To Own or to Rent?
The Effects of Transaction Taxes on Housing Markets*

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Abstract

Using transaction records on housing sales and leases, we estimate the effect of Toronto's imposition of a land transfer tax in 2008. We find three novel effects of an increase in land transfer tax: (1) a rise in buy-to-let transactions but a fall in owner-occupier transactions despite the tax applying to both, (2) a simultaneous fall in the price-to-rent ratio and the sales-to-leases ratio; and (3) a decline in the moving hazard for existing homeowners and an increase in the time on the market it takes to sell. We develop a housing search model with an owing or renting choice and entry of investors. It accounts for the three new facts by predicting an increase in demand in the rental market which generates a fall in the homeownership rate and a reduction in mobility within the ownership market. The implied deadweight loss as a percentage of tax revenue raised is large at 79%, with more than 40% due to the presence of the rental market.

JEL CLASSIFICATIONS: D83; E22; R21; R28; R31.

KEYWORDS: rental market, buy-to-let investors, homeownership rate, land transfer tax.

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1 Introduction

Transaction taxes have a long history that goes back to the 17th century to raise money for war. Over time, transaction taxes on stocks and other assets have been scrapped. However, transaction taxes on properties — also called land transfer taxes in North America and stamp duties in Europe — remain a prominent source of tax revenue in over twenty-seven OECD countries today.[1]

While a large and increasing literature has focused on the distortionary effects of transaction taxes within the owner-occupied market, little is known about how the tax distorts households' tenure choice and hence housing allocation across the owner-occupied and rental markets.[2] This is surprising because the rental sector typically comprises at least a third of the housing market and increasing homeownership has been a main policy target in many countries (Brueckner and Pereira, 1994, Gabriel and Rosenthal, 2005). Even less is known about how search frictions interact with the transaction taxes in affecting households' mobility decisions and tenure choices, despite the fact that search frictions represent a pronounced feature of housing markets and makes the transaction taxes more distortionary for housing than for other assets.

In this paper, our objective is to gain a comprehensive understanding of the effects of land transfer taxes on housing markets, along both the intensive margin, namely moving and transaction decisions, and the extensive margin, namely the decision of owing versus renting. The quantitative findings are hard to reconcile with the standard demand response due to higher taxes discouraging transactions with lower surplus, but consistent with a search-theoretic model where idiosyncratic match quality amplifies the distortions that the transaction tax causes to housing decisions on both margins and the associated welfare.

We use a unique dataset on housing sales transactions and leasing transactions from the Multiple Listing Service (MLS) transaction records in the Greater Toronto Area (GTA). The data allow us to identify property purchases made by buy-to-let investors as opposed to owner occupiers. We investigate the effects of a land transfer tax (LTT) on property sales prices paid by buyers that was introduced in Toronto in 2008 at the municipal level, but not in other parts of the GTA. The effective change in the tax rate is 1.3 percentage points. We estimate the effects of this LTT change on a rich array of housing market outcomes by comparing transactions before and after across treated and untreated neighbourhoods based on a regression discontinuity design.

We start by documenting a set of novel empirical facts. First, we find that the land transfer tax

[1] Taxes on property including property tax and land transfer tax are important sources of tax revenue in the OECD countries. The U.K, U.S., and Canada are among the highest with taxes on property amounting to 12–14% of tax revenue. This is substantial considering tax on personal income is about 30–40% in those three countries. See also discussion in Scanlon, Whitehead and Blanc (2017), Lenoel, Matsu and Naisbitt (2018).

[2] This includes but not limited to Besley, Meads and Surico (2014), Hilber and Lyytikäinen (2017), Best and Kleven (2018) for U.K.; Benjamin, Coulson and Yang (1993), Slemrod, Weber and Shan (2017), Kopczuk and Munroe (2015) for the U.S.; Dachis, Duranton and Turner (2012) for Canada; Eerola, Harjunen, Lyytikäinen and Saarimaa (2019), Määttänen and Terviö (2020) for Finland; Fritzsche and Vandrei (2019) for Germany; Davidoff and Leigh (2013) for Australia; Van Ommeren and Van Leuvensteijn (2005) for Netherland; Agarwal, Chau, Hu and Wan (2021) for Hong Kong and Huang, Li and Yang (2021) for Singapore.

(LTT) reduces the flows from both sides of the ownership market. On the one hand, the increase in the LTT reduces homeowners' moving hazard by 13%, implying a 14 months longer stay in her current home. On the other hand, it increases the time on the market by 27%, equivalent to one week longer in our sample market. These findings highlight how the LTT interact with two frictions in the housing market. First, match quality depreciating over time. The LTT increases the level of mismatch that existing homeowners tolerate, resulting in fewer homes being put on the market for sale. Second, the process of finding a buyer or a home is lengthy. By imposing additional transaction cost on buyers, the LTT makes them pickier in searching for a home. With existing homeowners staying in their current property longer and properties taking longer to sell, overall transactions should decline.

Turning to transactions, while the LTT indeed reduces overall sales, it has strikingly opposite effects on buy-to-own and buy-to-rent transactions despite the fact that the same LTT was imposed on both types of transactions. In particular, the LTT increase causes a 9.8% decline in home purchases but a 2.3% increase in buy-to-rent investor purchases. By definition, buy-to-rent investors are those who buy properties from owner-occupied markets and put them on the rental market to lease. Consistent with this, we find the LTT reduces the lease-to-sales ratio by 23.4%. Thus a careful evaluation of the LTT should account for flows between the owner-occupied and rental sectors. Intuitively, the imposition of the LTT makes owning a home less attractive relative to renting. The increase in demand for rental properties further encourages the entry of buy-to-let investors, which shifts properties from the owner-occupied sector to the rental sector, resulting in a fall in the homeownership rate.[3]

Third, the LTT increases the price-rent ratio (the inverse of rent yield) by 1.6. This is not surprising since the the burden of the LTT falls more lightly on the rental market compared to the market for owner-occupied properties. Interestingly, the 1.3 percentage-point LTT increase causes a larger 2% decline in the price paid by homeowners. Intuitively, with match quality depreciating over time, owner-occupiers expect to be subject to the LTT not only once, but each time they desire to move, both directly as buyers of subsequent properties, and indirectly as sellers hit by lower prices. As a result, price paid by homeowners declines more than the LTT.

Together, the empirical findings above provide two new insights that help guide the modeling approach for studying the full equilibrium effects of the LTT. First, the novel heterogeneous treatment effects of the LTT on the owner-occupied market versus the rental market reveal a rich picture about the flows of households and properties between rental and owner-occupied sectors. By choosing to rent, individuals save themselves from the burden of paying the increased LTT. Moreover, when tenants desire to move, the landlord does not need to sell and pay the LTT again. This mitigates the impact of the tax on overall house transactions and suggests an important role that the LTT plays

[3]The homeownership rate, measured by the fraction of properties lived in by their owners at a five-year frequency reported by the Statistics Canada, steadily increased from 51% to 54.5% during 1996–2006, followed by almost no growth and then a decline to 52.3% in 2016.

in shaping households' tenure choice. Second, the newly gained systematic evidence on the moving hazard, time on the market and sales price highlights the limitation of considering the LTT in isolation from search frictions. In particular, the LTT does not operate independently. It interacts with frictions when affecting the housing market outcomes. The idiosyncratic match quality is a key driver for the potential welfare losses associated with the LTT both along the intensive margin, when some households who would have been better matched with a new home are now stuck with their old home, and along the extensive margin, when some households who would have owned a home now enter the rental market.

To better understand the economic forces behind the rich housing market dynamics documented above and quantify the relative importance of the intensive-margin and extensive-margin changes on welfare, we develop and calibrate a model suitable to analyse the housing ownership and rental markets jointly. A crucial feature is that individuals can choose which market to participate in, subject to paying the cost of accessing credit to become a homeowner. These credit costs are heterogeneous across individuals. This gives rise to an entry decision on the buy side of the rental market. On the seller side, there is free entry of buy-to-let investors.[4] The equilibrium homeownership rate is the one consistent with the behaviour of both households and buy-to-rent investors.

Both ownership and rental markets are subject to search frictions where market tightness (the ratio of buyers to sellers) affects the probability of meeting, and idiosyncratic match quality affects the probability of transacting conditional on meeting. Match quality is a persistent variable subject to occasional idiosyncratic shocks, after which homeowners make a decision whether to move, doing so if match quality is below a moving threshold.

An increase in LTT in the model makes existing homeowners more tolerant of poor match quality, and thus reduces moving rates, lengthens holding periods, and reduces housing transactions. The higher LTT also makes it less attractive to be an owner-occupier relative to a renter, so some marginal homebuyers are dissuaded from paying the credit cost to enter the ownership market. Since they must still live somewhere, this raises demand in the rental market and attracts more buy-to-rent investors.

The model spells out two facets of the welfare cost of the LTT. First, the 'lock-in' effect of less moving within the ownership market. This gives rise to misallocation of properties among homeowners in that average match quality declines. Second, there is misallocation across the markets in that fewer people will pay the credit cost to access better match quality in the ownership market.

We calibrate the model to the city of Toronto housing market for 2006–2008. The homeownership rate was about 54% and the city has an active buy-to-let market.[5] We simulate the effects of

[4] An investor is anyone who owns a property but does not use a property to live in. Thus, an investor simply represents funds invested in housing; the identities of the investors do not matter.

[5] Some have argued that rental and ownership markets are distinct segments of the housing market using U.S. national data Glaeser and Gyourko (2007), Bachmann and Cooper (2014). However, when focusing on a market like the City of Toronto, these two segments are more interrelated. However, the fraction of buy-to-sell transactions remains stable at around 4% throughout most of the sample period. For this reason, we abstract from buy-to-sell transactions in this paper.

the 1.3 percentage point increase in the effective LTT. The model captures the decline in buy-to-own transactions, the increase in buy-to-rent transactions, a higher ratio of leases to sales, and the decline in the price-to-rent ratio. In this exercise, the model is calibrated to match the estimated change in the moving hazard.

The model's prediction for buy-to-own and buy-to-rent transactions are respectively a 14.6% decrease and 2% increase. The endogenous adjustment of households' choice of renting versus owning and the entry of investors imply the lease-to-sales ratio increases by 15.4%. In equilibrium, the price-to-rent ratio declines by 1.8% and the homeownership rate by 0.9 percentage points. Time on the market for sellers goes up by 8.6%.

The welfare costs of the LTT increase are substantial. Per dollar of tax revenue raised, the LTT generates a welfare loss equivalent to 79% of the increase in tax revenue. The welfare loss is due to distortions across and within the two housing markets, and both are significant. The distortion across the two markets generate a loss equivalent to 25% of the extra tax revenue. Within the markets, distortions to the rental market and ownership market generate losses of 5% and 49% of tax revenue respectively. Overall, the rental market is associated with 30% of the total loss relative to extra tax revenue, which is around 40% of the total loss.

Related literature During the last two decades, concerns about the economic costs of stamp duty have grown among policymakers and in academic research. Two prominent ones are the 'Henry Review' appointed by the Australian government and the 'Mirrlees Review' by the U.K. government. Both reviews found significant costs associated with stamp duties owing to reduced mobility and the distortions associated with ad valorem taxes. The reviews proposed reforms to replace stamp duty with a land value tax or tax on housing consumption (Henry, Harmer, Piggott, Ridout and Smith, 2009, Mirrlees, Adam, Besley, Blundell, Bond, Chote, Gammie, Johnson, Myles and Poterba, 2010).

These findings are confirmed by economists working on housing using data from Australia, Canada, Finland, Germany, the U.S. and the U.K. The majority of the literature has focused on the effects on mobility, transaction volumes, or house prices.[6] Among them, a few have also computed the welfare cost of public funds per unit of tax revenue: Dachis, Duranton and Turner (2012) for Toronto, Hilber and Lyytikäinen (2017) and Best and Kleven (2018) for the U.K., Eerola, Harjunen, Lyytikäinen and Saarimaa (2019) and Määttänen and Terviö (2020) for Finland, and Fritzsche and Vandrei (2019) for Germany. These losses are associated with the intensive margin through the reduction in transactions and mobility within the ownership market. But as Poterba (1992) wrote, "*finding the ultimate behavioral effects requires careful study of how tax parameters affect each household's decision of whether to rent or own as well as the decision of how much housing to*

[6]See, for example, Lundborg and Skedinger (1999), Van Ommeren and Van Leuvensteijn (2005), Davidoff and Leigh (2013), Besley, Meads and Surico (2014), Dachis, Duranton and Turner (2012), Kopczuk and Munroe (2015), Hilber and Lyytikäinen (2017), Best and Kleven (2018), Eerola, Harjunen, Lyytikäinen and Saarimaa (2019), Fritzsche and Vandrei (2019), and Määttänen and Terviö (2020).

consume conditional on tenure."[7]

We make two contributions to this literature. First, empirically we document the different response of buy-to-let investors and the effects on the price-to-rent ratio, which show the importance of the extensive margin. Second, we develop and quantify a housing search model featuring search in both ownership and rental markets with an endogenous moving decision within and across the two markets.

There is a large body of literature Wheaton (starting from 1990, and followed by many others) that studies frictions in the housing market using a search-and-matching model as done here. That extensive literature is surveyed by Han and Strange (2015).[8] Among those, Lundborg and Skedinger (1999) explicitly study the effect of transaction taxes on search effort in a version of the Wheaton (1990) model. Since they abstract from the rental market and the decision to move, their model cannot be used to study the impact of transaction taxes on homeownership and mobility.

While the majority of housing search models have abstracted from search in the rental market, recent papers by Halket, Pignatti and di Custoza (2015) and Ioannides and Zabel (2017) have explicitly taken into account search in both ownership and rental markets.[9] While their objectives are different from ours, with the former focusing on the relationship between rent-to-price ratios and homeownership across sub-markets, and the latter focusing on the Beveridge curve in the housing market, they abstract from the moving decision that is crucial in our setting in driving effects on both extensive and intensive margins.

The application of our theory is close to Dachis, Duranton and Turner (2012) in studying the effects of the 2008 Land Transfer Tax (LTT) in Toronto. We combine the transaction-level sales data with the rental data, both from the Multiple Listing Service transaction records. We examine an array of market outcomes above and beyond sales prices and volumes, which allows us to gain a comprehensive understanding of how the housing market reacts to the LTT. Using a partial equilibrium model for the ownership market, Dachis, Duranton and Turner (2012) computed the welfare loss of the LTT to be 13% of tax revenue. By considering a general-equilibrium search model with endogenous moving within and across the ownership and rental markets, we find a much larger loss of around 42% of tax revenue.

Two recent paper with a related objective to our are Kaas, Kocharkov, Preugschat and Siassi (2021) and Cho, Li and Uren (2021) in analyzing the effect of stamp duty on the homeownership rate and its implications for welfare in a model without search frictions. A key advantage of our paper is that we identify the differential effect of stamp duty on buy-to-let investors and owner-occupiers

[7]The importance of the rental market has been emphasized by Bracke (2015), Demers and Eisfeldt (2021).

[8]For recent examples, see Anenberg and Bayer (2020), Díaz and Jerez (2013), Gabrovski and Ortego-Marti (2019), Guren (2018) Head, Lloyd-Ellis and Sun (2014), Moen, Nenov and Sniekers (2021), Ngai and Tenreyro (2014), Ngai and Sheedy (2020), Piazzesi, Schneider and Stroebel (2020), and Genesove and Han (2012).

[9]There is also a literature on understanding changes in the homeownership rate using models without search, for example, Chambers, Garriga and Schlagenhauf (2009), Fisher and Gervais (2011), Sommer and Sullivan (2018), and Floetotto, Kirker and Stroebel (2016). See Goodman and Mayer (2018) for a survey of the determinants of homeownership.

using data on leasing and transaction records. On the theory side, we highlight the indivisible nature of housing and separately allow for buy-to-let investors in a search model to capture the differential effect we find empirically.

Section 2 presents the data and the estimation for the effects of land transfer tax. Section 3 develops a dual ownership and rental markets model of housing. Section 4 calibrates the model to the city of Toronto and derives the quantitative effects of land transfer tax and associated welfare loss due to distortions within each market and the reallocation across the two markets.

2 Data and Institutional Background

2.1 Data

Our data for housing sales transactions and leasing transactions come from the Multiple Listing Service (MLS) transaction records in the Greater Toronto Area (GTA), the fourth largest North American Metropolitan area. The data cover residential property transactions from 2000 to 2018 and lease transactions for 2006–2018. For each sale transaction, we observe house price, time on the market, transaction date, the exact address and neighbourhood. For each lease transaction, we observe listing date, lease date, monthly rent and lease term, and the exact address and neighbourhood. For transactions that occurred after 2006, we also observe detailed house characteristics such as the number of bedrooms, the number of washrooms, the number of kitchens, lot size (except for condominiums/apartments), house style, family room style, basement structure/style, and heating types/sources.

Since we observe detailed address and transaction dates, we can identify the properties that appear in both datasets by property address. Doing so allows us to generate two novel measures that link the rental and owner-occupied sectors in the housing market. First, if a sale of a property is followed by being listed on the rental market between 0 and 18 months after the sale, we identify it as a buy-to-rent transaction.[10] Alternatively, if a sale of property is followed by being listed for sale between 0 and 18 months after the original sale, we identify it as a buy-to-sell transaction.[11] The remaining sales transactions are considered home purchase transactions. Between 2006-2017, the fraction of buy-to-own transactions declines from 89% to 84% while the fraction of buy-to-rent transactions triples from 4% to 12%. In contrast, the fraction of buy-to-sell transactions remains stable at around 4% throughout most of the sample period. For this reason, we exclude those buy-to-sell transactions from our estimation sample.

Second, for buy-to-rent transactions, we impute the property-level price-rent ratio. This is differ-

[10] The recent rise in buy-to-rent transactions has been mirrored in other countries, including the U.S. and Norway (Mills, Molloy and Zarutskie, 2019, Bo, 2021).

[11] For robustness check, we change the 18 month threshold to 6, 12, and 24 months, respectively. The estimation results do not change significantly.

ent from the average price-rent ratio that is often used to measure housing market conditions (Shiller, 2007). This ratio ignores the fact that properties underlying average rent and properties underlying average sales price are often not comparable. For instance, Glaeser and Gyourko (2007) use the 2005 American Housing Survey and show that "the median owner-occupied unit is nearly double the size of the median rented housing unit," and rental units are more likely to be located near the city centre. Following Bracke (2015), we address this concern by deriving the price-rent ratio from the sales price and rent for the same property that was leased out within a year after a sale.[12]

Using these rich data, we examine housing market outcomes both at the market segment level and at the transaction level. A market segment is defined by property type × community × year × month. A community refers to a neighbourhood.[13] Property types include single-family-houses, townhouses, condominiums, and apartments. For each housing market segment, housing outcomes include the number of sales, buy-to-own (BTO) sales, buy-to-rent (BTR) sales, number of leases, leases-to-sales ratio, price-to-rent ratio, the average price paid by homebuyers and the average price paid by investors. At the transaction level, housing outcomes include sales price, time on the market for sellers and the length that a homeowner stays in her current home.

2.2 The 2008 Toronto Land Transfer Tax

Land transfer taxes (LTT) are common across Canada. Although called Land Transfer Tax, the tax is applied to the entire transaction price, paid by buyers. Before 2008, residential transactions in Ontario, including the city of Toronto, were subject to the provincial level land transfer tax. There were no additional municipality level land transfer taxes. Since 2000, the city of Toronto has experienced a persistent housing boom and usually well-balanced budget. Following a rather surprising budget shortfall in late 2007, the Toronto city council approved a land transfer tax for real estate transactions in the city of Toronto that closed after 1 February 2008.[14] At the same time, the rest of the Greater Toronto Area had the same provincial-level LTT as before. Appendix Table A1 summarizes the LTT schedule for the city of Toronto before and after February 2008. The effective LTT rate, measured by the mean of land transfer taxes relative to home sales prices across transactions, is 1.08% during the pre-policy year and 2.04% during the post-policy year for the city of Toronto. This implies a 0.96 percentage points increase in the effective LTT.

Table 1 presents descriptive statistics for the city of Toronto before and years after the LTT. To start, we compare transactions that occurred two years before and two years after the policy. The top rows show the number of buy-to-own transactions declines by 14% from 55,436 to 47,664 after the

[12] Using a similar approach, Rutherford, Rutherford and Yavas (2021) impute the price-rent ratio using the matched sales and rental data in Miami-Dade County.

[13] There are 296 communities in the Greater Toronto Area, including 140 communities in the city of Toronto. See https://www.toronto.ca/city-government/data-research-maps/neighbourhoods-communities/neighbourhood-profiles/.

[14] Dachis, Duranton and Turner (2012) provide detailed discussion and evidence, showing that the 2008 LTT in Toronto arrived in a mostly unanticipated way.

LTT. In contrast, buy-to-rent transactions increases by 7% from 3,144, to 3,370. For homeowners who have spent a complete spell in our sample period, on average they stay in their house for about 5-6 years. In addition, a property stays on the market for about 25 days before it is sold. These statistics indicate two sources of nonnegligible frictions in the housing market. First, match quality depreciates over time. Second, search is costly, hence the process of finding a buyer or a home is lengthy. To understand how these frictions interact with the LTT in affecting housing markets, we leverage the richness of our sales and lease transaction records and interpret the patterns through the lens of a search model featuring both ownership and rental markets.

Table 1. *Descriptive Statistics*

	City of Toronto			
	Pre-LTT	Post-LTT		Overall Sample
	2006/01-2008/01	2008/02-2010/02	2008/02-2012/02	2006/01-2018/02
# of BTO Sales	55,436	47,664	98,484	306,571
# of BTR Sales	3,144	3,370	7,788	29,276
Median Length of Stay (Days)	1,039	1,332.5	1,497	1,648
Mean Length of Stay (Days)	1,069.93	1,384.51	1,573.03	1,903.19
Median Time on Market (Days)	20	18	17	15
Mean Time on Market (Days)	30.54	28.80	27.05	25.40
Median Sale Price	318,000	343,000	369,900	419,990
Mean Sale Price	401,504.1	426,362.9	460,903.2	555,484
Median Average Rent	1,525	1,616.58	1,374.17	1,800
Mean Average Rent	1,802.59	1,905.74	1,947.21	2,086.53
Median Same-property Price-Rent ratio	13.45	14.94	15.99	15.02
Mean Same-property Price-Rent ratio	14.46	15.72	16.74	15.89

Notes: Data source is the Multiple Listing Service (MLS) residential transaction records from the city of Toronto (2006-2018).

3 Estimating the Effects of the Land Transfer Tax

Estimating the effect of the LTT on housing markets is challenging as the implementation of the LTT coincided with the initial stage of the 2008 financial crisis, which may confound the relationship between the LTT and changes in housing market outcomes. We note that the LTT caused two discrete changes in the housing market: one at the border of the city of Toronto, the other on the date the LTT was imposed. Together, this gives us an opportunity to establish the causal effects of the LTT by comparing the change in housing market outcomes for households that are 'treated' with the LTT to households in "untreated" suburban neighborhoods.

Our identification strategy resembles the regression discontinuity design in Dachis, Duranton and Turner (2012), who estimate the short-run (six months) effects of the LTT on transaction volume and sales price in the single family housing market. The innovation here is that we examine a rich array of market outcomes above and beyond sales price and volume. For example, we use individual households' moving hazard instead of total transactions to measure the impact of the

LTT on residential mobility. The resulting hazard estimates provide direct and precise micro-level mobility evidence that complements the aggregate level finding on transaction volumes. In addition, our estimation is not limited to the owner-occupied sector. Benefiting from our uniquely rich data, we are able to estimate not only the flows between owner-occupied and rental sectors but also the transaction and mobility decisions within each sector, which allows us to gain a comprehensive understanding of how the housing market reacts to the LTT. Finally, our estimation sample covers a longer time period and richer types of residential properties.

In the main estimation, we define January 2006 – January 2008 as the pre-policy period and February 2008 – February 2012 as the post-policy period.[15] We regress a set of housing market outcome variables on the interaction of a dummy indicating the city of Toronto and a dummy indicating the post-policy period. Given that the LTT was implemented for the city of Toronto in February 2008, we rely on the coefficient on the interaction of the dummies to capture the possible LTT effects. To ensure relatively homogeneous housing stock and neighbourhoods in our sample, we control for a rich set of time-varying house characteristics and further compare properties on the opposite sides of the city border — the geographic lines that determine whether the LTT is applicable. By limiting the sample to properties in close proximity to each other but on opposite sides of the borderline, we control for unobserved neighbourhood and housing stock differences. Importantly, the possibility that housing market outcome variables make a discrete jump at the border while neighbourhoods continue to change in a smooth manner allows us to isolate the relationship between the LTT and housing market outcomes. The estimation sample is depicted in Figure 4.

