International Diversification, Reallocation, and the Labor Share *

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Abstract

We develop a theory linking the labor share to international financial diversification. Firms choose labor facing aggregate uncertainty and hence the nature of aggregate risk affects the allocation of labor and both micro and macro labor shares. Increasing opportunities for international risk sharing leads to a reallocation of labor towards riskier/low labor share firms and a rise in the median (or within-firm) labor share, matching key micro-level facts. We use firm-level and cross-country data to document a number of empirical patterns consistent with our model, namely: (i) riskier firms have lower labor shares, (ii) international diversification is associated with reallocation towards risky firms and declines in the aggregate labor share, and (iii) industries with greater heterogeneity have higher sensitivity of their labor share to international diversification.

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1 Introduction

The last 40 years have witnessed a global decline in labor's share of income (Karabarbounis and Neiman, 2014) concurrent with a rapid deepening in international financial integration (Lane and Milesi-Ferretti, 2018). This paper explores the links between the international diversification of risk, risk-sharing between firms and workers, and the resulting labor share. The paper stresses the dual (and competing) effects of deeper international diversification on the aggregate labor share: a *within-firm* effect that increases micro-level and aggregate labor shares and a *reallocation* effect that shifts production towards lower labor share firms, which decreases the aggregate labor share. We derive and test empirically conditions for which the latter effect dominates, explaining the negative relationship between international financial integration and the aggregate labor share observed in the data.

Figure 1 displays the two main empirical patterns motivating our study:[1] over recent decades, measures of the aggregate labor have fallen substantially, while simultaneously, foreign holdings of domestic risky assets (e.g., equity) have increased dramatically. These patterns hold for both the US and globally.

(a) GDP-Weighted Averages (b) United States

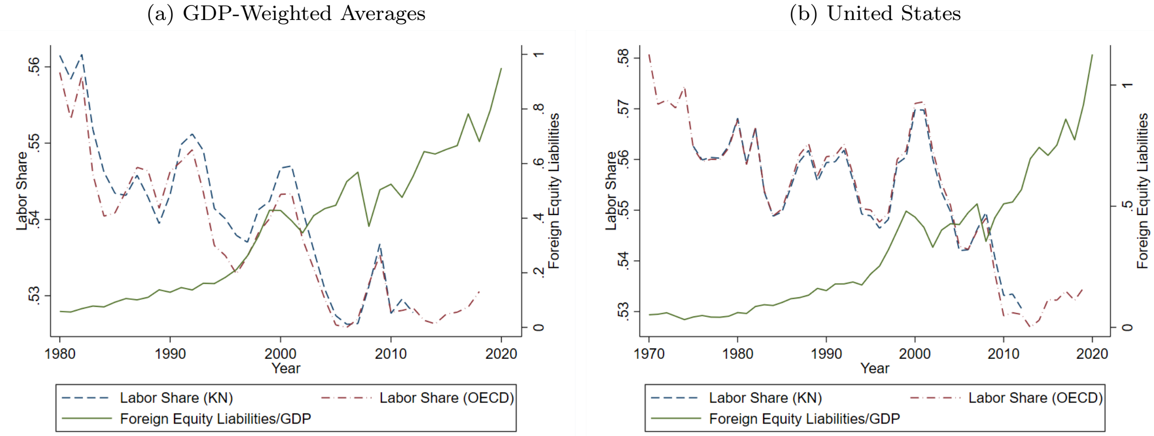

Figure 1: Trends in Labor Share and Equity Share of Foreign Investors

Notes: Figure displays the aggregate labor share (left-axis) and foreign investors' stock of domestic equity, divided by GDP (right-axis). Panel (a) displays GDP-weighted averages across countries; panel (b) displays statistics for the United States.

The paper proposes a simple yet novel framework linking aggregate risk – and opportunities for international risk-sharing – to the allocation of labor across firms and both micro and aggregate labor shares. Firms insure workers against market risk, but the price of such insurance depends on firms' exposure to that risk and their ability to diversify it away. International diversification reduces the price of risk and hence the implicit cost to workers of obtaining

[1]Details of the data are in Section 3.

insurance. Such diversification leads to an increase in micro-level labor shares within any given firm. At the same time, diversification leads riskier/low labor share firms that are more exposed to market risk to expand. The resulting reallocation can generate a reduction in the aggregate labor share even as the within-firm labor share increases. These predictions are consistent with recent evidence on micro and macro labor shares as forcefully documented in Hartman-Glaser et al. (2019) and Kehrig and Vincent (2021).

Empirically, the within firm insurance effect implies that firms that are more exposed to market risk display a lower labor share. This paper is the first to establish this robust fact using 47 years of data on US firms and controlling for a rich array of fixed effects and firm characteristics. We also provide empirical evidence of the reallocation effect by showing that riskier/low labor share firms expand their share of output as international diversification increases. Furthermore, using a panel of 25 countries over 38 years, we establish a robust negative link between the aggregate labor share and international diversification, suggesting that the reallocation effect dominates. These results indicate that international diversification can explain part of the decline in the labor share in a way that is consistent both with the within/between firm empirical decomposition of such decline and with a standard model of the labor share under market uncertainty. The paper also uses ORBIS cross-country firm-level data to show that industries in which there is greater dispersion in firm-level labor shares see their aggregate labor share fall by more in response to an increase in international diversification. This fact arises because there is more scope for reallocation from safer/ high labor share to risky/ low labor share firms within these industries.

Firms are subject to market risk and firm-specific risk when they choose their inputs. Our model captures the influence of these two sources of risk by introducing a standard heterogeneous firm production model under uncertainty. Market risk is introduced through a standard stochastic discount factor that is used to value firm cash flows, while firm-specific risk is introduced as a shock to firm productivity. Factor payments are determined before the shocks are realized.[2] In this framework, the firm-specific labor share depends both on the share of labor in the production function and on the covariance between market and firm-specific risk. As long as this covariance is non-zero, the equilibrium labor share will differ from the share of labor in production due to an insurance effect. In particular, if firm productivity is procyclical while market risk is countercyclical, as standard theory and empirics suggest, then the covariance term is negative, which reduces labor's share of expected income. Workers suffer reduced compensation in order to be insured. Firms that more exposed to aggregate risk, i.e. for which

[2] The environment extends recent work studying the impact of risk adjustments in the capital allocation (David et al., 2021; David and Zeke, 2022) to the allocation of labor. Donangelo et al. (2018) also study firm-level labor shares and risk premia.

the covariance between firm and market risk is more negative, display a lower labor share.

The model enables us to assess the consequences of a change in the nature of aggregate risk – more specifically, of diversification opportunities – that reduces the covariance between firm and market risk. Such a change induces a dual effect: (i) the decrease in the risk premium faced by firms leads to a decline in the wage discount and to an increase in the labor share within each individual firm. This is the within-firm insurance effect. On the other hand, (ii) diversification opportunities disproportionately affect firms with higher risk exposure and lower labor share. This leads to a reallocation of production towards these firms.

The aggregate consequences of risk diversification depend on whether the within-firm insurance effect or between firm reallocation effect dominates. We demonstrate that the impact of such diversification on the aggregate labor share takes a non-monotone U-shape: there is a unique threshold in the risk premium above (below) which a decrease in risk leads to a decline (increase) in the labor share. It is only in the extreme case in which market risk can be fully diversified that the labor share is fully determined by the relative importance of labor inputs in the production function.

This key novel result has important implications in the presence of opportunities for international risk-sharing. Investors diversify country-specific risk by exchanging equity shares with investors in other countries. Limits to diversification are captured by a cost of international trade in these assets. As barriers to international diversification are reduced, the extent of local country risk falls, thus generating the competing within and reallocation effects and the U-shaped pattern in the aggregate labor share. One striking implication of the theory is that increasing diversification does not lead to a perpetual decline in the labor share – there exists a threshold level of diversification such that increases beyond this point will lead to a reversal and the labor share begins to rise. At the limit, with full risk-sharing, labor share is completely determined by labor's share in the production function.

Our paper builds a bridge between two literatures: the literature on international financial integration and the literature on the global dynamics of labor share. That international integration favors risk-taking and growth has been demonstrated theoretically by Obstfeld (1994). Empirically, Thesmar and Thoenig (2011) show using French firm-level data that diversification in ownership leads to more risk-taking at the firm level. Levchenko et al. (2009) find sector-level volatility increases permanently following international financial liberalization suggesting an underlying risk-taking channel. Levchenko (2005) shows that the risk-sharing benefits of international financial integration may not be passed on to workers when access to the international insurance market is unevenly distributed and domestic risk-sharing is limited to self-enforcing contracts. Although the mechanism is quite different, our results share a similar theme – in our framework, in which only global "capitalists" trade internationally in financial assets, their endogenous risk-taking and reallocation of production towards risker firms can lead to a reduction in workers' share of national income.

Karabarbounis and Neiman (2014) document a global decline in the labor share and a vast ensuing literature has examined the causes of this decline, as recently summarized in Grossman and Oberfield (2021). Proposed candidates range from technical change and the relative price of capital goods (Karabarbounis and Neiman, 2014) to the rise of superstar firms (Autor et al., 2020; Lashkari et al., 2018), automation (Acemoglu and Restrepo, 2018, 2020; Autor and Salomons, 2018), increased trade globalization (Elsby et al., 2013) and changing market power of firms and workers (Barkai, 2020; Benmelech et al., 2020; Stansbury and Summers, 2020). Our paper provides a novel theory along with strong empirical support, namely, the role of financial globalization and its implications for the labor share due to international risk-diversification and its effects on risk-taking, worker insurance and wages. Perhaps closest to our work, Hartman-Glaser et al. (2019) emphasize the role of within-firm risk sharing between capitalists and workers in conjunction with increasing idiosyncratic volatility, whereas we study the effects of heterogeneous exposure to aggregate risk on the ex-ante distribution of expected labor shares and the implications of international diversification on the nature of this risk.[3] With these differences in mind, we view our empirical results and theoretical explanation as complementary to theirs.

In key empirical contributions, Kehrig and Vincent (2021) and Hartman-Glaser et al. (2019) decompose the labor share decline, in the case of the US, as the outcome of two opposing effects: a within-firm increase in labor share and a reallocation towards firms with lower labor share within each industry. The observed decline in the industry labor share results from the reallocation effect dominating the within firm effect. Our theoretical mechanism and empirical findings also stress these dual forces, relating both of them to changes in the nature of aggregate risk induced by increasing financial globalization.

The rest of the paper is organized as follows. Section 2 demonstrates how labor's share of income, both at the micro and macro level, are influenced by changes in the price of risk in a standard production environment with labor chosen under uncertainty. Section 3 develops a general equilibrium model of international diversification with heterogeneous firms and demonstrates how changes in the extent of international diversification can affect labor's share of income. Section 4 documents that several key implications of our model are supported by both cross-country and firm-level panel data. Section 5 concludes.

[3]There is significant evidence that workers are insured within the firm and especially so against temporary shocks (Guiso et al., 2005) and further that labor choices are made under considerable uncertainty (David, Hopenhayn, and Venkateswaran, 2016).

2 Risk, Input Allocations and the Labor Share

This section lays out our theory linking the labor share to the nature of aggregate risk and heterogeneous exposure to that risk across firms. For ease of notation we focus on a one-shot static problem and omit time subscripts, but the results extend to dynamic versions as well. We focus first on a closed-economy setting and derive sharp comparative statics with respect to changes in the nature of aggregate risk. We then embed this setup in a multi-country setting and link these changes to the degree of international financial diversification.

Multiple production technologies, i.e., "firms," produce a single homogeneous good according to:

$$Y_i = A_i K_i^{\alpha_1} L_i^{\alpha_2}, \quad \alpha_1 + \alpha_2 < 1 \tag{1}$$

Firms differ in their productivity, A_i. We focus on the Cobb-Douglas case in the text for tractability, but we show in Appendix A that the main insight extend to a broader class of production functions, including a more general CES function of capital and labor.

Input choices are made prior the realization of the shock and we assume that payments to factors of production cannot be state-contingent. In this sense, firms insure workers against the realization of shocks since wage payments are independent of these shocks and they are fully reflected in fluctuations in firm profits. Firms choose inputs to maximize the expected discounted value of cash flows, i.e.,:

$$\max_{L_i, K_i} \mathbb{E}\left[\Lambda\left(A_i K_i^{\alpha_1} L_i^{\alpha_2} - W L_i - R K_i\right)\right], \tag{2}$$

where Λ is a stochastic discount factor (SDF) used to price all cash flows.[4]

Firm-level labor shares. The optimality condition yields an expression for the share of expected sales paid to labor, which we refer to as *expected labor share*:

$$\frac{W L_i}{\mathbb{E}\left[Y_i\right]} = \alpha_2\left(1 - \kappa_i\right), \quad \text{where} \quad \kappa_i = -\text{cov}\left(\frac{\Lambda}{\mathbb{E}\left[\Lambda\right]}, \frac{A_i}{\mathbb{E}\left[A_i\right]}\right) \tag{3}$$

The expression shows that expected labor share is equal to the elasticity of the production function with respect to labor inputs, α_2, adjusted by a risk premium, which is captured by (the negative of) the covariance of firm-level productivity with the SDF. We can further decompose

[4] The exact timing of when payments to inputs are made is not crucial; only that these payments are not contingent on the realization of shocks.

the risk premium, κ_i, into the product of two terms:

$$\kappa_i = \underbrace{-\frac{\text{cov}\left(\mathcal{E}_i, \Lambda\right)}{\text{std}\left(\Lambda\right)}}_{\text{quantity of risk}} \times \underbrace{\frac{\text{std}\left(\Lambda\right)}{\mathbb{E}\left[\Lambda\right]}}_{\text{price of risk}}$$

where $\mathcal{E}_i = \frac{A_i}{\mathbb{E}[A_i]}$ captures unanticipated shocks to A_i. The quantity of risk is firm-specific and captures the "exposure" of each firm to the SDF. Importantly, it is exogenous and thus changes in the cross-section of risk premia stem solely from changes in the second term, the price of risk.

