Inflation, Skill Loss During Unemployment, and TFP in the Long Run*

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Abstract

We develop a search model with frictional goods and labor markets to study the long run relationship between inflation, unemployment, and TFP when workers lose skills during unemployment. As inflation increases, fewer jobs are created, workers experience longer unemployment durations and their skills deteriorate, causing TFP to decline. Our calibrated results show that transitioning from the Friedman rule to 10% annual inflation lowers TFP by 4.3%. A stochastic version of the model demonstrates that prolonged periods of inflation, such as the 1970s in the US, can have lasting negative effects on productivity.

JEL Classification: E24, J24, E40, D24

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1 Introduction

The relationship between inflation and labor market performance has been widely studied in macroeconomics, where labor market performance is typically measured by the unemployment rate. While this literature is vast and dates back to Phillips (1958), the connection between inflation and unemployment continues to be widely discussed as the Federal Reserve weighs the potential trade-offs of allowing inflation to run above the target inflation rate. Alongside this literature and policy discussions, a large body of evidence has been documented that workers lose skills during unemployment.[1] When workers lose skills during unemployment, the economy's productivity becomes endogenous as the aggregate stock of human capital is a function of labor market flows (Ortego-Marti, 2017b, 2020). From a macroeconomic perspective, skill loss during unemployment provides a channel through which inflation can impact the economy's aggregate human capital and TFP. The goal of this paper is to develop a framework which captures and quantifies the long run relationship between inflation, unemployment, and an additional measure of labor market performance, TFP, when workers lose skills during unemployment.

We build a microfounded model of money with frictional goods and labor markets as in Berentsen et al. (2011). The frictional labor market follows the Diamond (1982) and Mortensen and Pissarides (1994) model of equilibrium unemployment, whereas the monetary side of the model follows the New Monetarist literature (Lagos and Wright, 2005; Rocheteau and Wright, 2005; Lagos et al., 2017). Firms post vacancies in order to hire workers. Those firms who match with a worker can sell their output at a markup in a subsequent retail market where there is anonymity and lack of commitment, which makes a means of payment (fiat money) essential. As households use real balances to consume in the retail market, there is a direct linkage between the value of money and the expected revenue of filling a vacancy. The key ingredient in our baseline framework is that workers are heterogeneous in the skill level and that highly-skilled workers who do not find a job are subject to skill loss shocks. TFP, as measured by average labor productivity, becomes function of both the skill composition of unemployed workers and net output produced in the retail market.

An escalation in the inflation tax impacts TFP through two channels. First, as is standard in monetary search models, higher inflation causes households to hold less real balances, thereby reducing their consumption in the retail market. This reduction in demand decreases the expected productivity of a match, as the net output generated by selling output in the retail market falls. Second, as the expected productivity of a vacancy decreases, firms create less vacancies and workers' job finding probability decreases. When workers face longer unemployment durations, they are more exposed to skill loss shocks and the skill composition of the work force deteriorates. As the skill composition decays, the average production in the labor market also decreases. Ultimately, TFP falls due to declines in both the net output generated by the retail market and the average skill level of the workforce.

Next, motivated by evidence showing that occupations which require more skills experience higher rates of skill decay (Ortego-Marti, 2017a), we extend our model to include heterogeneous firms. Firms can create a simple or complex vacancy as in Albrecht and Vroman (2002), where there are two distinctions between simple and complex jobs. First, highly-skilled workers produce more output in complex jobs. Second, the productivity of highly-skilled workers who are hit with a skill loss shock declines more in complex than simple jobs. In this version of the model, the aggregate stock of vacancies and the composition of job complexity (i.e., the fraction of jobs that are complex) are endogenous and jointly determined with the value of money and skill composition of unemployed workers. Therefore, TFP becomes a function of the composition of job

[1] In Section 2.3, we review the empirical evidence documenting skill loss during unemployment.

complexity in addition to the skill composition of the unemployed and value of money. A hike in anticipated inflation now has an adverse impact on the creation of complex vacancies, as skill loss has a larger impact on productivity in complex jobs. Thus, the total effect of increasing inflation on TFP can be decomposed into three components: (i) a worsening of the skill distribution, (ii) a reduction in the net output generated by the retail market, and (iii) a shift in the composition of job complexity from complex to simple jobs.

We calibrate the model to the US economy between 1955-2017 to quantify the effects of anticipated inflation on unemployment, the skill composition of unemployed workers, composition of job complexity, and TFP. We also use the calibrated model to decompose changes in TFP into the aforementioned three channels. A key target in our calibration comes from the empirical literature which has estimated the effect of an additional month of unemployment duration on reemployment wages. We choose the skill loss parameters so that when we simulate employment histories, the effects of unemployment history on wages are consistent with the empirical evidence. We show that the model can match this evidence and other targets commonly used in the labor and monetary search literatures. Additionally, the calibrated model generates an upward sloping long-run Phillips curve as in Berentsen et al. (2011) and is consistent with the evidence presented in Section 2.1. The model also generates a negative relationship between anticipated inflation, the work force's average skill level, and the fraction of jobs which are complex.

In our main quantitative exercises, we estimate the productivity costs of inflation. To begin, we vary the nominal interest rate from its minimum value observed between 1955-2017 (a level that is consistent with 1% annual inflation) to its maximum (consistent with 11% annual inflation). The model implies that such an escalation in inflation reduces TFP by nearly 4%.[2] Our results show that the effect is non-linear, as TFP is most sensitive to changes in inflation at low rates of inflation. This is also illustrated by comparing an economy at the Friedman Rule to one with 10% annual inflation, where our results show that such an increase in inflation causes the unemployment rate to rise from 5.1% to 6.2% and TFP to decrease by 4.3%. When we decompose the productivity costs of inflation into the three channels, we find that shifts in the skill composition of workers accounts for nearly 75% of TFP losses, whereas the shift from complex to simple jobs accounts for 15% and the remaining 10% is attributable to the decline in retail market net production.

While our main quantitative exercises highlight the contribution of changes in the skill composition of unemployed workers to the productivity costs of inflation, these exercises also compare different steady-states. As a final exercise, we solve the version of the model where anticipated inflation (as measured by the nominal interest rate) follows a stochastic process and present the resulting simulations after feeding nominal interest rate data between 1955-2017 through the model.[3] The stochastic version of the model generates an increase in trend unemployment through the 1970s and 1980s as anticipated inflation increased in the US. At the same time, the skill composition of the unemployed decayed and composition of job complexity shifted from complex to simple jobs. We find that TFP reached its lowest level in 1985, with a 0.82% decline relative to its level in 1955. At this level, the contribution of the three channels are relatively equal showing that changes in the skill composition of the unemployed cause larger productivity losses and account for a higher share of productivity losses following a permanent increase in anticipated inflation. The simulations also show there are scarring effects of prolonged periods of high inflation, as TFP remains depressed even after the unemployment rate has begun to recover.

The rest of this paper is organized as follows. Section 1.1 discusses our contributions relative to the related

[2] We provide four alternative calibrations in Appendix C and show that the productivity costs of inflation are largely unchanged.

[3] A primary motivation for this exercise is that the economy may not quickly transition between steady-states following a change in the nominal interest rate, as it takes time for the skill composition of unemployed workers to evolve.

literature while Section 2 presents some motivating data. Section 3 introduces the baseline environment and Section 4 discusses the equilibrium. Section 5 provides an extension with heterogeneous firms. Section 6 performs the quantitative analysis. Finally, Section 7 concludes. All theoretical proofs, empirical details, and quantitative technicalities are delegated to the Appendix.

1.1 Related Literature

This paper is most closely related to Berentsen et al. (2011), who find a positive correlation between anticipated inflation and unemployment in the US between 1955-2005. To explain this relationship, the authors developed a microfounded framework which integrated the Diamond (1982) and Mortensen and Pissarides (1994) model of equilibrium unemployment with the monetary search framework of Lagos and Wright (2005).[4] The inflation tax causes households to carry less real balances across periods, which generates a decline in demand for firm's output and therefore a decline in the expected revenue of a filled job. Following the decline in revenue, firms post less vacancies and unemployment increases.[5] We build on Berentsen et al. (2011) in two ways. The first is to incorporate skill loss during unemployment, which allows for us to study the relationship between inflation, unemployment, and TFP. Further, we incorporate heterogeneous firms, allowing for a decomposition of the effect of the inflation tax on TFP into three channels: net output, the skill composition of the unemployed, and composition of job complexity.

More broadly, our paper contributes to the literature studying the effect of inflation on labor market performance. While this literature is large, our paper is most closely related to studies focusing on the long run relationship between anticipated inflation and labor market performance.[6] This literature dates back to Friedman (1977), who proposed that the long run Phillips curve may be upward sloping due to the distortionary inflation tax. More recently, Ait Lahcen et al. (2021) show that higher inflation increases both the level and volatility of unemployment. A vast majority of this literature measures labor market performance via the unemployment rate while emphasizing how additional payment and financial frictions interact with the effect of inflation on unemployment.[7] For example, Gu et al. (2019) focused on financial frictions faced by firms while Bethune et al. (2015) studied the relationship between liquidity, household borrowing, and unemployment. A recent exception is Gomis-Porqueras et al. (2020), who studied the long run relationship between inflation, unemployment, and capital. Our contribution to this literature is to focus on unemployment and TFP, as measured by average labor productivity.

Our paper builds on previous work on skill loss during unemployment. Two influential papers are Pissarides (1992) and Ljungqvist and Sargent (1998). Pissarides (1992) showed that unemployment is more persistent when unemployed workers lose skills during unemployment, whereas Ljungqvist and Sargent (1998) argued that generous UI benefits in conjunction with skill loss provides an explanation for high unemployment in Europe relative to the US. Kospentaris (2021) decomposed the effects of longer unemployment durations on job-finding rates and reemployment wages into unobserved heterogeneity and skill loss dur-

[4] See Rocheteau and Nosal (2017) and Lagos et al. (2017) and references therein for a survey of the New Monetarist approach.

[5] Additional, but closely frameworks developed to study the relationship between inflation and unemployment include Rocheteau et al. (2007) and Dong (2011), whose model of unemployment follows Rogerson (1988). Rocheteau et al. (2021) study equilibria in which it takes multiple periods for unemployed workers to reach their optimal holdings of real balances, thereby generating heterogeneity in money holdings across employment statuses. Lehmann (2012) extend the Mortensen and Pissarides (1994) to introduce payment frictions in the product market by assuming matches in the labor market on separate islands produce distinct goods which are non-storable, non-transportable, and are not consumed by their producers.

[6] There is an interesting and closely literature which has studied the relationship the provision of public liquidity on unemployment. See, for example, Rocheteau and Rodriguez-Lopez (2014) and Dong and Xiao (2019).

[7] A complementary study is Berentsen et al. (2012), who find that countries with higher inflation rates tend to have lower growth rates. While our focus is not on growth, our findings are related in the sense that we study a channel (skill loss during unemployment) through which inflation lowers productivity.

ing unemployment. Ortego-Marti (2017b, 2020) illustrated how loss of skill during unemployment impacts TFP while Doppelt (2019) proposed skill loss during unemployment as a mechanism to explain long-run relationship between growth and unemployment. However, none of these papers incorporate skill loss into a microfounded model of money and unemployment.

2 Motivating Evidence

This section reviews some motivating empirical evidence. Section 2.1 examines the slope of the long run Phillips curve. Section 2.2 discusses the relationship between low frequency trends in unemployment and productivity growth. Section 2.3 summarizes the empirical evidence on skill loss during unemployment. Section 2.4 studies the relationship between low frequency trends in inflation and productivity growth. Empirical details are delegated to Appendix B.

2.1 Long Run Phillips Curve

To begin, Figure 1 uses US data from 1955-2017 and presents scatter plots containing the relationship between average annual unemployment and anticipated inflation, as measured by nominal Aaa corporate bond interest rates. Following Berentsen et al. (2011), we remove higher frequency fluctuations by applying an HP filter with a larger smoothing parameter while the last panel presents five-year averages. Figure 1 clearly shows a positive relationship between the lower-frequency movements in unemployment and anticipated inflation, a correlation we aim to capture in our theory. Figure 1 in Appendix B.3 shows a similar relationship between unemployment and realized inflation. Further evidence supporting an upward sloping long run Phillips curve is provided by Haug and King (2014), who used a strategy centered around the band-pass filter, and Ait Lahcen et al. (2021) who use panel data from OECD countries.

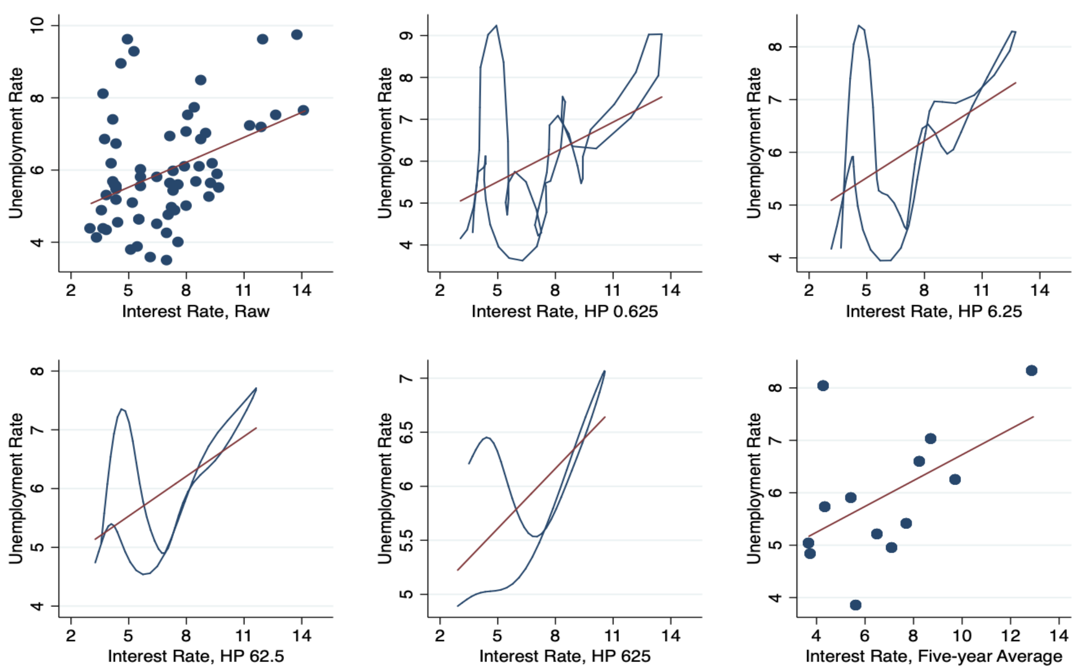


Figure 1: Interest Rate and Unemployment

2.2 Unemployment and TFP

The second fact which motivates our model is the relationship between unemployment and productivity growth at the macro level. Doppelt (2019) found a negative correlation between low frequency movements in unemployment and productivity growth. We replicate Doppelt (2019)'s finding in Figure 2 where we present the HP filtered trends in the unemployment rate and TFP growth rate.[8] The correlation between the two series is -0.5481. Further evidence on the negative relationship between unemployment and productivity growth can be found in cross-country studies. See, for example, Pissarides and Vallanti (2007).

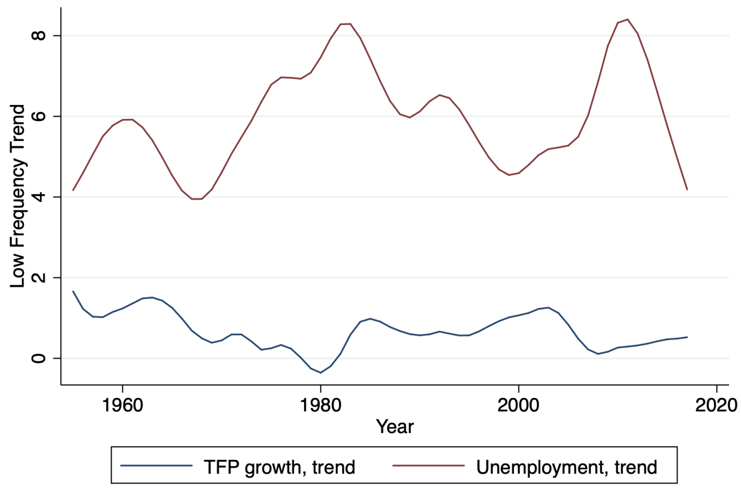


Figure 2: Low Frequency TFP Growth and Unemployment

Doppelt (2019) proposed skill loss during unemployment as a channel in generating this relationship. If workers lose skills during unemployment, then we would expect slower productivity growth during times of high unemployment, as the aggregate stock of human capital is lower. While intuitive, there is a large body of empirical evidence which documents skill loss during unemployment. We now briefly review this evidence.

2.3 Empirical Evidence on Skill Loss During Unemployment

Our objective is to summarize a few recent findings from the literature documenting skill loss during unemployment and the effect of job displacement on wages. For more thorough reviews of this vast literature, we refer the reader to Fallick (1996), Kletzer (1998), and Schmieder et al. (2016).[9] In series of papers, Ortego-Marti (2016, 2017a,b) estimated skill loss during unemployment using the Panel Study of Income Dynamics (PSID). An advantage of the PSID's panel structure is it allows for worker fixed effects to be controlled for. Moreover, one can study how wage losses depend on unemployment duration. Ortego-Marti (2016, 2017b) estimate that an additional month of unemployment is associated with a 1.22% wage loss, which is consistent with other estimates from the job displacement literature. For example, Schmieder et al. (2016) estimate the causal effect of unemployment duration on wages using German administrative data. Without restricting workers' prior experience, the authors estimate that an additional month of unemployment reduces wages

[8] Alternatively, one can study the low frequency movements between unemployment and labor productivity growth. We present this in Figure 15 in Appendix B.3. The correlation between these two series is -0.7279.

[9] Additionally, a few prominent studies in the job displacment literature include Couch and Placzek (2010), Davis and von Wachter (2011), Jacobson et al. (1993), Jarosch (2021), Schmieder et al. (2016), von Wachter et al. (2009), Ruhm (1991), Stevens (1997), Addison and Portugal (1989), Carrington (1993), and Farber (1997).

by 1.3%.[10] There is also a literature which provides more direct evidence of skill loss during unemployment. Edin and Gustavsson (2008), using test score data from Sweden, find that a full year of non-employment is associated with a loss of the equivalent of 0.7 years of schooling.[11]

Our theory assumes there are long-lasting effects of unemployment durations on human capital. This is supported by evidence which shows skill losses are extremely persistent. Ortego-Marti (2016) shows that a month of unemployment accumulated in the previous 5 years lowers wages by 1.61% whereas a month of unemployment experienced more than 5 years ago still decreases wages by 1.04%. Additionally, Davis and von Wachter (2011) and Jarosch (2021) find that wage losses follow workers for more than 20 years.

2.4 Inflation and TFP

Finally, Figure 3 presents scatter plots between the nominal Aaa corporate bond interest rate and the TFP growth rate. As we progressively remove higher-frequency observations, there is a negative relationship between lower-frequency changes in anticipated inflation and TFP growth.[12] This is consistent with evidence presented by Berentsen et al. (2012) showing that countries with higher inflation rates tend to have lower growth rates. Additionally, this relationship is consistent with the upward sloping Phillips curve and the literature cited in Section 2.2 showing a negative relationship between unemployment and productivity growth. Our theory aims to capture and quantify the impact of inflation on TFP through the skill loss channel which, as shown in Section 2.3, has a plethora of empirical support.

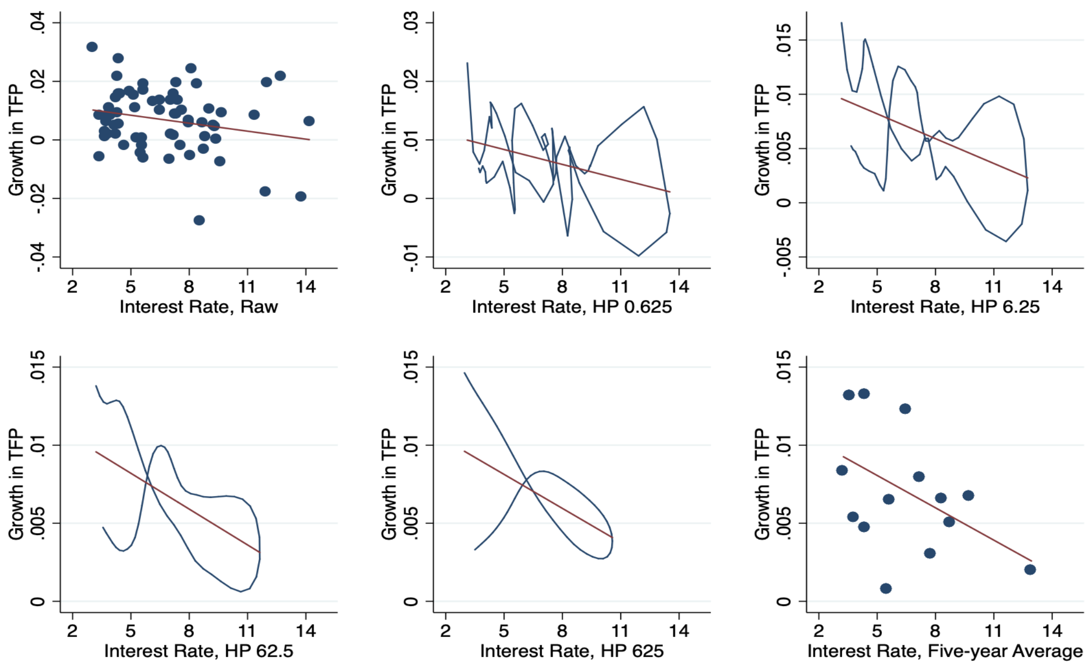


Figure 3: Interest Rate and TFP Growth

[10] Neal (1995) and Addison and Portugal (1989) estimate depreciation rates between $1.44 - 1.59\%$.

