_{ii'}(j))^{1/\tilde{v}}\right],\ j\in\mathcal{S}\\
&=\psi(j)\mathrm{E}\left[V^m(i,j,i)+\tilde{v}\ln\sum_{i'}\exp(V^m(i,j,i')-\tilde{\kappa}_{ii'}(j)-V^m(i,j,i))^{1/\tilde{v}}\right]
\end{aligned}$$

where the expectation is taken with respect to the labor endowment shock. Using the migration probability expression derived in Section B.2, this can be further re-written as

$$\mathrm{E}V(i,j)=\psi(j)\mathrm{E}\left[\overline{V}^{+}(i,j)-\tilde{v}\ln m(i,j)\right],\ j\in\mathcal{S}$$

where I have used the fact that $V^m(i,j,i) = \overline{V}^+(i,j)$. Note that

$$\overline{V}^+(i,j) = \int_j^{j+\Delta} e^{-\rho(\varsigma-j)} \ln(A_i c_\varsigma^{1-\eta} \mathbf{h}_\varsigma^\eta) d\varsigma + e^{-\rho\Delta} \mathrm{E}V(i,j+\Delta)$$

Substitute into the previous expression:

$$\mathrm{E}V(i,j) = \psi(j)\mathrm{E}\left[\int_j^{j+\Delta} e^{-\rho(\varsigma-j)} \ln(A_i c_\varsigma^{1-\eta} \mathbf{h}_\varsigma^\eta) d\varsigma - \tilde{\nu} \ln m(i,j) + e^{-\rho\Delta} V(i,j+\Delta)\right], \ j \in \mathcal{S}$$

which can be written as

$$\mathrm{E}V(i,j) = \psi(j)\mathrm{E}\left[\int_j^{j+\Delta} e^{-\rho(\varsigma-j)} \ln(A_i c_\varsigma^{1-\eta} \mathbf{h}_\varsigma^\eta m(i,j)^{-\tilde{\nu}/\tilde{\rho}}) d\varsigma + e^{-\rho\Delta} V(i,j+\Delta)\right], \ j \in \mathcal{S}$$

Iterate forward (and use $V(i,J) = 0$) to obtain

$$\mathrm{E}V(i,j) = \mathrm{E} \sum_{s\in\mathcal{S}|s\geq j} \frac{\Psi(s)}{\Psi(j)} \int_s^{s+\Delta} e^{-\rho(\varsigma-j)} \ln(A_i c_\varsigma^{1-\eta} \mathbf{h}_\varsigma^\eta m(i,s)^{-\tilde{\nu}/\tilde{\rho}}) d\varsigma, \ j \in \mathcal{S}$$

Note that the welfare effect definition (11) can be written as

$$\begin{aligned}
V'(i,j,\varepsilon,\kappa) &= \mathrm{E} \sum_{s\in j\cup\{z\in\mathcal{S}|z>j\}} \frac{\Psi(s)}{\Psi(j)} \int_s^{s+\Delta(s)} e^{-\rho(\varsigma-j)} [\ln([1+\omega(i,j)] A_\varsigma c_\varsigma^{1-\eta} \mathbf{h}_\varsigma^\eta) + \varepsilon_\varsigma - \kappa_\varsigma] d\varsigma \\
&= \mathrm{E}\left[\int_j^{j+\Delta(j)} e^{-\rho(\varsigma-j)} [\ln([1+\omega(i,j)] A_i c_\varsigma^{1-\eta} \mathbf{h}_\varsigma^\eta) + \varepsilon - \kappa] d\varsigma + e^{-\rho\Delta(j)} V'(i,j+\Delta(j))\right] \\
&= \mathrm{E}\left[\int_j^{j+\Delta(j)} e^{-\rho(\varsigma-j)} [\ln(A_i c_\varsigma^{1-\eta} \mathbf{h}_\varsigma^\eta) + \varepsilon - \kappa] d\varsigma + \tilde{\rho}(j) \ln(1+\omega(i,j)) + e^{-\rho\Delta(j)} V'(i,j+\Delta(j))\right]
\end{aligned}$$

where $\tilde{\rho}(j) \equiv (1 - e^{-\rho\Delta(j)})/\rho$. Substituting the expression for the value function at ages $j \in \mathcal{S}$ from above yields