The price of risk is common across firms. If agents are risk-neutral and hence the SDF constant, the risk premium is zero. If agents are risk averse and shocks to the SDF are correlated with shocks to productivity, this is no longer the case and expected labor share of income is not in general equal to its share in the production function. More concretely, if firm productivity is procyclical and the SDF countercyclical, as standard theory and empirics suggest, then $\text{cov}\left(\mathcal{E}_i, \Lambda\right) < 0$, which implies a positive risk premium, reducing the expected labor share. The risk premium is firm-specific and is larger (depressing expected labor share more) for firms with a higher quantity of risk, i.e., more procyclical firms that covary more negatively with the SDF. The magnitude of this effect is increasing in the price of risk.

The intuition for the result is as follows: firms insure workers and so bear the entirety of cash flow risk. The risk premium captures the cost of this insurance, which leads firms to hire fewer workers, given the wage rate and expected productivity, reducing the expected labor share. To see this clearly, we can derive the relative allocation of labor as:

$$\frac{L_i}{L} = \frac{\left(\mathbb{E}\left[A_i\right]\left(1-\kappa_i\right)\right)^{\frac{1}{1-\alpha_1-\alpha_2}}}{\sum_i \left(\mathbb{E}\left[A_i\right]\left(1-\kappa_i\right)\right)^{\frac{1}{1-\alpha_1-\alpha_2}}}. \tag{4}$$

Firms with high risk premiums have lower expected labor shares and a lower relative allocation of labor as compared to firms with low risk premiums, conditional on expected productivity.

Aggregate labor share. The aggregate expected labor share can be written as an (expected) output-weighted average of firm-level expected labor shares:

$$\frac{WL}{\mathbb{E}\left[Y\right]} = \sum_i \frac{\mathbb{E}\left[Y_i\right]}{\mathbb{E}\left[Y\right]} \frac{WL_i}{\mathbb{E}\left[Y_i\right]} \tag{5}$$

where output shares are given by

$$\frac{\mathbb{E}\left[Y_i\right]}{\mathbb{E}\left[Y\right]} = \frac{\mathbb{E}\left[A_i\right]^{\frac{1}{1-\alpha_1-\alpha_2}}\left(1-\kappa_i\right)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}}{\sum_h \mathbb{E}\left[A_h\right]^{\frac{1}{1-\alpha_1-\alpha_2}}\left(1-\kappa_h\right)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}}. \tag{6}$$

Notice that, in a key part of our analysis, both output shares and firm-level labor shares – and so both components that add up to the aggregate labor share in (5) – are functions of the risk premium.

Combining expressions, we can express the aggregate expected labor share as:

$$\frac{WL}{E\left[Y\right]} = \alpha_2 \frac{\sum_i \left(A_i\left(1-\kappa_i\right)\right)^{\frac{1}{1-\alpha_1-\alpha_2}}}{\sum_i A_i^{\frac{1}{1-\alpha_1-\alpha_2}}\left(1-\kappa_i\right)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}}, \tag{7}$$

In the absence of risk adjustments in the allocation, the aggregate expected labor share is equal to α_2. More generally, however, the aggregate expected labor share depends on the full set of risk premia across firms, as they affect both firm-level labor shares and firm-level shares of aggregate output.

For convenience, we have worked with the expected labor share; the realized labor share depends on this measure and additionally the realization of shocks, $\frac{\mathbb{E}[Y]}{Y}$, which is identical to (the inverse of) a TFP shock:[5]

$$\frac{WL}{Y} = \frac{\mathbb{E}\left[Y\right]}{Y}\frac{WL}{\mathbb{E}\left[Y\right]} \tag{8}$$

2.1 Changes in the Price of Risk

Consider a change in the pricing of risk. We can represent this in general as a change in the dynamics of the SDF. Formally, we can define a function, χ, which maps the set of exogenous shocks, $\{A_i\}$, to a value of the SDF, i.e., $\chi : \{A_i\} \rightarrow \Lambda$. More concretely, in our application below, increasing international diversification lowers domestic holdings of domestic assets, which reduces the exposure of domestic consumption and the SDF to the set of domestic shocks and thus the price of risk. A change in the function χ leads to a change in the risk premium, which induces the following effects on the aggregate expected labor share:

$$\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \chi} = \underbrace{\sum_i \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \chi}\frac{WL_i}{\mathbb{E}\left[Y_i\right]}}_{\text{reallocation effect}} + \underbrace{\sum_i \frac{\mathbb{E}\left[Y_i\right]}{\mathbb{E}\left[Y\right]}\frac{\partial \frac{WL_i}{\mathbb{E}[Y_i]}}{\partial \chi}}_{\text{within effect}}. \tag{9}$$

[5] $\frac{\mathbb{E}[Y]}{Y}$ depends on the realizations of firm-level productivity shocks and the output shares of firms; we provide an analytical characterization in Appendix A.

The first term in (9) is the *reallocation effect*: it captures the contribution of changes in the allocation of resources – and hence output – across firms, conditional on the distribution of firm-level labor shares. The second term is the *within effect* - it captures the contribution of changes in micro-level labor shares, conditional on the distribution of resources.

Examples. For more insight into how the pricing of risk affects labor share, we need to put more structure on the environment. We first consider an example in which there are two production technologies: a risky and a safe. We then consider a version where Λ and A_i are log-linear functions of an aggregate shock and the loading of A_i's on this shock are normally distributed across firms. In both of these cases, we show that following a fall in the price of risk, the reallocation effect decreases the aggregate labor share while the within effect increases it. Furthermore, we derive the conditions under which the reallocation effect dominates.

Example 1: One risky, one safe technology. Assume that there are two types of technologies, i.e., $i \in \{s, r\}$. The safe technology has productivity always equal to its expectation and the risky technology has productivity that is stochastic and negatively correlated with the SDF. The safe technology bears no risk premium and hence $\frac{WL_s}{\mathbb{E}[Y_s]} = \alpha_2$, i.e., the labor share of this firm is pinned down by the production technology. In contrast, the risky technology features a positive risk premium κ_r.

Proposition 1 formalizes the effects of a change in the price of risk on the aggregate expected labor share:

Proposition 1. *A decrease in the price of risk and thus κ_r implies:*

1. *The within effect increases labor share:* $\sum_i \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]} \frac{\partial \frac{WL_i}{\mathbb{E}[Y_i]}}{\partial \kappa_r} = -\alpha_2 \frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]} < 0.$
2. *The reallocation effect reduces labor share:* $\sum_i \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \kappa_r} \frac{WL_i}{\mathbb{E}[Y_i]} = \alpha_2 \frac{\alpha_1 + \alpha_2}{1 - \alpha_1 - \alpha_2} \frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]} \frac{\mathbb{E}[Y_s]}{\mathbb{E}[Y]} \frac{\kappa_r}{(1+\kappa_r)} < 0.$
3. *There exists a threshold $\overline{\kappa_r} > 0$ such that labor share is increasing in κ_r and the price of risk:* $\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \kappa_r} > 0$ *iff* $\kappa_r > \overline{\kappa_r}$.

Proof. See appendix A. □

Part (i) of the proposition shows that as the price of risk falls, so does the implicit cost of insuring workers' wages and hence the labor share within the risky technology increases. Part (ii) shows that the lower price of risk leads to a reallocation of inputs and output towards the risky sector, increasing its share of economic activity. These two forces act in opposing directions on the aggregate expected labor share: the *within effect* raises it while the *reallocation effect* lowers it. Part (iii) of the proposition shows that there exists a threshold, $\overline{\kappa_r}$, such that when the price of risk and thus κ_r are high enough, the reallocation effect dominates and the aggregate expected labor share falls with the price of risk.

We illustrate these patterns in panel A of Figure 2. At the micro-level, the risky firm's labor share is monotonically increasing in the price of risk while that of the safe firm is constant. The risky firm's share of inputs and output are decreasing in the price of risk. When the price of risk is above the threshold, i.e., $\kappa_r > \overline{\kappa_r}$, then the reallocation effect is larger than the within effect and a fall in the price of risk leads to a decline in the aggregate labor share of income; if $\kappa < \overline{\kappa_r}$, then the opposite holds. The figure also underscores a novel implication of the theory: due to the two competing forces at work, the aggregate expected labor share is not monotonic in the price of risk. If the price of risk continues to fall, the labor share reverses its decline at the threshold $\overline{\kappa_r}$ and then begins to rise, eventually stabilizing at its share in the production function, α_2. Thus to extent the observed decline in the labor share is related to a reduction in the price of risk, the theory in fact predicts an eventual recovery.

Example 2: Gaussian shocks and risk exposure. The same intuition applies in environments with richer cross-firm heterogeneity. Assume now that there are a continuum of technologies with heterogeneous exposures to a single aggregate shock and that the SDF is an affine function of the same shock, i.e., we have the following system (where lower-case denotes natural logs):

$$\begin{aligned} a_i &= \mathbb{E}\left[a_i\right] + \beta_i x, \qquad \beta_i \sim \mathcal{N}\left(1, \sigma_\beta^2\right), \quad x \sim \mathcal{N}\left(0, \sigma_x^2\right) \\ \lambda &= \mathbb{E}\left[\lambda\right] - \lambda_x x, \qquad \lambda_x > 0 \end{aligned}$$

The first expression shows that firms differ in their exposure to the aggregate shock, with the degree of this heterogeneity captured by the dispersion in β_i, σ_β^2. By definitition, the average exposure is unity. The SDF is decreasing in the aggregate shock, capturing the usual intuition that marginal utility is countercyclical. The price of risk is approximately equal to $\lambda_x \sigma_x$.

Proposition 2 proves an analog of Proposition 1:

Proposition 2. *A decrease in the price of risk and thus λ_w implies:*

1. *If the employment-weighted aggregate risk exposure is positive, the within effect increases labor share: if $\sum_i \beta_i \frac{L_i}{L} > 0$, then $\sum_i \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]} \frac{\partial \frac{WL_i}{\mathbb{E}[Y_i]}}{\partial \lambda_x} < 0$*
2. *The reallocation effect decreases labor share: $\sum_i \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \lambda_x} \frac{WL_i}{\mathbb{E}[Y_i]} > 0$*
3. *There exists a threshold $\overline{\lambda_x} = \frac{(1-\alpha_1-\alpha_2)^2}{1-(\alpha_1+\alpha_2)^2} \frac{1}{\sigma_\beta^2 \sigma_x^2}$ such that $\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \lambda_x} > 0$ iff $\lambda_x > \overline{\lambda_x}$*

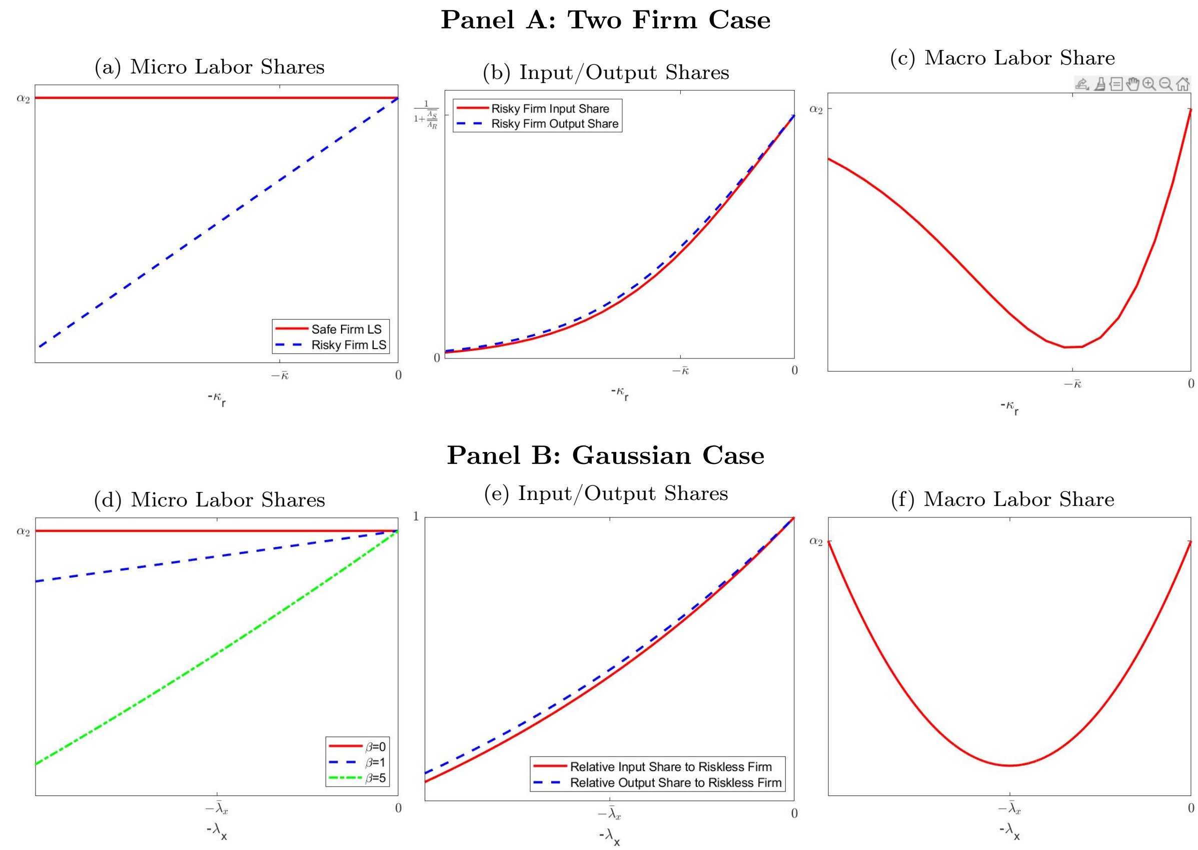

Figure 2: The Price of Risk and the Labor Share

Proof. See appendix A. □

The intuition is exactly the same as for Proposition 1. We illustrate the patterns in this case in Panel B of Figure 2. The left-hand plot shows that the within-firm labor share of risky firms increases as the price of risk falls and the slope is steeper the riskier (i.e., higher beta) is the firm. The middle plot shows as an example the relative share of inputs and output for a firm with $\beta = 1$ compared to a riskless firm with $\beta = 0$.[6] This relative share is decreasing in the price of risk. Finally, as before, the right-hand plot shows that the aggregate expected labor share first falls then increases with the price of risk.