The underlying assumption is that housing market outcomes for treated and untreated households experienced similar housing market trends in the absence of the LTT. The validity of our strategy rests on three assumptions. First, the real estate market did not anticipate the tax. Second, no other policy changes differentially affected the Toronto and suburban real estate market at the same time when the LTT was imposed. Third, there is no substantial sorting of buyers from inside to outside the border in response to the LTT. We relax these assumptions in robustness checks.

3.1 Homeowners' Moving Hazard

We start with estimating the effect of the LTT on individual homeowners' mobility. Unlike many previous studies that use housing transactions to measure mobility, we observe precisely when a homeowner puts the home on the market for sale. To provide a visual sense of the time patterns of mobility, we estimate the standard Kaplan-Meier (KM) estimator for the moving hazard. The KM estimator computes the conditional probability of putting a home on the market for sale, given the time since the homeowner initially moved in. As shown in Figure 3 in Appendix A.2, the median survival time that a homeowner stays in her home is 113 months. The hypothesis of homogeneity of hazard rates over time is not rejected at the 0.01 level. The hazard shape observed here provides

[15] In robustness checks, we change the sample period to 2006-2010 and 2006-2018, respectively.

a natural basis for estimating the transaction hazard using a Weibull model, where the monotonicity restriction on the hazard rate is imposed.

Specifically, a unit of observation is each month since a homeowner has bought her home and an event is whether she puts the home on the market for sale given that this has not occurred to that point. The hazard function for homeowner j in a given year-month t is parameterized as follows:

$$h(t|LTT_{jt}, \mathbf{x}_{jt}) = pt^{p-1} \exp(\alpha + \mathbf{x}_{jt}\beta + LTT_{jt}\gamma)$$

where p is the parameter indicating the exponential changes of risk and x_{jt} is a rich set of time-varying controls for house j. Throughout our analysis, we include an indicator for post LTT periods and an indicator for being in the city of Toronto. Their interaction is the LTT - the key variable of interest. To isolate the causal impact of the LTT on homeowners' mobility, we include a rich set of time-varying housing attributes, all interacting with property-type fixed effects. We also include a rich combination of fixed effects: Toronto $\times$ property-type fixed effects, year $\times$ property-type fixed effects, month $\times$ property-type fixed effects and community $\times$ property-type fixed effects. This allows us to control flexibly the differential evolution of housing market outcomes across different property types in different communities. In addition, we include the initial home value, measured by the purchase price a homeowner had previously paid. House value is typically considered as a proxy for the transfer tax in the previous literature on residential mobility. However, such a proxy is imperfect, as the non-tax-related moving costs are positively associated with the existing property's value, in both monetary and psychological terms (Ioannides and Hardman (1995); Han (2008)). Thus households who occupy a residence of higher property value are less likely to move, even in the absence of changes in the taxes. By controlling for the initial purchase price, we are able to separate the LTT effect on residential mobility from other transaction costs. Finally, the standard errors are cluster-corrected at the homeowner level in all specifications.

The results are presented in Table 2. In column (1), we restrict the sample to 3km on each side of the border. As shown in column (1), being subject to the LTT reduces the moving hazard by 7.4%, although imprecisely estimated. In addition, a 1% increase in the initial home value reduces the moving hazard by 10%. Thus, transaction taxes and non-tax transaction costs have almost equivalent dampening effects on residential mobility, suggesting that a careful evaluation of the LTT should separately consider these two costs. Moreover, $ln(p)$ is significantly greater than 0, indicating an increasing risk of moving over time.

One legitimate concern is that households may have anticipated that it is more costly to transact a home after the implementation of the LTT and therefore rushed to transact before the LTT. As discussed extensively in Dachis, Duranton and Turner (2012), such anticipation of the 2008 LLT in the Toronto market was quite limited, and if any, would occur within 3 months before and after the policy. In light of this, column (2) repeats estimation in column (1) but includes 6 monthly indicators for houses in Toronto from November 2007 to April 2008 to condition out the run-up in transactions

Table 2. *LTT Effects on Moving Hazard*

	Dependent Variable: The event of moving						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
LTT	-0.0704	-0.130**	-0.188**	-0.151**	-0.194***	-0.232**	-0.112
	(0.0635)	(0.066)	(0.080)	(0.051)	(0.054)	(0.087)	(0.078)
log(initial home value)	-0.0985**	-0.0949**	-0.170**	-0.0729**	-0.0758**	-0.103**	-0.0226
	(0.0460)	(0.0460)	(0.059)	(0.0345)	(0.0344)	(0.044)	(0.0548)
post-LTT	-0.204***	-0.206***	-0.128**	-0.193***	-0.193***	-0.130**	-0.264***
	(0.048)	(0.048)	(0.058)	(0.039)	(0.039)	(0.054)	(0.056)
Toronto	0.131**	0.191**	0.224**	0.239***	0.239***	0.300***	0.161**
	(0.056)	(0.058)	(0.071)	(0.045)	(0.045)	(0.061)	(0.069)
ln(p)	0.511***	0.513***	0.501***	0.523***	0.523***	0.519***	0.530***
	(0.010)	(0.010)	(0.012)	(0.007)	(0.007)	(0.010)	(0.012)
Observations	1,691,369	1,691,369	1,142,052	2,831,897	2,831,897	1,651,935	1,179,962
Distance threshold	3KM	3KM	3KM	5KM	5KM	5KM	2KM
House characteristics	YES	YES	YES	YES	YES	YES	YES
Indicators TO +-3 m.		YES	YES	YES	YES	YES	YES
Time trends TO		YES	YES	YES	YES	YES	YES
Distance LTT trends					Step	Step	
Donut hole			1KM			2KM	

Notes: A unit of observation is a household whose property is listed on MLS every year and month, from January 2006 to February 2012. Transactions that take place within 18 months will be discarded. A survival model with risk specified as Weibull distribution on the effect of LTT is with an indicator for post-LTT period, Toronto*Property Type Fixed Effects, Month*Property Type Fixed Effects, Community*Property Type Fixed Effects. Indicators TO +- 3m are six dummy variables for transactions inside Toronto during the last 3 months of 2007 and the first 3 of 2008. Time trends TO indicates the presence of separate time trends for transactions in and outside of Toronto. Distance threshold is the maximum distance to the Toronto border for a transaction to be included. Distance LTT trend denotes the inclusion of an interaction term between LTT and distance to the Toronto border. Robust standard errors in parentheses. *, **, ***: corresponding coefficient significant at 10, 5, 1%.

right before the policy and possible retention right after the policy.[16] We further add a linear time trend for households in Toronto, which allows for spatially differentiated trends in and outside the city of Toronto. The LTT reduces the transaction hazard by 13% and the estimate is significant at the 5% level. Given that this specification has the most extensive controls, we retain it as the main specification from now on.

Another potential sorting bias is that some buyers may switch from inside to outside the border in response to the LTT, making moving more likely to happen for homeowners outside of the border. This would violate the assumption that homeowners outside of the city border are unaffected by the tax change. Such substitution, if exists, would most likely happen right adjacent to the border. To mitigate this concern,we apply a "donut approach" by repeating the estimation in column (2) within the 3km ring but excluding homeowners within 1km on each side of the border. As shown in column (3), doing so increases the reduction in the moving hazard to 18.8%, which is not statistically

[16]Our strategy of dealing with the possible anticipation effects is also consistent with Berard and Trannoy (2014) and Benjamin, Coulson and Yang (1993), both of which explicitly estimate the anticipated effects associated with a real estate transfer tax. Using the French data, the former find that the anticipation effect is limited to one month right before the implementation of the reform while the post-policy retention effect lasts up to 3 months. Using the Philadelphia data, the latter find that the anticipation effect is very small and limited to two months before the tax implementation.

different from the estimate in column (2), suggesting that substitution across the border is unlikely to bias our main estimates.

In column (4), we replicate our preferred regression of column (2) but extend our sample of observations to consider all properties sold within five kilometres of the Toronto border instead of three. In column (5), we further add an interaction term between LTT and a dummy variable for Toronto households between two and a half and five kilometres away from the Toronto border. This allows homeowners to react to the LTT shock differently, depending on their distance to downtown. In column (6), we repeat the estimation in Column (5) but exclude homeowners within 2km on each side of the border. This allows us to address possible substitution across the border in a wider set of neighborhoods. In all these three specifications, the estimated LTT effects on moving hazard remain consistent and robust, in terms of both economic and statistical significance. In column (7), we replicate our preferred specification from column (2) but restrict our sample to all properties sold within two kilometres of the Toronto border instead of three. The coefficient is very close to that in our preferred specification but becomes less precise possibly due to the smaller sample size.

Based on our most preferred specification (column 2), the implementation of the LTT reduces the hazard rate of moving by 13%. Given that the mean length of stay for homeowners before the policy is 113 months, this implies that on average homeowners stay in their current home for 14 month longer after the LTT, indicating a substantial lock-in effect. Our findings are consistent with the evidence found in other countries.[17] We also repeat the moving hazard estimation for alternative sample periods: 2006-2010 and 2006-2018. As shown in Appendix Table A2, the estimated LTT effect remains robust to shorter and longer post-policy periods. This implies that the lock-in effect on residential mobility of transaction taxes is not only substantial but also permanent.

3.2 Time-on-Market

The substantive lock-in effect we estimated above not only points to a widening mismatch between existing homeowners and their current homes but also could potentially hamper the functioning of the transaction market. In particular, knowing that the LTT increases the length of her stay once a new owner moves in, buyers would become pickier *ex ante* in searching for a home. This could in turn increase the time it takes for a transaction to be completed. Interestingly, the distortions on home search has not been examined in the literature. To shed light on this, we now use the transaction-level sales data to estimate the causal effect of the LTT on time-on-market.

A unit of observation is a transaction. For a given transaction, we measure time-on-market as

[17] For example, using the Netherlands data, Van Ommeren and Van Leuvensteijn (2005) find that a 1 percentage-point increase in the value of transaction costs — as a percentage of the value of the residence — decreases residential mobility rates by 8.05-12.66 percent. Using the UK data, Hilber and Lyytikäinen (2017) find that a 2 percentage-point increase in the SDLT reduces the annual rate of mobility by 2.6 percentage points. While their analysis relies on the cutoff of home values as a proxy for the transfer tax, we benefit from a natural experimental setting that generates two discrete changes in the housing market: one at the border of the city of Toronto, the other on the date the LTT was imposed.

the number of days between the time when the property was initially listed and the time when a transaction agreement is made between buyers and sellers. Using the same regression discontinuity designed laid out above, we now regress the log of time-on-market on the LTT with a rich set of controls and specifications as specified in Table 3. Table 3 presents the estimates on time-on-market. Column (1) indicates that the implementation of the LTT is associated with a 27% increase in time-on-market, which is equivalent to a week longer based on the pre-policy sample mean. In the remaining columns, we condition out the anticipation effects, allow for spatially differentiated time trends, control for substitution across borders, change the sample from 3km to 5km and 2km on each side of the border, respectively. The estimated LTT effect on time-on-market is robust–both in terms of economic and statistical significance– to these alternative specifications and sample selection.

Table 3. *LTT on Days on Market (at the transaction level)*

	Dependent Variable: ln(Days on Market)						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
LTT	0.270***	0.280***	0.289***	0.301***	0.273***	0.224***	0.319***
	(0.032)	(0.034)	(0.041)	(0.026)	(0.029)	(0.049)	(0.044)
Observations	39,948	39,948	28,097	69,164	69,164	45,001	24,163
Distance threshold	3KM	3KM	3KM	5KM	5KM	5KM	2KM
House characteristics	YES	YES	YES	YES	YES	YES	YES
Indicators TO +-3 m.		YES	YES	YES	YES	YES	YES
Time trends TO		YES	YES	YES	YES	YES	YES
Distance LTT trends					Step	Step	
Donut hole			1KM			2KM	

Notes: A unit of observation is a transaction listed on MLS, from January, 2006 to February, 2012. Each cell represents a separate regression of a housing outcome variable (specified on the left column) on the LTT. All regressions are estimated with an indicator for post-LTT period, Toronto*Property Type Fixed Effects, Month*Property Type Fixed Effects, Community*Property Type Fixed Effects. Indicators TO +- 3m are six dummy variables for transactions inside Toronto during the last 3 months of 2007 and the first 3 of 2008. Time trends TO indicates the presence of separate time trends for transactions in and outside of Toronto. Distance threshold is the maximum distance to the Toronto border for a transaction to be included. Distance LTT trend denotes the inclusion of an interaction term between LTT and distance to the Toronto border. Robust standard errors in parentheses. *, **, ***: corresponding coefficient significant at 10, 5, 1%.

Our estimates in Table 3 provide the first evidence on how the LTT affects the liquidity aspect of the housing market, suggesting that search frictions are non-negligible in analyzing the effects of the LTT. Conditional on a home being listed, the LTT increases the time-on-the market by 27%. This substantial delay in completing a transaction is driven by two tax-related factors, both of which make buyers choosier in searching for a home. First, with the LTT, a buyer's valuation needs to exceed a seller's valuation by the amount of the additional transaction costs for a transaction to take place. Moreover, with the match quality depreciating over time, the level of the pickiness for a buyer further increases as she needs to reduce the need to move and pay the transaction cost again in the future.

Summarizing Tables 2 and 3, we find that the LTT reduces the flows from both sides of the ownership market. On the one hand, it increases the level of mismatch that existing homeowners will tolerate, resulting in fewer homes being put on the market for sale. On the other hand, it imposes additional transaction cost on home buyers, making them pickier in home shopping. Together, these two forces lead to fewer sales transactions, which we turn to in the next subsection.

3.3 Transactions: Buy-to-Own v.s. Buy-to-Rent; Sales v.s. Leases

In this subsection, we estimate the effects of the LTT on the transaction volume. We use the same regression discontinuity design as described above. A unit of observation now is a market segment defined by *community* $\times$ year $\times$ month for single-family-homes and the outcome variable is the log of the number of transactions.[18]

The literature has consistently shown that the implementation of the LTT reduces the total number of sales transactions and hence residential mobility.[19] We found similar evidence in our sample market. However, property sales alone can be an imperfect proxy for residential mobility even at the aggregate level, because sales can be also undertaken by investors and landlords. For this reason, we go beyond the literature by examining the composition effects of the LTT on (1) the number Buy-to-Own (BTO) transactions v.s. Buy-to-Rent (BTR) transactions and (2) the number of leases versus the number of sales.

We start with estimating the LTT effects on the BTO sales. The results are presented in row (1) of Table 4. Column (1) indicates that the implementation of the LTT is associated with a 4.7% decrease in sales. In column (2), we include 6 monthly indicators around the implementation of the LTT for neighbourhoods in Toronto to condition out the possible anticipation and retention effects. We further add a linear time trend for neighbourhoods in Toronto, allowing for the possibility of spatially differentiated housing market trends in and outside of Toronto. The estimated LTT effect changes to -9.8% and the estimate is significantly significant. In column (3), we apply a donut approach by excluding neighborhoods within 1km of each side of the border. As discussed earlier, this mitigates the possible concern of sorting buyers from inside to outside the border in response to the LTT. The estimated effect of the LTT changed slightly to -7.2%. In columns (4)-(6), we replicate these various specifications but extend the sample of observations to consider all neighborhoods within five kilometres of the Toronto border instead of three. In column (7), we replicate the most preferred specification from column (2) but restrict the sample to all neighborhoods within two kilometres of

[18] Multiple Listing Service data cover almost the entire universe of single-family-home transactions, but this is not always the case for condominiums. To mitigate the bias associated with the possible selection in the condominium sample, we restrict the sample to single-family-homes for transaction regressions in this section.

[19] Using the UK property transaction data, Best and Kleven (2018) find that a temporary 1 percentage-point cut in the tax rate — due to the 2008–9 stamp duty holiday on properties worth between £125,001 and £175,000 — led to a 20% increase in transactions. Using the German single-family home sales, Fritzsche and Vandrei (2019) find that a one-percentage-point increase in the transfer tax yields about 7% fewer transactions. Moreover, Dachis, Duranton and Turner (2012) show that the same LTT studied here caused a 15% decline in the sales volume using the postal code level data.

the Toronto border. The coefficient remains robust and consistent across all these specifications.

Table 4. *Effects of the LTT on Transaction Volumes in Sales and in Leasing*

Dependent Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ln(# BTO Sales)	-0.0474	-0.0982*	-0.0715	-0.100**	-0.100**	-0.0678	-0.155**
	(0.0364)	(0.0587)	(0.0669)	(0.0443)	(0.0443)	(0.0559)	(0.0729)
Observations	3748	3748	2839	6420	6420	3868	2552
ln(# BTR Sales)	0.0328*	0.0227*	0.078	0.0526*	0.0526	0.0758	0.0154
	(0.0175)	(0.0119)	(0.049)	(0.031)	(0.035)	(0.0904)	(0.135)
Observations	556	556	444	1093	1093	729	364
ln(# Leases/# Sales)	0.0740	0.234**	0.190	0.242**	0.242**	0.236**	0.280*
	(0.0727)	(0.117)	(0.133)	(0.0824)	(0.0824)	(0.100)	(0.146)
Observations	1355	1355	1104	2660	2660	1782	878
Distance threshold	3KM	3KM	3KM	5KM	5KM	5KM	2KM
Indicators TO +-3 m.		YES	YES	YES	YES	YES	YES
Time trends TO		YES	YES	YES	YES	YES	YES
Distance LTT trends					Step	Step	
Donut hole			1KM			2KM	

Notes: A unit of observation is a market segment defined by community X year X month for single-family–house transactions, from January 2006 to February 2012. Transactions that take place within 18 months are discarded. Each cell represents a separate regression of a housing outcome variable (specified on the left column) on the LTT. All regressions are estimated with an indicator for post-LTT period, Toronto*Property Type Fixed Effects, Calendar Month*Property Type Fixed Effects, Community*Property Type Fixed Effects. Indicators TO +- 3m are six dummy variables for transactions inside Toronto during the last 3 months of 2007 and the first 3 of 2008. Time trends TO indicates the presence of separate time trends for transactions in and outside of Toronto. Distance threshold is the maximum distance to the Toronto border for a transaction to be included. Distance LTT trend denotes the inclusion of an interaction term between LTT and distance to the Toronto border. Robust standard errors in parentheses. *, **, ***: corresponding coefficient significant at 10, 5, 1%.

Taking column (2) as our baseline specification, row (1) shows that the implementation of the LTT is associated with a roughly 9.8% decrease in the homeowner purchases. However, this is not the case for all transactions. Row (2) shows that the same LTT increases the buy-to-rent sales by 2.3%. Thus, while the LTT substantially reduces residential mobility, it does not discourage investors turnover.[20]

The contrast between how the LTT affects transactions by homeowners and by investors is quite striking. As noticed in Section 3, the same LTT was imposed on all buyers regardless of their purchase purpose. While investors and homebuyers may receive different treatments in mortgage markets and capital gain taxes, these differences would have been conditioned out with our differences-in-differences approach as there is no evidence that these treatments differ either across Toronto and suburban real estate markets or before and after the implementation of the LTT. One thing to note is that partial exemptions for the LTT were given to first-time homebuyers. Compared with buy-to-rent investors, buy-to-own buyers are more likely to be first-time homebuyers and hence would

[20]Given the dominance of home transactions in the housing market, the total sales volume dropped in response to the LTT.

benefit more from this partial exemption. If this was the case, we should expect that the dampening effect of LTT on buy-to-own buyers is less than that on buy-to-rent investors. However, the result shows the opposite, suggesting that the estimated differential impacts of the LTT on buy-to-own and buy-to-rent transactions are robust in their direction.

By construction, buy-to-rent investors are those who buy properties from owner-occupied markets and put them on the rental market to lease. An increase in buy-to-rent transactions should lead to an increase in the number of lease transactions relative to sale transactions. Consistent with this, row (3) indicates that the LTT is associated with a 23.4 percent increase in the lease-to-sales ratio. Intuitively, the imposition of the LTT makes owning a home less attractive relative to renting. The increase in demand for rental properties encourages the entry of landlords. This is consistent with an inflow of investors induced by the LTT as evidenced in row (2), contributing to a shift of properties from the owner-occupied sector to the rental sector, resulting in a fall in the homeownership rate.

3.4 Price and Rent

In an efficient housing market, we expect that the LTT be fully capitalized into the price. Appendix Table A1 summarizes the LTT schedule for the city of Toronto before and after February 2008. The effective LTT rate during the pre-policy year, measured by the mean of land transfer taxes relative to home sales prices across transactions, is 1.47% using the actual tax rate and 2.74% using the post-policy tax rate for the city of Toronto. This implies a 1.27-percentage-point increase in the effective LTT, which is equivalent to an 86% increase in the LTT. We now estimate how this translates into changes in the sales price and price-rent ratio.

Using the market segment level data, we apply the regression discontinuity design to the average sales price. The results are presented in row (1) of Table 5. Column (1) shows that the LTT reduces sales price by 1.76%. In column (2), the estimate changes slightly to -2% when we condition out possible anticipation effects and allow for spatially differentiated trends. The estimate remains robust when further excluding properties adjacent to the border and relaxing the 3-kilometres bands to either side of the border. When we further restrict the sample to the price paid by homeowners, as shown in row (2), the estimated price elasticity with respect to the LTT are consistent with row (1) across specifications. One potential concern is that the composition of housing stock might differ before and after the LTT. To address this, we further estimate the LTT effects on the sales price using the individual transaction level data for single family houses, controlling for a rich set of time-varying house characteristics. As shown in Table A3, the LTT reduces the sales price by 1.70%, consistent with the market-segment level estimates.

In Tables A3-A4, we repeat the transaction-level and market-segment level estimation on the sales price for the 2006-2010 and 2006-2018 periods, respectively. The fact that the estimated price effect is robust and persistent across different post-policy periods suggests that the housing market is efficient in responding to changes in tax rates. In an efficient market, prices fully and instantaneously

Table 5. *Effects of the LTT on Sales Price and Price-Rent Ratio*

Dependent Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ln(Average Price)	-0.0176***	-0.0200***	-0.0229***	-0.0174***	-0.0174***	-0.0125**	-0.0255***
	(0.005)	(0.005)	(0.006)	(0.004)	(0.004)	(0.005)	(0.007)
Observations	11169	11169	8688	19227	19227	11802	7425
ln(BTO Average Price)	-0.0176***	-0.0202***	-0.0226***	-0.0173***	-0.0173***	-0.0119**	-0.0260***
	(0.005)	(0.006)	(0.006)	(0.004)	(0.004)	(0.006)	(0.007)
Observations	10849	10849	8457	18703	18703	11534	7169
Price/Rent Ratio (Same Property)	-1.503**	-1.591**	-2.421**	-1.097**	-1.097**	-0.637	-2.078*
	(0.698)	(0.734)	(0.791)	(0.473)	(0.473)	(0.530)	(1.063)
Observations	1,165	1,165	964	2,555	2,555	1,842	695
Distance threshold	3KM	3KM	3KM	5KM	5KM	5KM	2KM
Indicators TO +-3 m.		YES	YES	YES	YES	YES	YES
Time trends TO		YES	YES	YES	YES	YES	YES
Distance LTT trends					Step	Step	
Donut hole			1KM			2KM	

Notes: A unit of observation is a market segment defined by community X property type X year X month, from January, 2006 to February, 2012. Each cell represents a separate regression of a housing outcome variable (specified on the left column) on the LTT. All regressions are estimated with an indicator for post-LTT period, Toronto*Property Type Fixed Effects, Year*Property Type Fixed Effects, Month*Property Type Fixed Effects, Community*Property Type Fixed Effects. Indicators TO +- 3m are six dummy variables for transactions inside Toronto during the last 3 months of 2007 and the first 3 of 2008. Time trends TO indicates the presence of separate time trends for transactions in and outside of Toronto. Distance threshold is the maximum distance to the Toronto border for a transaction to be included. Distance LTT trend denotes the inclusion of an interaction term between LTT and distance to the Toronto border. Robust standard errors in parentheses. *, **, ***: corresponding coefficient significant at 10, 5, 1%.

reflect all available relevant information. The housing market is known to be efficient in the long run as the housing supply is price elastic over a 10-year period (Baum-Snow and Han, 2021). Our estimates from 2006-2010 show that changes in tax rates are efficiently capitalized into house prices even in the short run. Intuitively, with the LTT, the buyer's valuation needs to exceed the seller's valuation by the amount of the additional transaction costs for a transaction to take place.

In row (3), we further estimate how the LTT affects the price-rent ratio. We choose to use the price-rent ratio derived from the same property listed in the sales market and rental market at approximately the same time. This allows us to address the usual concern with the average price-rent ratio since the composition of properties in the sales market is often different from that in the rental market. Column (1) shows that the LTT reduces the price-rent ratio by 1.5, representing a 10% deviation from the sample mean of the price-rent-ratio before the LTT. The estimate is robust to different specification checks and sample selections. Moreover, a null that the price decline is no greater than the rent decline is rejected at the 1% level, indicating that the LTT dampens price more than the rent. This, together with the earlier evidence that the LTT reduces sales more than leases, reveals heterogenous treatment effects of the LTT on the owner-occupied and rental sectors. In particular, by distorting matching of houses and households in the owner-occupied market, the LTT makes it less attractive to own a home, so some marginal homebuyers would be dissuaded from entering the ownership market. Since they must live somewhere, this raises the demand in the rental market, which explains the simultaneous decline in the price-rent ratio and in the sales-to-leases ratio.