$$\begin{aligned}
V'(i,j,\varepsilon,\kappa) = \mathrm{E}\Big[&\int_j^{j+\Delta(j)} e^{-\rho(\varsigma-j)} [\ln(A_i c_\varsigma^{1-\eta} \mathbf{h}_\varsigma^\eta) + \varepsilon - \kappa] d\varsigma + \tilde{\rho}(j) \ln(1+\omega(i,j)) \\
&+ \sum_{s\in\mathcal{S}|s>j} \frac{\Psi(s)}{\Psi(j)} \int_s^{s+\Delta} e^{-\rho(\varsigma-j)} \ln([1+\omega(i,j)] A_i c_\varsigma^{1-\eta} \mathbf{h}_\varsigma^\eta m(i,s)^{-\tilde{\nu}/\tilde{\rho}}) d\varsigma \Big] \\
&= V(i,j,\varepsilon,\kappa) + \left[\tilde{\rho}(j) + \tilde{\rho} \sum_{s\in\mathcal{S}|s>j} \frac{\Psi(s)}{\Psi(j)} e^{-\rho(s-j)}\right] \ln(1+\omega(i,j)) \\
&= V(i,j,\varepsilon,\kappa) + \left[\sum_{s\in j\cup\{z\in\mathcal{S}|z>j\}} \frac{\Psi(s)}{\Psi(j)} e^{-\rho(s-j)} \tilde{\rho}(s)\right] \ln(1+\omega(i,j))
\end{aligned}$$

As noted in Section 2,

$$V(i,j,\varepsilon,\kappa) = \overline{V}(i,j) + \tilde{\rho}(j)(\varepsilon-\kappa)$$

It follows that

$$\omega(i,j) = \exp\left(\frac{V'(i,j) - V(i,j)}{\sum_{s\in j\cup\{z\in\mathcal{S}|z>j\}} \frac{\Psi(s)}{\Psi(j)} e^{-\rho(s-j)}\tilde{\rho}(s)}\right) - 1$$

C Robustness and Extensions

C.1 Agglomeration and Congestion in Production

In the baseline model, there are neither congestion nor agglomeration forces in production. In this subsection, I examine how incorporating such forces alters the main results. To do this, I extend the baseline production function (5) to

$$Y_{it} = \overline{Z}_{it} L_{it}^{\alpha} K_{it}^{\lambda} F_{it}^{1-\alpha-\lambda}$$
$$\overline{Z}_{it} = z_{it} L_{it}^{\mu}$$

where F is a fixed factor of production, z_{it} is exogenous productivity, and μ is the elasticity of local productivity with respect to employment. I assume that firms take productivity as given when choosing employment. As in the baseline model, I also assume that factor prices are set competitively and that physical capital is freely mobile across locations. Under these assumptions, wages are

$$w_{it} = Z_{it} L_{it}^{\frac{\alpha+\lambda+\mu-1}{1-\lambda}}$$

where

$$Z_{it} \equiv \left[\alpha z_{it} \left(\frac{\lambda}{\alpha R_t}\right)^{\lambda} F_{it}^{1-\alpha-\lambda}\right]^{\frac{1}{1-\lambda}}$$

Following Herkenhoff et al. (2018), I set the labor and capital shares to $\alpha = 0.66$ and $\lambda = 0.29$, so that the fixed factor share is 0.05. I set the agglomeration parameter $\mu = 0.055$ to the midpoint of the range of $[0.03, 0.08]$ reported by Rosenthal and Strange (2004). In this case, the estimated welfare coefficient from equation (12) is $\hat{\beta} = 0.44$, which is nearly identical to the baseline estimate of $\hat{\beta} = 0.43$.

D Computational Appendix

D.1 Household's Problem

I solve the household's problem using the finite differences method described by Achdou et al. (2022). Voluntary wealth x is discretized using an unevenly spaced grid with 50 nodes. The i_{th} wealth gridpoint is set to $x_i = x_{\max} z_i^2$, where z_i is an evenly spaced grid on the interval $[0, 1]$. This generates a grid with lower bound 0, upper bound $x_{\max}$, and a higher concentration of nodes near the borrowing limit (where the curvature of the value and policy functions is highest). Owner-occupied housing h is discretized using a grid that includes 0 (for renters) and an evenly spaced grid with 25 nodes on the interval $[h_{\min}, h_{\max}]$. I verify ex-post that the upper bound of the wealth grid and the bounds of the housing grid do not bind in any of my exercises.

I discretize age using an evenly spaced grid with 80 nodes. Let j_n denote the discretized ages. Given the value function at age j_n, the value and policy functions at age j_{n-1} are computed as follows. I compute the discretized policy functions for c and h^r using the finite differences algorithm and the first-order conditions shown in Appendix B.1. Once this is done, I compute the discretized value function at age j_{n-1}, using the algorithm described by Kaplan et al. (2020a). This algorithm takes advantage of the fact that (ignoring the first two constraints) the HJB equation (1) can be re-written as

$$0 = \min\Big\{\rho\overline{V}_t(x,h,i,\ell,j) - \Big[\max_{c,h^r} u(A_i,c,\mathbf{h}) + \partial_x\overline{V}_t(x,h,i,\ell,j)\dot{x} + \dot{\overline{V}}_t(x,h,i,\ell,j)\Big]$$
$$,\ \overline{V}_t(x,h,i,\ell,j) - \overline{V}^*_t(x,h,i,\ell,j)\Big\}$$