2.2 International Diversification

The previous results illustrated the connection between the price of risk and labor's share of income both at the micro and macro levels. In this section we close the model in general equilibrium and link changes in the price to increasing opportunities for international diversification.

There are a continuum of islands, i.e., "countries," indexed by j. Consumption goods are

[6] The relative share of firms with $\beta_i \neq 1$ is equal to the relative shares of the $\beta = 1$ firm to the power of β_i.

homogeneous and fully mobile across countries (i.e., there are no frictions on trade). Labor is immobile across countries while financial assets are imperfectly mobile, described in more detail below.

In each country a continuum of firms operate one of two production technologies, a risky and a safe, indexed by $i \in \{s, r\}$, as outlined in Example 1 above. The safe technology has productivity always equal to its expectation, while the risky technology depends on the realization of a country-specific shock. For simplicity, we assume these shocks are uncorrelated across countries.

There are two types of households in each country: workers and capitalists. Workers provide labor for production and consume a portion of the final good; they cannot trade financial assets. Capitalists consume their portion of the final good and can trade a limited set of financial instruments: equity shares in firms, both domestic and foreign, and a risk-free bond. Timing works as follows: in the first period, firms decide the quantity of capital and labor to employ and capitalists receive an endowment that can be either consumed then or used for capital investment. Capitalists also make asset allocation decisions – how much of their endowment to sell and which financial assets to purchase. In the second period, production occurs, workers are paid their wages and capitalists their profits and both workers and capitalists consume the final good.

Worker and firm problems. Workers have utility over consumption and leisure, which they maximize subject to their budget constraint:

$$C_{wj} = W_j \sum_i L_{ij}$$

where C_{wj} denotes consumption of workers in country j. Because workers cannot trade financial assets they are hand-to-mouth and simply consume their labor income.

The firm's problem is identical to that in expression (2). In equilibrium, the domestic capitalist will always be a marginal investor and therefore we can use $\Lambda_j = \frac{U'(C_{ej1})}{U'(C_{ej0})}$ as the relevant SDF pricing the payouts of firms in country j, where $U'(\cdot)$ denotes the marginal utility of consumption for capitalists.[7]

Capitalist problem. The representative capitalist in country j receives an endowment of capital $K_{0,j}$ in the first period. They can either consume units of this endowment, sell it to firms who use it as productive capital or exchange it for financial assets. Purchasing a share

[7]Appendix B gives details of the firm problem and provides conditions for Λ_j to be the relevant SDF pricing country j assets in equilibrium. In brief, we assume there is an additional cost for firms that becomes overly foreign-owned. Because this cost is never incurred in equilibrium, we suppress it in our derivations here.

of a foreign firm in country h comes with a cost equal to a fraction τ_h of the amount invested. Thus there are limits to international diversification. This cost can be interpreted either as a literal tax on foreign investment or a simple reduced-form representation of informational or administrative costs of foreign investment. In the second period, capitalists receive the operating profits firms pay out to their shareholders and use those funds to purchase consumption goods.

The date-zero budget constraint of the capitalist is:

$$C_{ej0} = K_{0j} - K_{1j} - \sum_{i} S_{ij} P_{ij} - \sum_{h \neq j} (1 + \tau_h) S_{ijh} P_{ih} - P_{rf} S_{rf} + T_{0j} \tag{10}$$

and the date-one budget constraint:

$$C_{ej} = S_{ij} V_{ij} + \sum_{h \neq j} \sum_{i} S_{ijh} V_{ih} + S_{rf} + P_{kj} K_{1j} + T_j \tag{11}$$

where $V_{ij} = A_{ij} K_{ij}^{\alpha_1} L_{ij}^{\alpha_2} - K_{ij} P_{kj} - W_j L_{ij}$ is the value of firm i in country j after shocks are realized and P_{ij} its market value in period 0 before shocks are realized. The terms T_{0j} and T_j are lump sum transfers of the purchase prices of firms that are rebated to capitalists to ensure that consumption markets clear.

We assume that capitalists have CRRA preferences over consumption in each period.[8] They act to maximize the expected discount sum of utility subject to (10) and (11) and non-negativity constraints on consumption and firm equity shares (i.e., no shorting). An equilibrium is an allocation and set of prices that solve firms', workers', and capitalists' problems and satisfies the market clearing conditions in asset, goods, and input markets. Appendix B provides details of each agents' problem and more fully lays out the equilibrium conditions.

Diversification, the price of risk and the labor share. It is straightforward to verify that there exists a threshold, τ_j^{aut}, at which in equilibrium the shares of all country j firms are held by country j capitalists – we call this 'financial quasi-autarky' (the risk-free bond may still be traded by that country and/or that domestic capitalists may still hold foreign assets). In quasi-autarky, the domestic allocation only depends on domestic preferences, technology and the (common) risk-free rate. If $\tau_j < \tau_j^{aut}$, then in equilibrium the risky firm in country j is held in positive quantities both by domestic and foreign capitalists; the safe securities are never held by foreign capitalists when $\tau_j > 0$. In this case, the optimality conditions for domestic and foreign capitalists (from some country h) yield the following expressions for the share price of

[8]We make this assumption for simplicity. We can prove many of our results under the weaker restrictions that the utility function is a continuous, increasing and concave function of consumption.

the domestic risk firm, respectively:

$$P_{rj} \;\; = \;\; \mathbb{E}\left[\Lambda_j V_{rj}\right], \qquad P_{rj}\left(1+\tau_j\right) = \mathbb{E}\left[\Lambda_h V_{rj}\right] \tag{12}$$

Intuitively, if both foreign and domestic investors own shares of the risky firm then their cost-adjusted valuations of the firm must be equal. Note that the valuation of the domestic capitalist can be expressed as the riskless discounted value of the firm (by the common risk-free interest rate) less a risk premium due to the covariance of the domestic SDF with productivity. The foreign investor's valuation, on the other hand, is equal to the risk-free discounted value less the cost of foreign investment: the technology bears no risk premium for the foreign investor in country h because in equilibrium their SDF is independent of country j productivity.

Combining these expressions yields a condition showing that the risk premium in country j is pinned down by the cost of investment to foreign investors:

$$\kappa_{rj} = \mathrm{cov}\left(\frac{\Lambda_j}{\mathbb{E}\left[\Lambda_j\right]}, \frac{A_{rj}}{\mathbb{E}\left[A_{rj}\right]}\right) = \frac{\tau_j\left(1-\alpha_1-\alpha_2\right)}{1+\tau_j\left(1-\alpha_1-\alpha_2\right)} \tag{13}$$

As the cost to foreign investors falls, they increase their demand for risky firm shares. In equilibrium, domestic capitalists must reduce their holdings of these shares as a fraction of their portfolio and thus their consumption and SDF become less sensitive to the realization of productivity of the domestic risky firm.

We apply (13) to the results from section 2 to derive expressions for expected labor share and labor allocations are as function of τ_j for $\tau_j \in [0, \tau_j^{aut})$. At the micro-level, the risky firm expected labor share is given by

$$\frac{L_{rj} W_j}{E\left[Y_{rj}\right]} = \alpha_2 \frac{1}{1+\tau_j\left(1-\alpha_1-\alpha_2\right)}$$

and, as before, that of the safe firm is simply equal to α_2. As τ_j falls, so does the risk premium, increasing the expected labor share for the risky technology.

The allocation of labor to the risky firm satisfies

$$\frac{L_{rj}}{L_j} = \frac{\left(\mathbb{E}\left[A_{rj}\right]\frac{1}{1+\tau_j(1-\alpha_1-\alpha_2)}\right)^{\frac{1}{1-\alpha_1-\alpha_2}}}{\mathbb{E}\left[A_{sj}\right]^{\frac{1}{1-\alpha_1-\alpha_2}} + \left(\mathbb{E}\left[A_{rj}\right]\frac{1}{1+\tau_j(1-\alpha_1-\alpha_2)}\right)^{\frac{1}{1-\alpha_1-\alpha_2}}},$$

A fall in τ_j reduces the risk premium and leads to a reallocation of resources towards the risky technology. Thus, increasing diversification generates both the within and reallocation effects on the aggregate labor share.

Last, the aggregate expected labor share is given by:

$$\frac{W_j L_j}{\mathbb{E}\left[Y_j\right]} = \alpha_2 \frac{1 + \left(\frac{\mathbb{E}[A_{rj}]}{\mathbb{E}[A_{sj}]} \frac{1}{1+\tau_j(1-\alpha_1-\alpha_2)}\right)^{\frac{1}{1-\alpha_1-\alpha_2}}}{1 + \left(\frac{\mathbb{E}[A_{rj}]}{\mathbb{E}[A_{sj}]}\right)^{\frac{1}{1-\alpha_1-\alpha_2}} \left(\frac{1}{1+\tau_j(1-\alpha_1-\alpha_2)}\right)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}} \tag{14}$$

Proposition 3 formalizes the effects of changes in diversification opportunities, i.e., τ_j, on the aggregate expected labor share:

Proposition 3. *For $\tau_j < \tau_j^{aut}$, a fall in the cost of foreign investment, τ_j, implies:*

1. *The equilibrium price of risk decreases.*
2. *Domestic holdings of the risky firm fall if the real interest rate is non-negative.*
3. *The within effect increases labor share: $\sum_i \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]} \frac{\partial \frac{WL_i}{\mathbb{E}[Y_i]}}{\partial \tau_j} < 0$*
4. *The reallocation effect decreases labor share: $\sum_i \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \tau_j} \frac{WL_i}{\mathbb{E}[Y_i]} > 0$*
5. *There exists a threshold $\overline{\tau_j}$ such that $\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \tau_j} > 0$ iff $\tau_j > \overline{\tau_j}$.*[9]

Proof. See Appendix B. □

2.3 Empirical Predictions

The model developed in the previous sections has several testable empirical predictions. Here we lay these out and we investigate them in turn in the remainder of the paper.

Prediction 1: International diversification affects aggregate labor share. Proposition 3 implies that the cost for foreign investors in a country (τ_j) affects its labor share. In particular, a first-order Taylor expansion of the equation for aggregate labor share yields the following expression for labor's share of income:

$$\log \frac{W_j L_j}{Y_j} = \alpha_j + \gamma_\tau \tau_j + \gamma_{tfp} \left(tfp_j - \mathbb{E}\left[tfp_j\right]\right)$$

Our theory implies that if the reallocation effect dominates the within effect, then $\gamma_\tau > 0$; further, preset wages imply that the coefficient $\gamma_{tfp} < 0$. Although the transaction cost, τ_j

[9] In some parameterizations (i.e., if there is relatively little risk or capitalists are close to risk neutral) it is possible that $\overline{\tau_j} > \tau_j^{aut}$.

is not directly observable, in equilibrium, τ_j is monotonically decreasing in foreign investor's holdings of domestic firm equity, suggesting the following regression:

$$\log \frac{W_{jt} L_{jt}}{Y_{jt}} = \alpha_j + \gamma_{FEQ} FEQ_{jt} + \gamma_{tfp} \left(tfp_{jt} - \mathbb{E}_{t-1} \left[tfp_{jt} \right] \right) + \varepsilon_{jt} \tag{15}$$

where FEQ_j is a measure of foreign holdings of country j equity. Again, if the reallocation effect dominates the within effect, the implies that $\gamma_{FEQ} < 0$ - the aggregate labor share should be falling in international diversification.[10]

Prediction 2: Risky firms have lower labor shares. Our model also has implications for micro-level observables. In particular, and in a key building block of the theory, expression (3) shows that conditional on expected productivity, riskier firms should have lower labor shares.[11] The result suggests a regression of the form

$$\log LS_{it} = \gamma_\beta \beta_i + controls + \varepsilon_{it} \tag{16}$$

where the theory predicts $\gamma_\beta < 0$.

Prediction 3: International diversification is associated with a reallocation towards risky/low labor share firms. Proposition 3 shows that increasing international diversification leads to a reallocation of labor towards risky/low labor share firms. Appendix B shows that a similar reallocation occurs for capital and output as well. These results suggest regressions of the form

$$\log \frac{Z_{it}}{\overline{Z}_t} = \gamma_{\beta,FEQ} \beta_i \times FEQ_t + controls + \varepsilon_{it} \tag{17}$$

where $Z = Y, L, K$ denotes capital, labor or output, $\overline{Z}$ denotes industry totals and, as above, FEQ_t is a measure of foreign equity liabilities. The theory predicts $\gamma_{\beta,FEQ} > 0$. Similarly, the theory suggests an analog of (17) to test whether international diversification is associated with reallocation towards lower labor share firms:

$$\log \frac{Z_{it}}{\overline{Z}_t} = \gamma_{LS,FEQ} LS_{it} \times FEQ_t + controls + \varepsilon_{it}. \tag{18}$$

where we should find $\gamma_{LS,FEQ} < 0$.