[11] Studies documenting skill loss due to breaks in production include David and Brachet (2011); Hockenberry et al. (2008); Hockenberry and Helmchen (2014), Globerson et al. (1989); Bailey (1989) and Shafer et al. (2001).

[12] Appendix B.3 presents additional sets of results by examining the relationship between inflation and TFP growth and by also considering the relationship between inflation/interest rates and labor productivity growth.

3 Environment

Time is discrete and goes on forever. There are two types of agents indexed by $j \in \{h, f\}$: a measure 1 of households, h, and a large measure of firms, f, where the measure of active firms is endogenous.[13] Each period is divided into three stages. In stage 1, households and firms trade labor services and produce a general good in a decentralized labor market. In stage 2, households and firms trade specialized goods in a retail market. In stage 3, agents trade fiat money and the general good in a frictionless centralized market. The general good is taken as the numéraire. All goods are non-storable across time periods.[14] The sequence of markets within a representative time period is summarized in Figure 4.

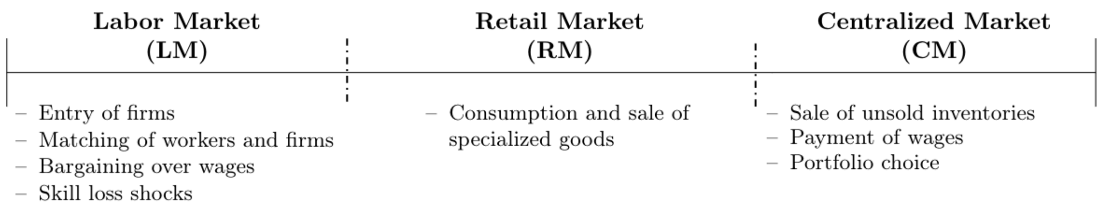


Figure 4: Timing of a representative period

Households are heterogeneous in their skill due to skill loss during unemployment. There are two levels of skill indexed by $\varepsilon \in \{L, H\}$: low (L) and high (H). Employed high skill workers produce y units of output per period. Low skill workers produce δy with $\delta \in (0, 1)$. Skill loss occurs as follows. At the beginning of each period, firms post vacancies and hire workers. After hiring takes place, high skill households who entered the period unemployed and did not find a job become permanently low-skilled with probability σ.

The household's lifetime discounted utility is given by

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t [\epsilon_t \upsilon(q_t) + x_t], \tag{1}$$

where $\beta = (1+\rho)^{-1} \in (0, 1)$ is a discount factor, $q_t \in \mathbb{R}_+$ is consumption of the specialized good, $\epsilon_t \in \{0, 1\}$ is a preference shock for the specialized good, and $x_t \in \mathbb{R}_+$ is consumption of the numéraire. We assume $\upsilon' > 0 > \upsilon''$, $\upsilon'(0) = \infty$, $\upsilon'(\infty) = 0$, and $\upsilon(0) = 0$. The preference shocks, $\{\epsilon_t\}_{t=0}^{\infty}$, are i.i.d. across agents and time with $\Pr[\epsilon_t = 1] = \alpha$ and $\Pr[\epsilon_t = 0] = 1 - \alpha$.

Households and firms meet each other in stage 1 according to a constant returns to scale meeting technology $\mathcal{N}(u, v)$ where u is the stock of unemployed workers and v is the stock of vacancies. The probability that an unemployed household meets a firm is given by $\xi_h = \mathcal{N}(u, v)/u = \mathcal{N}(1, \theta)$ where $\theta = v/u$ is labor market tightness. We assume that $\mathcal{N}(1, \theta)$ is strictly increasing in θ with $\lim_{\theta \to 0} \xi_h = 0$, and $\lim_{\theta \to \infty} \xi_h = 1$. Similarly, the probability that a firm meets a worker is given by $\xi_f = \mathcal{N}(u, v)/v = \mathcal{N}(1, \theta)/\theta$. We assume that $\mathcal{N}(1, \theta)/\theta$ is strictly decreasing in θ with $\lim_{\theta \to 0} \xi_f = 1$ and $\lim_{\theta \to \infty} \xi_f = 0$. Existing matches in the labor market are destroyed at the beginning of stage 1 with probability $\lambda \in [0, 1]$. Those whose match is destroyed in period t can not be matched again until period $t + 1$.

There is anonymity and lack of commitment in the retail market which makes a means of payment

[13] Throughout the analysis, we use 'household' and 'worker' interchangeably.

[14] This assumption is crucial in motivating money's role as a medium of exchange. If goods were storable, households could carry them into the next retail market and use them in quid pro quo exchanges. Hence, to focus on money's role as a medium of exchange, we assume all goods are non-storable.

essential (Kocherlakota, 1998). We further assume that fiat money is always recognizable and cannot be counterfeited, whereas counterfeit claims to real assets (e.g., claims on firms' profits) cannot be recognized, leaving money as the medium of exchange in the retail market.[15] Firms who produced in stage 1 can sell q units of their inventory in stage 2 at cost $c(q)$ where $c' > 0$ and $c'' > 0$.

In stage 3, unemployed households are paid an unemployment benefit b, where $b < \delta y$, while employed households are paid their wage.[16] All households pay lump-sum taxes and firm's profits are paid out as dividends to households. Vacant firms can pay k units of the numéraire to enter the next labor market with a vacancy and agents have the opportunity to accumulate real balances. To keep the distribution of skills stationary, we assume that between periods a fraction $\mu \in (0, 1)$ of workers leave the labor force and that a measure μ of workers enter the labor force as unemployed who are highly-skilled. We assume the real balances among those who leave the labor force are equally redistributed among the new entrants.

There is a government who finances government expenditures, G, and unemployment benefits b, by levying lump-sum taxes on households in stage 3 and by printing fiat money at rate π.

4 Equilibrium

Sections 4.1 and 4.2 describe the value functions and optimization problems agent faces throughout the three stages. Section 4.3 solves for the equilibrium in the retail market. Section 4.4 solves the wage bargaining problem in the labor market and describes the entry of firms. Section 4.5 characterizes the set of stationary equilibria. Finally, Section 4.6 discusses the relationship between inflation and TFP.

Throughout the analysis, we focus on stationary equilibria where aggregate real balances are constant across time. Letting ϕ_t denote the price of money in terms of the numéraire, it follows that $\phi_t = (1+\pi)\phi_{t+1}$ in a stationary equilibrium and the real gross rate of return of money is $1 + r = \phi_{t+1}/\phi_t = 1/(1 + \pi)$.

4.1 Households

As is common in monetary search models, we solve the model through backwards induction, beginning with stage 3.

Stage 3: Centralized Market

Consider a type $\Omega \in \{L, H\} \times \{0, 1\}$ household where $\Omega = (\varepsilon, 0)$ denotes an unemployed household of skill level ε and $\Omega = (\varepsilon, 1)$ denotes a worker of skill level ε who is employed. The value of household with real balances z in the centralized market is given by

$$W_\Omega(z) = \max_{x,z'} \left\{x + \bar{\beta} U_\Omega(z')\right\}, \tag{2}$$

$$\text{s.t.} \quad x + \frac{z'}{1+r} = w_\Omega + \Delta + z + T, \tag{3}$$

$$x \geq 0, \tag{4}$$

[15] See Rocheteau and Rodriguez-Lopez (2014) for an environment where agents trade claims on firm's profits. Further, Lester et al. (2012) provide a more formal analysis of how an asset's recognizability determines acceptability in exchange.

[16] As in Berentsen et al. (2011), we have wages paid in the centralized market to abstract from specifying whether the wage is paid in money or goods, as it does not make a difference how they are paid in the centralized market. See Gu et al. (2019) for an analysis where firms must pay wages in stage 1 using cash.

where x is consumption of the numéraire, z is current real balances, z' is real balances brought into the next period, w_Ω is labor market income, Δ is dividends, T is transfers net of taxes, $U_\Omega(z')$ is the continuation value of entering stage 1 with labor market status Ω and holding z' real balances, and $\bar{\beta} \equiv \beta(1-\mu)$ is the effective discount factor. Note that the non-negativity constraint, (4), will not bind if the unemployment benefits, b, are large enough. Assuming that (4) does not bind and substituting for x using the budget constraint, one obtains

$$W_\Omega(z) = I_\Omega + z + \max_{z'} \left\{ -\frac{z'}{1+r} + \bar{\beta} U_\Omega(z') \right\}, \tag{5}$$

where $I_\Omega = w_\Omega + \Delta + T$ is net income. From (5), the household's value function is linear in z and thus their choice of real balances, z', is independent of their current holdings of real balances. It will also be shown later that the household's choice of z' is independent of their labor market status, Ω. As a result, the distribution of real balances will be degenerate.

Stage 2: Retail Market

We consider a competitive retail market, where all agents take prices, p, as given. The value of a household with real balances z and labor market status Ω in the retail market, $V_\Omega(z)$, satisfies

$$V_\Omega(z) = \alpha \max_{pq \le z} \left\{ \upsilon(q) + W_\Omega(z - pq) \right\} + (1-\alpha) W_\Omega(z). \tag{6}$$

From equation (6), households are hit with a preference shock and consume with probability α, subsequently choose their consumption, q, to maximize their lifetime discounted utility, and enter the centralized market with $z - pq$ units of real balances. With probability $1-\alpha$ the household does not consume and enters stage 3 with z units of real balances.

Stage 1: Labor Market

Households enter the labor market with labor market status Ω and real balances z. The value of an unemployed household with high skills, $U_{H,0}(z)$, satisfies

$$U_{H,0}(z) = \xi_h V_{H,1}(z) + (1-\xi_h)\left\{ \sigma V_{L,0}(z) + (1-\sigma) V_{H,0}(z) \right\}. \tag{7}$$

From (7), a high skill household meets a firm and becomes employed with probability ξ_h. With probability $1-\xi_h$, however, the high skill household does not meet a firm and is susceptible to skill loss. If they do not find a job, they become low-skilled with probability σ and remain highly-skilled with probability $1-\sigma$.

The value of an unemployed household with low skills, $U_{L,0}$, satisfies

$$U_{L,0}(z) = \xi_h V_{L,1} + (1-\xi_h) V_{L,0}(z), \tag{8}$$

which has a similar interpretation as equation (7), except that low skill households are not susceptible to skill loss shocks.

The value function of a household of skill level ε who enters the labor market employed is given by

$$U_{\varepsilon,1}(z) = \lambda V_{\varepsilon,0}(z) + (1-\lambda) V_{\varepsilon,1}(z). \tag{9}$$

Substituting (6) into (7)-(9) and using the linearity of the $W_\Omega(z)$ gives the following value functions which summarizes all three markets:

$$U_{H,0}(z) = \alpha\big[\upsilon(q) + z - pq\big] + (1-\alpha)z + \xi_h W_{H,1}(0) + \\ (1-\xi_h)\big\{\sigma W_{L,0}(0) + (1-\sigma)W_{H,0}(0)\big\}, \tag{10}$$

$$U_{L,0}(z) = \alpha\big[\upsilon(q) + z - pq\big] + (1-\alpha)z + \xi_h W_{L,1}(0) + (1-\xi_h)W_{L,0}(0), \tag{11}$$

$$U_{\varepsilon,1}(z) = \alpha\big[\upsilon(q) + z - pq\big] + (1-\alpha)z + \lambda W_{\varepsilon,0}(0) + (1-\lambda)W_{\varepsilon,1}(0), \tag{12}$$

which can in turn be substituted into (5) to give:

$$W_\Omega(z) = I_\Omega + z + \max_{z'\geq 0}\left\{ -\frac{z'}{1+r} + \bar{\beta}\Big[\alpha\big[\upsilon(q) + z' - pq\big] + (1-\alpha)z' + \mathbb{E}W_{\Omega'}(0)\Big]\right\}, \tag{13}$$

where the expectation is taken with respect to next period's type, Ω'. From (13), the choice of real balances is independent of Ω and z. Therefore, all households enter next period holding the same amount of real balances, z'.

4.2 Firms

Stage 3: Centralized Market

Firms who are matched enter the centralized market with unsold inventory x and real balances z. As carrying money is costly, firms do not need to carry real balances into the next time period. It follows that the value function for a firm who is currently matched with a type ε household is given by

$$\Pi_{\varepsilon,1}(x,z) = x + z - w_\varepsilon + \bar{\beta}J_{\varepsilon,1}, \tag{14}$$

where $J_{\varepsilon,1}$ is the continuation value of entering next period's labor market in a match.

Firms who enter the centralized market unmatched have no inventory as they did not produce in the labor market. They have a choice of whether to pay k units of the numéraire to enter the next labor market with a vacancy. Therefore, the problem of a vacant firm is given by

$$\Pi_0(z) = z + \max\big\{0, -k + \bar{\beta}J_0\big\}. \tag{15}$$

Stage 2: Retail Market

Let $K_{\varepsilon,1}$ denote the value of a firm who enters the retail market having produced y_ε units of output in the labor market. It follows that

$$K_{\varepsilon,1} = \max_{q\leq y_\varepsilon} \Pi_{\varepsilon,1}(y_\varepsilon - c(q), pq). \tag{16}$$

From (16), firms sell q units of specialized goods in the retail market and enter the centralized market with $y_\varepsilon - c(q)$ units of unsold inventory and pq units of real balances. Finally, for unmatched firms, $K_0 = \Pi_0$ as they have no inventory to sell in the retail market.

Stage 1: Labor Market

Let φ denote the share of job seekers with low skills. The value function for a firm who enters the labor market with a vacancy, J_0, is given by

$$J_0 = \xi_f\big\{\varphi K_{L,1} + (1-\varphi)K_{H,1}\big\} + (1-\xi_f)K_0, \tag{17}$$

whereas the value function of an employed firm entering the labor market is given by

$$J_{\varepsilon,1} = \lambda K_0 + (1-\lambda)K_{\varepsilon,1}. \tag{18}$$

The firm's value functions can be simplified by substituting (14) and (18) into (16) to obtain

$$K_{\varepsilon,1} = R_\varepsilon - w_\varepsilon + \bar{\beta}\big[\lambda K_0 + (1-\lambda)K_{\varepsilon,1}\big], \tag{19}$$

where $R_\varepsilon = y_\varepsilon + pq - c(q)$ is the total revenue of a filled job.

There is free entry of firms so that the expected profits of creating a vacancy are equal to zero, i.e. $K_0 = \Pi_0 = 0$. It follows that the firm's entry decision becomes

$$\Pi_0 = \max\{0, -k + \bar{\beta}\xi_f\big[\varphi K_{L,1} + (1-\varphi)K_{H,1}\big]\big\}, \tag{20}$$

where, from equation (19), $K_{\varepsilon,1} = (R_\varepsilon - w_\varepsilon)/(1-\bar{\beta}(1-\lambda))$. It follows that firms post vacancies until the cost of creating a vacancy is equated to the expected profits of filling a vacancy:

$$k = \frac{\bar{\beta}\xi_f}{1-\bar{\beta}(1-\lambda)}\big\{\varphi(R_L - w_L) + (1-\varphi)(R_H - w_H)\big\}. \tag{21}$$

4.3 Retail Market Equilibrium

Recall that all households enter the retail market holding the same amount of real balances. Their problem is given by

$$\max_{q^D}\ \upsilon(q^D) - pq^D \ \text{ s.t. } pq^D \le z. \tag{22}$$

If cash is costly to hold, households will not accumulate more than they need to consume in the retail market and their budget constraint will bind. It follows that $q^D = z/p$.

The problem of a firm matched with a household of skill level ε is

$$\max_{q_\varepsilon^S}\ pq_\varepsilon^S - c(q_\varepsilon^S) \ \text{ s.t. } c(q_\varepsilon^S) \le y_\varepsilon. \tag{23}$$

According to (23), the firm maximizes revenue net of costs incurred to sell their inventory in the retail market. Assuming $\lim_{q_\varepsilon \to \delta y} c'(q_\varepsilon)$ is large enough, the inventory constraint will not bind for any firm. It follows that $c'(q_\varepsilon^S) = p$, which simply equates price with marginal costs, and that the firm's supply can be expressed as

$$q_\varepsilon^S = c'^{-1}(p). \tag{24}$$

From (24), it is straightforward to see a firm's quantity supplied in the retail market, q_ε^S, is independent of the worker's skill level, ε, i.e. $q_L^S = q_H^S = q^S$ where $q^S = c'^{-1}(p)$.

Prices are determined through market clearing. There is a measure one of households and a fraction α receive a preference shock, so aggregate demand is given by $Q^D = \alpha q^D$. Denoting u as the unemployment rate, the measure of firms with inventory to sell is $1-u$. Market clearing is given by

$$\alpha q^D = (1-u)c'^{-1}(p). \tag{25}$$

With the outcome of the retail market in hand, we revisit the household's choice of real balances. From equation (5), we have

$$\max_{z' \geq 0} \left\{ -\frac{z'}{1+r} + \bar{\beta}\big[\alpha v(q) + (1-\alpha)z'\big] \right\}, \tag{26}$$

where q is a function of z'. By the Fisher equation, $(1+i) = (1+\rho)(1+\pi)$, the household's portfolio choice is equivalent to

$$\max_{z' \geq 0} \big\{ -(1+i)z' + (1-\mu)\big[\alpha v(q) + (1-\alpha)z'\big] \big\}, \tag{27}$$

and the first-order condition is

$$i = (1-\mu)\left[\alpha v'(q)\frac{\partial q}{\partial z'} + (1-\alpha)\right] - 1. \tag{28}$$

As households are price takers in the retail market, we have $\partial q / \partial z' = 1/p$. Combining $c'(q^S) = p$ and market clearing conditions with (28) gives

$$i = (1-\mu)\left[\alpha \frac{v'(q)}{c'\big(\frac{\alpha q}{1-u}\big)} + (1-\alpha)\right] - 1. \tag{29}$$

We refer to equation (29) as the RM curve, which, given the unemployment rate u, determines q. As in Berentsen et al. (2011), the RM curve is downward sloping in the (u, q) space, as long as firms make positive profits in the retail market. The intuition is straightforward: as u increases, there are less firms with inventory to sell in the retail market, which increases the price, p. Therefore, households carry less real balances across periods, causing q to decrease. Proposition 1 summarizes.