To summarize, our findings suggest that the LTT may lead to the misallocation of the housing stock along both the intensive and extensive margins. On the intensive margin, some transactions that would have occurred in the current period are postponed or never happen; some households who would have been better matched with a new home are now stuck with their old home. On the extensive margin, some households who would have owned a home now enter the rental market; some properties that would have been bought by homeowners are now bought by investors. The newly gained evidence on time-on-market, moving hazard and price effects highlights the limitation of considering the effect of the LTT in isolation from the underlying market frictions, particularly idiosyncratic match quality between house and households. The uncovered heterogenous treatment effects of the LTT on the owner-occupied and rental sectors and for buy-to-own versus buy-to-rent buyers further reveal that a careful evaluation of the LTT should account for both household flows and property flows between owning and renting. To gain a comprehensive understanding of the rich housing market dynamics documented above and to derive the relevant welfare implications, we now build a search-theoretical model featuring not only simultaneous rental and owner-occupied markets but also endogenous moving arising from changing match quality.

4 A dual ownership and rental markets model of housing

There is a city with two housing markets: an ownership market and a rental market. There is a unit measure of ex-ante identical properties and a constant measure ψ of households. Time is continuous, and everyone has discount rate r for future payoffs. Households exit the city exogenously at rate ρ, who are replaced by an equal inflow of new households. There is free entry of investors, who become landlords and rent out properties. Investors simply represent funds invested in housing and could be living within the city or from elsewhere.

Properties are either up for sale, offered for rent, or not available in either market. They are owned either by those who live in them or by landlords. When not for sale or rent, properties are occupied by a renter or an owner-occupier. Some owners or renters are looking to move, and they choose whether to search in the ownership or rental market. Owner-occupiers looking to move put their property up for sale. Landlords choose whether to let or to sell the properties they own. At rate ρ_l, landlords receive a shock forcing them to sell their property, for example, for liquidity reasons.

The measure of buyers in the ownership market is b_o, comprising home-buyers b_h who will live in the property they buy, and investors b_k. The fraction of investors among buyers is denoted by ξ. Those looking to rent are b_l. On the other side of the two markets, properties available for sale are u_o and properties available for rent are u_l. The tightness of market i — the ratio of 'buyers' to 'sellers'

— is denoted by θ_i, where $i \in \{o, l\}$ indexes the ownership (o) or rental (l) market:

$$\xi = \frac{b_k}{b_o} \quad \text{and} \quad \theta_i = \frac{b_i}{u_i}, \quad \text{where} \quad b_o = b_h + b_k. \tag{1}$$

Search frictions place limits on meetings between participants in both markets. Meetings are viewings of properties that allow for offers to buy or to rent. Meeting rates are determined by constant-returns-to-scale meeting functions $\Upsilon^i(b_i, u_i)$. The rate $\Upsilon^i(b_i, u_i)/b_i$ at which buyers/renters view properties in market i is denoted by q_i. Constant returns to scale makes q_i a function of tightness θ_i:

$$q_i = \frac{\Upsilon^i(b_i, u_i)}{b_i} = \Upsilon^i\left(1, \theta_i^{-1}\right), \quad \text{and} \quad \frac{\Upsilon^i(b_i, u_i)}{u_i} = \Upsilon^i(\theta_i, 1) = \theta_i q_i \quad \text{for} \quad i \in \{o, l\}. \tag{2}$$

The meeting rate $\Upsilon^i(b_i, u_i)/u_i$ for sellers in market i is $\theta_i q_i$. The meeting function is increasing in both b_i and u_i, hence q_i decreases with θ_i, while $\theta_i q_i$ increases with θ_i. Intuitively, if there are more 'buyers' relative to 'sellers' in a particular market, the meeting rate is lower for those viewing properties but higher for those offering properties for sale or to let.

Owner-occupiers or renters living in a property receive a match-specific flow value ε. At the time of a meeting when a household views a property, match quality ε between the property and the household is drawn from distribution function $G_i(\varepsilon)$ for market i. The distribution of ε can differ across markets, for instance, allowing for a 'warm glow' effect of home-ownership where flow value are higher on average. From the perspective of an investor owning a property, all properties are ex-ante identical prior to being viewed by potential tenants or buyers.

Idiosyncratic match quality ε for those living in a property is a persistent variable subject to occasional shocks. These shocks represent life events that make a property less well matched to the household occupying than it originally was. Shocks arrive independently across households and time at rate a_i, which can differ by housing tenure $i \in \{o, l\}$. For owner-occupiers, the arrival of a shock reduces match quality from ε to $\delta_o\varepsilon$, where $\delta_o < 1$ is a parameter.[21] For renters, match quality ε is reduced to 0 following a shock — effectively $\delta_l = 0$.

Following a shock, owner-occupiers and renters decide whether to move and start searching for another property to live in, with owners putting their current property up for sale. Moving is endogenous and depends on how low match quality has become relative to expectations of match quality in a alternative property, though for renters, moving depends only on the arrival of a shock.[22]

Those who decide to move choose to buy or rent their next property by searching in the ownership market or the rental market. Households pay an idiosyncratic cost χ when they enter the ownership market for the first time. This can be thought of as household-specific factors affecting the cost or availability of mortgages, such as credit histories or wealth for downpayments. χ is independently

[21] The model has no shocks that increase match quality, however, these would not cause households to move house.

[22] It is possible to extend the model to have $\delta_l > 0$. However, it turns out that the endogeneity of moving is quantitatively unimportant for renters here, so the model is simplified by assuming $\delta_l = 0$.

drawn from a distribution $G_m(\chi)$ when a household arrives in the city and decides to buy or rent.

A household's credit cost persists over time, but while the household is in the rental market, χ redrawn with probability γ from the same probability distribution $G_m(\chi)$ if the arrival of an exogenous shock causes the household to move — either shocks to match quality or the landlord selling owing to an exit shock. Households exiting the city sell properties they own. When tenants choose to move or exit the city, their landlords decide whether to look for a new tenant or to sell.

4.1 The ownership market

Buyers in the ownership market are either home-buyers or investors. The expected value of owning a property is the same for all investors because they face the same expected rents when their property is let, while home-buyers put different values on properties because of idiosyncratic match quality.

After a buyer has met a seller and viewed a property, revealing the quality of the match to home-buyers, the buyer and seller negotiate a price and a transaction occurs if mutually agreeable. The land transfer tax is represented by proportional taxes levied on the transaction price paid by the buyer. Home-buyers and investors face tax rates τ_h and τ_k respectively, which in principle can differ.

The Bellman equation for the value K of being an investor who buys at price P_k is

$$rK = -F_k + q_o\,(U_l - (1+\tau_k)P_k - C_k - K) + \dot{K}\,, \tag{3}$$

where $\dot{K}$ is the derivative of the value K with respect to time t (the dependence of variables on time t is not indicated explicitly). There is a flow search cost F_k incurred by investors, $\tau_k P_k$ is the land transfer tax paid, and C_k is any other transaction costs paid by investors. Investors meet sellers at rate q_o, and because investors have no idiosyncratic match quality with a property, this is also the rate at which they are able to buy. After buying, investors make their properties available for rent and receive the common expected value U_l of being a landlord.

The Bellman equation for the value B_o of being a home-buyer is

$$rB_o = -F_h + q_o \int \max\left\{H(\varepsilon) - C_h - (1+\tau_h)P(\varepsilon) - B_o, 0\right\} \mathrm{d}G_o(\varepsilon) - \rho B_o + \dot{B}_o\,. \tag{4}$$

Buyers make viewings of properties at rate q_o, which reveal match quality ε drawn from a distribution $G_o(\varepsilon)$. The value of being an owner-occupier of a property where match quality is currently ε is $H(\varepsilon)$. After meeting a seller, the home-buyer negotiates a price $P(\varepsilon)$ if a deal is mutually beneficial and moves into the property. This occurs when match quality ε is sufficiently high. Home-buyers incur a flow search cost F_h while looking for properties. If a transaction goes ahead, $\tau_h P(\varepsilon)$ is the tax paid by the home-buyer, and C_h is other transaction costs such as moving costs. Home-buyers, like any other household, exogenously exit the city at rate ρ.

Since properties are ex ante identical, both owner-occupiers and landlords selling their properties have a common expected value U_o, which satisfies the Bellman equation

$$rU_o = -M + \theta_o q_o \Bigg((1-\xi) \int \max\{P(\varepsilon) - C_u - U_o, 0\} \, \mathrm{d}G_o(\varepsilon) + \xi \max\{P_k - C_u - U_o, 0\} \Bigg) + \dot{U}_o\,, \quad (5)$$

where M is the flow cost of maintaining a property paid by all owners and C_u is a transaction cost paid by sellers. Meeting with buyers occur at rate $\theta_o q_o$, and the probabilities the meeting is with a home-buyer or an investor are equal to the respective fractions $1-\xi$ and ξ of the pool of buyers made up of these two groups. The owner decides whether to sell, receiving price P_k if selling to an investor and $P(\varepsilon)$ if selling to a home-buyer with match quality ε.

The Bellman equation for the value $H(\varepsilon)$ of an owner-occupier with current match quality ε is

$$rH(\varepsilon) = \varepsilon - M + a_o\left(\max\{H(\delta_o\varepsilon), B_o + U_o\} - H(\varepsilon)\right) + \rho(U_o - H(\varepsilon)) + \dot{H}(\varepsilon)\,, \quad (6)$$

where ε is the flow utility derived from occupying a property when match quality is currently ε. Idiosyncratic shocks arrive at rate a_o, reducing match quality to $\delta_o\varepsilon$. The household then decides whether to remain in the property and receive value $H(\delta_o\varepsilon)$, or to move out and become both a seller and a home-buyer, which has combined value $B_o + U_o$.[23] Moving occurs if match quality $\delta_o\varepsilon$ after the shock has become sufficiently low.

4.2 The rental market

Participants on both sides of the rental market — potential tenants and landlords — are ex ante identical. When a household meets a landlord and views the property, match quality ε is drawn from distribution $G_l(\varepsilon)$. If mutually agreeable, the household moves in and becomes a tenant. There is no commitment and no long-term contract: either the tenant or the landlord can end the relationship at any subsequent time. Rents are determined by ongoing negotiation between the two parties.

The Bellman equation for the value U_l of a landlord having a property available to let is

$$rU_l = -M + \theta_l q_l \int \max\{L(\varepsilon) + \Pi(\varepsilon) - C_l - U_l, 0\} \mathrm{d}G_l(\varepsilon) + \rho_l(U_o - U_l) + \dot{U}_l\,. \quad (7)$$

The landlord meets potential tenants at rate $\theta_l q_l$. If a tenant with match quality ε moves in, the landlord incurs costs C_l and receives value $L(\varepsilon)$, which includes the ongoing rents that are negotiated. At the point of agreeing the tenant can move in, there is also negotiation over an initial one-off fee $\Pi(\varepsilon)$ paid by the tenant to the landlord. At any point in time, landlords exogenously exit at rate ρ_l and must sell their property, receiving value U_o.

[23] Homeowners cannot become landlords after deciding to move, implicitly because there is a sufficiently large credit cost of having two mortgages to retain ownership of their existing property as well as buying a new one.

The value of landlord whose property is currently occupied by a tenant with match quality ε is $L(\varepsilon)$. The Bellman equation for this value function is

$$rL(\varepsilon) = R(\varepsilon) - M - M_l + (a_l + \rho)\left(\max\{U_l, U_o\} - L(\varepsilon)\right) + \rho_l(U_o - L(\varepsilon)) + \dot{L}(\varepsilon)\,, \tag{8}$$

where $R(\varepsilon)$ is the rent negotiated between landlord and tenant, and M_l is an extra maintenance cost incurred by landlords when properties are let. Idiosyncratic shocks received by tenants cause them to move out of rental properties at rate $a_o + \rho$, either because match quality is reduced to zero or because the household must leave the city. After a tenant moves out, the landlord decides whether to look for another tenant or sell the property, thus receiving the maximum of U_l and U_o.

The value B_l of a household searching for a property to rent satisfies the Bellman equation

$$rB_l = -F_w + q_l \int \max\{W(\varepsilon) - \Pi(\varepsilon) - C_w - B_l, 0\}\, \mathrm{d}G_l(\varepsilon) - \rho B_l + \dot{B}_l\,, \tag{9}$$

where q_l is the rate at which properties are viewed, and F_w is the flow cost of searching for a rental property. If the household moves into a property with match quality ε as a tenant then value $W(\varepsilon)$ is received after paying the initial fee $\Pi(\varepsilon)$ to the landlord and incurring costs C_w. The Bellman equation for the value function $W(\varepsilon)$ is

$$\begin{aligned} rW(\varepsilon) = \varepsilon - R(\varepsilon) &+ \gamma(a_l + \rho_l)\left(G_m(Z)(B_o - \bar{\chi}) + (1 - G_m(Z))B_l - W(\varepsilon)\right) \\ &+ (1-\gamma)(a_l + \rho_l)(B_l - W(\varepsilon)) - \rho W(\varepsilon) + \dot{W}(\varepsilon)\,, \quad \text{with} \quad \bar{\chi} = E[\chi | \chi \leq Z]\,. \end{aligned} \tag{10}$$

The flow utility ε derived from occupying a rental property is the same as for an owner-occupied property with the same match quality ε, but the tenant pays rent $R(\varepsilon)$. Rent negotiations ensure the landlord and tenant are willing to remain matched until a shock makes it mutually agreeable to terminate the tenancy. Households, including tenants, receive exit shocks at rate ρ. Exit shocks for landlords with arrival rate ρ_l, or shocks that reduce tenants' match quality to zero with arrival rate a_l, also bring a tenancy to an end. In these cases, the tenant does not redraw the credit cost χ with probability $1-\gamma$, which means the household goes back to the rental market and obtains value B_l.

When a new value of the credit cost χ is drawn, either for tenants who move (with probability γ) or new entrants to the city, there is threshold Z for χ below which it is optimal to enter the ownership market and buy rather than rent. This has value B_o after paying the cost χ. If the cost is too high, the household goes to the rental market. Before the cost is drawn, the expected value is an average of $B_o - \bar{\chi}$ and B_l using the probabilities $G_m(Z)$ and $1 - G_m(Z)$ as weights, where $\bar{\chi}$ denotes the expected value of the cost χ conditional on actually paying it.

4.3 Stocks and flows into, across, and within the two markets

A property is in any one of four states: for sale (measure u_o), to let (measure u_l), or occupied by an owner or a renter ('occupying' in the sense that the property is currently neither available in the market for sale or rent). Owner-occupied and renter-occupied properties have measures h_o and h_l, respectively. These measures of the four states must sum to the unit measure of all properties:

$$h_o + h_l + u_o + u_l = 1\,. \tag{11}$$

Similarly, the total measure ψ of households is distributed over four possible states: home-buyers (b_h), those looking for a property to rent (b_l), owner-occupiers (h_o), and tenants (h_l). Although households can own multiple properties, a household occupies at most one property at a time, and households look either to buy or rent if and only if they do not currently occupy a property. Hence:

$$h_o + h_l + b_h + b_l = \psi\,. \tag{12}$$

The measure of buyers $b_o = b_h + b_k$ in the ownership market also includes a measure b_k of those looking to buy as investors. Given free entry of investors, b_k adjusts so that at all points in time the value of entry by further investors is zero:

$$K = 0\,. \tag{13}$$

The entry of households to the ownership market as home-buyers for the first time depends on the threshold Z for the credit cost χ. The marginal new entrant is indifferent between the two markets:

$$Z = B_o - B_l\,. \tag{14}$$

Credit costs are drawn from the distribution $G_m(\chi)$ by a fraction γ of tenants who move within the city because of shocks (either to their own match quality or landlord exit) and all households new to the city. If N_l denotes the flow of tenants who decide to move ($n_l = N_l/h_l$ is the moving rate for tenants h_l), γN_l redraw their credit cost χ. Of those, a fraction $G_m(Z)$ are below the threshold Z and so enter the ownership market as home-buyers. The same applies to the measure $\rho\psi$ of households who enter the city. The flow of first-time home-buyers is therefore $\Theta = (\gamma n_l h_l + \rho\psi)G_m(Z)$.

Those tenants not drawing a new credit cost when moving (probability $1-\gamma$), or those whose new credit cost is above Z (probability $1 - G_m(Z)$), search in the rental market for a new property. The flow of owner-occupiers who decide to move is N_o (the moving rate of those in h_o is $n_o = N_o/h_o$), and all enter the ownership market as home-buyers because they have paid the credit cost before.

Home-buyers b_h and households h_l searching for a rental property exit from this state by either completing a transaction or exiting the city. Viewings occur at rates q_i in the two markets $i \in \{o, l\}$, and suppose the probabilities that the match quality revealed by a viewing is sufficiently high for a mutually agreeable deal with the seller/landlord are π_o and π_l in the two markets. The flows of sales to home-buyers and leases agreed with tenants are

$$S_h = q_o \pi_o b_h \quad \text{and} \quad S_l = q_l \pi_l b_l \,. \tag{15}$$

The laws of motion for the stocks of home-buyers b_h and households looking to rent b_l are thus

$$\dot{b}_h = n_o h_o + (\gamma n_l h_l + \rho \psi) G_m(Z) - (q_o \pi_o + \rho) b_h \,, \quad \text{and} \tag{16}$$

$$\dot{b}_l = (1-\gamma) n_l h_l + (\gamma n_l h_l + \rho \psi)(1 - G_m(Z)) - (q_l \pi_l + \rho) b_l \,. \tag{17}$$

Investors b_k make viewings at rate q_o and are able to transact at this rate because they have no idiosyncratic match quality with properties. The flow of sales to investors is S_k, which added to S_h gives total transactions S_o in the ownership market. Let κ denote the fraction of sales to investors:

$$S_o = S_h + S_k \,, \quad \text{where} \quad S_k = q_o b_k \,, \quad \text{and} \quad \kappa = \frac{S_k}{S_o} = \frac{\xi}{\xi + (1-\xi)\pi_o} \,, \tag{18}$$

where the equation for κ in terms of the fraction of investors ξ follows from (1) and (15). The flow of revenue from the land transfer tax is $\tau_h S_h + \tau_k S_k$.

From the perspectives of sellers and landlords, the transaction rates in the two markets are

$$s_o = \frac{S_o}{u_o} = \theta_o q_o \left(\xi + (1-\xi)\pi_o\right) \,, \quad \text{and} \quad s_l = \frac{S_l}{u_l} = \theta_l q_l \pi_l \,, \tag{19}$$

and hence the laws of motion for properties for sale u_o and to let u_l are:

$$\dot{u}_o = (n_o + \rho) h_o + \rho_l (h_l + u_l) - s_o u_o \,, \quad \text{and} \tag{20}$$

$$\dot{u}_l = (a_l + \rho) h_l + \kappa s_o u_o - (s_l + \rho_l) u_l \,. \tag{21}$$

Properties come up for sale if owner-occupiers move within or exit the city, or landlords are hit by a liquidity shock (irrespective of whether their properties are currently occupied by tenants). Properties are offered to let if tenants are hit by a match quality shock or exit the city, or investor purchases make new rental properties available. Properties come off these markets with successful transactions, or in the case of the rental market, if landlords must exit because of a liquidity shock.

Finally, flows of properties onto and off the two markets imply the following laws of motion for the stocks of owner-occupiers h_o and tenants h_l:

$$\dot{h}_o = (1-\kappa) s_o u_o - (n_o + \rho) h_o \,, \quad \text{and} \tag{22}$$

$$\dot{h}_l = s_l u_l - (n_l + \rho) h_l \,. \tag{23}$$

The flows and stocks in the ownership and rental markets are summarized in Figure 1 for households and Figure 2 for properties.

Figure 1: *Flows and stocks for households*

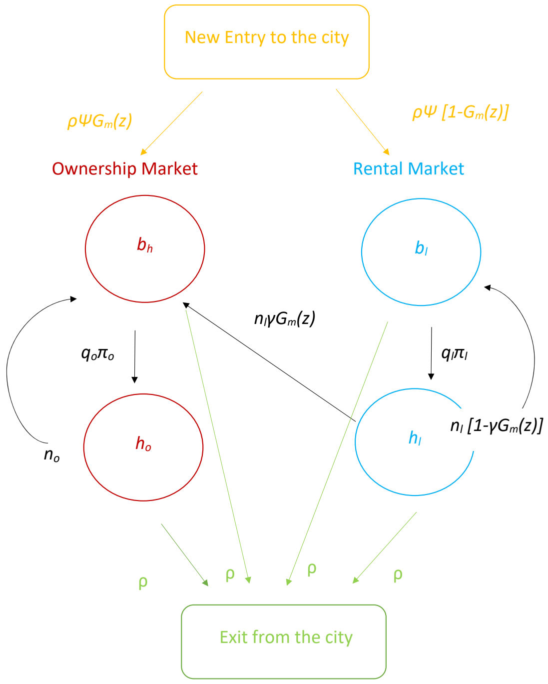
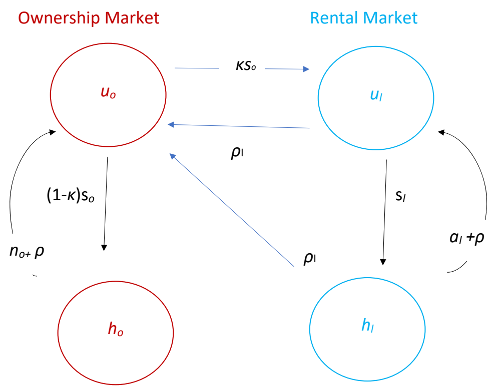


Figure 2: *Flows and stocks for properties*



4.4 Functional forms, parameter restrictions, and bargaining protocols

The meeting functions $\Upsilon^i(b_i, u_i)$ for $i \in \{o, l\}$ have Cobb-Douglas functional forms:

$$\Upsilon^i(b_i, u_i) = A_i b_i^{1-\eta_i} u_i^{\eta_i}, \quad \text{hence} \quad q_i = A_i \theta_i^{-\eta_i}, \tag{24}$$

where A_i is the efficiency with which viewings occur in market i, and η_i are the elasticities of buyers' and renters' viewing rates with respect to the market tightnesses θ_i (see 1 and 2). These parameters can differ across markets. New match qualities ε are drawn from Pareto distributions for $i \in \{o, l\}$:

$$G_i(\varepsilon) = 1 - \left(\frac{\varepsilon}{\zeta_i}\right)^{-\lambda_i}, \tag{25}$$

with ζ_i being the minimum possible draw in market i, and λ_i specifies the shape of the distribution, in particular, how compressed realizations of ε are towards the minimum. Expected match quality from a viewing in market i is $E_i[\varepsilon] = \zeta_i \lambda_i / (\lambda_i - 1)$ for $\lambda_i > 1$. Draws of the credit cost χ of homeownership are from a log Normal distribution with mean and standard deviation parameters μ and σ:

$$G_m(\chi) = \Phi\left(\frac{\log \chi - \mu}{\sigma}\right), \quad \text{implying} \quad \bar{\chi} = e^{\mu + \frac{\sigma^2}{2}} \frac{\Phi\left(\frac{\log Z - \mu - \sigma^2}{\sigma}\right)}{\Phi\left(\frac{\log Z - \mu}{\sigma}\right)}, \tag{26}$$

where $\Phi(\cdot)$ is the standard Normal CDF, and $\bar{\chi}$ is the expectation of χ conditional on $\chi \leq Z$.

A parameter restriction is imposed so that match-quality shocks in the ownership market are sufficiently large (δ_o is far enough below 1) that some, but not all, owner-occupiers require only one idiosyncratic shock to trigger moving. As has been stated, match-quality shocks to renters are sufficiently large ($\delta_l = 0$) that all tenant moves are exogenous.

The bargaining protocol in all meetings between agents over the terms of transactions (prices and rents) is Nash bargaining. Sellers (whoever they may be) have bargaining power ω_o when selling to a home-buyer and bargaining power ω_k when selling to an investor. Landlords have bargaining power ω_l in relation to tenants in both their initial meeting and in any subsequent rent negotiations.

4.5 Equilibrium in both housing markets

This section studies the equilibrium of the model for the allocation of properties and households across the two markets, and volumes of transactions and their terms (prices and rents) within each.