[10] If we were to extend our model to one with a CES aggregator, as in appendix A, (15) would change only in that we would have to add controls for the determinants of the aggregate K/L - for example the relative price of investment goods.

[11] The result does not depend on Cobb-Douglas production functions; Appendix B develops an analog of (3) under a more general CES production function.

Prediction 4: Industries with larger dispersion in labor shares should see larger declines in the industry-level labor share. With heterogeneity in risk exposures and thus labor shares across firms within the risky sector, we can link the sectoral-level labor share to the price of risk and the mean and dispersion of labor shares across firms. Consider the case of Gaussian risk exposures from example 2 in Section 2 and a given industry within the risky sector, denoted by s. In this case can express the elasticity of the change in the labor share of industry s to a change in the price of risk as:[12]

$$\frac{\partial \log \frac{W_{sjt}L_{sjt}}{E[Y_{sjt}]}}{\partial \lambda_{jt}} = \frac{1}{\lambda_{jt}} \left(\overline{LS}_{jst} - \log \alpha_{2s} + \text{var}\left(LS_{jst}\right) \frac{1-(\alpha_1+\alpha_2)^2}{(1-\alpha_1-\alpha_2)} \right)$$

where $\overline{LS}_{jst}$ and $\text{var}\left(LS_{jst}\right)$ denote the mean and cross-sectional variance of labor shares in industry-country-year sjt. Industries with greater more dispersion in risk exposures and hence, labor shares, have greater scope for reallocation in response to changes in the price of risk. Thus, the reallocation effect should be larger in those industries. In industries with higher mean risk exposure and hence, lower average labor share, the within effect should be smaller. The results suggests a regression of the following form:

$$\log LS_{sjt} = \gamma_\mu \overline{LS}_{sjt} \times FEQ_{jt} + \gamma_\sigma \text{var}\left(LS_{sjt}\right) Div_{j,t} + controls + \varepsilon_{it} \tag{19}$$

which tests whether the mean and dispersion of firm-level labor shares interacted with measures of international diversification, are associated with changes in industry-level labor shares. The theory predicts $\gamma_\mu < 0$ and $\gamma_\sigma < 0$.

3 Empirical Analysis

In this section we investigate the empirical predictions of the theory described in Section 2.3. We combine a number of datasets. We obtain cross-country panel data on aggregate labor shares from Karabarbounis and Neiman (2014), which covers 100 countries over the period 1975 to 2012. For robustness we also use aggregate labor shares from the OECD, which extends beyond the Karabarbounis and Neiman (2014) sample, which ends in 2012.[13] We measure international diversification by foreign equity liabilities, which represents the value of domestic equity liabilities held by foreign investors. We normalize this variable by dividing by GDP. The data are obtained from the External Wealth of Nations dataset of Lane and Milesi-Ferretti (2018), and include both foreign equity holdings via FDI and portfolio investment. We omit

[12]The derivation is in Appendix C.

[13]Data were obtained from Haver Analytics.

countries which are tax havens or have a disproportionate share of financial activity and focus on the remaining 25 advanced economies for our cross-country regressions.[14]

We use two micro-level datasets. First, US firm-level data from Compustat. This dataset enables us to compute measures of firm risk exposures from financial market data using high quality data with relatively good coverage (at least among large, publicly traded firms). Specifically, we proxy for these exposures using firms' CAPM betas. We estimate these betas using regressions of firm-level daily stock market returns on the daily market return for all trading days within a calendar year.[15] Note that, by definition, a weighted average of firm-level betas (by market capitalization) is always equal to one. In other words, if there is reallocation towards riskier firms, that makes the market portfolio itself riskier, lowering measured betas. This poses a challenge for our predictions relating to reallocation. To this end, we compute measures of firm *relative betas* by residualizing the estimated CAPM betas on industry-year fixed effects and computing the average of these residuals for each firm. From hereon, we use these relative betas as the main measure of firm risk exposures, β_i.

Measuring labor share in Compustat also presents challenges – while the number of employees is reported for most firms in Compustat, only a subset have data on labor compensation. To address this issue, we use three measures of firm labor share. The first is simply labor intensity, calculated as employees divided by sales, denoted $\frac{L}{Y}$. Because all of our analyses include industry fixed-effects, this is proportional to labor share if firms within an industry have the same average compensation per employee.[16] The second measure uses reported labor compensation divided by sales for the subset of firms that do report labor compensation in Compustat (roughly 9% of the sample), which we denote LS. Our third measure follows the approach laid out in Donangelo et al. (2018), which infers labor compensation per employee for firms with missing data from the industry-level average of firms that do report compensation, denoted ELS ('extended' labor share).

The second micro dataset we use is cross-country firm-level panel data from the ORBIS database. We construct measures of both the aggregate labor share for every industry-country-year, as well as the mean and standard deviation of (log) labor shares within each industry-country-year. We measure industries as 3 digit SIC codes and again omit countries that are tax havens or financial centers. We include only observations (industry-country-years) with at least 50 firms.

[14] Data were obtained from `https://www.brookings.edu/wp-content/uploads/2021/09/EWN-dataset_9.14.21.xlsx`.

[15] We obtain the daily market return from Ken French's data library.

[16] We focus primarily on differences in labor shares across firms within an industry since labor shares can vary across industries due to differences in production technologies. Moreover, previous work has documented that the labor share decline is foremost a within – rather than across – industry phenomenon.

(a) Karabarbounis and Neiman (2014) (b) OECD

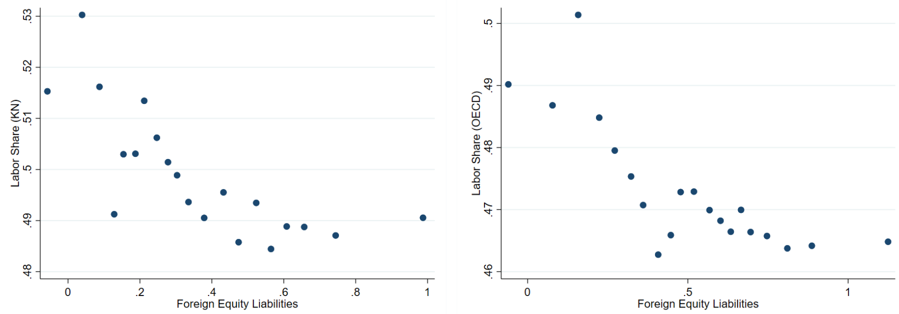

Figure 3: Foreign Equity Liabilities and the Labor Share

Notes: Figure displays bin-scatter plots of aggregate labor share and foreign equity liabilities, controlling for country fixed-effects. Each point in the figure is a bin of country-year observations.

3.1 International Diversification and Aggregate Labor Share

Prediction 1 links the extent of diversification to the aggregate labor share and in particular, suggests a negative relationship between the two if the reallocation effect dominates the within effect. Figure 3 displays a bin-scatter of country-level labor shares against foreign equity liabilities, as a percentage of GDP. We control for country-level characteristics by first removing country fixed-effects from both series – specifically, we residualize both variables on the fixed-effects and then add back in the unconditional means. The figures shows that when a country has larger foreign equity liabilities (as compared to its average level), it also tends to have a lower aggregate labor share.

Table 1 estimates the regression in expression (15) to more formally investigate these patterns. The results confirm prediction 1 - increases in foreign equity liabilities are associated with statistically significant reductions in the labor. Column (1) reports the results with country fixed-effects. Column (2) adds year fixed-effects. Column (3) controls for unexpected shocks to TFP, as implied by expression (15) and column (4) additionally controls for average hours per worker and the relative price of investment goods. These latter two variables are motivated by an extended version of our model with CES rather than Cobb-Douglas production (see Appendix A for details). Across all specifications, the coefficient on foreign equity liabilities remains negative and statistically significant. To gauge the economic magnitude, consider the estimated coefficient in column (4) of about -0.039. From 1975 to 2012 (the first and last periods of the KN database), foreign equity liabilities as a fraction of GDP increased from 4.3% to 56.8% in the United States. Putting these values together, the results imply a decline in the US labor share of about 1.1 percentage points over this period, about 35% of the total observed decline.

Table 1: International Diversification and the Labor Share

	(1)	(2)	(3)	(4)
Foreign Equity Liabilities	-0.0685**	-0.0491**	-0.0283**	-0.0387***
	(-2.51)	(-2.62)	(-2.30)	(-3.88)
TFP shock			-0.237	-0.397**
			(-1.17)	(-2.18)
Average hours				-0.127
				(-0.90)
Relative price of investment				0.102***
				(3.38)
Country fixed effects	yes	yes	yes	yes
Year fixed effects	no	yes	yes	yes
R^2	0.853	0.878	0.921	0.937
Observations	661	661	423	402

Notes: Table presents regressions of country-level labor shares on foreign equity liabilities as a percentage of GDP. Standard errors are two-way clustered by country and year. t-statistics in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

3.2 Firm-level Risk and Labor Share

In a key piece of our theory, prediction 2 states that firms which are more exposed to aggregate risk have, on average, lower labor shares. Figure 4 presents bin-scatter plots of firm betas – our measure of risk exposure – against the (log) labor share, after controlling for industry-by-time fixed-effects.[17] Panel (a) displays the results for labor intensity while panel (b) displays results using extended Labor Share.[18]

To confirm that this relationship is not only economically, but statistically significant and robust to standard controls, we estimate panel regressions of firm-level shares on betas as detailed in expression (16). Table 2 reports the results across the three measures of labor share and with a number of controls – firm profitability, age and size – and confirms that firms with higher risk exposure (i.e., beta) tend to have lower labor shares than low beta firms (within the same industry), even conditional on other observables. The effect is both statistically and economically significant. For example, the estimated coefficients imply that a firm with beta equal to one has, on average, a labor share that is more than 15% lower than a firm in the same industry with a beta equal to zero.

[17] To address issues of simultaneity, we use betas estimated over the previous calendar year.

[18] The other labor share measure from Donangelo et al. (2018), 'Labor Share', also has a negative relationship with firm market betas; however it is more volatile due to the small number of firms that report the measure within a typical industry.

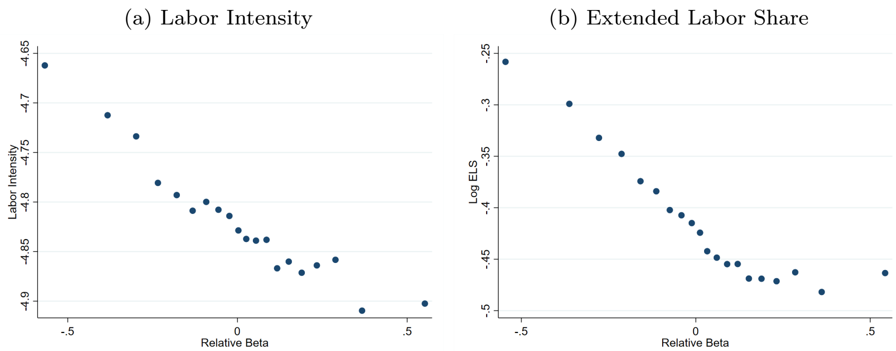

Figure 4: Exposure to Aggregate Risk and Labor's Share of Income

Notes: Figure displays bin-scatter plots of firm-level labor shares and betas, controlling for industry-year fixed-effects. Each point in the figure is a bin of firm-year observations.

Table 2: Firm-Level Labor Shares and Risk Exposures

	(1) $\log\left(\frac{L}{Y}\right)$	(2) $\log(ELS)$	(3) $\log(LS)$	(4) $\log\left(\frac{L}{Y}\right)$	(5) $\log(ELS)$	(6) $\log(LS)$
$\widehat{\beta}$	-0.238***	-0.241***	-0.105***	-0.336***	-0.176***	-0.166***
	(-12.57)	(-16.24)	(-3.16)	(-16.35)	(-14.62)	(-5.77)
Profitability				-1.019***	-1.514***	-1.470***
				(-24.10)	(-33.93)	(-20.92)
Age				-0.0275***	-0.00781***	-0.00738
				(-5.85)	(-2.73)	(-1.14)
Size				0.0609***	-0.00607***	0.0253***
				(14.69)	(-2.74)	(4.36)
Industry-year F.E.	yes	yes	yes	yes	yes	yes
R^2	0.677	0.405	0.718	0.716	0.510	0.797
Observations	153676	126730	11536	142760	118455	10039

Notes: Table presents regressions of (log) firm labor shares on firm betas and controls. Standard errors are two-way clustered by firm and year. t-statistics in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

3.3 Diversification and Reallocation

We now turn to our third prediction, namely, that increasing international diversification should lead to reallocation towards risky/low labor share firms. Figure 5 displays a bin-scatter plot of firm-level employment growth (relative to its industry) against its beta and two measures of labor share. The figure clearly illustrates that riskier/low labor share firms have grown substantially more than safer/high labor share ones, implying that there has been a reallocation of employment towards the former.