Proposition 1. *Suppose that $c'' > 0$. For all $i > 0$, the RM curve slopes downward in (u, q) space with $u = 0$ implying $q = \hat{q} \in (0, q^*)$, where $q^* \equiv \arg\max\{v(q) - c(q)\}$, and $u = 1$ implying $q = 0$. Further, the RM curve shifts down in i.*

4.4 Labor Market Equilibrium

Upon meeting in the labor market, the household's wage is determined through Nash bargaining. A type ε household's surplus of forming a match in the labor market is given by $S_\varepsilon^h = V_{\varepsilon,1}(z) - V_{\varepsilon,0}(z)$ whereas the firm's surplus is given by $S_{\varepsilon,1}^f = K_{\varepsilon,1}$. It follows that wages solve

$$w_\varepsilon \in \arg\max \big[V_{\varepsilon,1}(z) - V_{\varepsilon,0}(z)\big]^\gamma \big[K_{\varepsilon,1}\big]^{1-\gamma}, \tag{30}$$

where $\gamma \in [0,1]$ is the worker's bargaining power. Letting $S_\varepsilon = S_\varepsilon^h + S_\varepsilon^f$ denote the total surplus of a match between a type ε worker and firm, the solution to (30) gives the surplus sharing rules $S_\varepsilon^h = \gamma S_\varepsilon$; $S_\varepsilon^f = (1-\gamma)S_\varepsilon$. Using the Bellman equations, surplus sharing rules, and letting $\Delta_\sigma \equiv V_{H,0}(z) - V_{L,0}(z)$ denote the cost of skill loss, we have

$$S_L = \frac{R_L - b}{\bar{\beta}(\mu + \rho(1+\mu) + \lambda + \gamma\xi_h)}; \; S_H = \frac{R_H - b + \bar{\beta}(1-\xi_h)\sigma\Delta_\sigma}{\bar{\beta}(\mu + \rho(1+\mu) + \lambda + \gamma\xi_h)}. \tag{31}$$

See Appendix A.2 for a derivation of the cost of skill loss, Δ_σ. Substituting the surpluses into equation (21) gives the job creation condition:

$$\frac{k}{\xi_f} = (1-\gamma)\left[\frac{y[1-\varphi(1-\delta)] + (1-\varphi)\bar{\beta}(1-\xi_h)\sigma\Delta_\sigma + c'\left(\frac{\alpha q}{1-u}\right)\frac{\alpha q}{1-u} - c\left(\frac{\alpha q}{1-u}\right) - b}{\mu + \rho(1+\mu) + \lambda + \gamma\xi_h}\right]. \tag{32}$$

Equation (32) is the LM curve, as it determines entry of firms and thus, u, for a given q. It is straightforward to show the LM curve is also downward sloping in the (u, q) space, as a reduction in q decreases the expected revenue from a filled job, leading to less vacancies being created and for the unemployment rate to increase. Relative to Berentsen et al. (2011), the novelty in the LM curve is the entry of firms also depends on the skill composition of unemployed workers, φ, and the cost of skill loss, Δ_σ. As we show below, the skill composition, φ, is itself a function of market tightness. When more firms post vacancies, workers face shorter unemployment durations, and the skill composition of the unemployed improves, increasing the expected benefit of filling a vacancy. This complementarity is standard in search models with skill loss (see, e.g. Pissarides (1992)) and can lead to multiplicity of equilibria. This happens only under extreme parameter values. Thus, throughout the analysis we assume parameter values are such that, for a given q, there is a unique θ which solves (32). Proposition 2 summarizes.

Proposition 2. *Suppose that parameter values are such that there is a unique θ which solves (32). Further assume $k(\mu + \rho(1+\mu) + \lambda) < (1-\gamma)(\delta y - b)$. The LM curve slopes downward in (u, q) space. It passes through $(\underline{u}, q^*)$ with $\underline{u} \in (0, 1)$ and $(\bar{u}, 0)$ where $\bar{u} < 1$. If $\gamma = 0$, it shifts to the right with σ and shifts to the left with δ.*

4.5 General Equilibrium

To close the model, all that is left to do is to derive the stationary distribution of workers. Let u_ε and n_ε denote the measure of unemployed and employed households of skill level ε, respectively. It follows that

$$u_{L,t+1} = (1-\mu)\big[(1-\xi_h)[u_{L,t} + \sigma u_{H,t}] + \lambda n_{L,t}\big]; \; n_{L,t+1} = (1-\mu)\big[(1-\lambda)n_{L,t} + \xi_h u_{L,t}\big], \tag{33}$$

$$u_{H,t+1} = \mu + (1-\mu)\big[(1-\xi_h)(1-\sigma)u_{H,t} + \lambda n_{H,t}\big]; \; n_{H,t+1} = (1-\mu)\big[(1-\lambda)n_{H,t} + \xi_h u_{H,t}\big]. \tag{34}$$

From equations (33)-(34), it is straightforward to derive the stationary unemployment rate, $u = u_L + u_H$:

$$u = \frac{\mu + (1-\mu)\lambda}{\mu + (1-\mu)(\lambda + \xi_h)}, \tag{35}$$

and the fraction of unemployed workers who are less-skilled, φ:

$$\varphi = \frac{\sigma(1-\mu)(1-\xi_h)[1-(1-\mu)(1-\lambda)]}{\mu(1-\mu)\xi_h + [\mu + (1-\mu)(1-\xi_h)\sigma][1-(1-\mu)(1-\lambda)]}. \tag{36}$$

Definition 1. A stationary equilibrium is a vector $\{q, \theta, u, \varphi\}$ such that: retail market allocations, q, satisfies (29), market tightness, θ, satisfies (32), the unemployment rate, u, is given by equation (35), and the skill

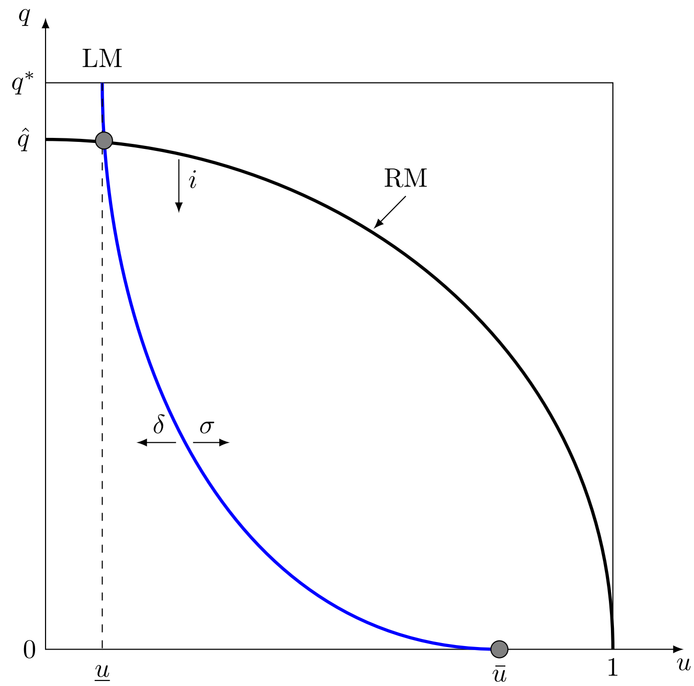


Figure 5: Steady-State Equilibrium

composition of unemployed workers, φ, is given by (36).

Figure 5 maps the RM and LM curves into the $\mathcal{B} = [0,1] \times [0, q^*]$ space where the intersection of the curves determines the equilibrium value of money, q, and unemployment rate, u. As discussed previously, the RM curve enters $\mathcal{B}$ at $(0, \hat{q})$ where $\hat{q} \leq q^*$ and exits at $(1, 0)$. The LM curve enters at $(q^*, \underline{u})$ and exits at $(0, \bar{u})$ (as long as k is small enough as shown in Proposition 2). There is always the possibility of a non-monetary equilibria and monetary equilibria. Additionally, the non-monetary and monetary equilibria need not be unique, as discussed previously due to the effect posting a vacancy has on the skill composition of unemployed workers. Proceeding under the assumption that parameters values are such that there is a unique θ and hence, u, for each value of q (which we verify in our quantitative analysis), and assuming workers' bargaining power is low, we have the following results regarding the skill loss parameters. An increase in the frequency of skill loss shocks, σ, or a decrease in δ, shifts the LM curve to the right, causing it to intersect the RM curve at a lower value of q and higher value of u. The intuition is straightforward: an increase in σ makes it more likely firms meet low skill workers, which causes less firms to enter and unemployment to increase. As less firms enter, the price of the retail market good increases, causing households to hold less real balances and q to decrease. Similarly, a decrease in δ means that skill loss lowers a workers' productivity by a larger amount, which decreases the expected value of creating a vacancy.

Consider a monetary equilibrium. If i increases, the opportunity cost of holding real balances increases, causing households to hold less real balances and the RM curve to shift down. As the RM curve shifts downward, demand for the specialized good in the retail market decreases, causing less firms to enter and for unemployment to increase. There is also a multiplier effect due to workers losing skills during unemployment: as less firms enter following an increase in i, the skill composition among the unemployed deteriorates, which further decreases the entry of firms and increases unemployment. Proposition 3 summarizes.

Proposition 3. *Assume that $k(\mu + \rho(1+\mu) + \lambda) < (1-\gamma)(\delta y - b)$ and $i > 0$. At least one monetary equilibrium with $q \in (0, q^*)$ and $u \in (0,1)$ exists. There also exists at least one non-monetary equilibrium with $q = 0$ and $u = \bar{u} \in (0,1)$. If the monetary equilibrium is unique and $\gamma = 0$, an increase in σ or decrease in δ reduces q and increases u. Finally, an increase in i reduces q and increases u.*

4.6 Inflation and TFP

We now focus on the relationship between monetary policy and TFP, where TFP is simply given by average labor productivity:

$$\text{TFP} = \overbrace{\varphi(\theta)\delta y + (1-\varphi(\theta))y}^{\text{Average production of numéraire}} + \underbrace{c'\left(\frac{\alpha q}{1-u(\theta)}\right)\frac{\alpha q}{1-u(\theta)} - c\left(\frac{\alpha q}{1-u(\theta)}\right)}_{\text{Net production in retail market}}. \tag{37}$$

The first two terms on the right side of (37) are the average production of the numéraire in the labor market, whereas the remaining terms are the net production of each firm in the retail market. Thus there are two channels through which a change in monetary policy impacts TFP. An increase to the nominal interest rate directly affects the entry of firms, the skill distribution of workers, and therefore the average production in the labor market. Secondly, as both q and the unemployment rate, u, react to the change in the nominal interest rate, so does the net output produced by each firm in the retail market. More formally,

$$\frac{d\text{TFP}}{di} = -y(1-\delta)\varphi'(\theta)\frac{\partial\theta}{\partial i} + g_1(q,\theta)\frac{\partial q}{\partial i} + g_2(q,\theta)\frac{\partial\theta}{\partial i}, \tag{38}$$

where $g(q,\theta) \equiv c'\left(\frac{\alpha q}{1-u(\theta)}\right)\frac{\alpha q}{1-u(\theta)} - c\left(\frac{\alpha q}{1-u(\theta)}\right)$. The effect of the nominal interest rate on TFP is three-fold. First, there is the decline in average production in the labor market following a shift in the skill composition of workers towards less-skilled workers. Second, the net output of each firm in the retail market declines as households hold less real balances. The third effect follows from the fact that as less firms post vacancies, each remaining firm will, conditional on q, supply a larger amount to the retail market. The first two effects decrease TFP, whereas the third effect increases TFP as each active firm may supply a larger amount of output to the retail market.

5 Heterogeneous Firms

We now extend the model to have heterogeneous firms. We do so by allowing firms to create one of two types of jobs indexed by $\chi \in \{s, c\}$: simple (s) and complex (c), which determines the specialized good the firm produces in the retail market. Employed high skill workers now produce y_χ units of output per period, where $y_s < y_c$. Low skill workers produce $\delta_\chi y_\chi$ with $\delta_\chi \in (0,1)$ and $\delta_c < \delta_s$. Thus, there are two distinguishing characteristics of complex jobs. The first is that high skill workers produce more output in complex jobs. Second, skill loss has a larger impact on a worker's productivity in complex jobs than in simple jobs, which is consistent with evidence that an additional month of unemployment history has a larger impact on wages in highly-skilled occupations (Ortego-Marti, 2017a).

Utility from consumption of specialized goods is given by $\upsilon(q_s, q_c)$, where $q_\chi \in \mathbb{R}_+$ is consumption of good χ, $\upsilon'(0, q_c) = \upsilon'(q_s, 0) = \infty$, $\upsilon'(\infty, q_c) = \upsilon'(q_s, \infty) = 0$, and $\upsilon(0,0) = 0$. The only other differences relative to the baseline model are that we assume the utility while unemployed is bounded by output in the lowest productivity match: $b < \min\{\delta_s y_s, \delta_c y_c\}$. We also allow for different vacancy posting costs across the two jobs, i.e. vacant firms can pay k_χ units of the numéraire to enter the labor market with a type χ vacancy.

A household's type is now given by $\Omega \in \{L, H\} \times \{0, s, c\}$ where $\Omega = (\varepsilon, 0)$ denotes an unemployed household of skill level ε and $\Omega = (\varepsilon, \chi)$ denotes a worker of skill level ε who is matched with a type χ job. Let p_χ denote the price of good χ in terms of the numéraire. The value function of household with real balances z and labor market status Ω in the retail market, $V_\Omega(z)$, is now given by

$$V_\Omega(z) = \alpha \max_{\sum_\chi p_\chi q_\chi \leq z} \big\{ \upsilon(q_s, q_c) + W_\Omega(z - p_s q_s - p_c q_c) \big\} + (1-\alpha) W_\Omega(z). \tag{39}$$

From equation (39), households choose their consumption of q_s and q_c to maximize their lifetime discounted utility and enter the centralized market with $z - \sum_\chi p_\chi q_\chi$ units of real balances.

Letting $\zeta = v_s/[v_s + v_c]$ denote the fraction of vacancies that are simple, the value of an highly-skilled and unemployed household is given by

$$U_{H,0}(z) = \xi_h \big\{ \zeta V_{H,s}(z) + (1-\zeta) V_{H,c}(z) \big\} + (1-\xi_h) \big\{ \sigma V_{L,0}(z) + (1-\sigma) V_{H,0}(z) \big\}. \tag{40}$$

Relative to the baseline model and equation (7), a high skill household can now meet a simple or complex job in the labor market. The value of an unemployed household with low skills now satisfies

$$U_{L,0}(z) = \xi_h \big\{ \zeta V_{L,s}(z) + (1-\zeta) V_{L,c}(z) \big\} + (1-\xi_h) V_{L,0}(z). \tag{41}$$

Through the same process as the baseline model, we can use the linearity of the $W_\Omega(z)$ to obtain value functions which summarizes all three markets, which can in turn be substituted into (5) to give

$$W_\Omega(z) = I_\Omega + z + \max_{z' \geq 0} \bigg\{ -\frac{z'}{1+r} + \bar{\beta} \Big[\alpha \big[\upsilon(q_s, q_c) + z' - \sum_\chi p_\chi q_\chi \big] + (1-\alpha) z' + \mathbb{E} W_{\Omega'}(0) \Big] \bigg\}. \tag{42}$$

From (42), the choice of real balances is still independent of current type, Ω, and real balances, z. Therefore, the distribution of real balances is degenerate.

Firms who enter the centralized market unmatched have a choice of whether to pay k_χ units of the numéraire to enter the next labor market with a type χ vacancy. A vacant firm's problem is now given by

$$\Pi_0(z) = z + \max \big\{ 0, -k_s + \bar{\beta} J_{0,s}, -k_c + \bar{\beta} J_{0,c} \big\}. \tag{43}$$

With free entry of firms, the expected profits of creating either type of vacancy are equal to zero. It follows that firms post type χ vacancies until

$$k_\chi = \frac{\bar{\beta} \xi_f}{1 - \bar{\beta}(1-\lambda)} \big\{ \varphi (R_{L,\chi} - w_{L,\chi}) + (1-\varphi)(R_{H,\chi} - w_{H,\chi}) \big\}, \tag{44}$$

where $R_{\varepsilon,\chi} = y_{\varepsilon,\chi} + p_\chi q_{\varepsilon,\chi} - c(q_{\varepsilon,\chi})$. Relative to the baseline model, there are two entry conditions, one for each type of job.

The household's problem in the retail market is now given by

$$\max_{q_s^D, q_c^D} \ \upsilon(q_s^D, q_c^D) - p_s q_s^D - p_c q_c^D; \ \text{ s.t. } \ p_s q_s^D + p_c q_c^D \leq z. \tag{45}$$

If cash is costly to hold, households will not accumulate more than they need to consume in the retail market and their budget constraint will bind. It follows that q_s^D and q_c^D satisfy

$$\frac{\upsilon_2\big(q_s^D, q_c^D\big)}{\upsilon_1\big(q_s^D, q_c^D\big)} = \frac{p_c}{p_s}, \tag{46}$$

$$\frac{z - p_c q_c^D}{p_s} = q_s^D. \tag{47}$$

The problem of type χ firm matched with a household of skill level ε is given by

$$\max_{q^S_{\varepsilon,\chi}} \; p_\chi q^S_{\varepsilon,\chi} - c(q^S_{\varepsilon,\chi}); \;\; \text{s.t. } c(q^S_{\varepsilon,\chi}) \leq y_{\varepsilon,\chi}. \tag{48}$$

Assuming again that the inventory constraint does not bind for any firm, the solution is given by $c'(q^S_{\varepsilon,\chi}) = p_\chi$ and the firm's supply can be expressed as $q^S_{\varepsilon,\chi} = c'^{-1}(p_\chi)$. It follows that a type χ firm's quantity supplied in the retail market, $q^S_{\varepsilon,\chi}$, is independent of the worker's skill level, i.e. $q^S_{L,\chi} = q^S_{H,\chi} = q^S_\chi$ where $q^S_\chi = c'^{-1}(p_\chi)$.

The total measure of firms with simple goods is given by $(1-u)\zeta$ whereas the measure of firms who supply complex goods is $(1-u)(1-\zeta)$. Market clearing conditions are given by

$$\alpha q_s^D = (1-u)\zeta c'^{-1}(p_s), \tag{49}$$

$$\alpha q_c^D = (1-u)(1-\zeta) c'^{-1}(p_c). \tag{50}$$

Equations (49)-(50) define a system of equations to determine retail market prices. Given the equilibrium prices, the quantities of consumption by each household and production by each firm satisfy the respective firm order conditions above.

The household's first-order condition for real balances is now given by

$$i = (1-\mu)\left[\alpha\left(\upsilon_1(q_s, q_c)\frac{\partial q_s}{\partial z'} + \upsilon_2(q_s, q_c)\frac{\partial q_c}{\partial z'}\right) + (1-\alpha)\right] - 1, \tag{51}$$

where $\partial q_s / \partial z' = \omega(z')/p_s$, $\partial q_c/\partial z' = (1-\omega(z'))/p_c$, and $\omega(z')$ is the fraction of an additional unit of real balances spent on simple goods. Combining $c'(q^S_\chi) = p_\chi$ and market clearing conditions with (46) and (51) gives

$$\frac{\upsilon_2\big(q_s, q_c\big)}{\upsilon_1\big(q_s, q_c\big)} = \frac{c'\big(\frac{\alpha q_c}{(1-u)(1-\zeta)}\big)}{c'\big(\frac{\alpha q_s}{(1-u)\zeta}\big)}, \tag{52}$$

$$i = (1-\mu)\left[\alpha\left(\frac{\upsilon_1(q_s, q_c)\omega(z')}{c'\big(\frac{q_s}{(1-u)\zeta}\big)} + \frac{\upsilon_2(q_s, q_c)(1-\omega(z'))}{c'\big(\frac{q_c}{(1-u)(1-\zeta)}\big)}\right) + (1-\alpha)\right] - 1, \tag{53}$$

which, given the unemployment rate, u, and composition of jobs, ζ, jointly determines q_s and q_c.

In the labor market, wages continued to be determined through Nash bargaining. Letting $S_{\varepsilon,\chi} = S^h_{\varepsilon,\chi} + S^f_{\varepsilon,\chi}$ denote the total surplus of a match between a type ε worker and type χ firm and combining the resulting surplus sharing rules with the Bellmans, we have

$$S_{L,s} = \frac{(R_{L,s} - b)(\mu + \rho(1+\mu) + \lambda) - \gamma\xi_h(1-\zeta)(R_{L,c} - R_{L,s})}{\bar{\beta}(\mu + \rho(1+\mu) + \lambda)(\mu + \rho(1+\mu) + \lambda + \gamma\xi_h)}, \tag{54}$$

$$S_{L,c} = \frac{(R_{L,c} - b)(\mu + \rho(1+\mu) + \lambda) - \gamma\xi_h\zeta(R_{L,s} - R_{L,c})}{\bar{\beta}(\mu + \rho(1+\mu) + \lambda)(\mu + \rho(1+\mu) + \lambda + \gamma\xi_h)}, \tag{55}$$

for the total match surpluses with low skill households and

$$S_{H,s} = \frac{(R_{H,s} - b + \bar{\beta}(1-\xi_h)\sigma\Delta_\sigma)(\mu + \rho(1+\mu) + \lambda) - \gamma\xi_h(1-\zeta)(R_{H,c} - R_{H,s})}{\bar{\beta}(\mu + \rho(1+\mu) + \lambda)(\mu + \rho(1+\mu) + \lambda + \gamma\xi_h)}, \tag{56}$$

$$S_{H,c} = \frac{(R_{H,c} - b + \bar{\beta}(1-\xi_h)\sigma\Delta_\sigma)(\mu + \rho(1+\mu) + \lambda) - \gamma\xi_h\zeta(R_{H,s} - R_{H,c})}{\bar{\beta}(\mu + \rho(1+\mu) + \lambda)(\mu + \rho(1+\mu) + \lambda + \gamma\xi_h)}, \tag{57}$$

for high skill households.[17] Note that the surpluses need not be positive. For example, consider $S_{H,S}$ from equation (56). If workers have high bargaining power, there are many complex jobs, and the difference in revenue a high-skilled worker generates in a complex job relative to a simple job is large, then the highly-skilled worker's reservation wage may be high enough to the point where they are better off forgoing a match with a simple job with the chance of matching with a complex job the next period. The fact that the differences in revenue, $R_{H,c} - R_{H,s}$, is an important determinant of the match surplus provides a channel for monetary policy to impact which types of jobs are created, as the relative profits of selling output in the retail market are tied to the value of money, and hence monetary policy. Proposition 4 provides a sufficient condition on fundamentals to ensure all matches generate a positive surplus, which is the class of equilibria we restrict our attention to.