4.5.1 Decisions made by homeowners and home-buyers

Suppose the seller of a property meets a home-buyer who draws match quality ε. If they were to agree to a sale at price $P(\varepsilon)$ then the home-buyer surplus would be $\Sigma_o^h(\varepsilon) = H(\varepsilon) - (1 + \tau_h)P(\varepsilon) -$

$C_h - B_o$ and the seller surplus $\Sigma_o^u(\varepsilon) = P(\varepsilon) - C_u - U_o$. The Nash bargaining problem is to choose $P(\varepsilon)$ to maximize $(\Sigma_o^u(\varepsilon))^{\omega_o}\left(\Sigma_o^h(\varepsilon)\right)^{1-\omega_o}$, where the surpluses of both must be non-negative for a transaction to go ahead. The first-order condition is $\Sigma_o^u(\varepsilon)/\Sigma_o^h(\varepsilon) = \omega_o/((1-\omega_o)(1+\tau_h))$, which determines how the joint surplus $\Sigma_o(\varepsilon) = \Sigma_o^h(\varepsilon) + \Sigma_o^u(\varepsilon)$ is to be shared.

In the absence of the transaction tax τ_h, the surplus would have been divided according to bargaining powers in line with the usual Nash rule. However, a positive transaction tax skews the division in favour of the buyer. Intuitively, the joint surplus $\Sigma_o(\varepsilon) = H(\varepsilon) - C_h - C_u - B_o - U_o - \tau_h P(\varepsilon)$ is increased by agreeing a lower price owing to the proportional tax, and a lower price increases the buyer's surplus. The resulting split is

$$\Sigma_o^h(\varepsilon) = (1-\omega_o^*)\Sigma_o(\varepsilon) \quad \text{and} \quad \Sigma_o^u(\varepsilon) = \omega_o^*\Sigma_o(\varepsilon), \quad \text{where} \quad \omega_o^* \equiv \frac{\omega_o}{1+\tau_h(1-\omega_o)}, \tag{27}$$

and the seller's share ω_o^* of the surplus is below her bargaining power ω_o. The price that delivers the division of the surplus in (27) is $P(\varepsilon) = C_u + U_o + \omega_o^*\Sigma_o(\varepsilon)$, which results in the joint surplus being

$$\Sigma_o(\varepsilon) = \frac{H(\varepsilon) - C_h - B_o - (1+\tau_h)(C_u + U_o)}{1+\tau_h\omega_o^*}. \tag{28}$$

As match quality ε is observable and surplus is transferable, transactions go ahead if $\varepsilon \geq y_o$, where y_o, the transaction threshold, is the level of match quality where the joint surplus is zero:

$$\Sigma_o(y_o) = 0. \tag{29}$$

Using (25), the proportion π_o of home-buyer viewings that lead to sales and the average transaction price P for home-buyer purchases are therefore

$$\pi_o = \int_{y_o} \mathrm{d}G_o(\varepsilon) = \left(\frac{y_o}{\zeta_o}\right)^{-\lambda_i}, \quad \text{and} \quad P = \frac{1}{\pi_o}\int_{y_o} P(\varepsilon)\mathrm{d}G_o(\varepsilon) = \frac{\omega_o^*\Sigma_o}{\pi_o} + C_u + U_o. \tag{30}$$

Prior to the realization of ε, the ex-ante joint surplus from a home-buyer viewing is denoted by:

$$\Sigma_o = \int_{y_o} \Sigma_o(\varepsilon)\mathrm{d}G_o(\varepsilon). \tag{31}$$

For existing owner-occupiers, there is a moving decision to be made when a match quality shock is received. Since the value function $H(\varepsilon)$ is increasing in ε, owner-occupiers decide to move the current level of match quality satisfies $\varepsilon < x_o$, where x_o, the moving threshold is the level of match quality such that the value of continuing to occupy the property equals the sum of the outside options B_o and U_o of being both a buyer and a seller in the ownership market:

$$H(x_o) = B_o + U_o. \tag{32}$$

The condition that some owner-occupiers require only one shock to trigger moving is $\delta_o y_o < x_o$.

The endogenous moving rate n_o is derived from the distribution of match quality over existing owner-occupiers together with the moving threshold x_o. The evolution over time of the distribution of owner-occupiers' match quality depends on idiosyncratic shocks and moving decisions. Surviving matches of households and properties differ along two dimensions: the initial level of match quality, and the number of shocks received since the match formed. By using the Pareto distribution (25) of new match quality, the appendix shows that the endogenous moving rate is

$$n_o = a_o - \frac{a_o \delta_o^{\lambda_o} x_o^{-\lambda_o}}{h_o} \int_{\upsilon \to -\infty}^{t} e^{-\left(\rho + a_o(1-\delta_o^{\lambda_o})\right)(t-\upsilon)} (1-\xi(\upsilon))\theta_o(\upsilon) q_o(\upsilon) u_o(\upsilon) \mathrm{d}\upsilon \,, \tag{33}$$

where $u_o(t)$ explicitly indicates that u_o depends on time t. Given the moving threshold x_o, the moving rate n_o displays history dependence due to persistence in the match-quality distribution.

4.5.2 Decisions made by landlords and tenants

For landlords and tenants, it is necessary to work backwards from ongoing rent negotiations to analyse their behaviour when they first meet during a viewing. Consider a tenant who has already moved into a property with match quality ε, so any transaction and moving costs are sunk. The tenant's surplus from remaining in the property is $\Lambda^w(\varepsilon) = W(\varepsilon) - B_l$, where the outside option is going back to the rental market because the tenant's cost χ of becoming a home-buyer does not change unless a shock occurs. The landlord's surplus from keeping the tenant is $\Lambda^l(\varepsilon) = L(\varepsilon) - U_l$, which assumes the outside option of putting the property back on the rental market is better than selling it ($U_l \geq U_o$), as will be confirmed. Both $W(\varepsilon)$ and $L(\varepsilon)$ depend on the rent $R(\varepsilon)$ paid.

The Nash bargaining problem has $R(\varepsilon)$ maximize $\left(\Lambda^l(\varepsilon)\right)^{\omega_l} \left(\Lambda^w(\varepsilon)\right)^{1-\omega_l}$, where ω_l is the landlord's bargaining power. There is no commitment to rent payments at any future date. The rent $R(\varepsilon)$ affects the surpluses through $L(\varepsilon)$ and $W(\varepsilon)$ in equations (8) and (10), noting that $\partial L(\varepsilon)/\partial R(\varepsilon) = -\partial W(\varepsilon)/\partial R(\varepsilon)$, so the first-order condition is $\Lambda^l(\varepsilon)/\Lambda^w(\varepsilon) = \omega_l/(1-\omega_l)$. The joint surplus $\Lambda(\varepsilon) = \Lambda^w(\varepsilon) + \Lambda^l(\varepsilon) = W(\varepsilon) + L(\varepsilon) - B_l - U_l$ is therefore divided according to the bargaining powers of the two parties as $\Lambda^l(\varepsilon) = \omega_l \Lambda(\varepsilon)$ and $\Lambda^w(\varepsilon) = (1-\omega_l)\Lambda(\varepsilon)$.

With rents negotiated in this way, tenants move out only after a match quality shock or if leaving the city, or if the landlord is forced to sell up. Tenants' moving rate n_l within the city is simply

$$n_l = a_l + \rho_l \,. \tag{34}$$

Now consider a landlord meeting a potential tenant during a viewing that reveals match quality ε. If the landlord agrees the tenant can move in after paying a fee $\Pi(\varepsilon)$ then the two parties incur costs C_l and C_w, respectively.[24] Note that the fee $\Pi(\varepsilon)$ is separate from the rent $R(\varepsilon)$, which is the

[24]The transaction costs C_w and C_l are a type of fixed matching cost, for example, the costs of finding out about the

subject of ongoing negotiation once the tenant moves in. The tenant's surplus is $\Sigma_l^w(\varepsilon) = W(\varepsilon) - \Pi(\varepsilon) - C_w - B_l$ and the landlord's surplus is $\Sigma_l^l(\varepsilon) = L(\varepsilon) + \Pi(\varepsilon) - C_l - U_l$.

If it is mutually agreeable for the tenant to move in then there is Nash bargaining over the fee $\Pi(\varepsilon)$ with the same bargaining powers ω_l and $1-\omega_l$ of landlord and tenant that apply in rent negotiations. The joint surplus $\Sigma_l(\varepsilon) = \Sigma_l^w(\varepsilon) + \Sigma_l^l(\varepsilon)$ is given by

$$\Sigma_l(\varepsilon) = W(\varepsilon) + L(\varepsilon) - B_l - U_l - C_w - C_l\,, \tag{35}$$

which is thus split according to $\Sigma_l^l(\varepsilon) = \omega_l \Sigma_l(\varepsilon)$ and $\Sigma_l^w(\varepsilon) = (1-\omega_l)\Sigma_l(\varepsilon)$. In terms of the surpluses $\Lambda^l(\varepsilon)$ and $\Lambda^w(\varepsilon)$ once the tenant has moved in, the surpluses on meeting can be expressed as $\Sigma_l^l(\varepsilon) = \Lambda^l(\varepsilon) + \Pi(\varepsilon) - C_l$ and $\Sigma_l^w(\varepsilon) = \Lambda^w(\varepsilon) - \Pi(\varepsilon) - C_w$. Since the bargaining problem for new rents is the same as for ongoing rents, $\Lambda^l(\varepsilon) = \omega_l\Lambda(\varepsilon)$ and $\Lambda^w(\varepsilon) = (1-\omega_l)\Lambda(\varepsilon)$, where $\Sigma_l(\varepsilon) = \Lambda(\varepsilon) - C_l - C_w$, it follows that Nash bargaining over the fee $\Pi(\varepsilon)$ gives

$$\Pi(\varepsilon) = \Pi = (1-\omega_l)C_l - \omega_l C_w\,, \tag{36}$$

which is independent of match quality ε. A lease is agreed if $\varepsilon \geq y_l$, where y_l, the leasing threshold, is the level of match quality ε where the joint surplus $\Sigma_l(\varepsilon)$ from (35) is zero:

$$\Sigma_l(y_l) = 0\,. \tag{37}$$

The proportion π_l of viewings of properties to let that lead to leases and the average rent R are

$$\pi_l = \int_{y_l} \mathrm{d}G_l(\varepsilon) = \left(\frac{y_l}{\zeta_l}\right)^{-\lambda_l}\,, \quad \text{and} \quad R = \frac{1}{\pi_l}\int_{y_l} R(\varepsilon)\mathrm{d}G_l(\varepsilon)\,. \tag{38}$$

Prior to the realization of ε, the ex-ante expected joint surplus from a rental-market viewing is

$$\Sigma_l \equiv \int_{y_l} \Sigma_l(\varepsilon)\mathrm{d}G_l(\varepsilon)\,. \tag{39}$$

4.5.3 The entry of investors

When an investor views a property to buy, the investor's surplus from a transaction at price P_k is $\Sigma_k^k = U_l - (1+\tau_k)P_k - C_k - K$ and the seller's surplus is $\Sigma_k^u = P_k - C_u - U_o$. If there are mutual gains from a deal, the price P_k is determined by Nash bargaining, where the seller has bargaining power ω_k when faced with an investor. The joint surplus $\Sigma_k = \Sigma_k^k + \Sigma_k^u$ is split according to $\Sigma_k^u/\Sigma_k^k = \omega_k/((1-\omega_k)(1+\tau_k))$, so the tax τ_k shifts the division of the surplus in favour of the investor:

$$\Sigma_k^k = (1-\omega_k^*)\Sigma_k \quad \text{and} \quad \Sigma_k^u = \omega_k^*\Sigma_k\,, \quad \text{where} \quad \omega_k^* \equiv \frac{\omega_k}{1+\tau_k(1-\omega_k)}\,. \tag{40}$$

tenant, as they are incurred before bargaining over the rent takes place (see Pissarides, 2009).

Since the joint surplus $\Sigma_k = U_l - C_k - C_u - U_o - K - \tau_k P_k$ is unaffected by considerations of match quality, either all investors are willing to buy or none, so an equilibrium with entry of investors requires non-negative Σ_k. When this is true, investors buy property at the rate q_o they meet sellers, and the price paid by all investors is

$$P_k = C_u + U_o + \omega_k^* \Sigma_k \,. \tag{41}$$

With this price, the joint surplus Σ_k from a meeting between an investor and a seller is

$$\Sigma_k = \frac{U_l - (1+\tau_k)U_o - (1+\tau_k)C_u - C_k}{1+\tau_k\omega_k^*} \,. \tag{42}$$

Note that a non-negative joint surplus Σ_k implies the value of having a property to let is always above the value of having a property for sale ($U_l \geq U_o$). Thus, after purchasing a property, an investor always prefers to keep it let out in the rental market.[25] Landlords sell properties only when hit by a liquidity shock, which arrive at rate ρ_l.

Given the free-entry condition (13), the Bellman equation (3) for investors' value K requires

$$\Sigma_k = \frac{F_k}{(1-\omega_k^*)q_o} \,, \tag{43}$$

which shows the surplus Σ_k rises with the tightness of the ownership market. Intuitively, the viewing rate q_o decreases when there are more buyers relative to sellers, so investors must be compensated by a higher surplus $(1-\omega_k^*)\Sigma_k$ for them to enter in equilibrium.

4.6 Welfare

Let Ω denote the present value of all flows of utility net of costs summed over all participants (households and investors) in the two housing markets. This is given by the differential equation

$$r\Omega = h_o Q_h + h_l Q_l - M - h_l M_l - b_h F_h - b_k F_k - b_l F_w - S_o((1-\kappa)C_h + \kappa C_k + C_u) \\ - S_l(C_l + C_w) - (\gamma n_l h_l + \rho\psi) G_m(Z)\bar{\chi} + \dot{\Omega} \,, \tag{44}$$

where Q_h and Q_l denote the average levels of current match quality ε across the h_o owner-occupiers and the h_l tenants. Prices and rents drop out from welfare Ω because these are just transfers among market participants. Maintenance costs, flow search costs, non-tax transaction costs, and credit costs are treated as resource costs that are deducted from welfare. This assumes that transaction costs reflect time and resources of market participants and intermediaries consumed in completing transactions. Likewise, credit costs, for example, interest-rate spreads on mortgages, are treated as

[25] In other words, pure 'flippers' — those who buy and sell shortly after — are not present in the model.

reflecting resources used up by banks. Tax revenue from the land transfer tax is not deducted because it is assumed this pays for public goods of equivalent value, or alternatively, that a given amount of public expenditure (of whatever resource cost and utility value) would otherwise need to be funded by alternative taxes on households.

The welfare measure (44) can be calculated using the existing equations augmented by a pair of differential equations for the average match qualities Q_h and Q_l:

$$\dot{Q}_h = \frac{(1-\kappa)s_o u_o}{h_o}\left(\frac{\lambda_o}{\lambda_o - 1}y_o - Q_h\right) - (a_o - n_o)\left(Q_h - \frac{\lambda_o}{\lambda_o - 1}x_o\right), \quad \text{and} \tag{45}$$

$$\dot{Q}_l = \frac{s_l u_l}{h_l}\left(\frac{\lambda_l}{\lambda_l - 1}y_l - Q_l\right), \tag{46}$$

which depend on differences between Q_h and Q_l and average new match qualities $\lambda_o y_o/(\lambda_o - 1)$ and $\lambda_l y_l/(\lambda_l - 1)$, and between Q_h and average surviving match quality $\lambda_o x_o/(\lambda_o - 1)$ after match-quality shocks faced by owner-occupiers.

4.7 Implications of the model in steady state

For constant tax rates τ_h and τ_k and other parameters, the model predicts the two housing markets converge to a steady state where the fractions of properties and households in various states $(h_o, h_l, u_o, u_l, b_h, b_k)$ are constant over time. This steady state also features a constant measure of investors b_k and the proportion ξ of buyers they account for, and constant market tightnesses θ_o and θ_l. The homeownership rate h is defined as the fraction of the population ψ who own a property they occupy h_o or are selling a property they occupied:

$$h = \frac{h_o + (1-\kappa)u_o}{\psi}, \tag{47}$$

where homeowners selling properties account for a fraction $1-\kappa$ of properties u_o on the market. The model also has implications for the demographics of owner-occupiers compared to tenants. The appendix shows that the average age difference α between the two groups is

$$\alpha = \left(1 + \frac{\rho}{\rho + n_l + q_l \pi_l}\right)\left(\frac{1}{\rho} - \frac{1}{\rho + \frac{\gamma n_l q_l \pi_l}{\rho + n_l + q_l \pi_l}}\right). \tag{48}$$

Among those occupying properties, there is a stationary distribution of match quality, which implies a constant moving rate n_o in (33) for owner-occupiers. The appendix shows that

$$n_o = a_o \left(\frac{\rho + a_o\left(1 - \delta_o^{\lambda_o}\right) - \rho \delta_o^{\lambda_o}\left(\frac{y_o}{x_o}\right)^{\lambda_o}}{\rho + a_o\left(1 - \delta_o^{\lambda_o}\right) + a_o \delta_o^{\lambda_o}\left(\frac{y_o}{x_o}\right)^{\lambda_o}}\right), \tag{49}$$

which is the steady-state moving rate within the city. Taking account of exit from the city, the expected lengths of occupation of a property by homeowners and tenants are

$$T_{mo} = \frac{1}{n_o + \rho} \quad \text{and} \quad T_{ml} = \frac{1}{n_l + \rho}, \tag{50}$$

where the moving rate within the city for tenants is from (34).

The average number of viewings v_o and v_l needed to sell or lease a property are respectively

$$v_o = \frac{1}{(1-\xi)\pi_o + \xi} \quad \text{and} \quad v_l = \frac{1}{\pi_l}, \tag{51}$$

and the expected times on the market for properties to sell T_{so} and lease T_{s_l}, and the expected times T_{bh} and T_{bl} for home-buyers and potential tenants successfully to find properties are

$$T_{so} = \frac{1}{s_o} \quad , \quad T_{sl} = \frac{1}{s_l} \quad \text{and} \quad T_{bh} = \frac{1}{q_o \pi_o} \quad , \quad T_{bl} = \frac{1}{q_l \pi_l}. \tag{52}$$

On average across buyers in the ownership market, the average time to complete a transaction is

$$T_{bo} = (1-\kappa)T_{bh} + \kappa T_{bk} = \frac{1}{q_o(\xi + (1-\xi)\pi_o)}, \quad \text{where} \quad T_{bk} = \frac{1}{q_o}. \tag{53}$$

Among purchases by home-buyers, the appendix shows the fraction ϕ of first-time buyers is

$$\phi = \frac{\rho\left(1 + \frac{n_o + \rho}{q_o \pi_o}\right)}{n_o + \rho\left(1 + \frac{n_o + \rho}{q_o \pi_o}\right)}. \tag{54}$$

These predictions are used to calibrate the model's parameters.

5 Quantitative results

In Toronto, both the homebuyers and buy-to-let investors face the same LTT when purchasing a property, thus τ_k is the same as τ_h before and after the LTT increase. Interestingly, as discussed in the empirical section 3, despite facing the same taxes and same increase, transactions rose for buy-to-let investors and fell for homebuyers. Such responses are tightly linked to the observed fall in price-to-rent ratio. The model illustrates the importance of allowing for the flows between the ownership and rental markets to understand these responses and we now turn to its quantitative implications.

We calibrate the model to match some key features of ownership and rental markets in the city of Toronto before the LTT change. The LTT for the city of Toronto was introduced in February 2008 so any transactions after such date have to pay the new Toronto tax in addition to the original provincial tax.[26] The effective LTT rate, measured by the mean of land transfer taxes relative to home sales prices across transactions, is 1.08% during the pre-policy period and 2.04% during the post-policy period for the city of Toronto, which implies an increase of 0.96%.

5.1 Calibration

The model is calibrated to the Toronto housing market before the LTT change, i.e. during the 2007–2008 period. The tax rate faced by both the home-buyer and buy-to-let investor are set to the effective LTT prior to the change, $\tau_k = \tau_h = 0.011$. The parameters of the model are calibrated to match a list of targets in Table 6 and the implied parameter values are reported in Table 7. The data source of all targets can be found in appendix A.6, and appendix A.7 provides the detailed calibration procedure. In summary, there are three broad sets of targets.

The first set of targets and parameters are set directly: $\left(\zeta_o, \psi, \omega_l, \frac{\omega_k}{\omega_o}, \frac{\omega_o}{\eta_o}, \frac{\omega_l}{\eta_l}\right)$. The minimum new match quality in the ownership market ζ_o is normalized to 1 and the measure of households is set to be the same as the measure of properties, $\psi = 1$. There is little information available for the bargaining power in the housing market. Our strategy is to assume equal bargaining power between landlord and tenant, $\omega_l = 0.5$. The bargaining power of the sellers is set to be the same whether he is facing a home-buyer or a buy-to-let investor, $\omega_k = \omega_o$. Finally the bargaining power of sellers (landlord) are set as the same as their corresponding elasticity in the matching functions of the two market, $\omega_o = \eta_o$ and $\omega_l = \eta_l$.

The second set of targets are set to match the search behavior and associated costs in the housing markets. The key targets for search behavior are viewings per sale v_o, viewings per lease v_l, time on the market for buyers and sellers T_{so}, T_{bo} in the ownership market and landlord's time on the rental market T_{sl}; and time-to-move for both homeowners and tenants T_{mo} and T_{ml}. The targets for costs in the ownership markets are maintenance cost for homeowner M, transaction costs excluding taxes for buyers and sellers (C_k, C_h, C_u), search flow costs of buyers F_k, F_h, all as a fraction of price. The target for costs in the rental markets include the extra maintenance costs M_l, transaction costs of landlords and renters (C_l, C_w) and search flow costs of tenant F_w, as a fraction of rent.

The last set of targets are related to the extensive margin across ownership and rental markets. They are the homeownership rate h, price-to-rent ratio P_k/R, the capitalized credit costs of marginal buyers relative to price Z/P, the fraction of first-time buyers ϕ, the difference in the average age of owner-occupiers and renters α, and the fraction of investors κ among total purchases. These targets put restriction on the parameters related to the exit rate from the city, the arrival rate of match quality shock in the two markets, the exit rate of the investors, the distribution of credit costs, the bargaining power of the seller, and the discount rate.

Finally, we choose the elasticity of the moving rate of owner-occupiers with respect to the tax

[26] See Appendix Table A1 for the rates. Dachis, Duranton and Turner (2012) provide a brief history of the introduction of LTT and argue that the LTT change was largely unanticipated.

Table 6. *Calibration targets*

Targets	Notation	Value
Directly imposed targets		
Ownership-market minimum new match quality normalization	ζ_o	1
Equal numbers of households and properties	ψ	1
Equal landlord and tenant bargaining powers	ω_l	0.5
Elasticities of matching functions equal to bargaining powers	$\omega_o/\eta_o = \omega_l/\eta_l$	1
Empirical targets		
Effective land transfer tax for buyers	$\tau_h = \tau_k$	0.011
Homeownership rate	h	0.54
Fraction of purchases by investors	κ	0.05
Fraction of first-time buyers	ϕ	0.4
Difference in average age of owner-occupiers and renters	α	8.3
Average price-rent ratio for same property	P_k/R	14.8
Price paid by investors relative to home-buyers	P_k/P	0.993
Capitalized credit costs of marginal buyer relative to price	Z/P	0.39
Ratio of credit costs of marginal and mean home-buyers	$Z/\bar{\chi}$	2.09
Transaction costs of buyers excluding tax relative to price	$C_h/P = C_k/P_k$	0
Maintenance cost relative to price	M/P	0.026
Landlords' extra maintenance/management costs relative to rent	M_l/R	0.08
Seller transaction costs relative to price	C_u/P	0.045
Landlord transaction costs relative to rent	C_l/R	0.083
Tenant transaction costs relative to rent	C_w/R	0.083
Flow search costs of home-buyers relative to price	F_h/P	0.040
Flow search costs of investors relative to home-buyers	F_k/F_h	1
Flow search costs of tenants relative to home-buyers	F_w/F_h	0.964
Sellers' time on the market	T_{so}	0.127
Buyers' time on the market	T_{bo}	0.162
Landlords' time on the rental market	T_{sl}	0.066
Viewings per sale	v_o	20.6
Viewings per lease	v_l	10.3
Average time between moves for owner-occupiers	T_{mo}	5.51
Average time between moves for tenants	T_{ml}	3.04
Elasticity of owner-occupier moving rate to tax change	β	10

Notes: See appendix A.6 for data sources and appendix A.7 for the calibration procedure.

increase, β, to match the decrease in transactions observed in the data.

5.2 Quantitative effects of transaction taxes

The effects of the rise in transaction taxes $\tau_h = \tau_k$ from 1.5% to 2.8% are reported in Table 8.