Table 3 reports results from regressions of firm shares of industry output, labor and capital

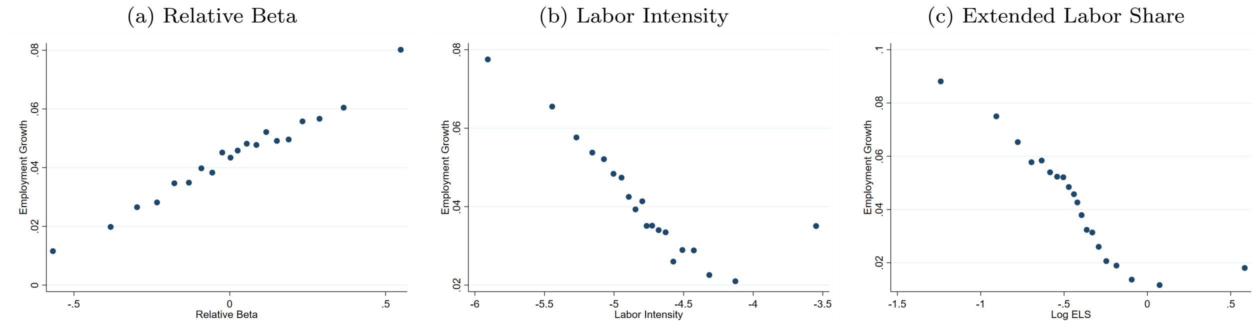

Figure 5: Trends in the Reallocation of Employment within Industry

Notes: Figure displays bin-scatter plots of firm-level labor shares and betas, controlling for industry-year fixed-effects. Each point in the figure is a bin of firm-year observations.

Table 3: International Diversification and Reallocation to Risky Firms

	(1) $\log(\frac{Y_i}{Y_{ind}})$	(2) $\log(\frac{L_i}{L_{ind}})$	(3) $\log(\frac{K_i}{K_{ind}})$	(4) $\log(\frac{Y_i}{Y_{ind}})$	(5) $\log(\frac{L_i}{L_{ind}})$	(6) $\log(\frac{K_i}{K_{ind}})$
$\widehat{\beta}$ × FEQ	2.318***	1.804***	2.372***	2.324***	1.829***	2.392***
	(11.29)	(9.11)	(9.44)	(10.59)	(8.82)	(9.02)
Profitability				1.479***	0.908***	1.058***
				(34.38)	(25.86)	(26.56)
Age				0.278***	0.247***	0.233***
				(21.46)	(18.69)	(14.33)
Industry-year F.E.	yes	yes	yes	yes	yes	yes
Firm F.E.	yes	yes	yes	yes	yes	yes
R^2	0.943	0.941	0.938	0.950	0.945	0.942
Observations	151652	150763	152913	146330	145021	147208

Notes: Table presents regressions of firm shares of industry output, labor and capital on the interaction of firm betas and US foreign equity liabilities as a fraction of GDP. Standard errors are two-way clustered by firm and year. t-statistics in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

on the interaction of US foreign equity liabilities with firm betas, as specified in expression (17). We include firm and industry-year fixed effects and controls for firm profitability and age.[19] The results show that increases in foreign holdings of US equity are associated with riskier firms composing a higher share of industry output and inputs, i.e., with a reallocation of resources towards those firms. The coefficient estimates imply that, on average, in response to the average annual increase in foreign equity liabilities for the United States of 1.7%, a firm with a beta one standard deviation above the mean increases its share of industry economic activity by about 1-1.5% more than a firm with the mean beta.

[19]The individual components of the interaction do not have to be controlled for since they are absorbed by the fixed-effects.

Table (4) reports the results of the analogous regression specified in (18) of firm share of industry output and inputs on the interaction of US foreign equity liabilities with firm labor shares. The table shows that increases in foreign holdings of US equity are associated with high labor share firms composing a smaller fraction of industry output and inputs.[20]

Table 4: International Diversification and Reallocation to Low Labor Share Firms

	(1) $\log(\frac{Y_i}{Y_{ind}})$	(2) $\log(\frac{L_i}{L_{ind}})$	(3) $\log(\frac{K_i}{K_{ind}})$	(4) $\log(\frac{Y_i}{Y_{ind}})$	(5) $\log(\frac{L_i}{L_{ind}})$	(6) $\log(\frac{K_i}{K_{ind}})$
$\log(ELS) \times$ FEQ	-0.328***	-0.256***	-0.406***	-0.398***	-0.311***	-0.447***
	(-5.65)	(-4.44)	(-6.08)	(-6.97)	(-5.69)	(-6.65)
Profitability				0.984***	0.948***	0.550***
				(19.19)	(18.56)	(8.74)
Age				0.237***	0.238***	0.251***
				(19.37)	(20.14)	(16.06)
Industry-year F.E.	yes	yes	yes	yes	yes	yes
Firm F.E.	yes	yes	yes	yes	yes	yes
R^2	0.954	0.942	0.947	0.957	0.945	0.949
Observations	119414	118338	118994	116579	115551	116180

Notes: Table presents regressions of firm shares of industry output, labor and capital on the interaction of firm labor share and US foreign equity liabilities as a fraction of GDP. Standard errors are two-way clustered by firm and year. t-statistics in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

3.4 Diversification, Heterogeneity and Industry-Level Labor Share

Prediction 4 suggests that industry-level labor share should fall more in response to increased diversification in industries with larger cross-firm dispersion in labor shares or a higher average labor share. We use the cross-country firm-level Orbis data to investigate this prediction. Table 5 reports results of panel regressions of industry-country-year labor shares on the interaction of foreign equity liabilities and the mean and standard deviation of (log) labor shares within that industry (we lag the labor shares measures to account for possible simultaneity bias) as implied by expression (19). We include rich fixed-effects in this regression (indeed, this is one of the main benefits of turning to the Orbis data), i.e., industry-year, country-year and industry-country effects (the unit of observation is industry-country-year), and include the interacted variables separately as controls. Columns (1) and (2) show that in line with the theory, country-industries with greater dispersion in firm-level labor shares experience larger declines in their labor share in response to increases in foreign holdings of domestic equity, as do country-industries with

[20] The results are qualitatively similar for the two other measures of labor share that we consider. We report these results in Appendix D.

higher average labor shares. Column (3) shows that the result is robust to controlling for the interaction of the mean and standard deviation of firm sales, which suggests that we are not simply picking up the effects of dispersion in firm size.

Table 5: Diversification, Heterogeneity and Industry-Level Labor Share

	(1)	(2)	(3)
Foreign Equity Liabilities × mean log(LS)	-0.127*	-0.0869**	-0.0940**
	(-1.86)	(-2.74)	(-2.62)
Foreign Equity Liabilities × stdev log(LS)	-0.0983*	-0.0513**	-0.0592**
	(-1.95)	(-2.40)	(-2.26)
Foreign Equity Liabilities × stdev log(sales)			-0.0099
			(-1.22)
Foreign Equity Liabilities × mean log(sales)			-0.0047
			(-1.07)
Fixed effects	no	yes	yes
R^2	0.485	0.791	0.804
Observations	71346	69431	57325

Notes: Table presents regression of industry-level labor shares on the mean and standard deviation of (log) labor shares within that industry interacted with foreign equity liabilities as a fraction of GDP. All specifications include the interacted variables individually as controls. Fixed effects, when included, consist of industry-year, country-year and industry-country effects. Standard errors are two-way clustered by industry-country and year. t-statistics in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

4 Conclusion

We develop a model of the allocation of inputs and labor's share of income in an environment with heterogeneous firms and where input decisions are made under uncertainty. In equilibrium, riskier firms have lower labor's share of income and allocated a smaller share of the inputs/outputs relative to their productivity, and the magnitude of this risk adjustment depends on the price of risk. In a model of international capital where the price of risk is endogenous to the degree of international risk sharing, we show that an increase in international diversification leads to lower prices of risk and leads to two competing effects on country labor shares of income. First, a fall in the price of risk leads to an increase in labor share for a given technology. Second, a fall in the price of risk leads to reallocation of labor and capital to riskier firms, who have lower labor shares. If the second effect dominates the first one, then an increase in international diversification can lead to a fall in aggregate labor's share of income, while within-technology labor's share of income rises. We document that the empirical predictions of the model are supported by both U.S. firm-level data, cross-country aggregate data, and cross-country firm-level data. Our empirical results imply that a sizable part of the observed decline in labor's share of income over the past several decades, both in the U.S. and internationally, could be explained by the concurrent increase in international diversification.

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Appendix

A Derivations and proofs for section 2

A.1 Derivations and additional results

Output shares Note that the output share of a given firm can be written as:

$$\frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]} = \frac{\mathbb{E}[A_i] K_i^{\alpha_1} L_i^{\alpha_2}}{\sum_h \mathbb{E}[A_h] K_h^{\alpha_1} L_h^{\alpha_2}}$$

Plugging in for input shares from (4) yields a characterization of output shares as a function of technology and covariance terms:

$$\frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]} = \frac{\mathbb{E}[A_i]^{\frac{1}{1-\alpha_1-\alpha_2}} \left(1 + \frac{Cov(\Lambda, A_i)}{\mathbb{E}[A_i]\mathbb{E}[\Lambda]}\right)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}}{\sum_h \mathbb{E}[A_h]^{\frac{1}{1-\alpha_1-\alpha_2}} \left(1 + \frac{Cov(\Lambda, A_h)}{\mathbb{E}[A_h]\mathbb{E}[\Lambda]}\right)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}}. \tag{20}$$

Realized labor share of income To derive the realized aggregate labor share, note that we can write:

$$\frac{WL}{Y} = \frac{\mathbb{E}[Y]}{Y} \frac{WL}{\mathbb{E}[Y]}$$

We can solve for the ratio of expected to realized output:

$$\begin{aligned}
\frac{\mathbb{E}[Y]}{Y} &= \frac{\sum_i \mathbb{E}[A_i] \left(\frac{K_i}{K}\right)^{\alpha_1} \left(\frac{L_i}{L}\right)^{\alpha_2}}{\sum_i A_i \left(\frac{K_i}{K}\right)^{\alpha_1} \left(\frac{L_i}{L}\right)^{\alpha_2}} \\
\Rightarrow \frac{\mathbb{E}[Y]}{Y} &= \frac{\sum_i \mathbb{E}[A_i] \left(\mathbb{E}[A_i] \left(1 + \frac{Cov(\Lambda, A_i)}{\mathbb{E}[\Lambda]\mathbb{E}[A_i]}\right)\right)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}}{\sum_i A_i \left(\mathbb{E}[A_i] \left(1 + \frac{Cov(\Lambda, A_i)}{\mathbb{E}[\Lambda]\mathbb{E}[A_i]}\right)\right)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}} \\
\Rightarrow \frac{\mathbb{E}[Y]}{Y} &= \frac{\sum_i \mathbb{E}[A_i]^{\frac{1}{1-\alpha_1-\alpha_2}} \left(1 + \frac{Cov(\Lambda, A_i)}{\mathbb{E}[\Lambda]\mathbb{E}[A_i]}\right)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}}{\sum_i \frac{A_i}{\mathbb{E}[A_i]} \mathbb{E}[A_i]^{\frac{1}{1-\alpha_1-\alpha_2}} \left(1 + \frac{Cov(\Lambda, A_i)}{\mathbb{E}[\Lambda]\mathbb{E}[A_i]}\right)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}} \qquad (21)
\end{aligned}$$

Note that this only depends on firm productivities and their covariance with the SDF. If TFP is properly measured ($TFP_t = \frac{Y_t}{K_t^{\alpha_1} L_t^{\alpha_2}}$) then

$$\frac{\mathbb{E}[Y]}{Y} = \frac{\mathbb{E}[TFP]}{TFP} \tag{22}$$

A.2 Proofs

Proposition 1

1. Within effect derivative Taking the derivative of the labor share of each technology yields:

$$\sum_i \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]} \frac{\partial \frac{WL_i}{\mathbb{E}[Y_i]}}{\partial \kappa_r} = -\alpha_2 \frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]} < 0 \tag{23}$$

2. Reallocation effect derivative Note plugging in the assumptions of the two firms into (6) yields:

$$\frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]} = \frac{\mathbb{E}[A_r]^{\frac{1}{1-\alpha_1-\alpha_2}} (1-\kappa_r)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}}{\mathbb{E}[A_s]^{\frac{1}{1-\alpha_1-\alpha_2}} + \mathbb{E}[A_r]^{\frac{1}{1-\alpha_1-\alpha_2}} (1-\kappa_r)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}}. \tag{24}$$

Taking the derivative w.r.t κ_r yields:

$$\frac{\partial \frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]}}{\partial \kappa_r} = -\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2} \frac{(1-\kappa_r)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}-1} \mathbb{E}[A_r]^{\frac{1}{1-\alpha_1-\alpha_2}} \mathbb{E}[A_s]^{\frac{1}{1-\alpha_1-\alpha_2}}}{\left(\mathbb{E}[A_s]^{\frac{1}{1-\alpha_1-\alpha_2}} + \mathbb{E}[A_r]^{\frac{1}{1-\alpha_1-\alpha_2}} (1-\kappa_r)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}\right)^2} \tag{25}$$

Which can be simplified as:

$$\frac{\partial \frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]}}{\partial \kappa_r} = -\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2} \frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]} \frac{\mathbb{E}[Y_s]}{\mathbb{E}[Y]} \frac{1}{1-\kappa_r} \tag{26}$$

Plugging in for labor's share of income implies that

$$\begin{aligned} \sum_i \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \kappa_r} \frac{WL_i}{\mathbb{E}[Y_i]} &= -\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2} \frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]} \frac{\mathbb{E}[Y_s]}{\mathbb{E}[Y]} \frac{1}{1-\kappa_r} (\alpha_2 (1-\kappa_r) - \alpha_2) \\ \Rightarrow \sum_i \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \kappa_r} \frac{WL_i}{\mathbb{E}[Y_i]} &= \alpha_2 \frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2} \frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]} \frac{\mathbb{E}[Y_s]}{\mathbb{E}[Y]} \frac{\kappa_r}{1-\kappa_r} \end{aligned} \tag{27}$$

If $\kappa_r \in (0,1)$ then clearly $\sum_i \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \kappa_r} \frac{WL_i}{\mathbb{E}[Y_i]} > 0$.[21]

3. Threshold $\overline{\kappa_r}$ Combining the results above yields:

$$\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \kappa_r} = -\alpha_2 \frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]} + \alpha_2 \frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2} \frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]} \frac{\mathbb{E}[Y_s]}{\mathbb{E}[Y]} \frac{\kappa_r}{1-\kappa_r} \tag{28}$$

Define $\overline{\kappa_r}$ such that:

$$1 + \frac{\mathbb{E}[A_r]^{\frac{1}{1-\alpha_1-\alpha_2}}}{\mathbb{E}[A_s]^{\frac{1}{1-\alpha_1-\alpha_2}}} (1-\overline{\kappa_r})^{\frac{1}{1-\alpha_1-\alpha_2}} - \frac{1}{1-\alpha_1-\alpha_2} \overline{\kappa_r} = 0 \tag{29}$$

It is easy to verify that $\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \kappa_r} = 0$ at $\kappa_r = \overline{\kappa_r}$.