Proposition 4. *Define $\bar{q} \equiv \arg\max\{c'(q)q - c(q)\}$. All matches generate a positive surplus if*

$$\frac{\delta_\chi y_\chi + [c'(\bar{q})\bar{q} - c(\bar{q})] - b}{\delta_{\chi'} y_{\chi'} - b} < \frac{\mu + \rho(1+\mu) + \lambda + \gamma}{\gamma} \text{ for } \chi \in \{s, c\}, \tag{58}$$

where $\chi' = c$ if $\chi = s$ and $\chi' = s$ if $\chi = c$.

With the surpluses in hand, we then have the job creation conditions

$$\frac{k_s}{\xi_f} = \bar{\beta}(1-\gamma)\big[\varphi S_{L,s} + (1-\varphi)S_{H,s}\big], \tag{59}$$

$$\frac{k_c}{\xi_f} = \bar{\beta}(1-\gamma)\big[\varphi S_{L,c} + (1-\varphi)S_{H,c}\big]. \tag{60}$$

Equation (59) is the entry condition for simple jobs while equation (60) is the entry condition for complex jobs. Assuming that (58) is satisfied, the flow equations of workers across their states are the same as in the baseline model.

Definition 2. A stationary equilibrium is a vector $\{q_s, q_c, \theta, \zeta, u, \varphi\}$ such that: retail market allocations, q_χ for $\chi \in \{s, c\}$, satisfy (52)-(53), market tightness and composition of vacancies, θ and ζ, satisfy (59)-(60), the unemployment rate, u, is given by equation (35), and the skill composition of unemployed workers, φ, is given by (36).

[17] See Appendix A.5 for a derivation of the cost of skill loss, Δ_σ, with heterogeneous firms.

Proposition 5. *Assume that $i > 0$ and*

$$k_\chi < \frac{(1-\gamma)\sigma(1-\mu)(\delta_\chi y_\chi - b)}{(\mu+\rho(1+\mu)+\lambda)(\mu+(1-\mu)\sigma)} \text{ for } \chi \in \{s, c\}. \tag{61}$$

At least one monetary equilibrium with $q_\chi > 0$ for $\chi \in \{s, c\}$, $u \in (0, 1)$, and $\zeta \in (0, 1)$ exists. There also exists at least one non-monetary equilibrium with $q_\chi = 0$ for $\chi \in \{s, c\}$, $u \in (0, 1)$, and $\zeta \in (0, 1)$.

As in the case with homogeneous firms, existence of an active equilibrium in which firms create a positive measure of vacancies is not guaranteed. Proposition 5 establishes that if each respective vacancy posting cost is low enough, then a positive measure of both simple and complex vacancies will be created, and hence $\theta > 0$ and $\zeta \in (0, 1)$. Also, the complementarity between vacancy posting decisions made by firms and the skill composition of unemployed workers is still present, giving rise to the possibility of multiplicity. The equilibrium can no longer be represented graphically, as the equilibrium is now comprised of four endogenous variables $(q_s, q_c, \theta, \zeta)$, as opposed to just (q, θ) in the case of homogeneous firms. While the model is somewhat more complicated with heterogeneous firms, there is now an additional channel through which inflation can impact TFP, which we discuss in the next section.

5.1 Inflation and TFP Revisited

With heterogeneous firms, TFP is now given by

$$\text{TFP} = \overbrace{\zeta\big[\varphi\delta_s y_s + (1-\varphi)y_s + c'(\hat{q}_s)\hat{q}_s - c(\hat{q}_s)\big]}^{\text{Average production in simple jobs}} + \underbrace{(1-\zeta)\big[\varphi\delta_c y_c + (1-\varphi)y_c + c'(\hat{q}_c)\hat{q}_c - c(\hat{q}_c)\big]}_{\text{Average production in complex jobs}}, \tag{62}$$

where $\hat{q}_s \equiv \alpha q_s/(1-u)\zeta$ and $\hat{q}_c \equiv \alpha q_c/(1-u)(1-\zeta)$. There are several differences relative to TFP in the case of homogeneous firms (see equation (37)). First, and most notably, is TFP now captures production from simple and complex jobs and is a function of the composition of job complexity, as measured by ζ. Second, the average production within each type of job now depends on several job specific characteristics. Production of the numéraire by low skill workers is determined by δ_χ, the job-specific production of less-skilled workers. Also, net production in the retail market varies across the two types of jobs and are determined by demand for the specialized good, q_χ, and the composition of job complexity.

There are now three channels through which inflation impacts TFP. The first two are the same as in the case with homogeneous firms: an increase in the nominal interest rate or money growth rate increases the opportunity cost of real balances and leads to a decline in demand, q_s and q_c. Second, as firms post less vacancies, workers experience prolonged unemployment durations and the skill composition of unemployed workers deteriorates, lowering average production in the labor market. It is at this point where a third channel is active, and that is a change in the composition of job complexity. While the model is now too complicated for analytical comparative statics, the idea is the following: a deterioration in the skill composition of the unemployed has an adverse impact on complex vacancies as the impact of skill loss on production in complex jobs is larger than in simple jobs (recall $\delta_c < \delta_s$). Thus, in addition to the skill composition of the unemployed worsening, the composition of vacancies can also shift away from complex jobs and towards simple jobs (i.e., ζ increases), which can cause a further decline in TFP as simple jobs are less productive than complex jobs. We revisit this in Section 6 when we quantitatively evaluate the contribution of these three channels to the productivity costs of inflation.

6 Quantitative Analysis

In this Section, we quantitatively evaluate the effect of inflation on TFP. Section 6.1 introduces our measure of job complexity while Section 6.2 details the calibration strategy. Section 6.3 introduces our main findings while Section 6.4 evaluates the contribution of the three channels to the effect of anticipated inflation on TFP. Finally, Section 6.5 presents the version of our model and resulting simulations when the nominal interest rate follows a stochastic process. Appendix B provides details on data used to calibrate the model.

6.1 Measuring Job Complexity

We measure job complexity by using abstract and manual task input measures constructed by Autor and Dorn (2013). The task requirements of each occupation are based on the US department of Labor's *Dictionary of Occupational Titles*, and hence merged with the census occupation classification. We then construct a normalized measure of job complexity for each occupation k, AM_k:

$$AM_k = \frac{(T^A_{k,1980} - T^M_{k,1980}) - \underline{AM}}{\overline{AM} - \underline{AM}}, \tag{63}$$

where $T^A_{k,1980}$ and $T^M_{k,1980}$ are the abstract and manual task input in each occupation k in 1980, and $\overline{AM}$ ($\underline{AM}$) is the maximum (minimum) of AM across all occupations.[18] See Appendix B.2 for a list of occupations with the highest and lowest complexity measures and more details on abstract and manual task measurements.

To map our measure of job complexity to the theory, we choose a cutoff value of AM where occupations below the cutoff are labelled as "simple" occupations and occupations above the cutoff are "complex" occupations. To identify the cutoff, Figure 6 plots the CDF of the average AM score between 1968-2017. Our baseline cutoff value of AM is shown by the red line in Figure 6, where there appears to be a discrete jump in the CDF. This corresponds to an AM score of 0.615. Therefore, as a baseline, we label complex occupations as those with an AM score above or equal to 0.615, and simple occupations as those with an AM score below 0.615. Under this cutoff, we find that an average of 52.5% of employed workers are in simple jobs, which establishes our target for the composition of job complexity. Clearly, the choice of the cutoff is arbitrary. We think it does well in separating what we think of as "simple" and "complex" occupations. For example, occupations such as salespersons and secretaries are just below the baseline cutoff, whereas bookkeepers and accounting clerks are right above it.[19] For robustness, we perform our quantitative exercises under alternative definitions of a simple and complex job where we instead use a lower cutoff (labelled "lower bound" in Figure 6) and a higher cutoff (labelled "upper bound" in Figure 6). See Appendix C for additional details and results.

[18]In contrast to Autor and Dorn (2013), our summary measure does not include routine input measure but instead focuses on abstract and manual input, as some occupations with a high abstract task input also have a high routine input. Therefore, we only take into account the abstract and manual inputs.

[19]Figure 13 in Appendix B.2 shows the occupations around the cutoff which distinguishes simple and complex jobs.

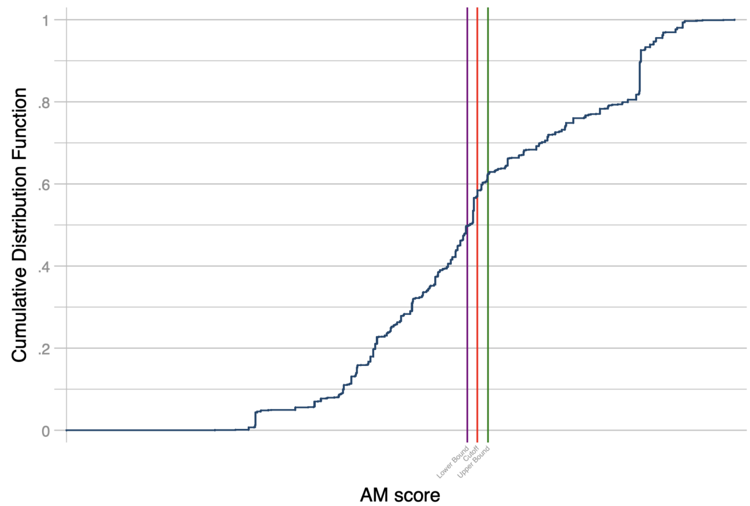


Figure 6: CDF of job complexity among workers between 1968-2017

6.2 Calibration Strategy

A unit of time is one month and we calibrate to US data covering 1955-2017. We set the rate of time preference to 1.68×10^{-3} to target a discount factor of $\beta = 0.98^{1/12}$. The probability of leaving the labor force is set to $\mu = 1/480$, which corresponds to being in the labor force on average for 40 years. The separation probability is set to $\lambda = 0.035$ following Shimer (2005). We assume a Cobb-Douglas matching technology: $\mathcal{N}(u, v) = Au^{\eta}v^{1-\eta}$. The elasticity of the matching function is set to $\eta = 0.5$, which is in line with the empirical evidence (Petrongolo and Pissarides, 2001). The worker's bargaining power is then set to $\gamma = 0.5$ to implement the Hosios (1990) condition. The matching efficiency, A, is set to target a steady-state unemployment rate of 5.9%. Combined with normalizing steady-state market tightness to one, we have $A = 0.5902$. We then normalize the output produced in matches between high-skill workers and complex jobs to $y_c = 1$ and choose the value of unemployment, b, so that the ratio of b to average labor productivity is equal to 0.79, which is between the common targets used by Hall and Milgrom (2008) and Hagedorn and Manovskii (2008). We find $b = 0.5542$. We then choose the entry costs, k_s and k_c, to target a market tightness of $\theta = 1$ as in Shimer (2005) and a composition of vacancies where $\zeta = 0.525$, which follows from our definition of simple and complex occupations. We find $k_s = 0.2454$ and $k_c = 0.5896$, indicating that complex jobs have significantly larger entry costs.

The remaining labor market parameters are the skill loss parameters $\{\delta_s, \delta_c, \sigma\}$. We calibrate these parameters to match the empirical evidence on the effect of unemployment duration on wages. As discussed by Laureys (2020), the empirical evidence on the effect of unemployment duration on wages can not be used to choose a unique value of σ, δ_s, and δ_c. Thus, we set $\sigma = 1/3$, which corresponds to skill loss taking 3 months on average and is well supported by the empirical evidence on how quickly skill loss occurs (Ortego-Marti, 2016). We then choose δ_s and δ_c to match the estimated effects of unemployment duration on wages. That is, we choose a value of δ_s and δ_c, and given the wages and transition probabilities between employment and unemployment, we simulate 10,000 employment histories and estimate the following regression:

$$\ln(wage_\chi) = \beta_0 + \beta_1 \times Unhis + \epsilon,$$

where $Unhis$ is the length of the unemployment spell in months and $\ln(wage_\chi)$ are log wages in type χ jobs.

For each simulation of employment histories, we compute β_1 for simple and complex jobs and repeat this process 100 times where we then have an average estimate of β_1 for each type of job. We vary δ_s and δ_c and repeat this exercise until our average estimate of β_1 is -0.0093 for simple jobs and -0.0193 for complex jobs.[20] Through this procedure, we find $\delta_s = 0.825$ and $\delta_c = 0.65$. Proceeding to the monetary side of the model, we target an average annual nominal interest rate of $i = 0.0689$, which corresponds to the average Aaa nominal corporate bond yield between 1955-2017. The cost to sell inventory in the retail market is $c(q) = q^{1.3}$, which generates mark-ups over average total costs of 30% (Faig and Jerez, 2005), and the retail market utility function is given by $\upsilon(q_s, q_c) = \varrho\sqrt{q_s q_c}$. The remaining parameters are y_s, the production of high-skilled workers in simple jobs, α, the frequency of preference shocks, and ϱ, the RM utility weight. We choose these parameters to target two moments common in the monetary search literature. The first is average money demand between 1955-2017 of $M/pY = 0.174$ where M is $M1$ plus money market deposit accounts and pY is nominal GDP. The corresponding money demand in the model is given by

$$\frac{M}{pY} = \frac{\frac{1}{1-u}\left[c'(\hat{q}_s)q_s + c'(\hat{q}_c)q_c\right]}{\zeta\left[\varphi\delta_s y_s + (1-\varphi)y_s + c'\hat{q}_s)\hat{q}_s - c(\hat{q}_s)\right] + (1-\zeta)\left[\varphi\delta_c y_c + (1-\varphi)y_c + c'(\hat{q}_c)\hat{q}_c - c(\hat{q}_c)\right]},$$

where $\hat{q}_s \equiv \alpha q_s/(1-u)\zeta$ and $\hat{q}_c \equiv \alpha q_c/(1-u)(1-\zeta)$. Second, we target the elasticity of money demand with respect to i of -0.383, which we obtain through estimating the following regression:

$$\log(MD_t) = \alpha + \beta\log(i_t) + \epsilon,$$

where MD_t is money demand in quarter t and i_t is the Aaa nominal corporate bond yield. We estimate this regression using data from 1955:Q1-2017:Q4. Through this process, we find $y_s = 0.7875$, $\alpha = 0.0501$, and $\varrho = 1.6932$. Table 1 summarizes the parameter values while Table 2 shows the model closely matches the targeted moments. For robustness, we perform four alternative calibrations. One is where markups are 20%, rather than 30%. The second is where it takes on average six months of unemployment to become low-skilled. The third and fourth target alternative values for the composition of job complexity. Parameter values and quantitative results under these alternative calibrations are presented in Appendix C.

6.3 Results

To begin, we present the effect of changes to anticipated inflation on equilibrium outcomes, where anticipated inflation is measured by the nominal interest rate. Figure 7 contains the results, where we compute the equilibrium outcomes at each value of the nominal interest rate observed in the data between 1955-2017.

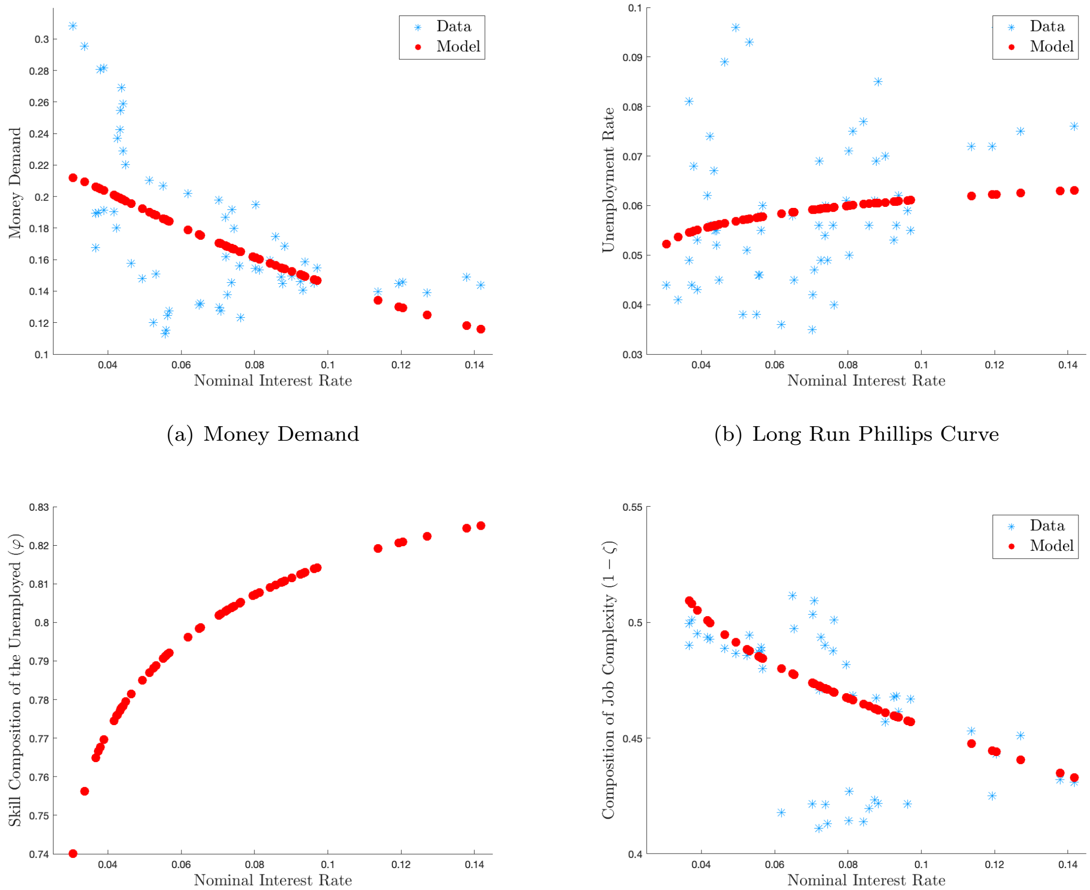
[20] Our target of -0.0093 for simple occupations follows Ortego-Marti (2017a) and is the average effect of unemployment duration on log wages in clerical, sales, craftsmen, foreman, and operator occupations. The target for complex jobs of -0.0193 also follows from Ortego-Marti (2017a) and is the average effect of unemployment duration on log wages in professional, technical, managers, and officials occupations.

Table 1: Parameter values

Parameter	Definition	Value
	Panel A: Assigned parameters	
ρ	Discount rate	1.68×10^{-3}
μ	Probability of exiting the labor force	2×10^{-3}
λ	Separation probability	0.035
η	Elasticity of matching function	0.50
γ	Worker's bargaining power	0.50
y_c	Productivity of high skill workers in complex jobs	1.00
σ	Probability of skill loss	1/3
i	Annual nominal interest rate	6.89×10^{-2}
a	Elasticity of cost function	1.30
	Panel B: Calibrated parameters	
A	Matching efficiency	0.590
y_s	Productivity of high skill workers in simple jobs	0.787
δ_s	Human capital decay in simple jobs	0.825
δ_c	Human capital decay in complex jobs	0.650
k_s	Vacancy posting cost: simple jobs	0.245
k_c	Vacancy posting cost: complex jobs	0.589
b	Value of unemployment	0.554
α	Pr. of consuming in RM	0.050
ϱ	RM utility weight	1.693

Table 2: Targeted moments

Moment	Data	Model
Unemployment rate	0.0590	0.0590
Fraction of jobs that are simple	0.5250	0.5250
Unemployment duration on wages in simple jobs (negative)	0.0093	0.0094
Unemployment duration on wages in complex jobs (negative)	0.0193	0.0191
Average Money demand	0.1740	0.1739
Elasticity of money demand (negative)	0.3830	0.3830



(a) Money Demand

(b) Long Run Phillips Curve

(c) Skill Composition of the Unemployed

(d) Composition of Job Complexity

Figure 7: Effects of a Change in Anticipated Inflation

Starting with Figure 7(a), we see the standard result that money demand is decreasing in the nominal interest rate, as an increase in the nominal interest rate increases the opportunity cost of holding real balances. Figure 7(b) shows the model generates the positive correlation between anticipated inflation and unemployment that is observed in US data over this period. The intuition follows from Berentsen et al. (2011): as money demand decreases (as shown in Figure 7(a)), so do the profits a firm can make by selling their inventory in the retail market, which thereby reduces the expected revenue of a filled job. This leads to less firms posting vacancies and for the unemployment rate to increase.