Table 7. *Calibrated parameters*

Parameter description	Notation	Value
Households per property	ψ	1
Discount rate	r	0.032
Households' exit rate from city	ρ	0.071
Investors' exit rate from city	ρ_l	0.011
Property maintenance cost	M	0.179
Landlords' extra maintenance/management costs	M_l	0.037
Minimum new match quality in ownership market	ζ_o	1
Minimum new match quality in rental market	ζ_l	0.741
Home-buyer new match quality distribution shape parameter	λ_o	26.1
Tenant new match quality distribution shape parameter	λ_l	42.0
Arrival rate of match quality shock in ownership market	a_o	0.154
Arrival rate of match quality shock in rental market	a_l	0.247
Match quality shock in ownership market	δ_o	0.876
Probability of drawing new credit cost after moving shock	γ	0.404
Mean of credit cost draw distribution	μ	1.64
Standard deviation of credit cost draw distribution	σ	1.48
Transaction costs of buyers excluding taxes	$C_k = C_h$	0
Transaction costs of sellers	C_u	0.310
Transaction costs of landlords	C_l	0.039
Transaction costs of tenants	C_w	0.039
Flow search costs of home-buyers and investors	$F_k = F_h$	0.274
Flow search costs in rental market	F_w	0.264
Meeting efficiency in ownership market	A_o	135.2
Meeting efficiency in rental market	A_l	195.7
Elasticity of ownership-market meetings w.r.t. sellers	η_o	0.250
Elasticity of rental-market meetings w.r.t. landlords	η_l	0.50
Bargaining power of sellers facing home-buyers	ω_o	0.250
Bargaining power of sellers facing investors	ω_k	0.240
Bargaining power of landlords in the rental market	ω_l	0.50

Notes: See appendix A.6 for data sources and appendix A.7 for the calibration procedure.

Consistent with the empirical estimates, the model predicts that BTO and BTR transactions go in different directions. Sales to home-buyers fall, while sales to investors rise, despite the two facing the same rise in tax rates. The model gets close quantitatively to matching the magnitudes of these changes in transactions.

There are three household behavioural responses to the higher LTT rates underlying the fall in BTO transactions. First, higher tax raises the cost of moving, which makes homeowners more tolerant of worse match quality (lower x_o). This is reflected in longer time-to-move for owner occupiers. Second, home-buyers become pickier (higher y_o). Because moving decisions are endogenous and

Table 8. *Simulations of the model following the increase in land transfer tax*

Variable	Model prediction	Data
Time-to-move for owner-occupiers	matched	13%
BTO transactions	−15%	−10%
BTR transactions	1.9%	3.2%
Time-to-sell	8.6%	28%
Lease-to-sales	15%	24%
Homeownership rate	−1.6% (−0.9 p.p.)	-
Price-to-rent ratio	−1.8%	−1.6%
BTO price	−1.9%	−2.0%
Tax revenue	46%	-
Effective tax rate	Increased 62% from 1.5% to 2.8%	

Notes: Percentage changes are log differences. The solution procedure to find the predictions of the model is described in appendix A.5. The effective tax rate is the mean tax rate computed using the actual LTT schedules before and after, but calculated for the same set of of properties that transacted between January 2007 and January 2008 in both cases.

match quality is persistent, home-buyers reduce the future incidence of moving — and thus reduce tax paid — by starting with better match quality. This results in longer time-to-sell. Finally, higher taxes reduce the joint surplus in the ownership market because part of the surplus is absorbed by the higher tax. This reduces renters' incentive to enter the ownership market.

Since investors face the same tax increase as home-buyers, the direct effect of the higher tax is to reduce entry of investors. However, there are two equilibrium effects at work. First, there is more demand for rental properties owing to households' reduced incentive to switch from renting to owning. Second, landlords do not have to sell their property and pay the LTT again when a renter moves, unlike a homeowners who have to buy again and pay the tax every time they move. This explains why the model predicts a rise in BTR transactions.

BTR transactions are a relatively small fraction of total transactions, so the overall effect is that transactions fall by 14%. Combined with the additional entry of investors, this means the ratio of leases to sales is higher. These changes also imply the homeownership rate falls by around one percentage point. Data on the homeownership rate is not available at the micro level and at high frequencies so the causal effect of the LTT change cannot be estimated. However, the empirical findings for BTR transactions and leases indicate that the homeownership rate would fall after the LTT increase, all else equal.[27]

In order for more investors to be willing to enter when faced with a higher tax, the price-to-

[27] Simply looking at the aggregate data on the homeownership rate in Toronto reveals a rising trend prior to the LTT increase and a flattening out afterwards. The period of stagnation in the homeownership rate coincides with a rising fraction of BTR transactions in the aggregate as seen in Figure 1.

rent ratio (calculated using the price paid by investors) must fall so that the free-entry condition is satisfied. The predicted change for this variable closely matches the empirical response.

The predicted price paid by home-buyers drops by 1.9%, matching the data almost exactly. Interestingly, the percentage change in price is larger than the percentage-point rise in the tax rate.[28] The impact on the average BTO price reflects the expectation that a given property will be subject to the tax each time it is sold, and thus the expected future incidence of tax is capitalized into the price.

Since sellers can sell to either home-buyers or investors they meet, the prices paid by the two groups are tightly linked, if not identical. Quantitatively, home-buyers are the dominant group of buyers, so the capitalization effect pushes down the BTR price. Hence, this is closely related to the earlier intuition for why more investors are willing to enter after the LTT increase.

The model predicts tax revenue rises by only 46% when the tax rate increases by 62%. The difference is explained by the erosion of the tax base: total transactions go down by 14%, and the average price drops by 2%, so the tax base shrinks by 16%.

Table 9. *Welfare effects of increase in land transfer tax predicted by model*

Variable	Result
Increase in deadweight loss as percentage of increase in tax revenue	79%
Loss as percentage of tax revenue attributable to across-market effects	25%
Loss as percentage of tax revenue attributable to within rental-market effects	5%
Loss as percentage of tax revenue attributable to within ownership-market effects	49%

5.3 Welfare effects of transactions taxes

The welfare costs of the LTT shown in Table 9 are substantial. Per dollar of tax revenue raised, the LTT generates a welfare loss equivalent to 79% of the increase in tax revenue. The welfare loss is due to distortions across and within the two housing markets, and both are substantial. The distortion across the two markets generate a loss equivalent to 25% of the extra tax revenue. Within the markets, distortions to the rental market and ownership market generate losses of 5% and 49% of tax revenue respectively. Overall, the rental market is associated with 30% of the total loss relative to extra tax revenue, which is around 40% of the total loss.

The welfare loss across the two market is due to the drop in homeownership rate given that average match quality is higher for owner-occupiers than renter-occupiers, and the increase in rental management costs. This loss is partly offset by the reduction in credit costs to become a homeowner.

[28] A simple analysis of tax incidence might suggest that prices should change by less than the tax rate rise because the buyer has bargaining power. See equation (27). The equation also shows the LTT as a proportional tax reduces the bargaining power of the seller, contributing to a lower price.

Within the ownership market, the loss is mainly due to the drop in match quality. The loss is partly offset by the reduction in search costs and non-tax transaction costs that are saved due to less moving taking place. The loss is much smaller within the rental market due to the small rise in rental activities. The source of welfare loss in the rental market is due to search and transaction costs.

The welfare loss we found is significantly larger than the 12.5% found by Dachis, Duranton and Turner (2012) for the Toronto LTT. We obtain a large welfare loss by explicitly modelling the rental market and the decision to own or to rent, and the decision to move within the ownership market.

6 Conclusions

Using a unique dataset on housing sales and leasing transactions, this paper documents two novel effects of a higher transaction tax. First, there is a rise in buy-to-let transactions and a fall in owner-occupier transactions despite the same tax applying to both. Second, there is a simultaneous fall in the price-to-rent ratio and in the sales-to-leases ratio. Both effects operate through the extensive margin across the ownership and rental market.

The paper builds a tractable model where households choose renting or owning where entry to the ownership market requires paying a cost of accessing credit. The higher transaction tax distorts allocation of properties across the two markets by reducing the homeownership rate and distorts the allocation within the ownership market by reducing mobility. The calibrated model implies a substantial welfare loss equivalent to 79% of the increase in tax revenue, with about 40% due to analysing the presence of a rental market.

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A Appendices

A.1 Data appendix

A.2 Kaplan-Meier Estimator for Moving

Figure 3: *Kaplan-Meier Estimator for Moving*

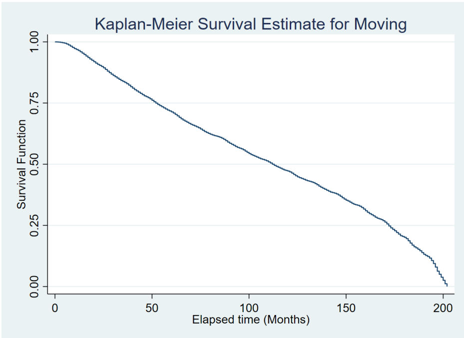

Figure 4: *Estimation Sample*

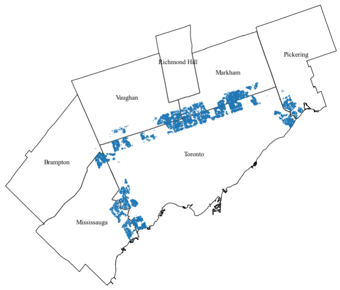

Table A1. *Land transfer taxes in Toronto*

City of Toronto LTT Tax Rate by Value ($) (Effective 1 February 2008)		Province of Ontario LTT Tax Rate by Value ($) (Effective 7 May 1997)	
0-55,000	0.5%	0-55,000	0.5%
55,000-400,000	1.0%	55,000-250,000	1.0%
400,000+	2.0%	250,000-400,000	1.5%
		400,000+	2.0%

Sources: Municipal Land Transfer Tax, City of Toronto, http://www.toronto.ca/taxes/mltt.htm; Provincial Land Transfer Tax, Historical Land Transfer Tax Rates, Province of Ontario. Reproduced from Dachis, Duranton and Turner (2012).

Note: For the Municipal LTT, exemptions are given to first-time homebuyers for the value of a purchase under $400,000 and for the provincial LTT exemptions are given to first-time home buyers for the value of a purchase under $227,500.

A.3 Additional robustness checks

Table A2. *Robustness Checks: Effects of the LTT on Moving Hazard*

Dependent Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)
2006-2012							
LTT	-0.0704	-0.130**	-0.188**	-0.151**	-0.194***	-0.232**	-0.112
	(0.0635)	(0.0657)	(0.0795)	(0.0507)	(0.0538)	(0.0865)	(0.0780)
2006-2010							
LTT	-0.0911	-0.156**	-0.176**	-0.178**	-0.218***	-0.243**	-0.147*
	(0.0720)	(0.0743)	(0.0892)	(0.0575)	(0.0634)	(0.110)	(0.0885)
2006-2018							
LTT	-0.0779	-0.125**	-0.169**	-0.162***	-0.179***	-0.213**	-0.0947
	(0.0584)	(0.0607)	(0.0737)	(0.0469)	(0.0481)	(0.0705)	(0.0719)
Distance threshold	3KM	3KM	3KM	5KM	5KM	5KM	2KM
House characteristics	YES	YES	YES	YES	YES	YES	YES
Indicators TO +-3 m.		YES	YES	YES	YES	YES	YES
Time trends TO		YES	YES	YES	YES	YES	YES
Distance LTT trends					Step	Step	
Donut Hole			1KM			2KM	

Notes: A unit of observation is a household whose property is listed on MLS every year and month. Transactions that take place within 18 months will be discarded. A survival model with risk specified as Weibull distribution on the effect of LTT is with an indicator for post-LTT period, Toronto*Property Type Fixed Effects, Month*Property Type Fixed Effects, Community*Property Type Fixed Effects. Indicators TO +- 3m are six dummy variables for transactions inside Toronto during the last 3 months of 2007 and the first 3 of 2008. Time trends TO indicates the presence of separate time trends for transactions in and outside of Toronto. Distance threshold is the maximum distance to the Toronto border for a transaction to be included. Distance LTT trend denotes the inclusion of an interaction term between LTT and distance to the Toronto border. Robust standard errors in parentheses. *, **, ***: corresponding coefficient significant at 10, 5, 1%.

Table A3. *Robustness Checks: Effects of the LTT on Sales Price (transaction level)*

2006-2010					
LTT	-0.0139**	-0.0172**	-0.0160**	-0.0140**	-0.0185**
	(0.00588)	(0.00625)	(0.00763)	(0.00595)	(0.00719)
Observations	8076	8076	5624	13550	5536
2006-2012					
LTT	-0.0165***	-0.0154**	-0.0103*	-0.0105**	-0.0223***
	(0.00477)	(0.00514)	(0.00610)	(0.00484)	(0.00598)
Observations	14702	14702	10088	24970	10161
2006-2018					
LTT	-0.0203***	-0.0196***	-0.0159**	-0.016**	-0.0244***
	(0.00427)	(0.00465)	(0.00547)	(0.008)	(0.00550)
Observations	32887	32887	22448	56756	22748
Distance threshold	3KM	3KM	3KM	5KM	2KM
Indicators TO +-3 m.		YES	YES	YES	YES
Distance LTT trends				Step	
Donut Hole			1KM		

Notes: A unit of observation is a transaction of single family houses listed on MLS, from January, 2006 to February, 2010/2012/2018. Each cell represents a separate regression of the logarithm of sales price (specified on the left column) on the LTT. All regressions are estimated with an indicator for post-LTT period, Toronto, Year Fixed Effects, Month Fixed Effects, Community Fixed Effects and Housing characteristics (heating, basement, family room, fireplace, # opf bedrooms/bathrooms/kitchens/rooms, lot size). Transactions take place within 18 months are dropped. Indicators TO +- 3m are six dummy variables for transactions inside Toronto during the last 3 months of 2007 and the first 3 of 2008. Time trends TO indicates the presence of separate time trends for transactions in and outside of Toronto. Distance threshold is the maximum distance to the Toronto border for a transaction to be included. Distance LTT trend denotes the inclusion of an interaction term between LTT and distance to the Toronto border. Robust standard errors in parentheses. *, **, ***: corresponding coefficient significant at 10, 5, 1%.

A.4 Deriving the equations of the model

A.4.1 Value functions and thresholds for homeowners and home-buyers

The value function $H(\varepsilon)$ from (6) is increasing in ε. Assuming $\delta_o y_o < x_o$ for all t, by taking ε in a neighbourhood above y_o or any value below, the Bellman equation (6) reduces to the following as $H(\delta_o\varepsilon) < B_o + U_o$:

$$rH(\varepsilon) = \varepsilon - M + a_o(B_o + U_o - H(\varepsilon)) + \rho(U_o - H(\varepsilon)) + \dot{H}(\varepsilon)\,.$$

This simplifies to

$$(r+\rho+a_o)H(\varepsilon) - \dot{H}(\varepsilon) = \varepsilon - M + a_o B_o + (\rho + a_o)U_o\,, \tag{A.1}$$

and by differentiating both sides with respect to ε in the restricted range:

$$(r+\rho+a_o)H'(\varepsilon) - \dot{H}'(\varepsilon) = 1\,.$$

For a given ε, this specifies a first-order differential equation in time t for $H'(\varepsilon)$. Since $H'(\varepsilon)$ is not a state variable, there exists a unique stable solution $H'(\varepsilon) = 1/(r+\rho+a_o)$, which is constant over time ($\dot{H}'(\varepsilon) = 0$). As $H'(\varepsilon)$ is independent of ε, integration over match quality ε shows the value function $H(\varepsilon)$ has the form

$$H(\varepsilon) = \underline{H} + \frac{\varepsilon}{r+\rho+a_o}\,, \quad \text{with} \quad \dot{H}(\varepsilon) = \dot{\underline{H}}\,, \tag{A.2}$$

Table A4. *Robustness Checks: Effects of the LTT on Sales Price (market level)*

Dependent Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)
2006-2012							
ln(Average Price)	-0.0176***	-0.0200***	-0.0229***	-0.0174***	-0.0174***	-0.0125**	-0.0255***
	(0.0050)	(0.0053)	(0.0055)	(0.0042)	(0.0042)	(0.0052)	(0.0069)
Observations	11,169	11,169	8,688	19,227	19,227	11,802	7,425
2006-2010							
ln(Average Price)	-0.0146**	-0.0186**	-0.0228***	-0.0172***	-0.0172***	-0.0124**	-0.0253**
	(0.0059)	(0.0061)	(0.0064)	(0.0049)	(0.0049)	(0.0061)	(0.0080)
Observations	7,519	7,519	5,833	12,946	12,946	7,953	4,993
2006-2018							
ln(Average Price)	-0.0173***	-0.0192***	-0.0230***	-0.0192***	-0.0192***	-0.0158**	-0.0252***
	(0.0046)	(0.0049)	(0.0052)	(0.0039)	(0.0039)	(0.0049)	(0.0065)
Observations	22,001	22,001	17,215	38,268	38,268	23,514	14,754
Distance threshold	3KM	3KM	3KM	5KM	5KM	5KM	2KM
Indicators TO +-3 m.		YES	YES	YES	YES	YES	YES
Time Trends TO		YES	YES	YES	YES	YES	YES
Distance LTT trends					Step	Step	
Donut hole			1KM			2KM	

Notes: A unit of observation is a market segment defined by community X property type X year X month, from January. Each cell represents a separate regression of a housing outcome variable (specified on the left column) on the LTT. All regressions are estimated with an indicator for post-LTT period, Toronto*Property Type Fixed Effects, Year*Property Type Fixed Effects, Month*Property Type Fixed Effects, Community*Property Type Fixed Effects. Indicators TO +- 3m are six dummy variables for transactions inside Toronto during the last 3 months of 2007 and the first 3 of 2008. Time trends TO indicates the presence of separate time trends for transactions in and outside of Toronto. Distance threshold is the maximum distance to the Toronto border for a transaction to be included. Distance LTT trend denotes the inclusion of an interaction term between LTT and distance to the Toronto border. Robust standard errors in parentheses. *, **, ***: corresponding coefficient significant at 10, 5, 1%.

where $\underline{H}$ is independent of ε, but may be time varying. This result is valid for ε in a neighbourhood above y_o and all values below. Substituting back into (A.1) shows that $\underline{H}$ satisfies the differential equation

$$(r+\rho+a_o)\underline{H}-\dot{\underline{H}} = a_oB_o+(\rho+a_o)U_o-M\,. \tag{A.3}$$

Since $x_o < y_o$, equation (32) together with (A.2) imply that

$$x_o = (r+\rho+a_o)(B_o+U_o-\underline{H})\,. \tag{A.4}$$

Equation (28) for the surplus and the definition of the transaction threshold (29) imply that y_o satisfies

$$H(y_o) = H(x_o)+C_h+(1+\tau_h)C_u+\tau_hU_o\,, \tag{A.5}$$

and combining (A.2) with (A.5) yields

$$y_o = x_o+(r+\rho+a_o)(C_h+(1+\tau_h)C_u+\tau_hU_o)\,. \tag{A.6}$$

The surplus $\Sigma_o(\varepsilon)$ is given in (28) and is divided according to (27). Equation (31) defines the expected surplus Σ_o. The Bellman equation for a buyer (4) can thus be expressed as the following differential equation:

$$(r+\rho)B_o-\dot{B}_o = (1-\omega_o^*)q_o\Sigma_o-F_h\,. \tag{A.7}$$

The surplus from trade with an investor and its division are given in (40) and (42). Together with the surplus from trade with a home-buyer, the Bellman equation of a seller (5) is the differential equation

$$rU_o-\dot{U}_o = \theta_oq_o\left(\omega_o^*(1-\xi)\Sigma_o+\omega_k^*\xi\Sigma_k\right)-M\,. \tag{A.8}$$

Using equations (25), (28), and (29), the expected surplus Σ_o in (31) can be written as

$$\Sigma_o = \int_{y_o}^{\infty} \lambda_o \zeta_o^{\lambda_o} \varepsilon^{-(\lambda_o+1)} \Sigma_o(\varepsilon) \mathrm{d}\varepsilon = \int_{y_o}^{\infty} \frac{\lambda_o \zeta_o^{\lambda_o} \varepsilon^{-(\lambda_o+1)} (H(\varepsilon) - H(y_o))}{1+\tau_h \omega_o^*} \mathrm{d}\varepsilon \,. \tag{A.9}$$

Make the following definition of $\bar{H}(\varepsilon)$ for an arbitrary level of match quality ε, and note the link with Σ_o:

$$\bar{H}(\varepsilon) = \int_{\upsilon=\varepsilon}^{\infty} \lambda_o \varepsilon^{\lambda_o} \upsilon^{-(\lambda_o+1)} (H(\upsilon) - H(\varepsilon)) \mathrm{d}\upsilon \,, \quad \text{where } \Sigma_o = \frac{\zeta_o^{\lambda_o} y_o^{-\lambda_o} \bar{H}(y_o)}{1+\tau_h \omega_o^*} \,. \tag{A.10}$$

Now restrict attention to ε such that $\delta_o \varepsilon < x_o$, so (6) implies $rH(\varepsilon) = \varepsilon - M + a_o(B_o + U_o - H(\varepsilon)) + \rho(U_o - H(\varepsilon)) + \dot{H}(\varepsilon)$. Since $\delta_o y_o < x_o$, this limits ε to a neighbourhood above y_o and all values below. Using (32):

$$\begin{aligned} r(H(\upsilon) - H(\varepsilon)) = (\upsilon - \varepsilon) + a_o \left(\max\{H(\delta_o \upsilon), H(x_o)\} - H(\upsilon)\right) - a_o(H(x_o) - H(\varepsilon)) \\ - \rho(H(\upsilon) - H(\varepsilon)) + (\dot{H}(\upsilon) - \dot{H}(\varepsilon)) \,, \end{aligned}$$

which holds for any $\upsilon \geq \varepsilon$. This simplifies to

$$(r+\rho+a_o)(H(\upsilon) - H(\varepsilon)) - (\dot{H}(\upsilon) - \dot{H}(\varepsilon)) = (\upsilon - \varepsilon) + a_o \max\{H(\delta_o \upsilon) - H(x_o), 0\} \,,$$

and multiplying both sides by $\lambda_o \varepsilon^{\lambda_o} \upsilon^{-(\lambda_o+1)}$, integrating over υ, and using the definition of $\bar{H}(\varepsilon)$ in (A.10):

$$(r+\rho+a_o)\bar{H}(\varepsilon) - \dot{\bar{H}}(\varepsilon) = \int_{\upsilon=\varepsilon}^{\infty} \lambda_o \varepsilon^{\lambda_o} \upsilon^{-(\lambda_o+1)} \left((\upsilon - \varepsilon) + a_o \max\{H(\delta_o \upsilon) - H(x_o), 0\}\right) \mathrm{d}\upsilon \,, \tag{A.11}$$

where the time derivative of $\bar{H}(\varepsilon)$ is obtained from (A.10):

$$\dot{\bar{H}}(\varepsilon) = \int_{\upsilon=\varepsilon}^{\infty} \lambda_o \varepsilon^{\lambda_o} \upsilon^{-(\lambda_o+1)} (\dot{H}(\upsilon) - \dot{H}(\varepsilon)) \mathrm{d}\upsilon \,.$$

In (A.11), the term in $(\upsilon - \varepsilon)$ integrates to $\varepsilon/(\lambda_o - 1)$ using the formula for the mean of a Pareto distribution. The second term is zero for $\upsilon < x_o/\delta_o$ because $H(\delta_o \upsilon)$ is increasing in υ. Hence, equation (A.11) becomes

$$(r+\rho+a_o)\bar{H}(\varepsilon) - \dot{\bar{H}}(\varepsilon) = \frac{\varepsilon}{\lambda_o - 1} + a_o \varepsilon^{\lambda_o} \int_{\upsilon=x_o/\delta_o}^{\infty} \lambda_o \upsilon^{-(\lambda_o+1)} (H(\delta_o \upsilon) - H(x_o)) \mathrm{d}\upsilon \,,$$

and with the change of variable $j = \delta_o \upsilon$ in the second integral, this can be written as

$$(r+\rho+a_o)\bar{H}(\varepsilon) - \dot{\bar{H}}(\varepsilon) = \frac{\varepsilon}{\lambda_o - 1} + a_o \delta_o^{\lambda_o} \varepsilon^{\lambda_o} \int_{j=x_o}^{\infty} \lambda_o j^{-(\lambda_o+1)} (H(j) - H(x_o)) \mathrm{d}j \,. \tag{A.12}$$

Make the following definition of a new variable X_o:

$$\begin{aligned} X_o(t) = (\lambda_o - 1)\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right) \int_{\upsilon=t}^{\infty} (r+\rho+a_o) e^{-(r+\rho+a_o)(\upsilon - t)} \Bigg(\\ \int_{\varepsilon=x_o}^{\infty} \lambda_o \varepsilon^{-(\lambda_o+1)} (H(\varepsilon, \upsilon) - H(x_o, \upsilon)) \mathrm{d}\varepsilon \Bigg) \mathrm{d}\upsilon \,. \end{aligned} \tag{A.13}$$

By differentiating with respect to time t this variable must satisfy the differential equation

$$\begin{aligned} (r+\rho+a_o)X_o - \dot{X}_o &= (\lambda_o - 1)(r+\rho+a_o)\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right) x_o^{-\lambda_o} \bar{H}(x_o) \\ &= (\lambda_o - 1)(r+\rho+a_o)\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right) \int_{\varepsilon=x_o}^{\infty} \lambda_o \varepsilon^{-(\lambda_o+1)} (H(\varepsilon) - H(x_o)) \mathrm{d}\varepsilon \,, \end{aligned} \tag{A.14}$$

which uses the definition of $\bar{H}(\varepsilon)$ in (A.10). Substituting into equation (A.12):

$$(r+\rho+a_o)\bar{H}(\varepsilon) - \dot{\bar{H}}(\varepsilon) = \frac{1}{\lambda_o - 1}\left(\varepsilon + \frac{a_o \delta_o^{\lambda_o} \varepsilon^{\lambda_o} \left((r+\rho+a_o)X_o - \dot{X}_o\right)}{(r+\rho+a_o)\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right)}\right) \,,$$

and by collecting terms this can be written as

$$(r+\rho+a_o)\left(\bar{H}(\varepsilon)-\frac{a_o\delta_o^{\lambda_o}\varepsilon^{\lambda_o}}{(\lambda_o-1)(r+\rho+a_o)\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right)}X_o\right)$$
$$-\left(\dot{\bar{H}}(\varepsilon)-\frac{a_o\delta_o^{\lambda_o}\varepsilon^{\lambda_o}}{(\lambda_o-1)(r+\rho+a_o)\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right)}\dot{X}_o\right)=\frac{\varepsilon}{\lambda_o-1}\,.$$

Since the right-hand side is time invariant and none of the variables are predetermined, it follows for each fixed ε there is a unique stable solution for $\bar{H}(\varepsilon)-a_o\delta_o^{\lambda_o}\varepsilon^{\lambda_o}X_o/((\lambda_o-1)(r+\rho+a_o)(r+\rho+a_o(1-\delta_o^{\lambda_p})))$ that is time invariant and equal to $\varepsilon/((\lambda_o-1)(r+\rho+a_o))$. This demonstrates that for any given ε in a neighbourhood above y_o or any value below it, the function $\bar{H}(\varepsilon)$ is given by

$$\bar{H}(\varepsilon)=\frac{1}{(\lambda_o-1)(r+\rho+a_o)}\left(\varepsilon+\frac{a_o\delta_o^{\lambda_o}\varepsilon^{\lambda_o}}{r+\rho+a_o(1-\delta_o^{\lambda_p})}X_o\right)\,. \tag{A.15}$$

Evaluating (A.15) at $\varepsilon=x_o$ and multiplying by $(\lambda_o-1)(r+\rho+a_o)(r+\rho+a_o(1-\delta_o^{\lambda_p}))x_o^{-\lambda_o}$:

$$(\lambda_o-1)(r+\rho+a_o)\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right)x_o^{-\lambda_o}\bar{H}(x_o)=\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right)x_o^{1-\lambda_o}+a_o\delta_o^{\lambda_o}X_o\,,$$

and substituting into (A.14) shows that X_o satisfies a differential equation in the moving threshold x_o:

$$\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right)X_o-\dot{X}_o=\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right)x_o^{1-\lambda_o}\,. \tag{A.16}$$

Finally, evaluating (A.15) at $\varepsilon=y_o$ and substituting into (A.10):

$$\Sigma_o=\frac{\zeta_o^{\lambda_o}}{(1+\tau_h\omega_o^*)(\lambda_o-1)(r+\rho+a_o)}\left(y_o^{1-\lambda_o}+\frac{a_o\delta_o^{\lambda_o}}{r+\rho+a_o(1-\delta_o^{\lambda_o})}X_o\right)\,. \tag{A.17}$$

In summary, (A.3), (A.4), (A.6), (A.7), (A.8), (A.16), and (A.17) form a system of differential equations in y_o, x_o, X_o, Σ_o, $\underline{H}$, B_o, and U_o, which take as given Σ_k, q_o and ξ.