Since the HJB equation is approximated by a system of linear equations in the finite differences algorithm, this can in turn be written as a linear complementarity problem. I solve this linear complementarity problem using the Newton-based method proposed by Fischer (1995) and code written by Yuval (2008). Since shocks arrive at discrete time intervals, the "intensity matrix" that contains the system of linear equations used to approximate the HJB is tridiagonal. I adapt Yuval's code to take advantage of this property, which speeds up the algorithm considerably. This is an important advantage of specifying the timing of idiosyncratic shocks as I do. One of the steps in this algorithm is to compute the value function conditional on adjusting owner-occupied housing, $\overline{V}^*_t(x,h,i,\ell,j)$. This requires evaluating the value function off of the discrete grids for x and h, which I do using linear interpolation. The solution to the linear complementarity problem also yields the optimal owner-occupied housing policy. This consists of an indicator for whether it is optimal to adjust $\mathbb{1}^{\text{adj}}_t(x,h,i,\ell,j)$, and the optimal owner-occupied housing quantity $h^*_t(x,h,i,\ell,j)$ conditional on adjusting.

I discretize the AR(1) process for the idiosyncratic component of labor endowments ℓ using Rouwenhorst's method (Rouwenhorst, 1995 and Kopecky and Suen, 2010) with 5 nodes. This method produces a transition matrix that I use to compute the expectation over labor endowment innovations at ages where idiosyncratic shocks occur. At these ages, I also take the expectation over idiosyncratic location preference and mortality shocks as shown in equation (2).

Starting with $\overline{V}_t(x,h,i,\ell,J) = 0$, I iterate backward on this procedure from age J to age 0. This yields discretized value and policy functions over the entire state space.

D.2 Density of State Variables

The density of state variables is discretized on the same grids as the policy and value functions. At age 0, the density of state variables is given exogenously (as described in Section 2). Given the density of state variables at age j_n, I compute the density of state variables at age j_{n+1} as follows. First, I compute the density conditional on not adjusting owner-occupied housing using the "implicit" updating method described by Achdou et al. (2022). I then construct a transition matrix that maps the discretized density of state variables prior to housing adjustments to the discretized density after adjustments. At gridpoints where adjustment is not optimal, the entire mass of that gridpoint is allocated to the same gridpoint post-adjustment. At gridpoints where adjustment is optimal, I allocate the mass post-adjustment to the gridpoints adjacent to the optimal choice (which in general will not lie on the grid) in inverse proportion to their relative distance from it. Multiplying the density conditional on no adjustments by this transition matrix yields the density of state variables at age j_{n+1}.

At ages where idiosyncratic shocks occur, I multiply the density by the survival probability $\psi(j_n)$ and apply the transition matrix for ℓ to account for mortality and labor endowment shocks. I then construct a transition matrix that accounts for migration. Similarly to agents who choose to make housing adjustments, the values of x and h after migration need not be on the discretized grids. As with the housing adjustment transition matrix, I allocate mass post-migration to the gridpoints adjacent to the optimal choice in inverse proportion to their relative distance from it.

Iterating forward on this procedure from the exogenous density at age 0 to the maximum age J yields the discretized density of state variables over the entire state space.

D.3 Transition Dynamics

I compute the transition dynamics between an initial ($t = 0$) and terminal ($t = \infty$) stationary equilibrium as follows. First, I discretize time using an evenly spaced grid with lower bound 0 and upper bound T. Denote the values on this grid by t_n. I start with an initial guess for prices $\{w^0_{it_n}, p^0_{it_n}\}_{i,t_n}$ in each location and discretized time. This guess should converge to the prices from the terminal steady state well before $t_n = T$.

I set the value function at time T equal to that of the terminal steady state. This is a good approximation as long as the economy has approximately converged to its terminal steady state by $t = T$ (I verify this ex-post, as described below). Given the value function at time t_n and prices at t_{n-1}, I compute the value and policy functions at time t_{n-1} as described in Section D.1. Since the value function is already known for every age at t_n, I use a parallelized loop over discretized ages to compute the value and policy functions at t_{n-1}. This is key to computing transition dynamics efficiently. Iterating backwards in time from $t_n = T$ to $t_n = 0$ yields the value and policy functions along the entire transition path.

The discretized density of state variables at $t_n = 0$ is that from the initial stationary equilibrium. Given the density of state variables at t_n, I compute the density at t_{n+1} as described in Section D.2. Iterating forward in time from $t_n = 0$ to $t_n = T$ yields the density of state variables along the entire transition path. Given these densities and parameters, I compute realized prices $\{w_{it_n}, p^r_{it_n}, p_{it_n}\}_{i,t_n}$ using equations (6)-(8).

I iterate on guesses for the time series of prices until the guess (which households expect to hold when making decisions) approximately equals the realized path. If convergence has not been achieved, I update the guess for prices using a linear combination of the old guess and realized prices. Once prices have converged, I verify that the density of state variables at $t_n = T$ is approximately equal to that from the terminal steady state. I find that $T = 60$ is a sufficiently long time for the economy to approximately converge to its terminal stationary equilibrium in all of my exercises.