Figure 7(c) demonstrates the effect of a change in anticipated inflation on the skill composition of the unemployed. As the nominal rate increases, money demand decreases, firms post less vacancies, and unemployed workers face longer unemployment durations. As a result, unemployed workers are more exposed to skill loss shocks and the skill distribution among the unemployed deteriorates. The model implies that an increase in the nominal interest rate from 3% (annual inflation rate of 0.98%) to 14% (annual inflation rate of 11.17%) causes the fraction of the unemployed who are less-skilled (φ) to increase from 0.74 to 0.83. Lastly, Figure 7(d) shows the effect of anticipated inflation on the composition of job complexity, as measured by the fraction of vacancies which are complex, $(1-\zeta)$. As discussed in Section 5.1, a deterioration of the skill distribution has an adverse impact on complex jobs, as skill loss causes a larger decline in productivity in complex than simple jobs. Figure 5.1 shows that as the nominal interest increases and more workers become less-skilled, the composition of vacancies shifts from complex jobs towards simple jobs (i.e., ζ increases).

6.4 Productivity Costs of Inflation

Turning now to the the productivity costs of inflation, Figure 8 shows the TFP generated by the model at values of the nominal interest rate observed in the data.[21] The first set of results we wish to discuss are the red circles, which represents TFP when all three channels described in Section 5.1 are active. The model shows a negative relationship between TFP and the nominal interest rate. Moreover, the quantitative effects are significant: an economy with a nominal interest rate of 14%, or an annual inflation rate of nearly 11%, is 3.9% less productive than an economy where the nominal interest rate is 3% and the annual inflation rate is 1%.

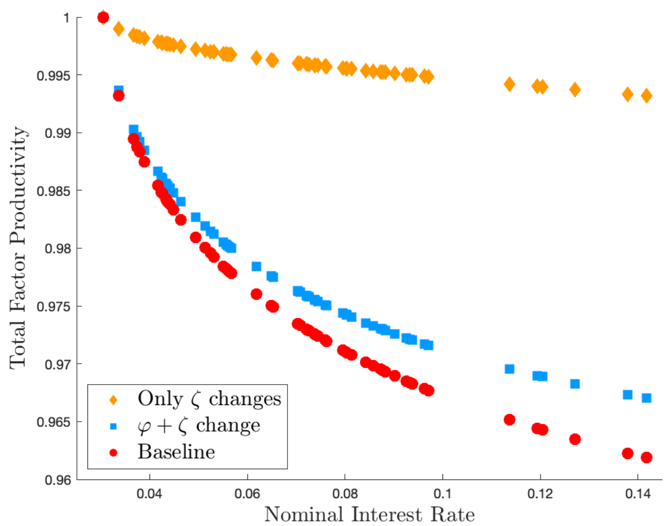


Figure 8: The Productivity Costs of Inflation

Our next goal is to quantify the contributions of the three respective channels through which a change to anticipated inflation affects TFP. As a first step, we compute the implied TFP while holding the net output produced by firms constant at the value corresponding to TFP $= 1$ while the skill composition among the unemployed and composition of job complexity change. This is represented by the blue squares in Figure 8, so that any differences between the blue squares and red circles are due to the change in retail market net production. As seen by comparing the two series, there is little difference between them, suggesting that the change in net production induced by a change in monetary policy does not contribute much to the aggregate effects on TFP. In a second step, we hold the skill composition of the unemployed fixed at its value where TFP $= 1$ (in addition to holding net production in the retail market fixed). This is represented by the orange diamonds in Figure 8, where differences between the orange diamonds and blue squares are due to changes in the skill composition of unemployed workers. The large gap between the two series indicates that the shift in the composition of the unemployed has a large quantitative contribution to the aggregate impact of a change in anticipated inflation on TFP. We will revisit this shortly. Finally, any difference between a TFP

[21] We normalize the highest level of TFP to 1 so that any respective differences are interpreted as percentage deviations.

level of 1 and the orange diamonds is due to a change in the composition of job complexity.

To put a precise number on each of the three channels, we compute the fraction of the total decline in TFP that is accounted for by a change in the net production in the retail market, skill composition of the unemployed, and composition of job complexity. First, we compute the total decline in TFP at a given nominal interest rate. We then compute the fraction of the total decline that is due to the change in net production by computing the ratio of the difference between the "Baseline" and "$\varphi + \zeta$" change series in Figure 8 and the total decline in TFP. The fraction of the total decline which is due to the change in the skill composition of the unemployed is simply the ratio of the difference between the "Only ζ changes" and "$\varphi + \zeta$ change" series in Figure 8 to the total decline in TFP. Finally, the fraction of the decline due to the composition of job complexity changing is simply the ratio of the total decline in TFP which is not yet accounted for and the total decline in TFP. Figure 9 contains the results.

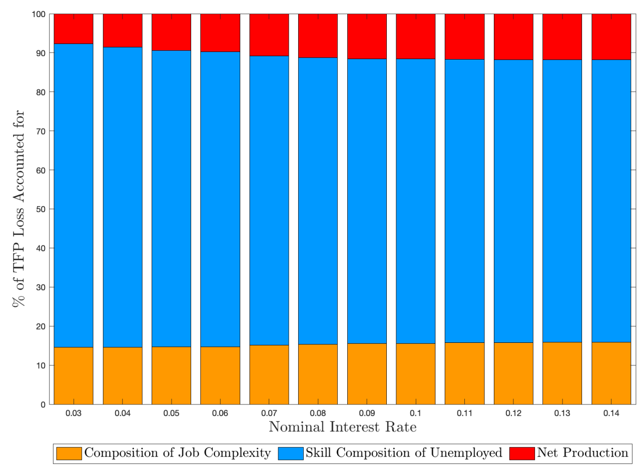


Figure 9: Decomposition of the Productivity Costs of Inflation

As seen in Figure 9, the fraction of the total decline in TFP due to a change in anticipated inflation which is due to a change in net output is around 10%, whereas the contribution of a change in the composition of job complexity is approximately 15%. This means that nearly 75% of the decline in TFP following an increase in anticipated inflation is due to the change in the skill composition of the unemployed. Moreover, these shares are relatively constant across different levels of the nominal interest rate.

Next, we carry out a quantitative experiment that is common among the New Monetarist literature. That is to quantitatively evaluate the effect of increasing the nominal interest rate from $i = 0$ (the Friedman rule) to a level which is consistent with an annual inflation rate of 10%. Table 3 contains our findings.

Table 3: From the Friedman Rule to 10% Annual Inflation

	Friedman rule	10% annual inflation
Unemployment rate	0.051	0.062
Fraction of jobs that are complex	0.536	0.440
TFP	1.000	0.957

From Table 3, increasing the nominal rate from 0 to a level which is consistent with an annual inflation rate of 10% increases the unemployment rate from 5.1% to 6.2%, an increase of 21.5%. As the unemployment rate increases and skills among the unemployed deteriorates, the percentage of jobs which are complex decreases from 53.6% to 44.0%, a decline of 17.9%. Finally, the last row shows that TFP decreases by 4.3%.

To provide some context for the productivity costs of inflation through the skill loss channel, we compare productivity costs generated by the model to TFP differences across low- and high-inflation OECD countries.[22] Table 4 presents the results.

Table 4: Comparison with TFP Differences Across OECD Countries

Country type	Inflation	TFP (data, normalized)	TFP (model, normalized)	% Explained
Low	5.45	1.000	1.000	–
High	8.20	0.938	0.994	9.67

From Table 4, low inflation OECD countries have an average annual inflation rate of 5.45%, whereas high inflation countries have an average annual inflation rate of 8.20% and 6.2% lower TFP. When we compute our model at two values of the nominal interest rate which correspond to 5.45% and 8.20% annual inflation rates, we find that TFP is 0.6% lower in the high inflation economies than the low inflation economies. Therefore, the model accounts for approximately 9.67% of the productivity differences observed in the data between low- and high-inflation OECD economies.

6.5 Stochastic Nominal Interest Rate

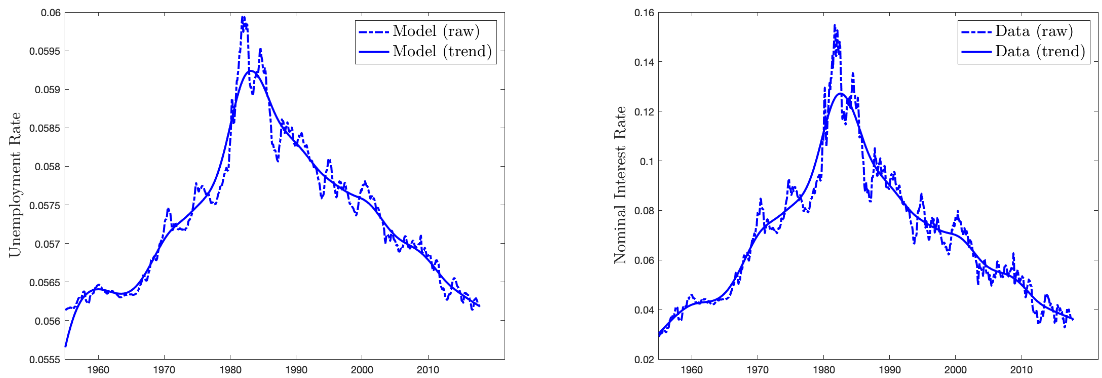
As a final quantitative exercise, we study the stochastic version of our model. In particular we assume the nominal interest rate follows an AR(1) process, solve the stochastic version of the model, and feed the nominal Aaa corporate bond yield series between 1955-2017 through the model.[23] Our motivation for doing so is twofold. First, Section 6.4 illustrated that the productivity costs of inflation can be sizeable and that the skill composition of unemployed workers accounts for nearly 75% of the productivity costs. However, these exercises compared different steady-states. If it takes time for the skill composition of unemployed workers to reach the new steady-state value following a change in the nominal interest rate, the steady-state experiments may overstate the contribution of the skill composition of unemployed workers changing to the productivity costs of inflation in an economy where the nominal interest rate evolves over time and is subject to shocks. Second, we can use the stochastic version of the model to evaluate how much low-frequency changes to anticipated inflation in the US between 1955-2017 impacted the unemployment rate, skill composition of the unemployed, composition of job complexity, and TFP. Figures 10-12 show the results.

Figure 10(a) demonstrates the counterfactual unemployment rate, which is generated only by changes in the nominal interest rate.[24] The model generates an increase in trend unemployment through the 1970s and 1980s following the increase in the nominal interest rate during this period. As anticipated inflation decreased following the early 1980s, the model generates a decrease in trend unemployment.

[22]We follow Berentsen et al. (2012)'s categorization of OECD countries as low- or high-inflation economies. Further, TFP data across OECD countries is from the Penn World Table 10.0 (Feenstra et al., 2015), downloadable at `https://www.rug.nl/ggdc/productivity/pwt/?lang=en`.

[23]See Appendix D for a definition of the stochastic recursive equilibrium.

[24]In Figures 10-12, the (trend) series are generated by applying an HP filter with smoothing parameter 129600 to the respective (raw) series.

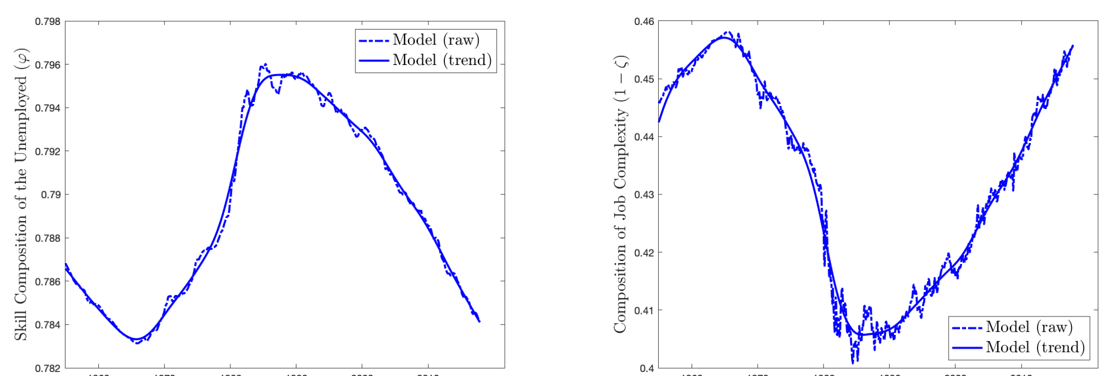


(a) Counterfactual Unemployment

(b) Nominal Interest Rate

Figure 10: Counterfactual Unemployment and Nominal Interest Rate

Turning to the compositions, Figure 11(a) shows the evolution of the skill composition of the unemployed. As the unemployment rate increases through the 1970s and 1980s, the skill composition of the unemployed degrades, as φ increased from 0.783 to 0.796 between 1966-1988. There are two additional features of φ to emphasize. First, the deterioration of the skill composition lags behind the unemployment rate. From Figure 10(a), the unemployment rate began to decline in 1983, whereas the skill composition only begins to recover after 1988. Second, the skill composition recovers at a slower rate than it takes to decay. It took 22 years for the skill composition to deteriorate between 1966-1988, whereas it required 29 years to recover and reach the levels observed in the late 1960s. Figure 11(b) presents the composition of the job complexity. As in the steady-state experiments, the fraction of jobs which are complex, $1-\zeta$, and fraction of unemployed workers who are less-skilled, φ, are inversely related. As the skill composition of unemployed workers was deteriorating through the 1970s and 1980s, so was the composition of job complexity. Following the decline in unemployment and improvement of the skill composition of unemployed, the fraction of jobs which are complex increases through the 1990s and 2000s.

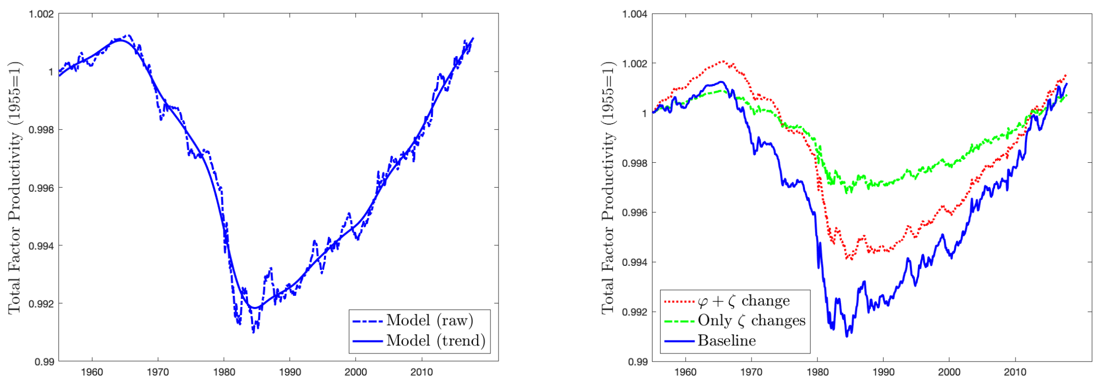


(a) Skill Composition of the Unemployed

(b) Composition of Job Complexity

Figure 11: Counterfactual Compositions

Next, Figure 12(a) displays the TFP series, which is normalized to 1 in 1955. As expected, TFP declines throughout the 1970s and 1980s as higher trend inflation caused the unemployment rate to increase, the skill composition of unemployed to worsen, and composition of job complexity to deteriorate. Our results show that trend TFP reaches its lowest value of 0.9918 in 1985, indicating a 0.82% decline in TFP relative to the 1955 level. Clearly this decline in TFP is smaller than the results in Section 6.4 which compared different steady-states and follows from the fact that it takes time for the skill composition of the unemployed to deteriorate following an increase in anticipated inflation. However, as we discussed above, it also takes time for the skill composition of the unemployed to recover. For this reason, the recovery in TFP lags behind the improvement in unemployment: the unemployment rate begins to recover in 1983, whereas TFP continues to decline until 1985.



(a) TFP

(b) Decomposition

Figure 12: Counterfactual TFP

Finally, Figure 12(b) carries out the same decomposition exercise as in Figure 8. We find that when TFP reaches its lowest value in the beginning of 1985, 35.7% of the decline in TFP relative to its 1955 level is due to the shift in the composition of job complexity, 33.24% is attributable to the change in the skill composition of unemployed workers, and the remaining 31.06% is a result of the decline in net production. Therefore, the contributions of the three channels are quantitatively similar in the stochastic version of the model. We attribute this result to the fact that it takes time for the skill composition of the unemployed to reach its new steady-state value following a change to anticipated inflation.

7 Conclusion

Economists have long been interested in the relationship between inflation and unemployment. Recently, models with both payment and labor frictions have been developed to analyze their relationship. Alongside this literature, a large body of evidence has shown workers' human capital decays when they are unemployed, generating a linkage between labor market flows and aggregate productivity. This paper has developed a framework to link these two large literatures and quantified the relationship between inflation, unemployment, and TFP when workers lose skills during unemployment. The model shows that the economy's labor market flows, skill composition of unemployed workers, and TFP are linked to the value of money and anticipated inflation. The quantitative exercises demonstrate that the productivity costs of inflation can be sizeable and that shifts in the skill composition of unemployed workers are a primary driver of these productivity costs. The business cycle version of the model illustrates that extended periods of high inflation, such as the 1970s in the US, can scar the economy as productivity lags behind the recovery in unemployment. We argue that this result is relevant for current policy discussions in the US around the potential trade-offs of allowing the economy to run above the inflation target for an extended period.

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Appendix

A Proofs and Derivations

A.1 Proof of Proposition 1

We first establish that the RM curve is downward sloping in the (u, q) space. Recall that the RM curve:

$$i = (1-\mu)\left[\alpha \frac{\upsilon'(q)}{c'\left(\frac{q}{1-u}\right)} + (1-\alpha)\right] - 1. \tag{A.1}$$

Suppose that u increases. It follows that the right hand side of (A.1) increases as $c'' > 0$. It follows that q must decrease to ensure (A.1) is satisfied, as $\upsilon'' < 0$.

We now establish that for $i > 0$ and $u = 0$ that $q = \hat{q} \in (0, q^*)$. If $i > 0$ and $u = 0$, it is straightforward to show $\hat{q}$ solves

$$\frac{i+\mu}{1-\mu} = \alpha\left[\frac{\upsilon'(\hat{q})}{c'(\hat{q})} - 1\right]. \tag{A.2}$$

As the left hand side of (A.2) is greater than zero, we require that $\upsilon'(q)/c'(q) > 1$. It follows that $\hat{q} < q^*$ as $\upsilon'(q^*)/c'(q^*) = 1$ and $\upsilon'(q)/c'(q)$ is decreasing in q.

Next we show that $q \to 0$ as $u \to 1$. We can see that as $u \to 1$, that $c'(q/(1-u)) \to \infty$ as $q/(1-u) \to \infty$. It follows that $q \to 0$ to ensure (A.1) is satisfied when $u \to 1$ as $\upsilon'(0) = \infty$.