Note that $\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \kappa_r}$ has the same sign as

$$-1 + \frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2} \frac{\mathbb{E}[Y_s]}{\mathbb{E}[Y]} \frac{\kappa_r}{1-\kappa_r} < 0 \tag{30}$$

note that the derivative of this w.r.t. κ_r is

$$\frac{\partial \left(-1 + \frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2} \frac{\mathbb{E}[Y_s]}{\mathbb{E}[Y]} \frac{\kappa_r}{1-\kappa_r}\right)}{\partial \kappa_r} = \frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2} \frac{\mathbb{E}[Y_s]}{\mathbb{E}[Y]} \frac{1}{(1-\kappa_r)^2} + \frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2} \frac{\partial \frac{\mathbb{E}[Y_s]}{\mathbb{E}[Y]}}{\partial \kappa_r} \frac{\kappa_r}{1-\kappa_r} \tag{31}$$

Both of these terms are positive if $\kappa_r \in (0,1)$, since $\frac{\partial \frac{\mathbb{E}[Y_s]}{\mathbb{E}[Y]}}{\partial \kappa_r} = -\frac{\partial \frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]}}{\partial \kappa_r}$ and we derived $\frac{\partial \frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]}}{\partial \kappa_r}$ above.

It immediately follows that if $\kappa_r > \overline{\kappa_r}$, $\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \kappa_r} > 0$. Similarly, if $\kappa_r < \overline{\kappa_r}$, $\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \kappa_r} < 0$.

Proposition 2

1. Within effect derivative Note that solving for output shares and firm labor shares yields:

$$\frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]} = \frac{\mathbb{E}[A_i]^{\frac{1}{1-\alpha_1-\alpha_2}} \left(e^{-\lambda_x \beta_i \sigma_x^2}\right)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}}{\sum_i \mathbb{E}[A_i]^{\frac{1}{1-\alpha_1-\alpha_2}} \left(e^{-\lambda_x \beta_i \sigma_x^2}\right)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}} \tag{32}$$

[21] Note that if $\kappa > 1$ would imply negative expected labor's share of income for the risky firm, which would be impossible. In practice, that would mean that the extent of risk aversion is so high that the firm would choose no labor in equilibrium, and therefore we would not have an interior solution.

$$\frac{WL_i}{\mathbb{E}\left[Y_i\right]} = \alpha_2 e^{-\lambda_x \beta_i \sigma_x^2} \tag{33}$$

Solving for the within risk premium derivative yields the expression:

$$\sum_i \frac{\mathbb{E}\left[Y_i\right]}{\mathbb{E}\left[Y\right]} \frac{\partial \frac{WL_i}{\mathbb{E}[Y_i]}}{\partial \lambda_x} = -\left(\sum_i \left(\beta_i \sigma_x^2\right) \frac{L_i}{L}\right) \frac{WL}{E\left[Y\right]} \frac{1}{\alpha_2} \tag{34}$$

This is clearly negative if the input-share weighted risk exposure, $\left(\sum_i \left(\beta_i\right) \frac{L_i}{L}\right)$, is positive.

2. Reallocation effect derivative Using similar algebra, we derive an expression the reallocation component as:

$$\sum_i \frac{WL_i}{\mathbb{E}\left[Y_i\right]} \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \lambda_x} = \frac{\alpha_1 + \alpha_2}{1 - \alpha_1 - \alpha_2} \sigma_x^2 \left(\lambda_x \sigma_x^2 \sigma_\beta^2\right) e^{-\lambda_x \sigma_x^2 \mu_\beta + \frac{1}{2}\sigma_\beta^2 \left(-\lambda_x \sigma_x^2\right)^1 \left(\left(\frac{1}{1-\alpha_1-\alpha_2}\right)^2 - \left(\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}\right)^2\right)} \tag{35}$$

Which is clearly positive if $\lambda_x > 0$.

3. Threshold $\overline{\kappa_r}$ Note that we can write:

$$\frac{WL}{E\left[Y\right]} = \alpha_2 \frac{\sum_i \mathbb{E}\left[A_i\right]^{\frac{1}{1-\alpha_1-\alpha_2}} \left(e^{-\lambda_x \beta_i \sigma_x^2}\right)^{\frac{1}{1-\alpha_1-\alpha_2}}}{\sum_i \mathbb{E}\left[A_i\right]^{\frac{1}{1-\alpha_1-\alpha_2}} \left(e^{-\lambda_x \beta_i \sigma_x^2}\right)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}} \tag{36}$$

Evaluating the integrals, and given the assumed independence of β_i and $\mathbb{E}\left[A_i\right]$ yields:

$$log\left(\frac{WL}{E\left[Y\right]}\right) = log\left(\alpha_2\right) - \mu_\beta \lambda_x \sigma_x^2 + \frac{1}{2}\sigma_\beta^2 \left(\lambda_x \sigma_x^2\right)^2 \frac{1 - \left(\alpha_1 + \alpha_2\right)^2}{\left(1 - \alpha_1 - \alpha_2\right)} \tag{37}$$

Which has derivative w.r.t. λ_x:

$$\frac{\partial log\left(\frac{WL}{E[Y]}\right)}{\partial \lambda_x} = \sigma_x^2 \left(-\mu_\beta + \lambda_x \sigma_\beta^2 \sigma_x^2 \frac{1 - \left(\alpha_1 + \alpha_2\right)^2}{\left(1 - \alpha_1 - \alpha_2\right)}\right) \tag{38}$$

From which the condition for the sign of $\frac{\partial \frac{WL}{E[Y]}}{\partial \lambda_x}$ immediately follows.

A.3 Alternate production functions

While we consider Cobb-Douglas production functions in our baseline model, the insights of our model on how risk apply more broadly. First, consider a generic production function that produces output using labor, capital, and possibly other inputs. If we assume that labor is chosen in advance, the optimization problem of the owner of this technology would yield:

$$\mathbb{E}\left[\Lambda\left(MRPL_i\right)\right] = \mathbb{E}\left[\Lambda\right] W \tag{39}$$

We can rearrange (39) to yield:

$$\mathbb{E}\left[MRPL_i\right]\left(1 + \frac{Cov\left(\Lambda, MRPL_i\right)}{\mathbb{E}\left[\Lambda\right]\mathbb{E}\left[MRPL_i\right]}\right) = W \tag{40}$$

This tells us that firms do not equalize the realized (or expected) Marginal revenue product of labor equal to their wage rates, but rather that this is adjusted by a covariance term which depends on the co-movement of their marginal revenue product of labor with the stochastic discount factor. Note that we can write labor's share of expected income as follows:

$$\frac{WL_i}{\mathbb{E}\left[Y_i\right]} = \frac{\mathbb{E}\left[MRPL_i\right]L_i}{\mathbb{E}\left[Y_i\right]}\left(1 + \frac{Cov\left(\Lambda, MRPL_i\right)}{\mathbb{E}\left[\Lambda\right]\mathbb{E}\left[MRPL_i\right]}\right) \tag{41}$$

With the Cobb-Douglas production function, $\frac{\mathbb{E}[MRPL_i]L_i}{\mathbb{E}[Y_i]}$ is equal to α_2.

A.3.1 CES production function

Here we derive analogues of the main expressions in section 2, the production function were instead to be a CES production function of capital and labor:

There are multiple production technologies that produce a homogeneous output as follows:

$$Y_i = A_i\left(K_i^{\rho}\left(1-\theta\right) + \theta L_i^{\rho}\right)^{\frac{\nu}{\rho}} \tag{42}$$

This yields an analogue of (3)

$$\frac{WL_i}{\mathbb{E}\left[Y_i\right]} = \frac{\nu\theta}{\left(\frac{K}{L}\right)^{\rho}\left(1-\theta\right)+\theta}\left(1 + \frac{Cov\left(\Lambda, A_i\right)}{\mathbb{E}\left[A_i\right]\mathbb{E}\left[\Lambda\right]}\right), \tag{43}$$

where $\frac{K}{L} = \frac{\sum_i K_i}{\sum_i L_i}$ is the aggregate ratio of capital to labor. We can therefore derive input shares:

$$\frac{L_i}{L} = \frac{K_i}{K} = \frac{\left(\mathbb{E}\left[A_i\right]\left(1 + \frac{Cov(\Lambda, A_i)}{\mathbb{E}[A_i]\mathbb{E}[\Lambda]}\right)\right)^{\frac{1}{1-\nu}}}{\sum_h \left(\mathbb{E}\left[A_h\right]\left(1 + \frac{Cov(\Lambda, A_h)}{\mathbb{E}[A_h]\mathbb{E}[\Lambda]}\right)\right)^{\frac{1}{1-\nu}}} \tag{44}$$

Note that this is identical to (4), except that returns to scale with CES is denoted by ν instead of $\alpha_1 + \alpha_2$.

We can write aggregate expected output shares as:

$$\frac{\mathbb{E}\left[Y_i\right]}{\mathbb{E}\left[Y\right]} = \frac{\mathbb{E}\left[A_i\right]^{\frac{1}{1-\nu}}\left(1 + \frac{Cov(\Lambda, A_i)}{\mathbb{E}[A_i]\mathbb{E}[\Lambda]}\right)^{\frac{\nu}{1-\nu}}}{\sum_h \mathbb{E}\left[A_h\right]^{\frac{1}{1-\nu}}\left(1 + \frac{Cov(\Lambda, A_i)}{\mathbb{E}[A_i]\mathbb{E}[\Lambda]}\right)^{\frac{\nu}{1-\nu}}} \tag{45}$$

and plug these into (5) (which is an identity and continues to hold) to yield an expression for the aggregate labor's share of expected income:

$$\frac{WL}{\mathbb{E}\left[Y\right]} = \frac{\nu\theta}{\left(\frac{K}{L}\right)^{\rho}(1-\theta) + \theta} \sum_i \frac{\mathbb{E}\left[A_i\right]^{\frac{1}{1-\nu}}\left(1 + \frac{Cov(\Lambda, A_i)}{\mathbb{E}[A_i]\mathbb{E}[\Lambda]}\right)^{\frac{1}{1-\nu}}}{\sum_h \mathbb{E}\left[A_h\right]^{\frac{1}{1-\nu}}\left(1 + \frac{Cov(\Lambda, A_i)}{\mathbb{E}[A_i]\mathbb{E}[\Lambda]}\right)^{\frac{\nu}{1-\nu}}} \tag{46}$$

Note that (8) and (9) are identities and also still hold. Given that our expressions for labor shares and output shares are closely related to the Cobb-Douglas case, it can easily be verified that versions of Propositions 1 and 2 also hold, though the exact threshold at which labor share is rising/falling in the price of risk differ slightly.

B Derivations and proofs for section 2.2

B.1 Firm's problem and the marginal investor

A firm wants to maximize the market value of their shares. This optimization is complicated by the fact that there are multiple possible investors.

If there is no penalty for firms being "more foreign" than other firms of the same risk level in the same island, then we end up with no equilibrium with representative firms, in which each risky firm has an incentive to deviate.[22]

[22]More precisely, there may be an equilibrium here in which these risky firms are either wholly owned by domestic investors or foreign investors, which each firm making input choices using the corresponding SDF.

We can formally set up the firm's problem as:

$$\max_{L_{i,j},K_{i,j}} \max \left\{ \mathbb{E}_j \left[\Lambda_j \left(A_{i,j} K_{i,j}^{\alpha_1} L_{i,j}^{\alpha_2} - W_j L_{i,j} - P_{k,j} K_{i,j} \right) \right], \max_{h \neq j} \frac{\mathbb{E}_h \left[\Lambda_h \left(A_{i,j} K_{i,j}^{\alpha_1} L_{i,j}^{\alpha_2} - W_j L_{i,j} - P_{k,j} K_{i,j} \right) \right]}{1 + \tau_j + \tau_{i,j}^*(S_{i,j})} \right\}$$

In the absence of the cost $\tau_{i,j}^*(S_{i,j})$, consider an equilibrium in which (1) all firms of a certain risk profile in island j make the same choice, and (2) are jointly owned by domestic and (some) foreign investors. If input decisions are made according to any SDF, firms have incentives to deviate. If they are made using the domestic SDF, then they can increase their valuation to foreign investors by changing their choices to be more risk-neutral. If they are made using the foreign (risk-neutral) SDF, then they can increase their valuation by domestic investors by changing their choices to be more risk-averse. If they are not following either, then deviations towards the direction suggested by either SDF can increase their value. It is easy to verify that there is no such equilibrium.