A.4.2 Moving rate in the ownership market

The flow of owner-occupiers who move within the city is denoted by N_o, and the moving rate is $n_o=N_o/h_o$. The group of existing homeowners h_o is made up of matches that formed at various points in the past and have survived to the present. Moving requires that homeowners receive an idiosyncratic shock, which has arrival rate a_o independent of history. A measure a_oh_o of households thus decide whether to move.

All matches began as a viewing with some initial match quality ε. The flow of viewings v_h done by home-buyers in the ownership market at a point in time is

$$v_h=q_ob_h=(1-\xi)\theta_oq_ou_o\,. \tag{A.18}$$

Initial match quality drawn in viewings is from a Pareto(ζ_o,λ_o) distribution (see 25). This match quality distribution has been truncated when transaction decisions were made and possibly when subsequent idiosyncratic shocks have occurred. Consider a group of surviving homeowners where initial match quality has been previously truncated at $\underline{\varepsilon}$. This group constitutes a fraction $\zeta_o^{\lambda_o}\underline{\varepsilon}^{-\lambda_o}$ of the initial measure of viewings, and the distribution of ε conditional on survival is Pareto$(\underline{\varepsilon},\lambda_o)$. Among this group, consider those whose current match quality is a multiple Δ of original match quality ε, where Δ is equal to δ_o raised to the power of the number of past shocks received.

Now consider a new idiosyncratic shock. Current match quality becomes $\varepsilon'=\delta_o\Delta\varepsilon$ in terms of initial match quality ε. Moving is optimal if $\varepsilon'<x_o$, so only those with initial match quality $\varepsilon\geq x_o/(\delta_o\Delta)$ survive. Since $\delta_o<1$ and $\delta_oy_o<x_o$, there is a range of variation in thresholds y_o and x_o that ensures $x_o/(\delta_o\Delta)>\underline{\varepsilon}$. Given the Pareto distribution, the proportion of the surviving group that does not move after the new shock is $\underline{\varepsilon}^{\lambda_o}(x_o/(\delta_o\Delta))^{-\lambda_o} = x_o^{-\lambda_o}\delta_o^{\lambda_o}\Delta^{\lambda_o}\underline{\varepsilon}^{\lambda_o}$. Since that surviving group is a fraction $\zeta_o^{\lambda_o}\underline{\varepsilon}^{-\lambda_o}$ of the original set of viewings, those that do not move after the new shock are a fraction $x_o^{-\lambda_o}\delta_o^{\lambda_o}\Delta^{\lambda_o}\underline{\varepsilon}^{\lambda_o} \times \zeta_o^{\lambda_o}\underline{\varepsilon}^{-\lambda_o} = (\zeta_o^{\lambda_o}x_o^{-\lambda_o}\delta_o^{\lambda_o}) \times \Delta^{\lambda_o}$ of that set of viewings. This is independent of any past truncation thresholds $\underline{\varepsilon}$ owing to the properties of the Pareto distribution.

The measure of the group choosing not to move after a new shock does depend on the total accumulated size Δ of past idiosyncratic shocks. Let Ξ_o be the integral of Δ^{λ_o} over the measure of current and past viewings done by households who have not yet exited the city. Since the size of the group choosing not to move is a common multiple $\zeta_o^{\lambda_o}x_o^{-\lambda_o}\delta_o^{\lambda_o}$ of Δ^{λ_o}, the measure of those choosing not to move after a new shock is $a_o\zeta_o^{\lambda_o}x_o^{-\lambda_o}\delta_o^{\lambda_o}\Xi_o$. Therefore, the size of the group of movers is

$$N_o = a_o h_o - a_o\zeta_o^{\lambda_o}x_o^{-\lambda_o}\delta_o^{\lambda_o}\Xi_o\,. \tag{A.19}$$

Since the arrival of idiosyncratic shocks is independent of history, a fraction a_o of the group used to define Ξ_o have Δ^{λ_o} reduced to $\delta_o^{\lambda_o}\Delta^{\lambda_o}$. Exit from the group occurs at rate ρ, and new viewings occur that start from $\Delta^{\lambda_o} = 1$ with measure v_h from (A.18). The differential equation for Ξ_o is thus

$$\dot{\Xi}_o = v_h + a_o(\delta_o^{\lambda_o}\Xi_o - \Xi_o) - \rho\Xi_o\,. \tag{A.20}$$

Define the following weighted average of current and past levels of home-buyer viewings v_h:

$$\bar{v}_h(t) = \int_{\upsilon\to-\infty}^{t} (\rho + a_o(1-\delta_o^{\lambda_o}))e^{-(\rho+a_o(1-\delta_o^{\lambda_o}))(t-\upsilon)}v_h(\upsilon)\mathrm{d}\upsilon\,,$$

and note that it satisfies the differential equation

$$\dot{\bar{v}}_h + (\rho + a_o(1-\delta_o^{\lambda_o}))\bar{v}_h = (\rho + a_o(1-\delta_o^{\lambda_o}))v_h\,. \tag{A.21}$$

A comparison of (A.20) and (A.21) shows that $\Xi_o = \bar{v}_h/(\rho + a_o(1-\delta_o^{\lambda_o}))$, and substituting this into (A.19) yields an equation for the moving rate $n_o = N_o/h_o$:

$$n_o = a_o - \frac{a_o\zeta_o^{\lambda_o}\delta_o^{\lambda_o}x_o^{-\lambda_o}\bar{v}_h}{(\rho + a_o(1-\delta_o^{\lambda_o}))h_o}\,. \tag{A.22}$$

A.4.3 Value functions and thresholds in the rental market

Adding together the Bellman equations (8) and (10) for the landlord and tenant value functions:

$$\begin{aligned} r(L(\varepsilon)+W(\varepsilon)) = \varepsilon - M - M_l + (\rho + a_l)(U_l - L(\varepsilon)) + \rho_l(U_o - L(\varepsilon)) + (1-\gamma)n_l(B_l - W(\varepsilon)) \\ + \gamma n_l(G_m(Z)(B_o - \bar{\chi}) + (1-G_m(Z))B_l - W(\varepsilon)) - \rho W(\varepsilon) + \dot{L}(\varepsilon) + \dot{W}(\varepsilon)\,. \end{aligned}$$

Letting $J(\varepsilon) = L(\varepsilon) + W(\varepsilon)$ denote the joint value, this can be rearranged and simplified, noting $B_o - B_l = Z$ and $n_l = a_l + \rho_l$ from (14) and (34):

$$(r+\rho+n_l)J(\varepsilon) = \varepsilon - M - M_l + (\rho + a_l)U_l + \rho_l U_o + n_l B_l + \gamma n_l G_m(Z)(Z - \bar{\chi}) + \dot{J}(\varepsilon)\,. \tag{A.23}$$

Differentiating with respect to ε leads to the differential equation

$$(r+\rho+n_l)J'(\varepsilon) = 1 + \dot{J}'(\varepsilon)\,.$$

This differential equation has a unique non-explosive solution for $J'(\varepsilon)$ for any given value of ε:

$$J'(\varepsilon) = \frac{1}{r+\rho+n_l}\,.$$

This time-invariant solution ($\dot{J}'(\varepsilon) = 0$) implies the solution for $J(\varepsilon)$ takes the following form:

$$J(\varepsilon) = \underline{J} + \frac{\varepsilon}{r+\rho+n_l}\,, \tag{A.24}$$

where $\underline{J}$ can be time varying in general. Substituting back into (A.23) and noting $\dot{J}(\varepsilon) = \dot{\underline{J}}$ shows that $\underline{J}$ satisfies the differential equation

$$(r+\rho+n_l)\underline{J} = n_l B_l + (\rho + a_l)U_l + \rho_l U_o - M - M_l + \gamma n_l G_m(Z)(Z - \bar{\chi}) + \dot{\underline{J}}\,. \tag{A.25}$$

The joint rental surplus from (35) is linked to $J(\varepsilon)$ by

$$\Sigma_l(\varepsilon) = J(\varepsilon) - C_l - C_w - B_l - U_l\,, \tag{A.26}$$

and together with (A.24), the definition of the rental transaction threshold y_l in (37) implies

$$y_l = (r + \rho + n_l)(B_l + U_l - \underline{J} + C_l + C_w)\,. \tag{A.27}$$

Using (37), (A.24), and (A.26), it follows that $\Sigma_l(\varepsilon) = (\varepsilon - y_l)/(r + \rho + a_l)$. The Pareto distribution in (25) then implies the expected rental surplus from (39) is

$$\Sigma_l = \frac{\zeta_l^{\lambda_l} y_l^{1-\lambda_l}}{(\lambda_l - 1)(r + \rho + n_l)}\,. \tag{A.28}$$

Using $\Sigma_l^l(\varepsilon) = L(\varepsilon) + \Pi(\varepsilon) - C_l - U_l = \omega_l \Sigma_l(\varepsilon)$, (35), and (39), the Bellman equation (7) for U_l becomes

$$(r + \rho_l)U_l - \dot{U}_l = \omega_l \theta_l q_l \Sigma_l - M + \rho_l U_o\,, \tag{A.29}$$

and similarly, with $\Sigma_l^w(\varepsilon) = W(\varepsilon) - \Pi(\varepsilon) - C_w - B_l = (1 - \omega_l)\Sigma_l$, the Bellman equation (9) for B_l becomes

$$(r + \rho)B_l - \dot{B}_l = (1 - \omega_l)q_l \Sigma_l - F_w\,. \tag{A.30}$$

The credit cost threshold Z satisfies (14). In summary, equations (A.25), (A.27), (A.28), (A.29), (A.30), and (14) determine y_l, Z, Σ_l, $\underline{J}$, B_l, and U_l.

The Bellman equation (8) can be written as follows:

$$(r + \rho + n_l)(L(\varepsilon) - U_l) = R(\varepsilon) - M - M_l - (r + \rho_l)U_l + \rho_l U_o + \dot{L}(\varepsilon)\,,$$

and substituting from (A.29) implies that rents $R(\varepsilon)$ are

$$R(\varepsilon) = M_l + \omega_l \theta_l q_l \Sigma_l + (r + \rho + n_l)(L(\varepsilon) - U_l) - (\dot{L}(\varepsilon) - \dot{U}_l)\,.$$

Since $\Lambda^l(\varepsilon) = L(\varepsilon) - U_l$, which entails $\dot{L}(\varepsilon) - \dot{U}_l = \dot{\Lambda}^l(\varepsilon)$, the division of the surplus $\Lambda^l(\varepsilon) = \omega_l \Lambda(\varepsilon)$ implies

$$R(\varepsilon) = M_l + \omega_l \theta_l q_l \Sigma_l + \omega_l \left((r + \rho + n_l)\Lambda(\varepsilon) - \dot{\Lambda}(\varepsilon)\right)\,.$$

By substituting from equation (35) and noting $\Sigma_l(\varepsilon) = \Lambda(\varepsilon) - (C_l + C_w)$, the expression for rents becomes

$$R(\varepsilon) = M_l + \omega_l(r + \rho + n_l)(C_w + C_l) + \omega_l \theta_l q_l \Sigma_l + \omega_l \left((r + \rho + n_l)\Sigma_l(\varepsilon) - \dot{\Sigma}_l(\varepsilon)\right)\,.$$

Noting that $\dot{\Sigma}_l(\varepsilon) = -\dot{y}_l/(r + \rho + n_l)$ for all ε, and using the definition of average rents R from (38):

$$R = M_l + \omega_l(r + \rho + n_l)(C_w + C_l) + \omega_l \theta_l q_l \Sigma_l + \omega_l \left((r + \rho + n_l)\frac{\Sigma_l}{\pi_l} + \frac{\dot{y}_l}{r + \rho + n_l}\right)\,,$$

which can be written as

$$R = M_l + \omega_l(r + \rho + n_l)(C_l + C_w) + \omega_l(r + \rho + n_l + \theta_l q_l \pi_l)\frac{\Sigma_l}{\pi_l} + \frac{\omega_l}{r + \rho + n_l}\dot{y}_l\,. \tag{A.31}$$

A.4.4 Market tightness

The total measure of properties in (11) and households in (12) together with the definitions of the fraction of investors and market tightness from (1) imply

$$((1 - \xi)\,\theta_o - 1)\,u_o + (\theta_l - 1)\,u_l = \psi - 1\,, \tag{A.32}$$

which yields a relationship between the market tightnesses θ_o and θ_l across the two markets given stocks of properties for sale u_o and properties for rent u_l, and the fraction ξ of investors among buyers.

A.4.5 Average match quality

Let Ψ_h denote the integral of ε over all current owner-occupiers. There is a flow of $v_h \pi_o$ of new owner-occupier matches, and using (1), (18), and (A.18), the size of this flow can be expressed as $(1 - \kappa)s_o u_o$. Since the transaction threshold is y_o, the Pareto distribution (25) implies the average value of ε in these new matches is $\lambda_o y_o/(\lambda_o - 1)$, so these new matches add to Ψ_h at rate $(1-\kappa)s_o u_o \lambda_o y_o/(\lambda_o - 1)$ over time.

Matches are destroyed (sending the contribution to Ψ_h to zero) if households exit the city or match-quality shocks arrive and households choose to move. Households exit the city at rate ρ, reducing Ψ_h by $\rho\Psi_h$. Match quality shocks arrive randomly at rate a_o for the measure h_o of owner-occupiers, leading to a flow N_o of movers out of the group $a_o h_o$ receiving a shock, which reduces the contribution to Ψ_h of those N_o to zero. For the group of size $a_o h_o - N_o$ that receives a shock but does not move, the conditional distribution of surviving match quality ε is truncated at x_o and is a Pareto distribution with shape parameter λ_o across all cohorts within that group. The mean of the truncated distribution is therefore $\lambda_o x_o/(\lambda_o - 1)$. Putting together all these effects on Ψ_h, the following differential equation must hold:

$$\dot{\Psi}_h = (1-\kappa)s_o u_o \frac{\lambda_o y_o}{\lambda_o - 1} + \left(N_o \times 0 + (a_o h_o - N_o) \times \frac{\lambda_o x_o}{\lambda_o - 1} - a_o \Psi_h\right) - \rho \Psi_h .$$

Average match quality among owner-occupiers is $Q_h = \Psi_h/h_o$, and thus $\dot{Q}_h = \dot{\Psi}_h/h_o - (\dot{h}_o/h_o)Q_h = \dot{\Psi}_h/h_o - (((1-\kappa)s_o u_o/h_o) - (n_o + \rho))Q_h$, where the second equation uses the differential equation for h_o in (22). Together with the equation for $\dot{\Psi}_h$ above and the definition of the moving rate $n_o = N_o/h_o$, average match quality Q_h must satisfy the differential equation (45).

Let Ψ_l denote the equivalent summation of surviving match quality in the rental market. Rental viewings occur at rate $v_l = q_l b_l$, and with leasing threshold y_l for match quality add to Ψ_l at rate $q_l \pi_l \lambda_l y_l b_l/(\lambda_l - 1)$ over time. Using (1) and (19), the flow increment to Ψ_l is $s_l u_l \lambda_l y_l/(\lambda_l - 1)$. Matches are destroyed if the household exits the city (rate ρ), the landlord must sell up (rate ρ_l), or match quality falls to zero (rate a_l). The differential equation for Ψ_l is thus $\dot{\Psi}_l = s_l u_l(\lambda_l y_l/(\lambda_l - 1)) - (a_l + \rho_l + \rho)\Psi_l$. Average match quality for tenants is $Q_l = \Psi_l/h_l$, hence $\dot{Q}_l = (\dot{\Psi}_l/h_l) - (\dot{h}_l/h_l)Q_l$, and by substituting $\dot{h}_l/h_l = (s_l u_l/h_l) - (n_l + \rho)$ from (23), the differential equation for Q_l is (46), which uses $n_l = a_l + \rho_l$ from (34).

A.5 Existence of a steady state and solution method

Equations for the steady state In a steady state where $\dot{B}_o = 0$ and $\dot{U}_o = 0$, the Bellman equations (A.7) and (A.8) become

$$(r+\rho)B_o = -F_h + (1-\omega_o^*)q_o\Sigma_o , \quad \text{and} \tag{A.33}$$

$$rU_o = \theta_o q_o((1-\xi)\omega_o^*\Sigma_o + \xi\omega_k^*\Sigma_k) - M . \tag{A.34}$$

Substituting from (A.8) into (A.6):

$$y_o = x_o + (r+\rho+a_o)\left(C_h + C_u + \tau_h\left(C_u - \frac{M}{r} + \frac{\theta_o q_o((1-\xi)\omega_o^*\Sigma_o + \xi\omega_k^*\Sigma_k)}{r}\right)\right). \tag{A.35}$$

The joint surplus $\Sigma_k = F_k/(1-\omega_k^*)q_o$ from selling to an investor comes from equation (43). In a steady state with $\dot{\underline{H}} = 0$, (A.3) implies that $(r+\rho+a_o)\underline{H} = a_o B_o + (\rho + a_o)U_o - M$. Substituting into (A.4) implies $x_o = (r+\rho+a_o)(B_o+U_o) - a_o B_o - (\rho+a_o)U_o + M$ and hence

$$x_o = M + (r+\rho)B_o + rU_o .$$

Then substituting the values of B_o and U_o from (A.33) and (A.34) yields

$$x_o + F_h = (1-\omega_o^* + (1-\xi)\omega_o^*\theta_o)\,q_o\Sigma_o + \theta_o q_o \xi\omega_k^*\Sigma_k . \tag{A.36}$$

With $\dot{X}_o = 0$ in steady state, equation (A.16) shows that $X_o = x_o^{1-\lambda_o}$. Substitution into (A.17) implies the expected joint surplus is

$$\Sigma_o = \frac{\zeta_o^{\lambda_o}}{(r+\rho+a_o)(\lambda_o-1)(1+\tau_h\omega_o^*)}\left(y_o^{1-\lambda_o} + \frac{a_o\delta_o^{\lambda_o}x_o^{1-\lambda_o}}{r+\rho+a_o\left(1-\delta_o^{\lambda_o}\right)}\right). \tag{A.37}$$

The average transaction price P from (30) can be written as follows by using equation (A.34) for U_o:

$$P = \left(\frac{r + \theta_o q_o (1-\xi)\pi_o}{r}\right)\left(\frac{\omega_o^* \Sigma_o}{\pi_o}\right) + \frac{\theta_o q_o \xi \omega_k^* \Sigma_k}{r} + C_u - \frac{M}{r}. \tag{A.38}$$

With $\dot{B}_l = 0$ and $\dot{U}_l = 0$, the Bellman equations (A.29) and (A.30) become

$$rB_l = -F_w + (1-\omega_l) q_l \Sigma_l - \rho B_l\,, \quad \text{and} \tag{A.39}$$

$$(r + \rho_l) U_l = \omega_l \theta_l q_l \Sigma_l - M + \rho_l U_o\,. \tag{A.40}$$

In steady state, $\dot{\underline{J}} = 0$, which yields $(r + \rho + n_l)\underline{J} = n_l B_l + (\rho + a_l) U_l + \rho_l U_o - M - M_l + \gamma n_l G_m(Z)(Z - \bar{\chi})$ using (A.25). Substituting into (A.27) and using $n_l = a_l + \rho_l$ implies

$$y_l = M + M_l + (r+\rho) B_l + (r + \rho_l) U_l - \rho_l U_o + (r + \rho + n_l)(C_l + C_w) - \gamma n_l G_m(Z)(Z - \bar{\chi})\,,$$

and by using (A.39) and (A.40) this becomes

$$y_l = M_l - F_w + (r + n_l + \rho)(C_w + C_l) - \gamma n_l G_m(Z)(Z - \bar{\chi}) + (1 - \omega_l + \omega_l \theta_l) q_l \Sigma_l\,. \tag{A.41}$$

The rent equation (A.31) in steady state is

$$R = M_l + \omega_l (r + \rho + n_l)(C_l + C_w) + \omega_l (r + \rho + n_l + \theta_l q_l \pi_l)\frac{\Sigma_l}{\pi_l}\,. \tag{A.42}$$

Multiplying both sides of (42) by $r + \rho_l$ and substituting for $(r+\rho_l)U_l$ from (A.40) leads to

$$\omega_l \theta_l q_l \Sigma_l = M - \rho_l U_o + (r + \rho_l)(1 + \tau_k) U_o + (r + \rho_l)\left((1+\tau_k) C_u + C_k + (1 + \tau_k \omega_k^*)\Sigma_k\right).$$

Using $(r + \rho_l)(1 + \tau_k) U_o - \rho_l U_o = (1 + \tau_k (1 + (\rho_l / r)) r U_o$ and substituting from (A.34) implies:

$$\omega_l \theta_l q_l \Sigma_l = \left(1 + \tau_k \left(1 + \frac{\rho_l}{r}\right)\right) \theta_o q_o \left((1-\xi)\omega_o^* \Sigma_o + \xi \omega_k^* \Sigma_k\right) \\ + (r + \rho_l)\left((1+\tau_k) C_u + C_k + (1 + \tau_k \omega_k^*)\Sigma_k\right) - \tau_k \left(1 + \frac{\rho_l}{r}\right) M\,. \tag{A.43}$$

By substituting B_o and B_l from (A.33) and (A.39) into (14):

$$(1 - \omega_o^*) q_o \Sigma_o - (1 - \omega_l) q_l \Sigma_l = (r + \rho) Z + F_h - F_w\,. \tag{A.44}$$

The price paid by investors in equilibrium is obtained from (41) and (A.34):

$$P_k = C_u + \frac{\theta_o q_o \left((1-\xi)\omega_o^* \Sigma_o + \xi \omega_k^* \Sigma_k\right) - M}{r} + \omega_k^* \Sigma_k\,. \tag{A.45}$$

Imposing a steady state $\dot{Q}_h = 0$ and $\dot{Q}_l = 0$ in the match quality equations (45) and (46) implies

$$Q_h = \frac{\lambda_o}{\lambda_o - 1}\left(\frac{n_o + \rho}{a_o + \rho} y_o + \frac{a_o - n_o}{a_o + \rho} x_o\right), \quad \text{and}$$

$$Q_l = \frac{\lambda_l}{\lambda_l - 1} y_l\,,$$

which also make use of $\dot{h}_o = 0$, $\dot{h}_l = 0$, and (22) and (23).