Finally, suppose that i increases. It follows that the left hand side of (A.1) increases, meaning that the right hand side must also increase. Holding fixed the unemployment rate, u, it is straightforward to see that q must decrease so that (A.1) is satisfied, as $\upsilon'(q)/c'(q/(1-u))$ is decreasing in q. Thus, following an increase in i, the equilibrium value of q at each unemployment rate as determined through the RM curve decreases and the curve shifts downward. ■

A.2 The Cost of Skill Loss: Homogenous firms

The cost of skill loss is defined by $\Delta_\sigma = V_{H,0}(z) - V_{L,0}(z)$, which after substituting the relevant value functions into (6) is given by

$$\Delta_\sigma = \frac{\bar{\beta}\gamma\xi_h(S_H - S_L)}{1 - \bar{\beta}(1-(1-\xi_h)\sigma)}. \tag{A.3}$$

From (A.3), the cost of skill loss is the discounted sum of the additional surplus a highly-skilled worker obtains in the labor market. Combining equation (A.3) with S_L and S_H from (31) and solving for Δ_σ gives

$$\Delta_\sigma = \frac{\gamma\xi_h y(1-\delta)}{(\mu+\rho(1+\mu)+\lambda+\gamma\xi_h)[1-\bar{\beta}(1-(1-\xi_h)\sigma)] - \gamma\xi_h\bar{\beta}(1-\xi_h)\sigma}. \tag{A.4}$$

A.3 Proof of Proposition 2

Our first objective is to show that there exists at least one θ (and hence u) that satisfies the LM curve for all $q \in [0, q^*]$. Recall the job creation condition:

$$\frac{k}{\xi_f} = (1-\gamma)\left[\frac{y[1-\varphi(1-\delta)] + (1-\varphi)\bar{\beta}(1-\xi_h)\sigma\Delta_\sigma + c'\left(\frac{\alpha q}{1-u}\right)\frac{\alpha q}{1-u} - c\left(\frac{\alpha q}{1-u}\right) - b}{\mu+\rho(1+\mu)+\lambda+\gamma\xi_h}\right]. \tag{A.5}$$

We know that if $\theta \to \infty$, then $\xi_f \to 0$ and the left hand side of (A.5) approaches ∞. The right hand side, however goes to

$$(1-\gamma)\left[\frac{y[1-\varphi(1-\delta)]+c'(\alpha q)\alpha q-c(\alpha q)-b}{\mu+\rho(1+\mu)+\lambda+\gamma}\right]<\infty. \tag{A.6}$$

Therefore, there exists at least one $\theta > 0$ which satisfies the job creation condition as long as the left hand side of (A.5) is less than the right hand side when $\theta \to 0$. As $\theta \to 0$, the left hand side of (A.5) approaches k. It is then straightforward to show that there is at least one $\theta > 0$ which satisfies (A.5) if

$$k<(1-\gamma)\left[\frac{y[1-\varphi(1-\delta)]+(1-\varphi)\bar{\beta}\sigma\Delta_\sigma-b}{\mu+\rho(1+\mu)+\lambda}\right]. \tag{A.7}$$

The right hand side of (A.7) is clearly decreasing in φ. To establish a sufficient condition for the existence of $\theta > 0$, we suppose that $\varphi \to 1$. It follows that the left hand side of (A.5) is less than the right hand side as $\theta \to 0$ and there exists at least one $\theta > 0$ which satisfies (A.5) if

$$k(\mu+\rho(1+\mu)+\lambda)<(1-\gamma)(\delta y-b). \tag{A.8}$$

For the rest of the proof, we assume that (A.8) holds and further assume that parameters are such that there is a unique θ which satisfies (A.5). This need not always be the case as an increase in θ improves the skill distribution among the unemployed (φ decreases), which means the right hand side of (A.5) can be upward sloping. This typically only occurs under extreme parameters quantitatively.

We proceed to show that the LM curve slopes downward in the (u, q) space. Suppose that q increases within the interval $[0, q^*]$. It follows that the profits a firm obtains from selling inventory in the retail market, $c'\left(\frac{\alpha q}{1-u}\right)\frac{\alpha q}{1-u} - c\left(\frac{\alpha q}{1-u}\right)$, increase, causing the right hand side of the job creation condition to increase. Therefore, market tightness, θ, must increase so that the left hand side of (A.5) also increases. As market tightness increases, the unemployment rate decreases, and hence, the LM curve is downward sloping in the (u, q) space. The lowest level of the unemployment rate, $\underline{u} = (\mu + (1-\mu)\lambda)/(\mu + (1-\mu)(\lambda + \xi_h(\bar{\theta}))$, is reached when $q = q^*$ and $\bar{\theta}$ solves (A.5) with $q = q^*$. Moreover, the highest level of the unemployment rate, $\bar{u} = (\mu + (1-\mu)\lambda)/(\mu + (1-\mu)(\lambda + \xi_h(\underline{\theta}))$ where $\underline{\theta}$ solves (A.5) when $q = 0$. Clearly, $\underline{\theta} > 0$ and therefore $\bar{u} < 1$.

Now suppose that $\gamma = 0$. Our last step is to establish that the LM curve shifts to the right with σ and to the left with δ. The job creation condition is given by

$$\frac{k}{\xi_f}=\frac{y[1-\varphi(1-\delta)]+c'\left(\frac{\alpha q}{1-u}\right)\frac{\alpha q}{1-u}-c\left(\frac{\alpha q}{1-u}\right)-b}{\mu+\rho(1+\mu)+\lambda}, \tag{A.9}$$

where the skill distribution among the unemployed is given by

$$\varphi=\frac{\sigma(1-\mu)(1-\xi_h)[1-(1-\mu)(1-\lambda)]}{\mu(1-\mu)\xi_h+[\mu+(1-\mu)(1-\xi_h)\sigma][1-(1-\mu)(1-\lambda)]}. \tag{A.10}$$

Suppose that δ increases. The right hand side of (A.9) increases. Therefore, market tightness must increase to satisfy the job creation condition. It then follows that for each q, market tightness increases and the unemployment rate decreases, i.e. the LM curve shifts to the left. Now consider an increase to σ. From (A.10), φ increases. As φ increases, the right hand side of (A.9) decreases. Therefore, market tightness decreases and the economy exhibits a higher unemployment rate at each value of q. This establishes that the LM curve shifts to the right following an increase in σ. ■

A.4 Proof of Proposition 3

We first establish the existence of at least one monetary equilibrium with $q \in (0, q^*)$ and $u \in (0, 1)$. Following the Proofs of Propositions 1 and 2, both the RM and LM curves are downward sloping in the (u, q) space. However, we know that at $u = 0$, the RM curve is below the LM curve and as $u \to 1$, the RM curve is above the LM curve as shown in Figure 5, indicating that the curves must cross at least once at a point where $q > 0$, $q \in (0, q^*)$, and $u \in (0, 1)$. In general, the monetary equilibrium may not be unique as there may be multiple θ which satisfy the job creation condition, as discussed in the Proof of Proposition 2.

Second, there also exists at least one non-monetary equilibrium as $q = 0$ is always an equilibrium outcome to the household's portfolio choice, irrespective of the unemployment rate. Additionally, as established in the Proof of Proposition 2, $u = \bar{u}$ where $\bar{u} < 1$ when $q = 0$ and (A.8) is satisfied.

Turning to the comparative statics and focusing on a monetary equilibrium. If $\gamma = 0$, then by the Proof of Proposition 2, an increase in σ or decrease in δ both cause the LM curve to shift to the right. It follows that the new LM curve intersects the RM curve at a lower value of q and higher unemployment rate, u. Finally, if i increases, then the RM curve shifts outward (following the Proof of Proposition 1) and the RM curve intersects the LM curve at a lower q and higher level of the unemployment rate, u. ■

A.5 The Cost of Skill Loss: Heterogeneous firms

As in the case with homogeneous firms, the cost of skill loss is given by $\Delta_\sigma = V_{H,0} - V_{L,0}$ and after substituting stage 1 and 3 value functions into (39) and solving for Δ_σ, we have

$$\Delta_\sigma = \frac{\bar{\beta}\gamma\xi_h\{\zeta(S_{H,s} - S_{L,S}) + (1-\zeta)(S_{H,c} - S_{L,c})\}}{1 - \bar{\beta}(1 - (1-\xi_h)\sigma)}. \tag{A.11}$$

Equation (A.11) has the same interpretation as equation (A.3): the cost of skill loss is the discounted sum of the additional surplus a highly-skilled worker earns in the labor market. In the version of the heterogeneous firms, the additional surplus obtained by a highly-skilled worker is weighted by the composition of job complexity. Combining equation (A.11) with equations (54)-(57) and solving for Δ_σ gives

$$\Delta_\sigma = \frac{\gamma\xi_h\{\zeta[y_s(1-\delta_s)\rho_1 - \gamma\xi_h(1-\zeta)(y_c(1-\delta_c) - y_s(1-\delta_s))] + (1-\zeta)[y_c(1-\delta_c)\rho_1 - \gamma\xi_h\zeta(y_s(1-\delta_s) - y_c(1-\delta_c))]\}}{\rho_1\rho_2[1 - \bar{\beta}(1-(1-\xi_h)\sigma)] - \rho_1\gamma\xi_h\bar{\beta}(1-\xi_h)\sigma}, \tag{A.12}$$

where $\rho_1 = \mu + \rho(1+\mu) + \lambda$ and $\rho_2 = \mu + \rho(1+\mu) + \lambda + \gamma\xi_h$.

A.6 Proof of Proposition 4

To establish sufficient conditions for all match surpluses to be positive, we only need to establish sufficient conditions for match surpluses with low skill workers to be positive. This is because match surpluses with high skill workers are always higher due to the fact that highly-skilled workers are more productive and forming a match saves them the risk of skill loss, captured by the term $\bar{\beta}(1-\xi_h)\sigma\Delta_\sigma$ in equations (56)-(57). From equation (54), $S_{L,s} > 0$ if

$$\frac{R_{L,s} - b}{R_{L,c} - b} > \frac{\gamma\xi_h(1-\zeta)}{\mu + \rho(1+\mu) + \lambda + \gamma\xi_h(1-\zeta)}. \tag{A.13}$$

The right hand side of (A.13) is decreasing in ζ and increasing in ξ_h. Thus, we set $\xi_h = 1$ and $\zeta = 0$ to find a sufficient condition on only fundamentals for $S_{L,s} > 0$:

$$\frac{R_{L,s} - b}{R_{L,c} - b} > \frac{\gamma}{\mu + \rho(1+\mu) + \lambda + \gamma}. \tag{A.14}$$

Further suppose that $q_s = 0$ and $q_c = \bar{q}$ where $\bar{q} \equiv \arg\max\{c(q)q - c(q)\}$. It follows that (A.14) and, therefore, (A.13), are satisfied if

$$\frac{\delta_s y_s - b}{\delta_c y_c + [c'(\bar{q})\bar{q} - c(\bar{q})] - b} > \frac{\gamma}{\mu + \rho(1+\mu) + \lambda + \gamma}, \tag{A.15}$$

which establishes the sufficient condition for $S_{L,s} > 0$.

Turning to matches between less-skilled workers and complex jobs, from (55), $S_{L,c} > 0$ if

$$\frac{R_{L,c} - b}{R_{L,s} - b} > \frac{\gamma \xi_h \zeta}{\mu + \rho(1+\mu) + \lambda + \gamma \xi_h \zeta}. \tag{A.16}$$

Following the same logic as above, it is straightforward to show that $S_{L,c} > 0$ if

$$\frac{\delta_c y_c - b}{\delta_s y_s + [c'(\bar{q})\bar{q} - c(\bar{q})] - b} > \frac{\gamma}{\mu + \rho(1+\mu) + \lambda + \gamma}. \tag{A.17}$$

Equations (A.15) and (A.17) establish sufficient conditions for all matches to generate a positive surplus. ■

A.7 Proof of Proposition 5

To show the existence of at least one monetary equilibrium, we first derive sufficient conditions to ensure firms post vacancies. Following the same steps as in the Proof of Proposition 2 to derive equation (A.8), it is straightforward to show that firms will always post a positive measure of type χ vacancies if

$$k_\chi < \frac{(1-\gamma)\sigma(1-\mu)(\delta_\chi y_\chi - b)}{(\mu + \rho(1+\mu) + \lambda)(\mu + (1-\mu)\sigma)}. \tag{A.18}$$

Therefore, if equation (A.18) holds for both $\chi = \{s, c\}$, then a positive measure of both types of vacancies are always created. It follows that $\theta > 0$, $u < 1$, and $\zeta \in (0,1)$.

We proceed by assuming (A.18) holds for all $\chi \in \{s, c\}$. It is straightforward to show the existence of a monetary equilibrium. First, from equations (46)-(47), there is a unique $q_\chi^D > 0$ for $\chi \in \{s, c\}$ which solves the household's optimization problem in the retail market for a given level of real balances, z, and prices p_s and p_c. Moreover, from equation (51), the marginal benefit to an additional unit of real balances is strictly decreasing in z'. Therefore, for a given $u \in (0,1)$ and $\zeta \in (0,1)$, there exists $q_\chi > 0$ for $\chi \in \{s, c\}$ which solves the household's portfolio choice and utility maximization problem within the retail market.

Finally, as in the case of homogeneous firms, there can be both a non-monetary equilibrium as $q_\chi = 0$ for $\chi \in \{s, c\}$ is always a solution to the household's portfolio choice. ■

B Empirical Appendix

B.1 Data Sources and Construction

This section describes the data used in the empirical motivation and quantitative exercises. All data used in Section 2 and Appendix B.3 were downloaded directly from the Federal Reserve Bank of St. Louis' FRED database.[25] The following are the data series we download from the FRED database, covering 1955-2017.

- Unemployment rate.
  - Series title: Civilian unemployment rate.
  - Series ID: UNRATE.
- Interest rate.
  - Series title: Moody's Seasoned Aaa Corporate Bond Yield.
  - Series ID: AAA.
- TFP.
  - Series title: Total Factor Productivity at Constant National Prices for United States (as constructed by Feenstra et al. (2015)).
  - Series ID: RTFPNAUSA632NRUG.
- CPI.
  - Series title: Consumer Price Index for All Urban Consumers: All Items.
  - Series ID: CPIAUCSL.
- Labor Productivity.
  - Series title: Nonfarm Business Sector: Real Output Per Hour of All Persons.
  - Series ID: PRS85006091.

We then use the data to construct the following series.

- TFP growth is the percentage growth in TFP between year $t-1$ and year t.
- Inflation is the percentage growth in CPI between year $t-1$ and year t.

The calibration uses several additional series downloaded from the FRED database.

- M1 (1955-1958).
  - Series title: M1 Money Stock.
  - Series ID: M1SL.
- M1 (1959-2017).

[25]Data from the FRED database can be directly downloaded from `https://fred.stlouisfed.org/`.

- Series title: M1 Money Stock.
- Series ID: M1SA.

- Money market deposit accounts.
  - Series title: Money Market Funds; Total Time and Savings Deposits.
  - Series ID: BOGZ1FL633030000Q.

- Nominal GDP.
  - Series title: Gross Domestic Product.
  - Series ID: GDP.

We then construct the series for money demand as follows.

- Construct the total money supply by adding together the M1 and money market deposit accounts.
- Calculate money demand by taking the ratio of the total money supply and nominal GDP. Note that this is the series used to construct the average money demand and elasticity of money demand in the calibration strategy outlined in Section 6.2.

B.2 Task Scores and Job Complexity

Measures of tasks are created by Autor and Dorn (2013), where they derived the abstract, routine and manual tasks using the US Department of Labor's *Dictionary of Occupational Titles* (DOT). The original five task measures of Autor et al. (2003) are collapsed into three categories, following Autor et al. (2006). The manual task measurement, $T^{M}_{k,1980}$, is the DOT variable for an occupation's demand for "eye-hand-foot coordination" in 1980. The abstract task measure, $T^{A}_{k,1980}$, is the average of the DOT's variables for "direction control and planning" which measures managerial and interactive tasks and "GED Math", measuring mathematical and formal reasoning requirement in 1980. More details could be found in Autor et al. (2003) Appendix Table 1. Table 5 shows the occupations with the twenty highest and lowest AM measures.

After constructing the AM measures for each occupation, we then match the AM scores by occupation to the Annual Social and Economic Supplement (ASEC) of the Current Population Survey (CPS) in order to calculate the fraction of total employment above and below our cutoff that distinguishes simple and complex jobs.[26] To merge the AM scores into the ASEC data, we used the 1990 Census Bureau occupational classification. That is, we use the "occ1990" variable in the ASEC data and merge with AM occupation scores using the crosswalk developed by Autor and Dorn (2013).[27] We then restrict our sample to individuals between ages 25-65.

Figure 13 shows several occupations around the cutoff between simple and complex occupations. The vertical axis is the occupation's average level of employment between 1968-2017. While the red line represents our baseline cutoff between simple and complex jobs, we also consider alternative cutoffs as shown by the purple and green vertical lines. See Appendix C for more details.

[26] We download the ASEC data directly from IPUMS CPS which is available at https://cps.ipums.org/cps/.

[27] The "occ1990" variable reports the respondent's primary occupation, i.e. the occupation where the respondent has received the highest income from.

Table 5: Occupation with the highest and lowest AM

Highest 20	*AM*	Lowest 20	*AM*
Physical Scientist	1	Dancers	0
Chemical Engineers	0.983	Parking Lot Attendant	0.222
Chemists	0.952	Paving and surfacing equipment operators	0.253
Actuaries	0.944	Operating Engineers of construction equipment	0.273
Dietitians and Nutritionists	0.942	Fire Fighting	0.273
Metallurgical and Materials Engineers	0.926	Excavating and Loading Machine Operators	0.281
Mechanical Engineers	0.926	Bus Driver	0.283
Funeral Directors	0.924	Truck, Delivery, and Tractor Drivers	0.283
Accountants and Auditors	0.922	Taxi Cab Driver	0.285
Petroleum, Mining and Geological Engineers	0.921	Roofer and Slaters	0.291
Managers of Medicine	0.914	Crane, derrick, winch, and hoist operators	0.291
Financial Managers	0.911	Structural Metal Workers	0.302
Aerospace Engineer	0.897	Plasterers	0.306
Atmospheric and Space Scientists	0.895	Textile and Sewing Machine Operator	0.343
Other Financial Specialist	0.893	Garbage and Recyclable Material Collector	0.343
Subject Instructor (HS/College)	0.892	Driller of Earth	0.361
Managers and Specialists in Marketing, Advertising, and Public relations	0.883	Railroad brake, coupler, and switch operators	0.362
Biological Scientists	0.882	Millwrights	0.370
Computer Software Developer	0.879	Carpenter	0.371

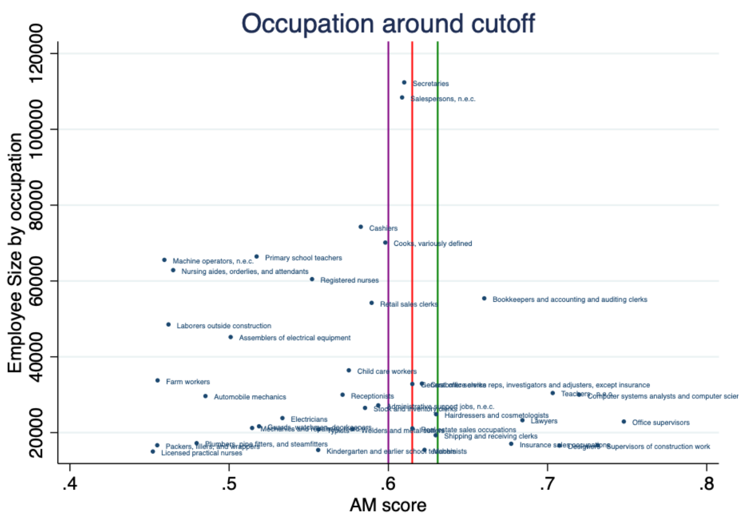


Figure 13: Occupations close to the cutoff between simple and complex jobs

B.3 Supplementary Figures

Figure 14 presents the scatter plots for the relationship between low frequency trends in CPI inflation and the unemployment rate using annual US data from 1955-2017. As in the main text, each data point is the annual average for a particular year. Figure 15 shows the relationship between low frequency movements in labor productivity growth and the unemployment rate. The correlation between the two series is -0.7279. Figure 16 presents low frequency trends in CPI inflation and the growth rate of TFP using annual US data from 1955-2017.