However, if we add the cost $\tau_{i,j}^*(S_{i,j})$ and assume that the cost if the firm is all foreign-owned (when other firms in its continuum are not) is large enough, then the incentive to deviate is eliminated, and firms make input choices using the domestic SDF.

B.2 Definition of an Equilibrium

An equilibrium consists of

- Physical allocations $L_{i,j}, K_{i,j}, Y_{i,j}, C_{E,j}, C_{W,j}, C_{E,j,0}, Y_j$
- Prices $W_j, P_k, P_{i,j}, \Lambda_j, P_{rf}, V_{i,j}$
- Asset holdings $S_{i,j}, S_{i,h,j}^*, S_{rf,j}, K_{1,j}$

Such that

$$
\begin{aligned}
&\Lambda_j = \frac{U_2'\left(C_{E,j}\right)}{U_1'\left(C_{E,j,0}\right)} \\
&\mathbb{E}\left[\Lambda_j\right] = P_{rf} \\
&\left(P_{i,j} - \mathbb{E}\left[\Lambda_j\left(A_{i,j}F\left(L_{i,j},K_{i,j}\right) - L_{i,j}W_j - K_{i,j}P_{k,j}\right)\right]\right)S_{i,j} = 0 \\
&\left(P_{i,j}\left(1+\tau_j\right) - \mathbb{E}\left[\Lambda_h\left(A_{i,j}F\left(L_{i,j},K_{i,j}\right) - L_{i,j}W_j - K_{i,j}P_{k,j}\right)\right]\right)S_{i,h,j} = 0 \\
&\alpha_2\mathbb{E}\left[\Lambda_j A_{i,j}K_{i,j}^{\alpha_1}L_{i,j}^{\alpha_2-1}\right] = \mathbb{E}\left[\Lambda_j\right]W_j \\
&\alpha_1\mathbb{E}\left[\Lambda_j A_{i,j}K_{i,j}^{\alpha_1-1}L_{i,j}^{\alpha_2}\right] = \mathbb{E}\left[\Lambda_j\right]P_{k,j} \\
&Y_{i,j} = A_{i,j}K_{i,j}^{\alpha_1}L_{i,j}^{\alpha_2} \\
&V_{i,j} = \left(A_{i,j}K_{i,j}^{\alpha_1}L_{i,j}^{\alpha_2} - K_{i,j}P_{k,j} - W_jL_{i,j}\right) \\
&Y_j = \sum_i Y_{i,j} \\
&C_{W,j} = W_j\sum_i L_{i,j} \\
&C_{E,j,0} = K_{0,j} - K_{1,j} - \sum_i S_{i,j}P_{i,j} - \sum_{h\neq j}\left(1+\tau_h\right)S_{i,j,h}^*P_{i,h} - P_{rf}S_{rf,j} + T_{0,j} \\
&C_{E,j} = S_{i,j}V_{i,j} + \sum_{h\neq j}\sum_i S_{i,j,h}^*V_{i,h} + S_{rf,j} + P_{k,j}K_{1,j} + T_j \\
&S_{i,j} + \sum_{h\neq j}S_{i,h,j}^* = 1 \\
&\sum_j S_{rf,j} = 0 \\
&\frac{\partial U_j^W}{\partial L_j} + W_j\mathbb{E}\left[\frac{\partial U_j^W}{\partial C_j}\right] = 0 \\
&1 = \mathbb{E}\left[\Lambda_j\right]P_{k,j} \\
&\sum_j K_{1,j} = \sum_j\sum_i K_{i,j}
\end{aligned}
$$

Where $T_{0,j}, T_j$ are chosen to make the clearing conditions hold and govern how the proceeds of purchasing the shares are distributed. For instance, we could set:

$$
\begin{aligned}
&T_{0,j} = \sum_i S_{i,j}P_{i,j} + \sum_{h\neq j}\left(1+\tau_j\right)S_{i,h,j}^*P_{i,j} \\
&T_j = 0
\end{aligned}
$$

B.3 Proofs to Proposition 3

Points 3-5 follow directly from proposition 1, since the 'risk premia' term κ_r can be written as an increasing function of τ_j: $\kappa_r = \frac{\tau_j(1-\alpha_1-\alpha_2)}{1+\tau_j(1-\alpha_1-\alpha_2)}$. The proofs for (1) and (2) are below:

1. The equilibrium price of risk decreases (13) implies that $Cov\left(\frac{\Lambda_j}{\mathbb{E}[\Lambda_j]}, \frac{A_{r,j}}{\mathbb{E}[A_{r,j}]}\right)$, which is a function of the quantity of technology r, j risk and price of risk, decreases in magnitude as τ_j falls. We can see this clearly if we decompose the risk adjustment, $Cov\left(\frac{\Lambda_j}{\mathbb{E}[\Lambda_j]}, \frac{A_{r,j}}{\mathbb{E}[A_{r,j}]}\right)$, into the quantity and price of risk:

$$Cov\left(\frac{\Lambda}{\mathbb{E}[A_r]}, \frac{A_r}{\mathbb{E}[\Lambda]}\right) = \sigma^2_{log(A_{r,j})} \frac{Cov\left(\frac{\Lambda_j}{\mathbb{E}[\Lambda_j]}, \frac{A_{r,j}}{\mathbb{E}[A_{r,j}]}\right)}{\sigma^2_{log(A_{r,j})}} \tag{47}$$

where $-\frac{Cov\left(\frac{\Lambda_j}{\mathbb{E}\left[\Lambda_j\right]}, \frac{A_{r,j}}{\mathbb{E}\left[A_{r,j}\right]}\right)}{\sigma^2_{log\left(A_{r,j}\right)}}$ is the price of risk: the (negative) elasticity of the island j capitalist's SDF with respect the the productivity of the risky technology in island j. Since the quantity of risk is exogenous, as τ_j gets smaller the price of risk is falling.

2. The fraction of shares in the risky firm, $S_{r,j}$, held by domestic investors falls Note that we can express the covariance term as:

$$\kappa_{r,j} \equiv \frac{\tau_j(1-\alpha_1-\alpha_2)}{1+\tau_j(1-\alpha_1-\alpha_2)} = -\frac{Cov\left(U_2'(C_{E,j}), A_{r,j}\right)}{\mathbb{E}[A_{r,j}]\,\mathbb{E}[U_2'(C_{E,j})]} \tag{48}$$

For analytic tractability, we will therefore show that $\frac{\partial S_{r,j}}{\partial \kappa_{r,j}} > 0$, which from the above will show that $\frac{\partial S_{r,j}}{\partial \tau_j} > 0$

Expression for consumption in the second period Taking the equilibrium conditions for consumption

$$\begin{aligned} C_{E,j} &= \sum_i S_{i,j}V_{i,j} + \sum_{h\neq j}\sum_i S^*_{i,j,h}V_{i,h} + S_{rf,j} + P_{k,j}K_{1,j} + T_j \\ C_{E,j,0} &= K_{0,j} - K_{1,j} - \sum_i S_{i,j}P_{i,j} - \sum_{h\neq j}\left(1+\tau_h+\tau^*_{i,h}(S_{i,h})\right)S^*_{i,j,h}P_{i,h} - P_{rf}S_{rf,j} + T_{0,j} \end{aligned}$$

Note that we know the value of firm shares in the second period conditional on shocks:

$$V_{r,j} = K_{r,j}^{\alpha_1} L_{r,j}^{\alpha_2} \left(A_{r,j} - \mathbb{E}\left[A_{r,j}\right](\alpha_1+\alpha_2)\left(1 + Cov\left(\frac{\Lambda_j}{\mathbb{E}\left[\Lambda_j\right]}, \frac{A_{r,j}}{\mathbb{E}\left[A_{r,j}\right]}\right)\right)\right)$$
$$V_{s,j} = K_{s,j}^{\alpha_1} L_{s,j}^{\alpha_2} A_{s,j}\left(1-(\alpha_1+\alpha_2)\right)$$

and that in equilibrium domestic investors hold only infinitesimal shares of each foreign security and are thus fully diversified:

$$\sum_{h\neq j}\sum_{i} S_{i,j,h}^{*} V_{i,h} = \sum_{h\neq j} S_{r,j,h}^{*} K_{r,j}^{\alpha_1} L_{r,j}^{\alpha_2} \mathbb{E}\left[A_{r,h}\right]\left(1-(\alpha_1+\alpha_2)\left(1+Cov\left(\frac{\Lambda_h}{\mathbb{E}\left[\Lambda_h\right]}, \frac{A_{r,h}}{\mathbb{E}\left[A_{r,h}\right]}\right)\right)\right)$$

Further the prices of these firm shares in the first period can be expressed as:

$$\begin{aligned} P_{s,j} &= \mathbb{E}\left[\Lambda_j\right] K_{s,j}^{\alpha_1} L_{s,j}^{\alpha_2} A_{s,j}\left(1-(\alpha_1+\alpha_2)\right) \\ P_{r,j} &= K_{r,j}^{\alpha_1} L_{r,j}^{\alpha_2}\left(\mathbb{E}\left[\Lambda_j A_{r,j}\right] - \mathbb{E}\left[\Lambda_j\right]\mathbb{E}\left[A_{r,j}\right](\alpha_1+\alpha_2)\left(1+\frac{Cov\left(\Lambda_j, A_{r,j}\right)}{\mathbb{E}\left[A_{r,j}\right]\mathbb{E}\left[\Lambda_j\right]}\right)\right) \end{aligned}$$

plugging these into $C_{E,j}, C_{E,j,0}$ are simplifying yields:

$$C_{E,j} = S_{r,j} K_{r,j}^{\alpha_1} L_{r,j}^{\alpha_2}\left(A_{r,j} - \mathbb{E}\left[A_{r,j}\right]\left(1+Cov\left(\frac{\Lambda_j}{\mathbb{E}\left[\Lambda_j\right]}, \frac{A_{r,j}}{\mathbb{E}\left[A_{r,j}\right]}\right)\right)\right) + \left(T_j + \frac{T_{0,j}}{\mathbb{E}\left[\Lambda\right]} + \frac{K_{0,j}}{\mathbb{E}\left[\Lambda\right]}\right) - \frac{C_{E,j,0}}{\mathbb{E}\left[\Lambda_j\right]} \tag{49}$$

Note further that $C_{E,j,0}$ has a relation with $C_{E,j,0}$ via the expectation of the SDF; This SDF is equated across capitalists as they can freely trade a risk-free bond.

$$\mathbb{E}\left[\Lambda\right] = \frac{\mathbb{E}\left[U_2'\left(C_{E,j}\right)\right]}{U_1'\left(C_{E,j,0}\right)} \tag{50}$$

Derivative of $C_{E,j,0}$ Note that (50) yields:

$$C_{E,j,0} = U_1^{'(-1)}\left(\frac{\mathbb{E}\left[U_2'\left(C_{E,j}\right)\right]}{\mathbb{E}\left[\Lambda\right]}\right) \tag{51}$$

and therefore

$$\frac{\partial \frac{C_{E,j,0}}{\mathbb{E}[\Lambda]}}{\partial \kappa_{r,j}} = \frac{U_1^{'(-1)'}\left(\frac{\mathbb{E}\left[U_2'\left(C_{E,j}\right)\right]}{\mathbb{E}[\Lambda]}\right)}{\mathbb{E}\left[\Lambda\right]^2}\frac{\partial \mathbb{E}\left[U_2''\left(C_{E,j}\right)\right]}{\partial \kappa_{r,j}} \tag{52}$$

Note that (49) implies:

$$\mathbb{E}\left[U_2'\left(C_{E,j}\right)\right]=\mathbb{E}\left[U_2'\left(S_{r,j}K_{r,j}^{\alpha_1}L_{r,j}^{\alpha_2}\left(A_{r,j}-\mathbb{E}\left[A_{r,j}\right]\left(1-\kappa_{r,j}\right)\right)+\left(T_j+\frac{T_{0,j}}{\mathbb{E}\left[\Lambda\right]}+\frac{K_{0,j}}{\mathbb{E}\left[\Lambda\right]}\right)-\frac{U_1'^{(-1)}\left(\frac{\mathbb{E}\left[U_2'\left(C_{E,j}\right)\right]}{\mathbb{E}[\Lambda]}\right)}{\mathbb{E}\left[\Lambda\right]}\right)\right]$$

and thus

$$\frac{\partial\mathbb{E}\left[U_2'\left(C_{E,j}\right)\right]}{\partial\kappa_{r,j}}=\frac{K_{r,j}^{\alpha_1}L_{r,j}^{\alpha_2}\mathbb{E}\left[U_2''\left(C_{E,j}\right)\left(\frac{\partial S_{r,j}}{\partial\kappa_{r,j}}\left(A_{r,j}-\mathbb{E}\left[A_{r,j}\right]\left(1-\kappa_{r,j}\right)\right)+\mathbb{E}\left[A_{r,j}\right]S_{r,j}\right)\right]}{1+\mathbb{E}\left[U_2''\left(C_{E,j}\right)\frac{U_1'^{(-1)'}\left(\frac{\mathbb{E}\left[U_2'\left(C_{E,j}\right)\right]}{\mathbb{E}[\Lambda]}\right)}{\mathbb{E}[\Lambda]^2}\right]} \tag{53}$$