The solution method is to reduce the problem to a numerical search over the fraction ξ of investors among buyers and ownership-market tightness θ_o to find the roots of two equations representing equilibrium in the ownership and rental markets.

Ownership-market transaction threshold Conditional on ξ and θ_o, within this search, there is also a numerical search to find the transaction threshold y_o in the ownership market. Equation (43) implies $q_o \Sigma_k = F_k / (1 - \omega_k^*)$ and equation (A.36) implies $q_o \Sigma_o = (x_o + F_h - \xi \theta_o q_o \omega_k^* \Sigma_k)/(1 - \omega_o^* + (1-\xi)\omega_o^* \theta_o)$. Together, it follows that

$$\theta_o q_o \left((1-\xi)\omega_o^* \Sigma_o + \xi \omega_k^* \Sigma_k\right) = \frac{\omega_o^* \theta_o}{1 - \omega_o^* + (1-\xi)\omega_o^* \theta_o}\left((1-\xi)(x_o + F_h) + \xi \frac{(1-\omega_o^*)\omega_k^*}{\omega_o^* (1 - \omega_k^*)} F_k\right).$$

Taking a value of y_o, the moving threshold x_o must satisfy (A.35), and substituting the expression above yields a linear equation for x_o which can be solved in terms of y_o:

$$x_o = \frac{y_o - (r+\rho+a_o)\left(C_h + (1+\tau_h)C_u - \tau_h \frac{M}{r} + \tau_h \frac{\theta_o \omega_o^*}{1-\omega_o^* + (1-\xi)\omega_o^*\theta_o}\left(\frac{(1-\xi)F_h}{r} + \frac{\xi(1-\omega_o^*)\omega_k^* F_k}{\omega_o^*(1-\omega_k^*)r}\right)\right)}{1 + \tau_h\left(\frac{(1-\xi)\omega_o^*\theta_o}{1-\omega_o^*+(1-\xi)\omega_o^*\theta_o}\right)\left(\frac{r+\rho+a_o}{r}\right)}\,. \quad \text{(A.46)}$$

Now combine equations (A.36), (A.37), (43), and substitute $q_o = A_o\theta_o^{-\eta_o}$ from (24):

$$x_o + F_h - \frac{(1-\omega_o^* + (1-\xi)\omega_o^*\theta_o)A_o\theta_o^{-\eta_o}\zeta_o^{\lambda_o}}{(1+\tau_h\omega_o^*)(r+\rho+a_o)(\lambda_o-1)}\left(y_o^{1-\lambda_o} + \frac{a_o\delta_o^{\lambda_o}x_o^{1-\lambda_o}}{r+\rho+a_o(1-\delta_o^{\lambda_o})}\right) - \frac{\xi\theta_o\omega_k^* F_k}{1-\omega_k^*} = 0\,. \quad \text{(A.47)}$$

Observe that the left-hand side of (A.47) is strictly increasing in x_o and y_o. As the value of x_o implied by (A.46) is strictly increasing in y_o, it follows that any solution of (A.46) and (A.47) for x_o and y_o is unique. Since the left-hand side of (A.47) is sure to be positive for large y_o and x_o, existence of a solution can be confirmed by checking whether the left-hand side is negative at $y_o = \zeta_o$, the minimum value of y_o.

Ownership-market variables Once y_o is found, the transaction probability in the ownership market conditional on a viewing is $\pi_o = (\zeta_o/y_o)^{\lambda_o}$. This yields κ from (18) given the value of ξ. Moreover, given that $q_o = A_o\theta_o^{-\eta_o}$ is known conditional on θ_o, the sales rate are s_o is found using (19). The moving threshold x_o is obtained from (A.46), and it can be verified whether $\delta_o y_o < x_o$ is satisfied. With x_o and y_o, the moving rate n_o is obtained from (49). The surplus Σ_o is found by substituting the thresholds into (A.37), and $\Sigma_k = F_k/((1-\omega_k^*)q_o)$ comes from (43).

A steady state has $\dot{u}_o = 0$ and $\dot{h}_o = 0$, so (20) and (22) require

$$s_o u_o = (n_o+\rho)h_o + \rho_l(h_l+u_l)\,, \quad \text{and} \quad \text{(A.48)}$$

$$(1-\kappa)s_o u_o = (n_o+\rho)h_o\,. \quad \text{(A.49)}$$

Since (11) implies $h_l + u_l = 1 - h_o - u_o$, dividing both sides of (A.48) by $\rho_l > 0$ and substituting implies $u_o + h_o - ((n_o+\rho)/\rho_l)h_o + (s_o/\rho_l)u_o = 1$. Equation (A.49) implies $h_o = ((1-\kappa)s_o/(n_o+\rho))u_o$, and substituting into the previous equation for u_o and solving:

$$u_o = \frac{1}{1 + \frac{(1-\kappa)s_o}{n_o+\rho} + \frac{\kappa s_o}{\rho_l}}\,, \quad \text{and} \quad h_o = \frac{(1-\kappa)s_o}{n_o+\rho}u_o\,. \quad \text{(A.50)}$$

This yields the homeownership rate h using (47).

Rental-market variables The moving rate $n_l = a_l + \rho_l$ in the rental market is given by parameters according to (34). Conditional on θ_o and ξ, there is also a numerical search for the transaction threshold y_l in the rental market. Given a value of y_l, the implied transaction probability from (38) is $\pi_l = (\zeta_l/y_l)^{\lambda_l}$. Using the formula (A.28) for the rental-market surplus:

$$\Sigma_l = \frac{\pi_l y_l}{(\lambda_l-1)(r+\rho+n_l)}\,.$$

Observe that $\omega_l\theta_l q_l\Sigma_l = \omega_l y_l s_l/((\lambda_l-1)(r+\rho+n_l))$, where $s_l = \theta_l q_l \pi_l$ is the letting rate from (19). By using this to substitute for the left-hand side of the equation (A.43), the letting rate implied by y_l is

$$s_l = \frac{(\lambda_l-1)(r+\rho+n_l)}{\omega_l y_l}\Bigg(\left(1+\tau_k\left(1+\frac{\rho_l}{r}\right)\right)\theta_o q_o\left((1-\xi)\omega_o^*\Sigma_o + \xi\omega_k^*\Sigma_k\right)$$
$$+ (r+\rho_l)\left((1+\tau_k)C_u + C_k + (1+\tau_k\omega_k^*)\Sigma_k\right) - \tau_k\left(1+\frac{\rho_l}{r}\right)M\Bigg)\,, \quad \text{(A.51)}$$

where the surpluses Σ_o and Σ_k were obtained when the ownership-market variables were found. Equation (2) gives the meeting rate $q_l = A_l\theta_l^{-\eta_l}$, and hence the letting rate $s_l = \theta_l q_l \pi_l$ satisfies $s_l = A_l\pi_l\theta_l^{1-\eta_l}$. The implied market tightness in the rental market is

$$\theta_l = \left(\frac{s_l}{A_l \pi_l}\right)^{\frac{1}{1-\eta_l}}, \tag{A.52}$$

and this also gives $q_l = A_l \theta_l^{-\eta_l}$.

In steady state, $\dot{u}_l = 0$ and $\dot{h}_l = 0$, hence equations (21) and (23) require

$$(s_l + \rho_l)u_l = (a_l + \rho)h_l + \kappa s_o u_o\,, \quad \text{and} \tag{A.53}$$

$$s_l u_l = (n_l + \rho)h_l\,. \tag{A.54}$$

Equation (11) and (A.54) imply $h_l + u_l = 1 - h_o - u_o$ and $h_l = (s_l/(n_l+\rho))u_l$. Combining these two equations and using the known values of h_o and u_o:

$$u_l = \frac{1 - h_o - u_o}{1 + \frac{s_l}{n_l + \rho}}\,, \quad \text{and} \quad h_l = \frac{s_l}{n_l + \rho} u_l\,. \tag{A.55}$$

Since (11) will hold and (A.48), (A.49), and (A.54) have been imposed, equation (A.53) holds automatically.

The steady state also has $\dot{b}_h = 0$ and $\dot{b}_l = 0$, which means that the following must hold:

$$(q_o \pi_o + \rho)b_h = n_o h_o + (\gamma n_l h_l + \rho \psi)G_m(Z)\,, \quad \text{and} \tag{A.56}$$

$$(q_l \pi_l + \rho)b_l = (1-\gamma)n_l h_l + (\gamma n_l h_l + \rho \psi)(1 - G_m(Z))\,. \tag{A.57}$$

Since (1) implies $b_h = (1-\xi)\theta_o u_o$, which is known, the value of $G_m(Z)$ is obtained by rearranging (A.56):

$$G_m(Z) = \frac{(\rho + q_o \pi_o)(1-\xi)\theta_o u_o - n_o h_o}{\gamma n_l h_l + \rho \psi}\,,$$

and it can be checked that $G_m(Z)$ is a well defined probability. Given that (12) will hold along with (A.49), (A.54), and (A.56), equation (A.57) is satisfied automatically. The threshold Z is obtained by inverting equation (26) with the known probability $G_m(Z)$:

$$Z = e^{\mu + \sigma \Phi^{-1}(G_m(Z))}\,,$$

and the average cost $\bar{\chi}$ follows immediately from (26) using Z. Finally, with all these variables known conditional on y_l, the value of y_l itself can be found by searching for a solution of equation (A.41). It can be checked whether the solution satisfies $y_l > \zeta_l$.

Criteria for the fraction of investors and market tightness Finally, two equations are needed to pin down ownership-market tightness and the fraction of investors among buyers. Conditional on each pair of values of θ_o and ξ, the steps above show how θ_l, u_o, and u_l can be calculated. With these, the first criterion to be checked is equation (A.32). The second criterion is the indifference threshold condition (A.44), where q_o, q_l, Σ_o, Σ_l, and Z can be obtained as above for given θ_o and ξ. Searching over values of θ_o and θ_l that satisfy these two criteria, the equilibrium is found.

A.6 Calibration targets

The land transfer tax is the main transaction cost paid by buyers in the ownership market. The effective LTT rate, measured by the mean of land transfer taxes relative to sales prices across transactions, is 1.08% during the pre-policy period (January 2006 – January 2008), so $\tau_h = \tau_k = 0.011$. The parameters of the model are calibrated to match the pre-policy period in Toronto.

Transaction costs in the ownership market Excluding the land transfer tax, buyers in the ownership market may pay a home inspection cost of about \$500, which is very small relative to the average house price. So it is assumed that other than the LTT, buyers in the ownership market do not pay other transaction costs, so $C_h = C_k = 0$.

Turning to the seller side, the primary cost for the seller of a property is real estate agent commissions. Using Multiple Listing Service sales data, the average commission rate is about 4.5%. There are some other costs such as legal fees of around $1,000, which are negligible, and some sellers may spend roughly $2,500 on staging but in some cases the seller's agent covers this expense as part of their commission, so not all sellers pay for staging out of their own pocket. Thus, C_u is set to be 4.5% of the average house price.

Maintenance costs The maintenance cost as an owner, M, is set so that in equilibrium it is 2.6% of the average house price. This cost is made up of a 2% maintenance cost and a 0.6% property tax in Toronto. The extra maintenance cost of being a landlord, M_l, is set to be 8% of the average rent. This cost includes two parts: approximately 5–7% that the landlord uses to hire a property manager; and approximately 1% that the landlord uses to hire someone to take out garbage, shovel the snow, salt the walkways, etc.

Transaction costs in the rental market In Toronto, landlords typically pay one month's rent to real estate agents to lease their property. So C_l is set to equal $1/12$ of annual rent. Tenants in Toronto do not typically pay a monetary transaction cost when renting a property. This implies the fee Π in the model is equal to zero, thus it follows from equation (36) that tenants' own costs are $C_w = ((1-\omega_l)/\omega_l)C_l$.

Flows in the housing market The flows in the housing market are affected by time-to-move, time-to-sell, and viewings-per-sale. The information on time-to-move and time-to-sell is based on the Toronto MLS on sales and rental transactions during the pre-policy period. The average duration of stay before a homeowner moves out is 2,010 days, so $T_{mo} = 2010/365$. The average duration of stay for a tenant is 1,109 days, so $T_{ml} = 1109/365$. The time-to-sell for homebuyers is 24 days and the average time-to-rent is 18.7 days. During this period, the fraction of withdrawals out of total for-sale listings is 48% and that out of total for-lease listings is 22%. So we set $T_{so} = (24/365)/(1-0.48)$ and $T_{sl} = (18.7/365)/(1-0.22)$ in the model to incorporate the withdrawal rate in the data. This is because time-to-sell in the data is calculated from the final successful listing without accounting for earlier unsuccessful attempts, so the true time-to-sell is longer.

Using the 'Profile of Buyers and Sellers' survey collected by NAR, Genesove and Han (2012) report that for the period 2006–2009, the ratio of average time-to-buy to the the average time-to-sell is 1.28 and the average number of homes visited is 10.7. Thus, we set $T_{bo} = 1.28 \times T_{so}$ and $v_o = 10.7/(1-0.48)$ in the model to incorporate the withdrawal rate in the data. The idea is that viewings of properties that have been withdrawn from the market are not counted, so actual viewings are larger than the reported viewings. We do not have information on the number of homes that renters visit on average. According to an industry expert, renters visit fewer homes than buyers, so we assume it is half the viewings per sale by setting $v_l = v_o/2$. Note that the time taken for a household to find a rental property, T_{bl}, is not an independent target and it is pinned down by the steady-state condition linking the homeownership rate, time-to-move, and time-to-sell in the two markets.

Flow search costs There are no direct estimates of the flow costs of searching F_k, F_h, and F_w. The approach taken here is to base an estimate of search costs on the opportunity cost of the time spent searching. More specifically, for buyers in the ownership market, assume one property viewing entails the loss of a day's income, so the value of $F_k = F_h$ can be calibrated by adding up the costs of making the expected number of viewings. With viewings per sale equal to the average number of viewings made by a buyer, the total search cost should be equated to $0.5 \times v_o(Y/365)$, where Y denotes average annual income and assuming a viewing takes up half a day. Thus, the calibration sets $T_{bo}F_h = v_oY/365$, and by dividing both sides by PT_{bo}, this implies $F_h/P = 0.5 \times (1/365)(Y/P)(v_o/T_{boo})$. Taking the median household-level income from Stats Canada, we obtain a price-to-income ratio in Toronto of $P/Y = 5.6$ in 2007. Given the value of v_o/T_{bo}, the implied buyer's flow search cost, $F_h = F_k$ is 2.9% of the average house price.

Using the same logic for the flow search cost for tenants in the rental market, the ratio of flow search cost for a tenant relative to a homebuyer F_w/F_h is set to $0.5 \times (v_l/v_o) \times (T_{bo}/T_{bl})$, where it is assumed that viewing a rental properties takes half the time needed to view a property to buy.

Targets related to movement across the two housing markets Using the MLS data on sales and rental transactions in 2007, the price-to-rent ratio is 14.8 for the same property in Toronto. Based on the 2006 City of Toronto Profile Report, the homeownership rate is $h = 54\%$, the average age of homeowners is 53.3 and the average age of tenants is 45.0. Hence the difference between the average ages of homeowners and renters is $\alpha = 8.3$. There is no survey that specifically captures the proportion of first-time buyers. The Canadian Associated of Accredited Mortgage Professionals (now called Mortgage Professionals Canada) undertook a survey in 2015 and found that the fraction is as high as 45% of purchases, which is consistent with the 44% found in the 2018 Canadian Household Survey for the GTA area. On the other hand, the Canada Mortgage and Housing Corporation suggests that the fraction of first-time buyers is about one third. Based on this information, the calibration sets $\phi = 0.4$. Finally, the fraction of purchases by investors is about 5% during this period so $\kappa = 0.05$.

Mortgage costs The cost χ of becoming a homeowner is computed from a comparison of the mortgage rate r_c the household would face relative to the risk-free interest rate r_f on government bonds. The interest rates r_c and r_f are real interest rates. There is a spread between them due to unmodelled financial fricitions. The risk-free real rate r_f used to discount future cashflows need not be the same as the discount rate r applied to future utility from owning property (allowing for an unmodelled housing risk premium between r and r_f). All these interest rates are expected to remain constant over the mortgage term.

Suppose the household buys a property at price P at date $t = 0$ by taking out a mortgage with loan-to-value ratio ℓ. Assume the mortgage has term T_c and a constant real repayment I over its term. Let $D(t)$ denote the outstanding mortgage balance at date t, which has initial condition $D(0) = \ell P$ and terminal condition $D(T_c) = 0$. The mortgage balance evolves over time according to the differential equation:

$$\dot{D}(t) = r_c D(t) - I \quad \text{and hence} \quad \frac{\mathrm{d}(e^{-r_c t} D(t))}{\mathrm{d}t} = -I e^{-r_c t}.$$

Solving this differential equation using the initial condition $D(0) = \ell P$ implies:

$$D(t) = e^{r_c t} \ell P - \frac{I}{r_c}(e^{r_c t} - 1).$$

The terminal condition $D(T_c) = 0$ requires that the constant real repayment I satisfies:

$$I = \frac{r_c \ell P}{1 - e^{-r_c T_c}}.$$

Homeowners exit at rate ρ, in which case it is assumed they repay their mortgage in full (using the proceeds of selling their property). Hence, there is a probability $e^{-\rho t}$ that the date-t repayment I will be made, and a probability $\rho e^{-\rho t}$ that the whole balance $D(t)$ is repaid at date t. The credit cost χ is the present value of the expected stream of repayments discounted at rate r_f minus the amount borrowed (which would equal the present value of the repayments if $r_c = r_f$ in the absence of credit-market imperfections):

$$\chi = \int_{t=0}^{T_c} e^{-r_f t} e^{-\rho t} I \mathrm{d}t + \int_{t=0}^{T_c} e^{-r_f t} e^{-\rho t} \rho D(t) \mathrm{d}t - \ell P.$$

To derive a formula for χ, first observe that

$$\int_{t=0}^{T_c} e^{-r_f t} e^{-\rho t} \mathrm{d}t = \frac{1 - e^{-(r_f + \rho) T_c}}{r_f + \rho} \quad \text{and} \quad \int_{t=0}^{T_c} e^{-r_f t} e^{-\rho t} e^{r_c t} \mathrm{d}t = \frac{1 - e^{-(r_f + \rho - r_c) T_c}}{r_f + \rho - r_c}.$$

Together with the formulas for $D(t)$ and I, the credit cost can thus be written as follows:

$$
\begin{aligned}
\chi &= \frac{\left(I+\frac{\rho I}{r_c}\right)}{(r_f+\rho)}(1-e^{-(r_f+\rho)T_c}) + \frac{\rho\left(\ell P - \frac{I}{r_c}\right)}{(r_f+\rho-r_c)}(1-e^{-(r_f+\rho-r_c)T_c}) - \ell P \\
&= \left(\frac{(r_c+\rho)(1-e^{-(r_f+\rho)T_c})}{(r_f+\rho)(1-e^{-r_cT_c})} + \frac{\rho\left(1-\frac{1}{1-e^{-r_cT_c}}\right)(1-e^{-(r_f+\rho-r_c)T_c})}{(r_f+\rho-r_c)} - 1\right)\ell P \\
&= \frac{\left((r_c+\rho)(1-e^{-(r_f+\rho)T_c}) - \frac{\rho(r_f+\rho)}{r_f+\rho-r_c}(e^{-r_cT_c}-e^{-(r_f+\rho)T_c}) - (r_f+\rho)(1-e^{-r_cT_c})\right)\ell P}{(r_f+\rho)(1-e^{-r_cT_c})} \\
&= \frac{\left((r_c-r_f) + \frac{\rho(r_f+\rho)-(r_c+\rho)(r_f+\rho-r_c)}{r_f+\rho-r_c}e^{-(r_f+\rho)T_c} - \frac{(r_f+\rho)(r_f+\rho-r_c)-\rho(r_f+\rho)}{r_f+\rho-r_c}e^{-r_cT_c}\right)\ell P}{(r_f+\rho)(1-e^{-r_cT_c})},
\end{aligned}
$$

and dividing both sides by price P and simplifying:

$$
\frac{\chi}{P} = \left(1 + \frac{r_c}{r_f+\rho-r_c}e^{-(r_f+\rho)T_c} - \frac{r_f+\rho}{r_f+\rho-r_c}e^{-r_cT_c}\right)\frac{(r_c-r_f)\ell}{(r_f+\rho)(1-e^{-r_cT_c})}.
$$

This equation is used to determine calibration targets for the marginal credit cost Z relative to the average property price P, and for the marginal credit cost Z relative to the mean credit cost $\bar{\chi}$ conditional on becoming a homeowner.

A mortgage term of 25 years is assumed ($T_c = 25$), and an average loan-to-value ratio of 80% ($\ell = 0.8$). Focusing on interest rates fixed for five years as a typical mortgage product, the 5-year conventional mortgage rate from Stats Canada was 7.07% in 2007. Given an inflation rate of 2.14%, the implied real mortgage rate is set to 4.93% for an average buyer. Information on mortgage spreads is then used to compute credit costs for the marginal buyer. Based on micro-level mortgage data from the Bank of Canada, the average contract mortgage rate during 2017–2018 was around 3.11%. Borrowers with low credit scores and therefore unqualified for major banks could obtain mortgages from Trust Companies or private lenders at mortgage rates of around 6.15%. Hence, a mortgage spread of 3% is assumed, implying the real mortgage rate for the marginal buyer is 7.93%. Since the mean mortgage cost is based on 5-year fixed rates, the equivalent risk-free rate is derived from 5-year government bonds. These had a yield of 4% in 2007, so the real risk-free rate is set to 1.86%.

In summary, $z = Z/P$ is derived from the formula for χ/P using $T_c = 25$, $\ell = 0.8$, $r_f = 1.86\%$, $r_c = 7.93\%$ (marginal), and the value of ρ obtained from the whole calibration method. The value of $Z/\bar{\chi}$ is obtained by taking the ratio of χ/P for $r_c = 7.93\%$ (marginal) and $r_c = 4.93\%$ (mean), with the other terms the same.

A.7 Calibration method

Fraction of investors among buyers Combining equations (18) and (51), the fraction of purchases made by investors is $\kappa = \xi v_o$, where ξ is the fraction of investors among all buyers (see 1) and v_o is average viewings per sale. Given empirical targets for κ and v_o, the required fraction ξ is

$$
\xi = \frac{\kappa}{v_o}. \tag{A.58}
$$

Transaction probabilities and selling and letting rates Using equations (51) and (52) and the value of ξ from (A.58), the targets for V_o, V_l, T_{so}, T_{sl} give the values of:

$$
\pi_o = \frac{v_o^{-1}-\xi}{1-\xi} \quad , \quad \pi_l = \frac{1}{v_l} \quad \text{and} \quad s_o = \frac{1}{T_{so}} \quad , \quad s_l = \frac{1}{T_{sl}}. \tag{A.59}
$$

Uses of the housing stock Equations (50), (52), (A.49), and (A.54) imply $u_o = (T_{so}/((1-\kappa)T_{mo}))h_o$ and $u_l = (T_{sl}/T_{ml})h_l$. The homeownership rate h in (47) satisfies $h_o + (1-\kappa)u_o = \psi h$, and substituting for u_o in terms of h_o yields $(1+T_{so}/T_{mo})h_o = \psi h$. This can be solved for h_o in terms of targets for h, ψ, and the time to move and time on the market. Similarly, by substituting for u_l, $h_l + u_l = (1+T_{sl}/T_{ml})h_l$, and (11) implies $h_l + u_l = 1-(h_o+u_o)$. Putting these equations together yields h_o, u_o, h_l, and u_l:

$$h_o = \frac{\psi h}{1+\frac{T_{so}}{T_{mo}}} \quad , \quad u_o = \frac{T_{so}}{(1-\kappa)T_{mo}} h_o \quad , \quad h_l = \frac{1-(h_o+u_o)}{1+\frac{T_{sl}}{T_{ml}}} \quad , \quad u_l = \frac{T_{sl}}{T_{ml}} h_l \,. \tag{A.60}$$

Exit rate of investors Using equation (A.48), $s_o u_o = (n_o+\rho)h_o + \rho_l(h_l+u_l)$, and solving for ρ_l yields $\rho_l = (s_o u_o - (n_o+\rho)h_o)/(h_l+u_l)$. With $(1-\kappa)s_o u_o = (n_o+\rho)h_o$ from (A.49), it follows that:

$$\rho_l = \frac{\kappa s_o u_o}{h_l+u_l} \,, \tag{A.61}$$

which can be calculated using (A.59) and (A.60).