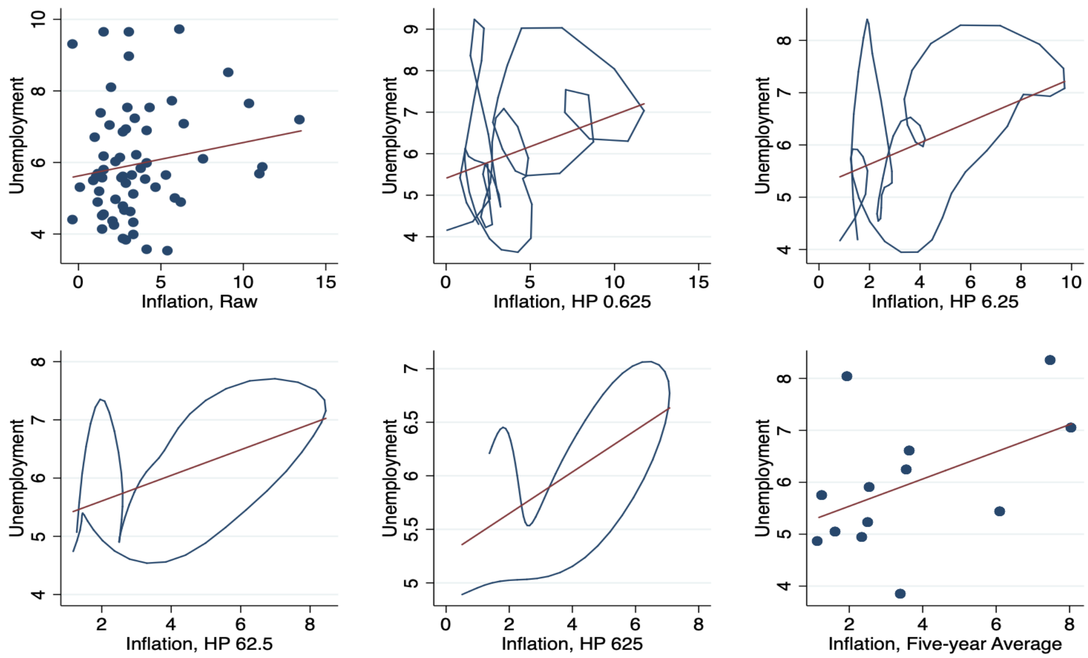


Figure 14: Inflation and Unemployment

Figure 15: Low Frequency Labor Productivity Growth and Unemployment

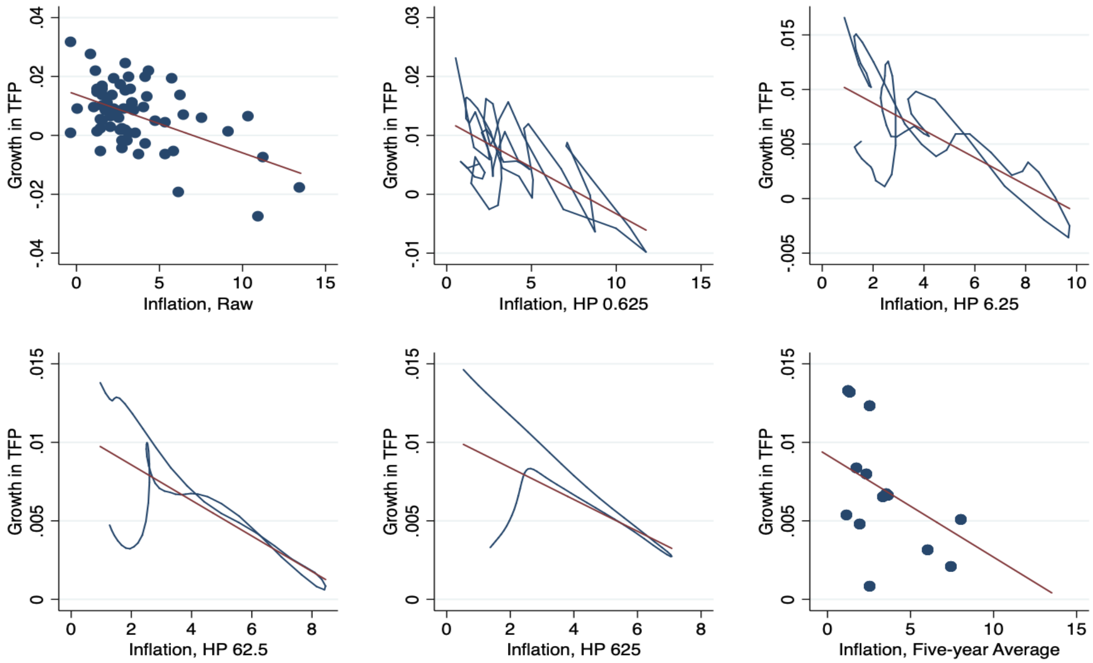


Figure 16: Inflation and TFP Growth

Figures 17 and 18 present scatter plots illustrating the relationship between lower-frequency movements between measures of anticipated inflation and the growth rate of labor productivity. The figures use quarterly US data from 1955-2017. Interest rate in Figure 17 is the nominal Aaa corporate bond rate, whereas inflation in Figure 18 is CPI inflation.

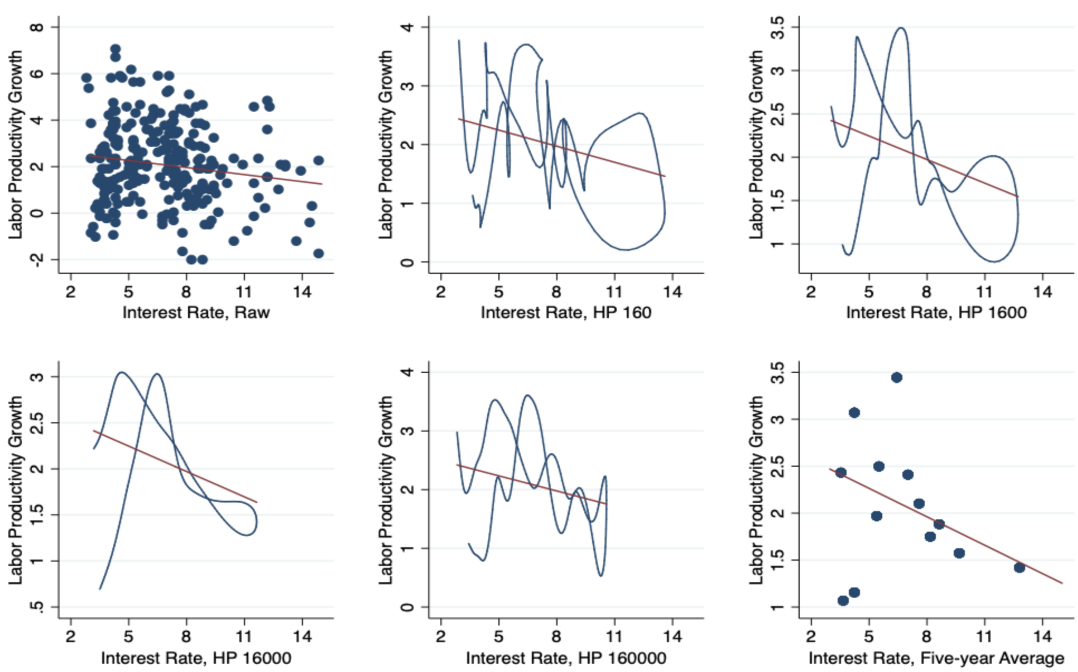


Figure 17: Interest Rate and Labor Productivity Growth

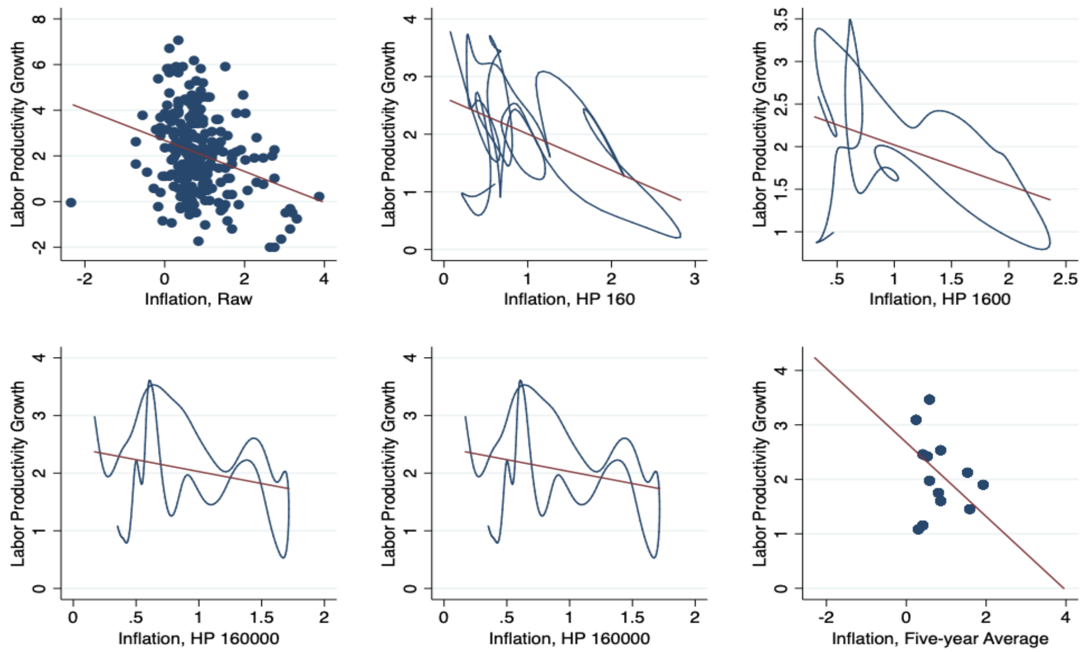


Figure 18: Inflation and Labor Productivity Growth

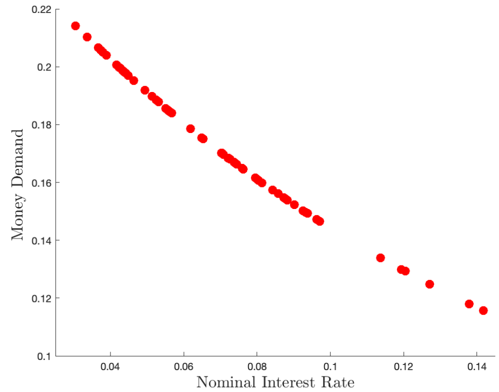
C Additional Quantitative Results

In this section, we present four alternative calibration strategies. The first one, "Markup", targets a lower level of markups in the retail sector (20%), as opposed to 30% in the baseline calibration. The second strategy, "Skill loss", assumes that skill loss takes six months to occur ($\sigma = 1/6$) rather than three months ($\sigma = 1/3$). The third strategy targets $\zeta = 0.50$ instead of $\zeta = 0.52$, where $\zeta = 0.50$ is the composition of job complexity when we use an AM score of 0.60 as the lower bound for a complex occupation. This is the AM score labelled as "Lower Bound" in Figure 6. The last strategy targets $\zeta = 0.62$, which corresponds to an AM score of 0.631 as being the lower bound for a complex occupation. In Figure 6, this is the cutoff labelled "Upper Bound". Table 6 presents parameter values under our baseline and four alternative strategies.

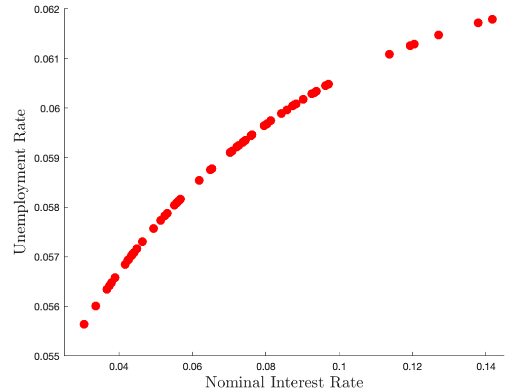
In the rest of this section, we present the quantitative results under the alternative strategies. Figures 19-21 contain the results for the markup calibration, Figures 22-24 for the skill loss calibration, Figures 25-27 for the strategy which targets $\zeta = 0.50$, and Figures 28-30 for the strategy which targets $\zeta = 0.62$.

Table 6: Parameter values under alternative calibrations

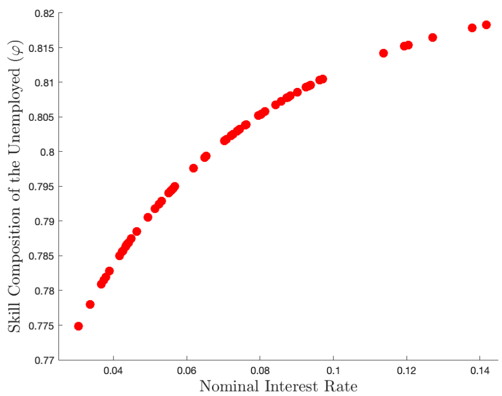
Parameter	Baseline	Markup	Skill loss	$\zeta = 0.50$	$\zeta = 0.62$
Panel A: Assigned parameters					
ρ	1.68×10^{-3}	1.68×10^{-3}	1.68×10^{-3}	1.68×10^{-3}	1.68×10^{-3}
μ	2×10^{-3}	2×10^{-3}	2×10^{-3}	2×10^{-3}	2×10^{-3}
λ	0.035	0.035	0.035	0.035	0.035
η	0.50	0.50	0.50	0.50	0.50
γ	0.50	0.50	0.50	0.50	0.50
y_c	1.00	1.00	1.00	1.00	1.00
σ	1/3	1/3	1/6	1/3	1/3
i	6.89×10^{-2}	6.89×10^{-2}	6.89×10^{-2}	6.89×10^{-2}	6.89×10^{-2}
a	1.30	1.20	1.30	1.30	1.30
Panel B: Calibrated parameters					
A	0.590	0.590	0.590	0.590	0.590
y_s	0.787	0.770	0.760	0.775	0.797
δ_s	0.825	0.826	0.839	0.825	0.825
δ_c	0.650	0.648	0.642	0.648	0.641
k_s	0.245	0.195	0.202	0.224	0.273
k_c	0.589	0.635	0.853	0.602	0.591
b	0.554	0.547	0.570	0.550	0.554
α	0.050	0.073	0.051	0.049	0.057
ϱ	1.693	1.847	1.705	1.686	1.731



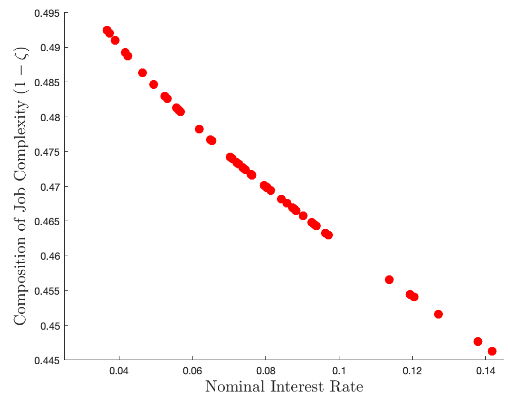
(a) Money Demand



(b) Long Run Phillips Curve



(c) Skill Composition of the Unemployed



(d) Composition of Job Complexity

Figure 19: Effects of a Change in Anticipated Inflation (markup calibration)

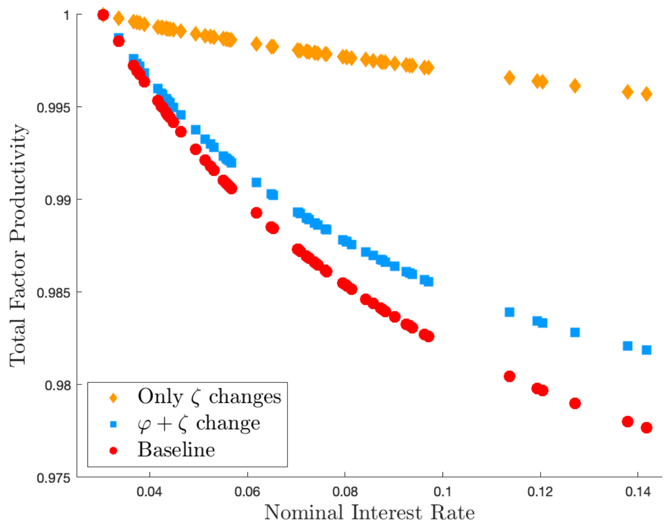


Figure 20: The Productivity Costs of Inflation (markup calibration)

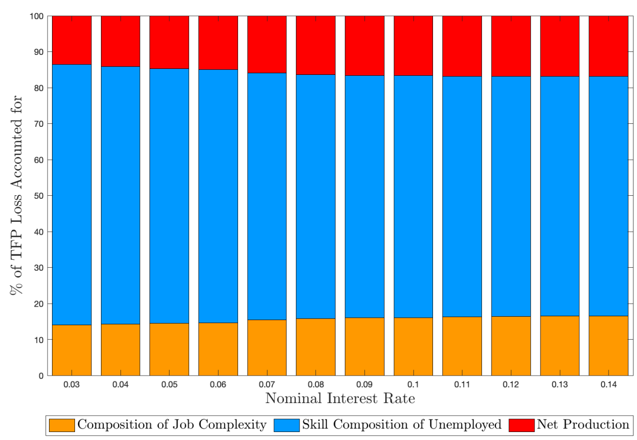
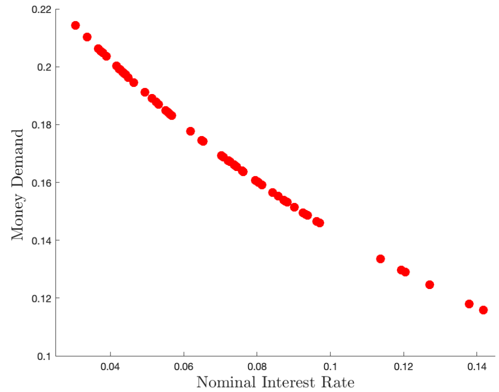
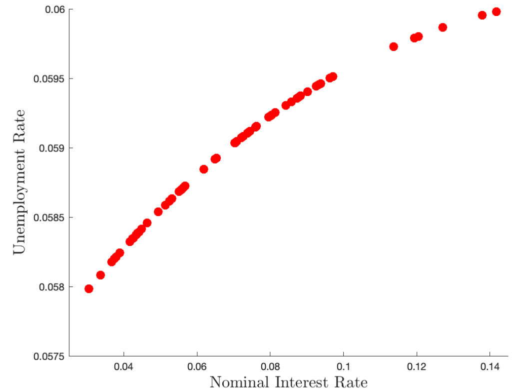


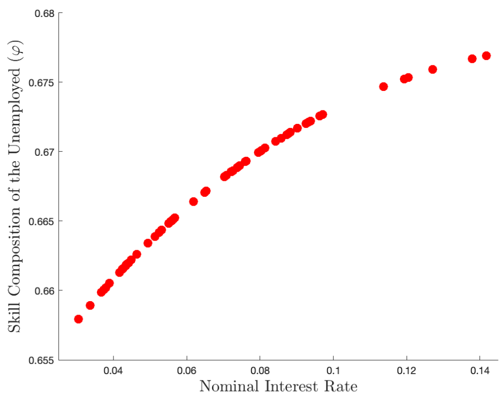
Figure 21: Decomposition of the Productivity Costs of Inflation (markup calibration)



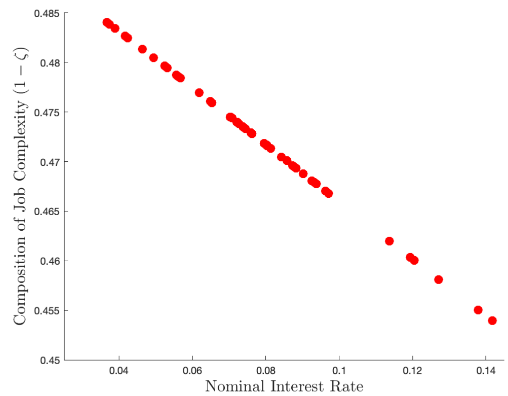
(a) Money Demand



(b) Long Run Phillips Curve



(c) Skill Composition of the Unemployed



(d) Composition of Job Complexity

Figure 22: Effects of a Change in Anticipated Inflation (skill loss calibration)

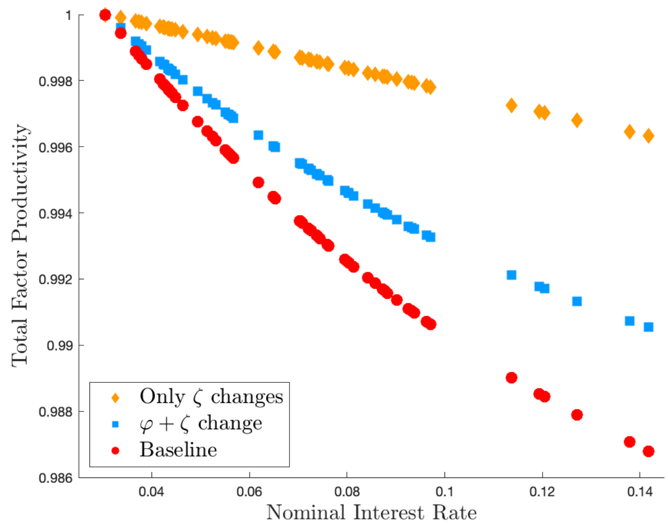


Figure 23: The Productivity Costs of Inflation (skill loss calibration)