Plugging in CRRA preferences yields:

$$\frac{\partial\frac{C_{E,j,0}}{\mathbb{E}[\Lambda]}}{\partial\kappa_{r,j}}=\frac{\frac{\gamma}{\gamma_0}\frac{\left(\frac{\mathbb{E}\left[C_{E,j}^{-\gamma}\right]}{\mathbb{E}[\Lambda]}\right)^{-\frac{1}{\gamma_0}-1}}{\mathbb{E}[\Lambda]^2}\left(\frac{\partial S_{r,j}}{\partial\kappa_{r,j}}\frac{1}{S_{r,j}}\left(\mathbb{E}\left[C_{E,j}^{-\gamma}\right]-\left(\overline{C}+C_0\right)\mathbb{E}\left[C_{E,j}^{-\gamma-1}\right]\right)+\mathbb{E}\left[A_{r,j}\right]S_{r,j}K_{r,j}^{\alpha_1}L_{r,j}^{\alpha_2}\mathbb{E}\left[C_{E,j}^{-\gamma-1}\right]\right)}{1+\frac{\gamma}{\gamma_0}\mathbb{E}\left[C_{E,j}^{-\gamma-1}\frac{\left(\frac{\mathbb{E}\left[C_{E,j}^{-\gamma}\right]}{\mathbb{E}[\Lambda]}\right)^{-\frac{1}{\gamma_0}-1}}{\mathbb{E}[\Lambda]^2}\right]} \tag{54}$$

Showing $E\left[x^{-\gamma}\right]^2-E\left[x^{-\gamma-1}\right]E\left[x^{-\gamma+1}\right]<0$ Note that we can write

$$\begin{aligned}E\left[x^{-\gamma}\right]^2-E\left[x^{-\gamma-1}\right]E\left[x^{-\gamma+1}\right] &= \int_x\int_y(xy)^{-\gamma}\left(1-\frac{y}{x}\right)f(x)f(y)\,dydx\\ &= \int_x\int_{y>x}(xy)^{-\gamma}\left(2-\left(\frac{y}{x}+\frac{x}{y}\right)\right)f(x)f(y)\,dydx\end{aligned} \tag{55}$$

note that for $x,y\geq 0$, $\left(\frac{y}{x}+\frac{x}{y}\right)\geq 2$. Thus $E\left[x^{-\gamma}\right]^2-E\left[x^{-\gamma-1}\right]E\left[x^{-\gamma+1}\right]<0$

Finishing proof Note that plugging in (49) into (48) and taking the derivative w.r.t $\kappa_{r,j}$ yields:

$$\frac{\partial S_{r,j}}{\partial\kappa_{r,j}}=\left(-\frac{\mathbb{E}\left[C_{E,j}^{-\gamma}\right]^2\mathbb{E}\left[A_{r,j}\right]S_{r,j}K}{\mathbb{E}\left[C_{E,j}^{-\gamma}\right]^2-\mathbb{E}\left[C_{E,j}^{-\gamma-1}\right]\mathbb{E}\left[C_{E,j}^{-\gamma+1}\right]}\frac{1}{\gamma}+\mathbb{E}\left[A_{r,j}\right]S_{r,j}K+\frac{\partial C_0}{\partial\kappa_{r,j}}\right)\frac{S_{r,j}}{\overline{C}+C_0} \tag{56}$$

Plugging in (54) and simplifying yields:

$$\frac{\partial S_{r,j}}{\partial \kappa_{r,j}} = \frac{-\frac{\mathbb{E}\left[C_{E,j}^{-\gamma}\right]^2 \mathbb{E}[A_{r,j}] S_{r,j} K_{r,j}^{\alpha_1} L_{r,j}^{\alpha_2}}{\mathbb{E}\left[C_{E,j}^{-\gamma}\right]^2 - \mathbb{E}\left[C_{E,j}^{-\gamma-1}\right] \mathbb{E}\left[C_{E,j}^{-\gamma+1}\right]} \frac{1}{\gamma} \left(1 + \mathbb{E}\left[C_{E,j}^{-\gamma-1}\right] \frac{\left(\frac{\mathbb{E}\left[C_{E,j}^{-\gamma}\right]}{\mathbb{E}[\Lambda]}\right)^{-\frac{1}{\gamma}-1}}{\mathbb{E}[\Lambda]^2}\right) + \mathbb{E}\left[A_{r,j}\right] S_{r,j} K_{r,j}^{\alpha_1} L_{r,j}^{\alpha_2}}{\left(T_j + \frac{T_{0,j}}{\mathbb{E}[\Lambda]} + \frac{K_{0,j}}{\mathbb{E}[\Lambda]}\right) + \left(\frac{1}{\mathbb{E}[\Lambda]} - 1\right) \left(\frac{\mathbb{E}\left[C_{E,j}^{-\gamma}\right]}{\mathbb{E}[\Lambda]}\right)^{-\frac{1}{\gamma}}}$$

We have shown above that $E\left[x^{-\gamma}\right]^2 - E\left[x^{-\gamma-1}\right] E\left[x^{-\gamma+1}\right] < 0$, therefore the numerator is positive. So $\frac{\partial S_{r,j}}{\partial \kappa_{r,j}} > 0$ (and therefore $\frac{\partial S_{r,j}}{\partial \tau_j} > 0$) iff

$$\left(T_j + \frac{T_{0,j}}{\mathbb{E}\left[\Lambda\right]} + \frac{K_{0,j}}{\mathbb{E}\left[\Lambda\right]}\right) + \left(\frac{1}{\mathbb{E}\left[\Lambda\right]} - 1\right) \left(\frac{\mathbb{E}\left[C_{E,j}^{-\gamma}\right]}{\mathbb{E}\left[\Lambda\right]}\right)^{-\frac{1}{\gamma}} > 0$$

The first term in parenthesis is the present value of transfers and the endowment, which are positive by assumption. The second term is nonnegative if $\mathbb{E}\left[\Lambda\right] \leq 1$, which is equivalent to saying that the risk free rate is non-negative.

C Empirical Appendix

C.1 Implications for cross-industry heterogeneity

Let us extend our model to one in which there are several industries s, and within each industry risk exposures are normally distributed with mean $\mu_{\beta_{s,j}}$ and variance $\sigma^2_{s,j}$ for industry s in country j. Then (38) implies that we can write the change in industry labor share as:

$$\frac{\partial log\left(\frac{W_{s,j} L_{s,j}}{E[Y_{s,j}]}\right)}{\partial \lambda_j} = \sigma^2_{s,j} \left(\mu_{\beta,s,j} + \lambda_j \sigma^2_{\beta,s,j} \sigma^2_{s,j} \frac{1 - (\alpha_1 + \alpha_2)^2}{(1 - \alpha_1 - \alpha_2)}\right) \tag{57}$$

This suggests a regression of the form:

$$\Delta log\left(LS_{s,j,t}\right) = \alpha_t + \gamma_\mu \mu_{\beta,s,j} \Delta Div_{j,t} + \gamma_\sigma \sigma^2_{\beta,s,j} \Delta Div_{j,t} + controls + \varepsilon_{i,t} \tag{58}$$

where each observation is an industry-country-year.

Note that if there is risk aversion (ie $\lambda_j < 0$), then our model implies $\gamma_\mu > 0$ and $\gamma_\sigma < 0$.

Note that in many datasets we cannot measure $\mu_{\beta,s,j}$ and $\sigma^2_{\beta,s,j}$, however firm-level labor shares are more readily observable:

$$log\left(\mathbb{E}\left[LS_{j,s,i,t}\right]\right) = log\left(\alpha_{2,s}\right) + \lambda_j \beta_i \sigma_{j,s}^2 \tag{59}$$

plugging this in yields:

$$\frac{\partial log\left(\frac{W_{s,j}L_{s,j}}{\mathbb{E}[Y_{s,j}]}\right)}{\partial \lambda_j} = \frac{1}{\lambda_j}\left(\left(\overline{log(\mathbb{E}\left[LS_{j,s,t}\right])} - log\left(\alpha_{2,s}\right)\right) + \sigma^2_{\mathbb{E}LS,j,s,t}\frac{1-\left(\alpha_1+\alpha_2\right)^2}{\left(1-\alpha_1-\alpha_2\right)}\right)$$

where $\overline{log(\mathbb{E}\left[LS_{j,s,t}\right])}$ is the mean of log labor share for industry s in country j at time t and $\sigma_{\mathbb{E}LS,j,s,t}$ is the cross-sectional standard deviation of labor shares in industry s in country j at time t. If firm betas are This suggests regression of the form:

$$\Delta log\left(LS_{s,j,t}\right) = \alpha_t + \gamma_\mu \mu_{LS,s,j,t}\Delta Div_{j,t} + \gamma_\sigma \sigma^2_{LS,s,j,t}\Delta Div_{j,t} + controls + \varepsilon_{i,t} \tag{60}$$

Where $\mu_{LS,s,j,t}$ and $\sigma^2_{LS,s,j,t}$ are the mean and variance of firm labor shares within the industry s in country j at time t.

D Additional Tables and Figures

D.1 Cross-Country Regressions with OECD Data

Table 6: Labor's share of income on international diversification

	(1)	(2)	(3)	(4)
Foreign Equity Liabilities/GDP	-0.0551**	-0.0627**	-0.0394**	-0.0467**
	(-2.72)	(-2.57)	(-2.14)	(-2.12)
TFP shock			-0.365*	-0.413**
			(-1.96)	(-2.09)
Average hours				0.00130
				(0.01)
Relative price of investment				0.0222
				(0.75)
Country fixed effects	yes	yes	yes	yes
Year fixed effects	no	yes	yes	yes
R^2	0.849	0.905	0.920	0.927
Observations	755	754	567	529

Notes: This table presents a regression of country labor share on foreign investors holding of domestic equity for advanced economics, excluding tax havens and small financial centers. T statistics are reported in parenthesis below the coefficient estimates, computed from standard errors clustered two ways by country and year. Significance levels denoted by: * $p < .10$ ** $p < 0.05$, *** $p < 0.01$.

D.2 Reallocation in Changes

Table 7: Reallocation, International Diversification, and Labor Share

	(1) $\Delta\log(\frac{Y_i}{Y_{ind}})$	(2) $\Delta\log(\frac{L_i}{L_{ind}})$	(3) $\Delta\log(\frac{Y_i}{Y_{ind}})$	(4) $\Delta\log(\frac{L_i}{L_{ind}})$
$\widehat{\beta}\times$ Δ FEQ	0.508***	0.292***	0.521***	0.270***
	(4.01)	(2.94)	(4.03)	(2.72)
$\widehat{\beta}$	0.0574***	0.0492***	0.0748***	0.0825***
	(10.67)	(11.33)	(10.34)	(14.23)
Profitability			0.0293**	0.241***
			(2.23)	(17.80)
Age			-0.0361***	-0.0259***
			(-13.73)	(-12.69)
Size			-0.00451***	-0.0142***
			(-5.04)	(-15.31)
Industry-year F.E.	yes	yes	yes	yes
R^2	0.204	0.169	0.228	0.211
Observations	137742	136188	127373	127141

Notes:

D.3 Reallocation Interaction with Alternate Labor Share Measures

Table 8: Reallocation, International Diversification, and Labor Share

	(1) $\log(\frac{Y_i}{Y_{ind}})$	(2) $\log(\frac{L_i}{L_{ind}})$	(3) $\log(\frac{K_i}{K_{ind}})$	(4) $\log(\frac{Y_i}{Y_{ind}})$	(5) $\log(\frac{L_i}{L_{ind}})$	(6) $\log(\frac{K_i}{K_{ind}})$
$\log(LS) \times$ FEQ	-0.882*	-1.179***	-1.343**	-1.123***	-1.075***	-1.711***
	(-1.90)	(-3.41)	(-2.47)	(-3.34)	(-3.50)	(-4.41)
Profitability				0.300	0.609**	-0.360
				(1.38)	(2.65)	(-1.23)
Age				0.261***	0.220***	0.240***
				(6.74)	(5.12)	(4.60)
Industry-year F.E.	yes	yes	yes	yes	yes	yes
Firm F.E.	yes	yes	yes	yes	yes	yes
R^2	0.979	0.971	0.974	0.981	0.972	0.975
Observations	10060	9849	9641	9858	9660	9435

Notes: This table presents the results of a regression of a firm's share of industry outputs & inputs on the interaction of firm labor share and foreign investors holdings of U.S. equity, scaled by GDP. T statistics are reported in parenthesis below the coefficient estimates, computed from standard errors clustered two ways by firm and year. Significance levels denoted by: * p < .10 ** p < 0.05, *** p < 0.01.

Table 9: Reallocation, International Diversification, and Labor Share

	(1) $\log(\frac{Y_i}{Y_{ind}})$	(2) $\log(\frac{L_i}{L_{ind}})$	(3) $\log(\frac{K_i}{K_{ind}})$	(4) $\log(\frac{Y_i}{Y_{ind}})$	(5) $\log(\frac{L_i}{L_{ind}})$	(6) $\log(\frac{K_i}{K_{ind}})$
$\log\left(\frac{L}{Y}\right) \times$ FEQ	-0.190***	-0.219***	-0.290***	-0.167***	-0.187***	-0.244***
	(-3.17)	(-4.17)	(-4.64)	(-2.79)	(-3.70)	(-3.85)
Profitability				1.241***	1.300***	1.154***
				(29.87)	(31.48)	(26.49)
Age				0.244***	0.249***	0.231***
				(18.40)	(19.04)	(13.14)
Industry-year F.E.	yes	yes	yes	yes	yes	yes
Firm F.E.	yes	yes	yes	yes	yes	yes
R^2	0.948	0.942	0.940	0.952	0.948	0.943
Observations	144566	143370	143770	140047	138809	139300

Notes: This table presents the results of a regression of a firm's share of industry outputs & inputs on the interaction of firm labor share and foreign investors holdings of U.S. equity, scaled by GDP. T statistics are reported in parenthesis below the coefficient estimates, computed from standard errors clustered two ways by firm and year. Significance levels denoted by: * p < .10 ** p < 0.05, *** p < 0.01.