Market tightness Using equations (19), (52), (53) it follows that $T_{bo} = \theta_o T_{so}$, so θ_o can be deduced from targets for T_{bo} and T_{so}. The definitions in (1) imply that $b_h = (1-\xi)\theta_o u_o$ and $b_l = \theta_l u_l$, and hence equation (12) can be solved for θ_l by substituting for b_o and b_l:

$$\theta_o = \frac{T_{bo}}{T_{so}} \,, \quad \theta_l = \frac{\psi - h_o - h_l - (1-\xi)\theta_o u_o}{u_l} \,, \quad \text{and} \quad T_{bl} = \theta_l T_{sl} \,, \tag{A.62}$$

where the final equation gives the value of T_{bl} using (19), (52), and θ_l and T_{sl}, which cannot be chosen freely given the other targets. With these variables known, the viewing rates for home-buyers and renters follow from (51), (52), and (53):

$$q_o = \frac{v_o}{T_{bo}} \,, \quad q_l = \frac{v_l}{T_{bl}} \,, \quad \text{and} \quad T_{bh} = \left(\frac{1-\xi}{1-\kappa}\right) T_{bo} \,, \tag{A.63}$$

where the final equation is the time-to-buy T_{bh} from (52) for home-buyers implied by the other targets using (18) and (53).

Transitions to homeownership Let ϕ denote the fraction of first-time buyers among home-buyers. Using the law of motion for home-buyers (16), the value of ϕ in a steady state with $\dot{b}_h = 0$ is

$$\phi = \frac{(\gamma n_l h_l + \rho\psi)G_m(Z)}{n_o h_o + (\gamma n_l h_l + \rho\psi)G_m(Z)} = \frac{(q_o\pi_o+\rho)b_h - n_o h_o}{(q_o\pi_o+\rho)b_h} \,.$$

This can be calculated from the ratio of inflows of buyers because all home-buyers transact at the same rate conditional on entering the stock b_h. The second expression for ϕ follows because b_h is a steady state. In steady state, (A.48) implies $(n_o+\rho)h_o = (1-\kappa)s_o u_o$, and (1), (18), and (19) imply $(1-\kappa)s_o u_o = q_o\pi_o b_h$. Dividing numerator and denominator of the expression for ϕ by h_o and substituting $q_o\pi_o b_h/h_o = n_o+\rho$:

$$\phi = \frac{\left(1+\frac{\rho}{q_o\pi_o}\right)(n_o+\rho) - n_o}{\left(1+\frac{\rho}{q_o\pi_o}\right)(n_o+\rho)} \,.$$

Rearranging yields the formula for ϕ in (54), and this can be written in terms of the time to move T_{mo} and home-buyers' time on the market T_{bh} from (50) and (52):

$$\phi = \frac{\rho\left(1+\frac{T_{bh}}{T_{mo}}\right)}{\frac{1}{T_{mo}} + \rho\frac{T_{bh}}{T_{mo}}} \,.$$

This can be rearranged to give the value of ρ in terms of ϕ and other known targets, and with this, the implied value of n_o can also be found from $n_o = (1/T_{mo}) - \rho$:

$$\rho = \frac{\phi}{T_{mo} + (1-\phi)T_{bh}} \,, \quad \text{and} \quad n_o = \frac{(1-\phi)(T_{mo}+T_{bh})}{T_{mo}(T_{mo}+(1-\phi)T_{bh})} \,. \tag{A.64}$$

Using (50), observe that $n_l+\rho = T_{ml}^{-1}$. Taking the value of ρ from (A.64), $n_l = T_{ml}^{-1}-\rho$, and it can be checked whether this is positive. With (34) and ρ_l from (A.61), the parameter $a_l = n_l-\rho_l$ is obtained immediately.

Let g_{ho}, g_{hl}, g_{bh}, and g_{bl} be the average ages of the household heads of those in h_o, h_l, b_h, and b_l, and g_h and g_l the average ages of those in h_o+b_h and h_l+b_l. The calibration target for the difference in the average ages of homeowners and renters is $\alpha = g_h-g_l$. Furthermore, let g_e and g_f denote the average age of new entrants to the city and first-time buyers respectively. Taking the group in h_o+b_h, exit occurs at rate ρ with first-time buyers of measure $\rho(h_o+b_h)$ arriving in steady state. The differential equation for the average age is thus $\dot{g}_h = 1-\rho g_h+\rho g_f$. A steady-state age distribution therefore has $g_h = g_f+\rho^{-1}$. It is convenient to consider all average ages relative to average age at first entry to the city, which are denoted by $\alpha_h = g_h-g_e$, $\alpha_l = g_l-g_e$, and similarly for the other groups. The definition of α and the average homeowner versus first-time buyer result imply:

$$\alpha = \alpha_h-\alpha_l\,, \quad \text{and} \quad \alpha_h = \alpha_f+\rho^{-1}\,. \tag{A.65}$$

Now consider the group h_l. There is exit at rate $n_l+\rho$ and entry $q_l\pi_l b_l/h_l = (n_l+\rho)$ from b_l as a proportion of the group h_l (see A.54 with $q_l\pi_l b_l = s_l u_l$) where the average age at entry is g_{bl}. Thus, $1 = (n_l+\rho)(g_{hl}-g_{bl})$ and hence:

$$\alpha_{hl} = \alpha_{bl}+(n_l+\rho)^{-1}\,. \tag{A.66}$$

Since $g_l = (h_l/(h_l+b_l))g_{hl}+(b_l/(h_l+b_l))g_{bl}$ by definition, it follows that $g_{hl}-g_l = (b_l/(h_l+b_l))(g_{hl}-g_{bl})$, and by using (50) and (A.66):

$$\alpha_{hl} = \alpha_l+\frac{b_l}{h_l+b_l}T_{ml}\,. \tag{A.67}$$

For the group b_l, given (A.57), there are outflows at rate $q_l\pi_l+\rho$, and inflows of proportion $\rho\psi(1-G_m(Z))/b_l$ from outside the city (average age g_e) and of proportion $(1-\gamma G_m(Z))n_l h_l/b_l$ from h_l (average age g_{hl}), hence:

$$1+\frac{\rho\psi(1-G_m(Z))}{b_l}g_e+\frac{n_l(1-\gamma G_m(Z))h_l}{b_l}g_{hl} = (q_l\pi_l+\rho)g_{bl}\,.$$

Using $\rho\psi(1-G_m(Z)) = (q_l\pi_l+\rho)b_l-(1-\gamma G_m(Z))n_l h_l$ from (A.57), this becomes $b_l = (1-\gamma G_m(Z))n_l h_l\alpha_{hl} = (q_l\pi_l+\rho)b_l\alpha_{bl}$. Substituting (A.66) and using (A.57) again leads to $\rho\psi(1-G_m(Z))\alpha_{hl} = b_l+(q_l\pi_l+\rho)b_l T_{ml}$, noting (50). With $\theta_l = b_l/u_l$, $s_l = \theta_l q_l\pi_l$, and $s_l u_l = h_l/T_{ml}$ from (50) and (A.53), $(q_l\pi_l+\rho)b_l T_{ml} = (h_l/T_{ml})T_{ml}+\rho b_l T_{ml} = h_l+\rho b_l T_{ml}$, and by putting these equations together:

$$\alpha_{hl} = \frac{(h_l+b_l)+\rho b_l T_{ml}}{\rho\psi(1-G_m(Z))}\,. \tag{A.68}$$

Finally, consider the ages of first-time buyers. Using (16), a fraction $\gamma n_l h_l G_m(Z)/((\gamma n_l h_l+\rho\psi)G_m(Z))$ come from h_l, and a fraction $\rho\psi G_m(Z)/((\gamma n_l h_l+\rho\psi)G_m(Z))$ are new entrants to the city. Therefore, $g_f = (\gamma n_l h_l/(\gamma n_l h_l+\rho\psi))g_{hl}+(\rho\psi/(\gamma n_l h_l+\rho\psi))g_e$, which can be written as:

$$\alpha_f = \alpha_{hl}-\frac{\rho\psi}{\gamma n_l h_l+\rho\psi}\alpha_{hl} = \alpha_{hl}-\frac{(h_l+b_l)+\rho b_l T_{ml}}{(\gamma n_l h_l+\rho\psi)(1-G_m(Z))}\,, \tag{A.69}$$

where the second expression substitutes from (A.68). Using (A.53) and (A.57) again to write $(\gamma n_l h_l+\rho\psi)(1-G_m(Z)) = (q_l\pi_l+\rho)b_l-(1-\gamma)n_l h_l = s_l u_l+\rho b_l-n_l(1-\gamma)h_l = (n_l+\rho)h_l+\rho b_l-(1-\gamma)n_l h_l = \rho(h_l+b_l)+\gamma n_l h_l$. Substituting this and (A.67) into (A.69):

$$\alpha_f = \alpha_l+\frac{b_l T_{ml}}{h_l+b_l}-\frac{(h_l+b_l)+\rho b_l T_{ml}}{\rho(h_l+b_l)+\gamma n_l h_l} = \alpha_l+\frac{T_{bl}T_{ml}}{T_{ml}+T_{bl}}-\frac{1+\rho\frac{T_{bl}T_{ml}}{T_{ml}+T_{bl}}}{\rho+\gamma n_l\frac{T_{ml}}{T_{ml}+T_{bl}}}\,, \tag{A.70}$$

where the second expression makes use of (A.60) and (A.62). Combining this formula with the two equations in (A.65) and simplifying yields the difference in average ages:

$$\alpha = \left(1+\rho\frac{T_{ml}T_{bl}}{T_{ml}+T_{bl}}\right)\left(\frac{1}{\rho}-\frac{1}{\rho+\gamma n_l\frac{T_{ml}}{T_{ml}+T_{bl}}}\right).$$

This is confirms the expression for α in (48) with reference to (50) and (52). Since ρ is known from earlier, this can be rearranged to give an equation for γ in terms of the targets:

$$\gamma = \frac{\alpha\rho^2(T_{ml}+T_{bl})^2}{((1-\alpha\rho)(T_{ml}+T_{bl})+\rho T_{bl}T_{ml})n_l T_{ml}}. \tag{A.71}$$

Furthermore, the targets pin down the value of $G_m(Z)$. Since $(\gamma n_l h_l + \rho\psi)(1-G_m(Z)) = \rho(h_l+b_l)+\gamma n_l h_l$ as shown above, the value of $G_m(Z)$ must satisfy:

$$G_m(Z) = \frac{\rho\psi - \rho(h_l+b_l)}{\gamma n_l h_l + \rho\psi}, \tag{A.72}$$

and all the terms in this expression are known.

Discount rate and bargaining powers The methodology here is to search over values of the discount rate r to solve one equation. Conditional on r, the bargaining powers ω_o and ω_k can be found as follows.

Dividing both sides of the price equation (A.38) by P, and both sides of the rent equation (A.42) by R and rearranging:

$$\frac{\omega_o^*\Sigma_o}{\pi_o P} = \frac{(1-c_u)r+m-\theta_o q_o\xi\frac{\omega_k^*\Sigma_k}{P}}{r+\theta_o q_o(1-\xi)\pi_o}, \quad \text{and} \quad \frac{\omega_l\Sigma_l}{\pi_l R} = \frac{1-m_l-\omega_l(r+n_l+\rho)(c_l+c_w)}{r+n_l+\rho+\theta_l q_l\pi_l}, \tag{A.73}$$

where $c_u = C_u/P$, $m = M/P$, $m_l = M_l/R$, $c_l = C_l/R$, and $c_w = C_w/R$ are known targets. Using equations (A.38) and (A.45) it follows that $P - P_k = (\omega_o^*\Sigma_o/\pi_o) - \omega_k^*\Sigma_k$, and hence $p_k = P_k/P$ satisfies the following equation:

$$1-p_k = \frac{\omega_o^*\Sigma_o}{\pi_o P} - \frac{\omega_k^*\Sigma_k}{P}, \quad \text{with} \quad \frac{\omega_k^*\Sigma_k}{P} = \frac{f_{kh}f_h}{q_o}\frac{\omega_k^*}{1-\omega_k^*}, \tag{A.74}$$

where the expression for $\omega_k^*\Sigma_k/P$ comes from (43) and the definitions of the targets $f_h = F_h/P$ and $f_{kh} = F_k/F_h$. Substituting for $\omega_o^*\Sigma_o/(\pi_o P)$ from (A.73) in the first equation of (A.74) implies $(r+\theta_o q_o(\xi+(1-\xi)\pi_o))(\omega_k^*\Sigma_k/P) = (1-c_u)r+m+(p_k-1)(r+\theta_o q_o(1-\xi)\pi_o)$, and then using the second part of (A.74):

$$\frac{\omega_k^*}{1-\omega_k^*} = \frac{q_o}{f_{kh}f_h}\frac{(1-c_u)r+m+(p_k-1)(r+\theta_o q_o(1-\xi)\pi_o)}{r+\theta_o q_o(\xi+(1-\xi)\pi_o)}. \tag{A.75}$$

This can be calculated using r, the targets, and other variables known so far. Since (40) implies $\omega_k^*/(1-\omega_k^*) = (\omega_k/(1-\omega_k))/(1+\tau_k)$, the implied seller bargaining power when facing an investor is $\omega_k = (\omega_k^*/(1-\omega_k^*))/((1/(1+\tau_k))+(\omega_k^*/(1-\omega_k^*)))$.

With ω_k and ω_k^* known conditional on r, substituting the second equation from (A.74) into (A.73) yields:

$$\frac{\omega_o^*\Sigma_o}{\pi_o P} = \frac{(1-c_u)r+m-\theta_o\xi f_{kh}f_h\frac{\omega_k^*}{1-\omega_k^*}}{r+\theta_o q_o(1-\xi)\pi_o}, \quad \text{and} \quad \frac{\omega_l\Sigma_l}{\pi_l R} = \frac{1-m_l-\omega_l(r+T_{ml}^{-1})(c_l+c_w)}{r+T_{ml}^{-1}+T_{sl}^{-1}}, \tag{A.76}$$

where the formula for $\omega_l\Sigma_l/(\pi_l R)$ uses (19), (50), and (52). Conditional on r, both expressions above are now known for given targets. Dividing both sides of the marginal first-time buyer indifference condition (A.44) by average price P gives $((1-\omega_o^*)/\omega_o^*)q_o\pi_o(\omega_o^*\Sigma_o/(\pi_o P)) = ((1-\omega_l)/\omega_l)(q_l\pi_l p_k/p_r)(\omega_l\Sigma_l/(\pi_l R)) + (r+\rho)z + f_h - f_{wh}f_h$, where $p_r = P_k/R$ is the price-rent ratio, and $z = Z/P$, and $f_{wh} = F_w/F_h$ are other targets. Noting that $(1-\omega_o^*)/\omega_o^* = (1+\tau_h)(1-\omega_o)/\omega_o$ is implied by (27) and using (52), this equation can be solved for $\omega_o/(1-\omega_o)$ with ω_l being a calibration target and by using the known values of (A.76):

$$\frac{\omega_o}{1-\omega_o} = \frac{\frac{(1+\tau_h)}{T_{bh}}\left(\frac{\omega_o^*\Sigma_o}{\pi_o P}\right)}{\frac{p_k}{p_r T_{bl}}\left(\frac{1-\omega_l}{\omega_l}\right)\left(\frac{\omega_l\Sigma_l}{\pi_l R}\right)+(r+\rho)z+(1-f_{wh})f_h}. \tag{A.77}$$

Hence, sellers' bargaining power when faced with a home-buyer is given by $\omega_o = (\omega_o/(1-\omega_o))/(1+(\omega_o/(1-\omega_o)))$.

Next, taking the free entry condition (A.43) and dividing both sides by P:

$$\frac{\theta_l q_l \pi_l p_k}{p_r}\left(\frac{\omega_l \Sigma_l}{\pi_l R}\right) = \left(1+\tau_k\left(1+\frac{\rho_l}{r}\right)\right)\theta_o q_o\left((1-\xi)\pi_o\left(\frac{\omega_o^* \Sigma_o}{\pi_o P}\right)+\xi\left(\frac{\omega_k^* \Sigma_k}{P}\right)\right)$$
$$+(r+\rho)\left((1+\tau_k)c_u + c_k p_k + (1+\tau_k\omega_k^*)\frac{\Sigma_k}{P}\right) - \tau_k\left(1+\frac{\rho_l}{r}\right)m\,,$$

noting the definition $c_k = C_k/P_k$. Substituting for Σ_k/P using (A.74) and solving for the price-rent ratio p_r:

$$p_r = p_k \theta_l q_l \pi_l\left(\frac{\omega_l \Sigma_l}{\pi_l R}\right)\left(\left(1+\tau_k\left(1+\frac{\rho_l}{r}\right)\right)\theta_o q_o(1-\xi)\pi_o\frac{\omega_o^*\Sigma_o}{\pi_o P} + (r+\rho)\left((1+\tau_k)c_u + c_k p_k\right)\right.$$
$$\left. + \left(r+\rho+\left(1+\tau_k\left(1+\frac{\rho_l}{r}\right)\right)\frac{\theta_o q_o \xi\,\omega_k}{1+\tau_k}\right)\frac{f_{kh} f_h}{(1-\omega_k)q_o} - \tau_k\left(1+\frac{\rho_l}{r}\right)m\right)^{-1}, \quad \text{(A.78)}$$

which uses $\omega_k^*/(1-\omega_k^*) = (\omega_k/(1-\omega_k))/(1+\tau_k)$ and $(1+\tau_k\omega_k^*)/\omega_k^* = (1+\tau_k)/\omega_k$. The formula for p_r depends on known calibration targets and r, and as p_r is itself a target, equation (A.78) can be solved numerically to determine the discount rate r.

Meeting functions With ω_o and ω_l known, the meeting function elasticities η_o and η_l are derived from the calibration targets for ω_o/η_o and ω_l/η_l. Since market tightnesses θ_o and θ_l are already determined in (A.62) and the viewing rates in (A.63), the meeting function efficiency parameters A_o and A_l are those satisfying (24):

$$A_o = q_o \theta_o^{\eta_o} \quad \text{and} \quad A_l = q_l \theta_l^{\eta_k}\,.$$

Ownership-market match quality distribution and idiosyncratic shocks All payoffs can be scaled without loss of generality so that the minimum new match quality in the ownership market is 1, hence $\zeta_o = 1$. A new variable β_o is introduced at this stage, which is defined as follows in terms of the other parameters and endogenous variables of the model:

$$\beta_o = \frac{\lambda_o a_o \delta_o^{\lambda_o}\left(\frac{y_o}{x_o}\right)^{\lambda_o}}{\rho + a_o\left(1-\delta_o^{\lambda_o}\right)}\,. \quad \text{(A.79)}$$

Suppose for now there is a target value of β_o alongside the other targets. At the final stage of the calibration, the econometric evidence on the response of transactions to the LTT change is used to determine β_o.

There is a numerical procedure to determine the arrival rate a_o of idiosyncratic shocks. Equation (49) implies the steady-state moving rate n_o can be written in terms of a_o, β_o, and λ_o:

$$n_o = \frac{a_o - \rho\frac{\beta_o}{\lambda_o}}{1+\frac{\beta_o}{\lambda_o}}\,.$$

Conditional on a value of a_o, the value of λ_o is found by solving this equation:

$$\lambda_o = \frac{(n_o+\rho)\beta_o}{a_o - n_o}\,, \quad \text{(A.80)}$$

using the provisional target for β_o and the values of ρ and n_o from (A.64). Next, take equation (A.36) and divide both sides by P, and make use of the second equation in (A.74):

$$\frac{x_o}{P} = \frac{(1-\omega_o^* + (1-\xi)\omega_o^*\theta_o)q_o\pi_o}{\omega_o^*}\left(\frac{\omega_o^*\Sigma_o}{\pi_o P}\right) + \frac{\omega_k^*}{(1-\omega_k^*)}\xi\,\theta_o f_{kh} f_h - f_h\,. \quad \text{(A.81)}$$

Similarly, dividing both sides of (A.35) by P:

$$\frac{y_o}{P} = \frac{x_o}{P} + (r+\rho+a_o)\left(\frac{\tau_h}{r}\left((1-\xi)\theta_o q_o \pi_o \frac{\omega_o^*\Sigma_o}{\pi_o P} + \xi\,\theta_o f_{kh} f_h \frac{\omega_k^*}{(1-\omega_k^*)}\right)\right.$$
$$\left. + c_h + (1+\tau_h)c_u - \tau_h \frac{m}{r}\right), \quad \text{(A.82)}$$

where (A.74) has been used again. Together, (A.81) and (A.82) give $y_o/x_o = (y_o/P)/(x_o/P)$ in terms of a_o and the calibration targets. With λ_o from (A.80) and y_o/x_o, equation (A.79) can be rearranged to solve for the idiosyncratic shock size parameter δ_o:

$$\delta_o = \left(\frac{\left(1+\frac{\rho}{a_o}\right)\beta_o}{\beta_o + \lambda_o\left(\frac{y_o}{x_o}\right)^{\lambda_o}}\right)^{\frac{1}{\lambda_o}}.$$

Using $\pi_o = (\zeta_o/y_o)^{\lambda_o}$ from (30), it follows that $y_o = \zeta_o/\pi_o^{1/\lambda_o}$, so y_o is known given ζ_o, λ_o, and π_o from (A.59). By using the ratio y_o/P in (A.82), the price P is obtained, and hence the cost parameters $C_h = c_h P$, $C_u = c_u P$, $C_k = c_k p_k P$, $F_h = f_h P$, $F_k = f_{kh} F_h$, and $M = mP$ are found. Furthermore, x_o is derived from (A.81) and Σ_o from the known value of $\omega_o^*\Sigma_o/(\pi_o P)$ in (A.76). Since these parameters are all computed conditional on a conjectured value of a_o, the value of a_o is verified by a numerical search to check where the equation for Σ_o in (A.37) holds. The requirement $\delta_o y_o < x_o$ can also be verified at this stage.

Distribution of credit costs The value of $G_m(Z)$ has already been determined in (A.72). Using (26), the marginal credit cost Z and the parameters μ and σ of the probability distribution satisfy:

$$\frac{\log Z - \mu}{\sigma} = \Phi^{-1}(G_m(Z)).$$

Using (26) to obtain an equation for $\log\bar{\chi}$ and subtracting this from $\log Z = \mu + \sigma\Phi^{-1}(G_m(Z))$:

$$\log\left(\frac{Z}{\bar{\chi}}\right) = \log G_m(Z) - \log\Phi\left(\Phi^{-1}(G_m(Z)) - \sigma\right) + \sigma\Phi^{-1}(G_m(Z)) - \frac{\sigma^2}{2},$$

noting that μ cancels out. Using the known value of $G_m(Z)$ and the target for $Z/\bar{\chi}$, this equation can be solved numerically to find the standard deviation parameter σ. Note that $Z = zP$ using the known value of P and the target for z. Together with σ consistent with the equation above, the value of the mean parameter is $\mu = \log Z - \sigma\Phi^{-1}(G_m(Z))$. The implied value of $\bar{\chi}$ follows from the target $Z/\bar{\chi}$ and Z.

Rental-market parameters Since P was determined earlier, the target for p_k determines the price paid by investors $P_k = p_k P$, and the target for p_r determines the average rent $R = P_k/p_r$. The targets for m_l, c_l, and c_w then imply values of the cost parameters $M_l = m_l R$, $C_l = c_l R$, and $C_w = c_w R$. The target for f_{wh} gives $F_w = f_{wh} F_h$ using the value of F_h obtained earlier.

With π_l known from (A.59), the value of Σ_l can be deduced from (A.76) using R and the calibration target ω_l. Equation (A.41) then implies $y_l = M_l - F_w + (r+n_l+\rho)(C_l + C_w) - \gamma n_l G(Z)(Z-\bar{\chi}) + (1-\omega_l+\omega_l\theta_l)q_l\Sigma_l$. Since $\pi_l = (\zeta_l/y_l)^{\lambda_l}$ from (38), equation (A.28) can be rearranged to solve for λ_l in terms of y_l, π_l, and Σ_l:

$$\lambda_l = 1 + \frac{\pi_l y_l}{(r+\rho+n_l)\Sigma_l}.$$

Knowing λ_l allows the rental-market minimum new match quality parameter to be deduced from the equation for π_l as $\zeta_l = y_l \pi_l^{1/\lambda_l}$.

Response of transactions to the land transfer tax Conditional on a value of β_o from (A.79), all of the other targets have been matched. A numerical search over β_o values is then used to match the model's predicted response of transactions to the land transfer tax with the econometric evidence.