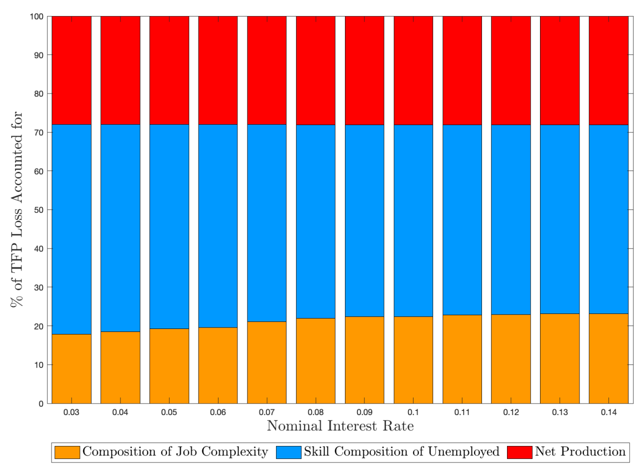
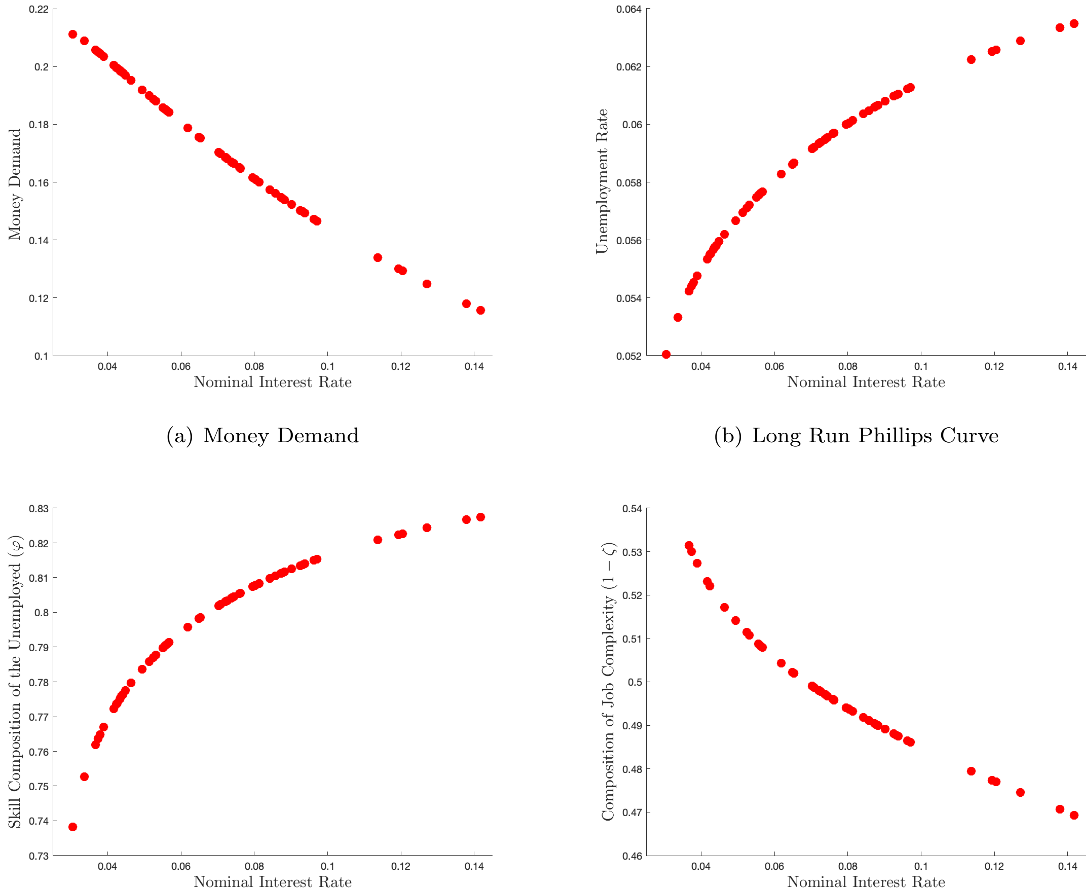


Figure 24: Decomposition of the Productivity Costs of Inflation (skill loss calibration)



(a) Money Demand

(b) Long Run Phillips Curve

(c) Skill Composition of the Unemployed

(d) Composition of Job Complexity

Figure 25: Effects of a Change in Anticipated Inflation (low ζ calibration)

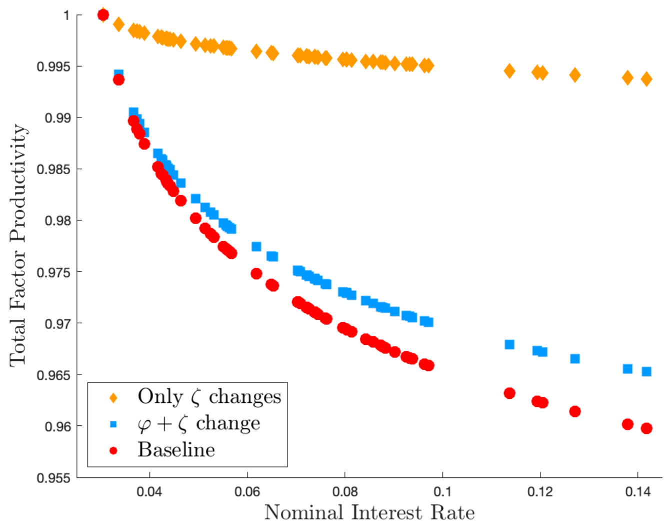


Figure 26: The Productivity Costs of Inflation (low ζ calibration)

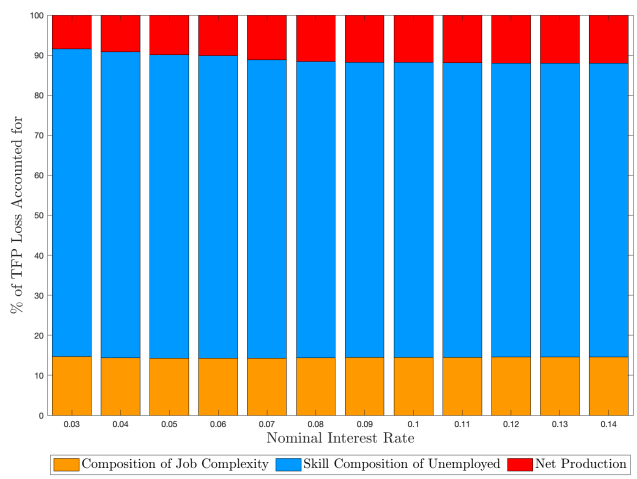
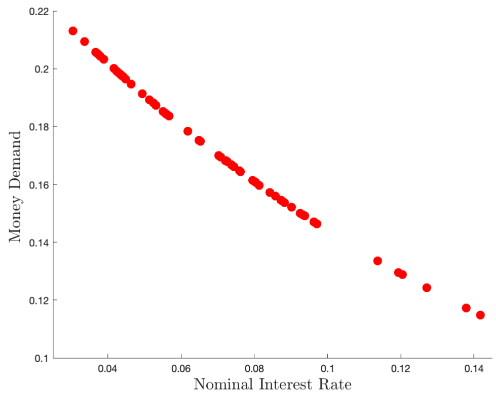
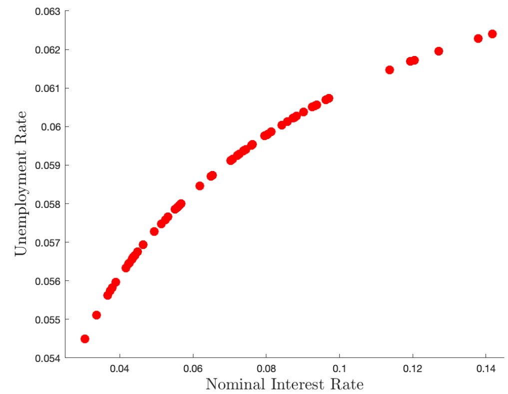


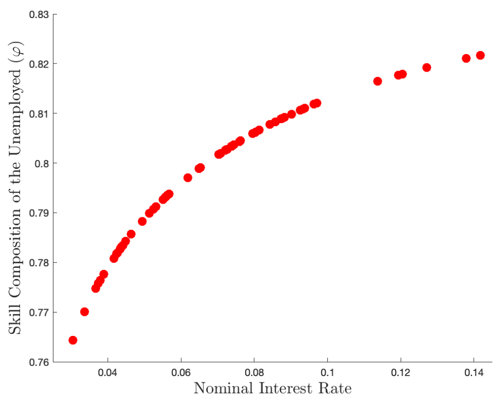
Figure 27: Decomposition of the Productivity Costs of Inflation (low ζ calibration)



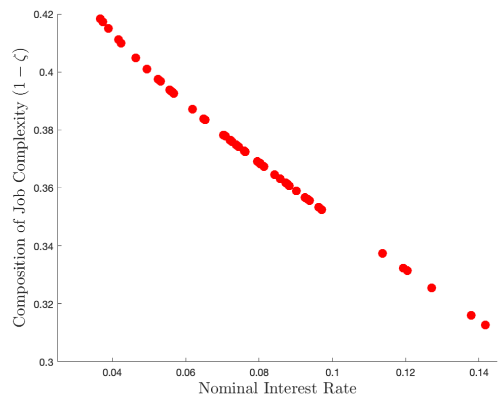
(a) Money Demand



(b) Long Run Phillips Curve



(c) Skill Composition of the Unemployed



(d) Composition of Job Complexity

Figure 28: Effects of a Change in Anticipated Inflation (high ζ calibration)

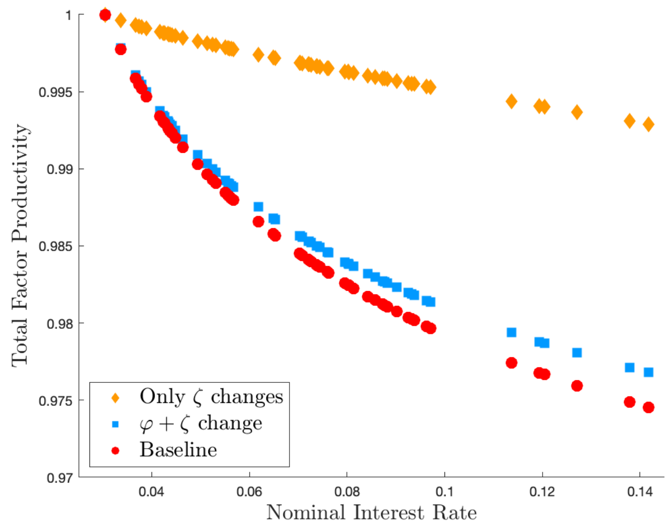


Figure 29: The Productivity Costs of Inflation (high ζ calibration)

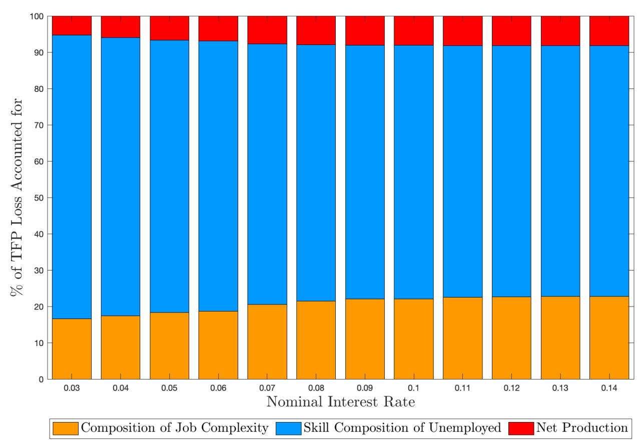


Figure 30: Decomposition of the Productivity Costs of Inflation (high ζ calibration)

D Stochastic Version of the Model

In the stochastic version of the model, the state vector is given by $\psi = (u_L, u_H, n_L, i)$, where the nominal interest rate follows a stochastic process that is given by:

$$\hat{i} = \bar{i} + \rho_i(i - \bar{i}) + \varepsilon_i, \ \varepsilon_i \sim N(0, \sigma_i), \tag{D.1}$$

and the laws of motion for u_L, u_H, and n_L are given by

$$\hat{u}_L(\psi) = (1-\mu)\big[\big(1-\xi_h(\theta(\psi))\big)[u_L + \sigma u_H]\big], \tag{D.2}$$

$$\hat{u}_H(\psi) = \mu + (1-\mu)\big[\big(1-\xi_h(\theta(\psi))\big)(1-\sigma)u_H + \lambda(1-u_L-u_H-n_L)\big], \tag{D.3}$$

$$\hat{n}_L(\psi) = \mu + (1-\mu)\big[(1-\lambda)n_L + \xi_h(\theta(\psi))u_L\big]. \tag{D.4}$$

We assume that the shock to the nominal interest rate is realized in stage 3 before households make their portfolio choice and firms make their vacancy posting decision.

Denoting $P(\hat{\psi};\psi)$ as the transition probability function between ψ and ψ', a recursive equilibrium is a list of functions $\{\theta(\psi), \zeta(\psi), R_{\varepsilon,\chi}(\psi), \Delta_\sigma(\psi), S_{\varepsilon,\chi}(\psi), q_\chi(\psi), P(\hat{\psi};\psi)\}$ for $\varepsilon \in \{L,H\}$ and $\chi \in \{s,c\}$ such that:

$$\begin{aligned}\frac{k_s}{\xi_f(\theta(\psi))} = \bar{\beta}(1-\gamma)\Bigg[&\varphi(\psi)R_{L,s}(\psi) + (1-\varphi(\psi))R_{H,s}(\psi) - b + \bar{\beta}\mathbb{E}\Bigg\{\frac{(1-\lambda-\gamma\xi_h(\theta(\hat{\psi}))\zeta(\hat{\psi}))k_s}{\xi_f(\theta(\hat{\psi}))\bar{\beta}(1-\gamma)} \\ &- \frac{\xi_h(\theta(\hat{\psi}))(1-\zeta(\hat{\psi}))\gamma k_c}{\xi_f(\theta(\hat{\psi}))\bar{\beta}(1-\gamma)} + (1-\varphi(\psi))(1-\xi_h(\theta(\hat{\psi})))\sigma\Delta_\sigma(\hat{\psi})\Bigg\}\Bigg],\end{aligned} \tag{D.5}$$

$$\begin{aligned}\frac{k_c}{\xi_f(\theta(\psi))} = \bar{\beta}(1-\gamma)\Bigg[&\varphi(\psi)R_{L,c}(\psi) + (1-\varphi(\psi))R_{H,c}(\psi) - b + \bar{\beta}\mathbb{E}\Bigg\{\frac{(1-\lambda-\gamma\xi_h(\theta(\hat{\psi}))(1-\zeta(\hat{\psi})))k_c}{\xi_f(\theta(\hat{\psi}))\bar{\beta}(1-\gamma)} \\ &- \frac{\xi_h(\theta(\hat{\psi}))\zeta(\hat{\psi})\gamma k_s}{\xi_f(\theta(\hat{\psi}))\bar{\beta}(1-\gamma)} + (1-\varphi(\psi))(1-\xi_h(\theta(\hat{\psi})))\sigma\Delta_\sigma(\hat{\psi})\Bigg\}\Bigg],\end{aligned} \tag{D.6}$$

$$\begin{aligned}\Delta_\sigma(\psi) = \bar{\beta}\mathbb{E}\Big[&\xi_h(\theta(\hat{\psi}))\gamma\big\{\zeta(\hat{\psi})[S_{H,s}(\hat{\psi}) - S_{L,s}(\hat{\psi})] + (1-\zeta(\hat{\psi}))\big[S_{H,c}(\hat{\psi}) - S_{L,c}(\hat{\psi})\big]\big\} \\ &+ \big(1-(1-\xi_h(\theta(\hat{\psi})))\sigma\big)\Delta_\sigma(\hat{\psi})\Big],\end{aligned} \tag{D.7}$$

$$S_{L,s}(\psi) = R_{L,s}(\psi) - b + \bar{\beta}\mathbb{E}\big[(1-\lambda-\gamma\xi_h(\theta(\hat{\psi}))\zeta(\hat{\psi}))S_{L,s}(\hat{\psi}) - \xi_h(\theta(\hat{\psi}))(1-\zeta(\hat{\psi}))\gamma S_{L,c}(\hat{\psi})\big], \tag{D.8}$$

$$S_{L,c}(\psi) = R_{L,c}(\psi) - b + \bar{\beta}\mathbb{E}\big[(1-\lambda-\gamma\xi_h(\theta(\hat{\psi}))(1-\zeta(\hat{\psi})))S_{L,c}(\hat{\psi}) - \xi_h(\theta(\hat{\psi}))\zeta(\hat{\psi})\gamma S_{L,s}(\hat{\psi})\big], \tag{D.9}$$

$$\begin{aligned}S_{H,s}(\psi) = R_{H,s}(\psi) - b + \bar{\beta}\mathbb{E}\big[&(1-\lambda-\gamma\xi_h(\theta(\hat{\psi}))\zeta(\hat{\psi}))S_{H,s}(\hat{\psi}) - \xi_h(\theta(\hat{\psi}))(1-\zeta(\hat{\psi}))\gamma S_{H,c}(\hat{\psi}) \\ &+ (1-\xi_h(\theta(\hat{\psi})))\sigma\Delta_\sigma(\hat{\psi})\big],\end{aligned} \tag{D.10}$$

$$\begin{aligned}S_{H,c}(\psi) = R_{H,c}(\psi) - b + \bar{\beta}\mathbb{E}\big[&(1-\lambda-\gamma\xi_h(\theta(\hat{\psi}))(1-\zeta(\hat{\psi})))S_{H,c}(\hat{\psi}) - \xi_h(\theta(\hat{\psi}))\zeta(\hat{\psi})\gamma S_{H,s}(\hat{\psi}) \\ &+ (1-\xi_h(\theta(\hat{\psi})))\sigma\Delta_\sigma(\hat{\psi})\big],\end{aligned} \tag{D.11}$$

where $\varphi(\psi) = u_L(\psi)/(u_L(\psi) + u_H(\psi))$ and the revenues of a filled job, $R_{\varepsilon,\chi}(\psi)$, are given by

$$R_{L,S}(\psi) = \delta_s y_s + c'\left(\frac{\alpha q_s(\psi)}{\zeta(\psi)(1 - \hat{u}_L(\psi) - \hat{u}_H(\psi))}\right)\frac{\alpha q_s(\psi)}{\zeta(\psi)(1 - \hat{u}_L(\psi) - \hat{u}_H(\psi))} - c\left(\frac{\alpha q_s(\psi)}{\zeta(\psi)(1 - \hat{u}_L(\psi) - \hat{u}_H(\psi))}\right), \tag{D.12}$$

$$R_{H,S}(\psi) = y_s + c'\left(\frac{\alpha q_s(\psi)}{\zeta(\psi)(1 - \hat{u}_L(\psi) - \hat{u}_H(\psi))}\right)\frac{\alpha q_s(\psi)}{\zeta(\psi)(1 - \hat{u}_L(\psi) - \hat{u}_H(\psi))} - c\left(\frac{\alpha q_s(\psi)}{\zeta(\psi)(1 - \hat{u}_L(\psi) - \hat{u}_H(\psi))}\right), \tag{D.13}$$

$$R_{L,C}(\psi) = \delta_c y_c + c'\left(\frac{\alpha q_c(\psi)}{(1-\zeta(\psi))(1 - \hat{u}_L(\psi) - \hat{u}_H(\psi))}\right)\frac{\alpha q_c(\psi)}{(1-\zeta(\psi))(1 - \hat{u}_L(\psi) - \hat{u}_H(\psi))} - c\left(\frac{\alpha q_c(\psi)}{(1-\zeta(\psi))(1 - \hat{u}_L(\psi) - \hat{u}_H(\psi))}\right), \tag{D.14}$$

$$R_{H,C}(\psi) = y_c + c'\left(\frac{\alpha q_c(\psi)}{(1-\zeta(\psi))(1 - \hat{u}_L(\psi) - \hat{u}_H(\psi))}\right)\frac{\alpha q_c(\psi)}{(1-\zeta(\psi))(1 - \hat{u}_L(\psi) - \hat{u}_H(\psi))} - c\left(\frac{\alpha q_c(\psi)}{(1-\zeta(\psi))(1 - \hat{u}_L(\psi) - \hat{u}_H(\psi))}\right), \tag{D.15}$$

$(q_s(\psi), q_c(\psi))$ solve

$$\frac{\upsilon_2(q_s(\psi), q_c(\psi))}{\upsilon_1(q_s(\psi), q_c(\psi))} = \frac{c'\left(\frac{\alpha q_c(\psi)}{(1-\zeta(\psi))(1-\hat{u}_L(\psi)-\hat{u}_H(\psi))}\right)}{c'\left(\frac{\alpha q_s(\psi)}{\zeta(\psi)(1-\hat{u}_L(\psi)-\hat{u}_H(\psi))}\right)}, \tag{D.16}$$

$$i = (1-\mu)\left[\alpha\left(\frac{\upsilon_1(q_s(\psi), q_c(\psi))\omega(\psi)}{c'\left(\frac{\alpha q_s(\psi)}{\zeta(\psi)(1-\hat{u}_L(\psi)-\hat{u}_H(\psi))}\right)} + \frac{\upsilon_2(q_s(\psi), q_c(\psi))(1-\omega(\psi))}{c'\left(\frac{\alpha q_c(\psi)}{(1-\zeta(\psi))(1-\hat{u}_L(\psi)-\hat{u}_H(\psi))}\right)}\right) + (1-\alpha)\right] - 1, \tag{D.17}$$

and, finally, $P(\hat{\psi}; \psi)$ is consistent with the laws of motion of (u_L, u_H, n_L, i) as defined by equations (D.1)-(D.4). The model is solved numerically in several steps. First, we approximate the stochastic process for the nominal interest rate using the Rouwenhorst (1995) method, where we estimate the first-order autocorrelation in the monthly nominal Aaa corporate bond rate to be 0.9965 and a standard deviation of 0.0021. Second, we estimate the policy functions $\big(\theta(\psi), \zeta(\psi)\big)$ through a projection algorithm as in Petrosky-Nadeau and Zhang (2017). More details are available upon request.