Homelessness*

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Abstract

For a sizable fraction of the homeless population, homelessness is a temporary state often triggered by shocks to income. In this paper, we examine economic policies that may help reduce the flow to homelessness for these individuals. We construct a model economy that is calibrated to especially capture the lower tail of the income distribution where bad enough shocks to income leave some people homeless. We use the model economy to examine the effectiveness of several policies such as rental subsidies, housing vouchers, and relaxation of borrowing constraints in reducing the flow to homelessness at the steady state. Our findings indicate that rent subsidies directed at particular sized units result in the lowest rate of homelessness while relaxing borrowing constraints results in the highest rate of homelessness.

Keywords: Inequality, Housing, Default, Income Shock, General Equilibrium
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1 Introduction

In this paper, we construct an economic model that generates homelessness and use it to examine policies that may help reduce the flow to homelessness. The model economy possesses some of the important properties of the United States (U.S.) economy such as the income distribution, the fraction of individuals whose rent is more than 30-50% of their income, and the fraction of homeowners versus renters. In this framework, some individuals become homeless involuntarily due to bad shocks to their income, while some others voluntarily choose to be homeless due to their economic conditions and search costs. We examine the effectiveness of several policies such as rental subsidies, housing vouchers, and relaxation of borrowing constraints in reducing the flow to homelessness.

In order to properly calibrate the model and assess the possibility of economic incentives in reducing the incidence of homelessness, we need a good understanding of the characteristics of the homeless population. In the U.S., the Department of Housing and Urban Development (HUD) provides two types of measure of homelessness. The first one, Homelessness Management Information Systems (HMIS), provides unduplicated counts of homeless people in shelters over the course of a year. The second one, Point-in-Time (PIT) measure, provides a snapshot of homelessness of both sheltered and unsheltered individuals on a single night. According to HMIS, there were 1,446,000 million people experiencing sheltered homelessness in 2018. According to PIT, there were 567,715 people homeless on a given night in 2019 where about 2/3 were in homeless shelters while 1/3 of them were unsheltered.[1]

Summarizing the characteristics of the homeless population precisely is complicated since detailed data is scant. Clearly, the homeless population is quite diverse, with an important fraction suffering from severe mental illnesses and/or drug abuse.[2] However, it is also clear that for a significant number of the homeless, homelessness is a temporary state often triggered by shocks to income. According to Culhane et al. (2007) who use administrative data on four jurisdictions, most shelter spells are short (80% of adults experienced short-term, temporary homelessness), and very few shelter spells last more than 2 years (10% of single adults who become homeless stayed in shelters over extended periods).[3] According to the greater Los Angeles homeless count in 2019, of the 60,000 homeless, 23% were first-time homeless and 71% did not have a serious mental illness. In San Francisco, in 2017, 13% of the homeless reported part-time of full-time work and many were receiving some sort of income. According to the 1996 National Survey of Homeless Assistance Providers and Clients, conducted by the U.S. Census Bureau (1996), 44% of the homeless did some work for pay in the month before being surveyed and 13% held a regular job. Ellen and O'Flaherty (2010) argue that most homelessness spells are due to bad luck, stochastic shocks that are hard to predict that may involve

[1] The ratio of sheltered versus unsheltered homeless varies significantly across states where, for example, in California 2/3 of the homeless were unsheltered during the same time.

[2] According to the Substance Abuse and Mental Health Services Administration, 20 to 25% of the homeless population in the United States suffers from some form of severe mental illness. For a discussion of the changes in federal and state laws that resulted in the deinstitutionalization of people with mental illness, see, for example Yohanna (2013).

[3] See also Link et. al 1994; Culhane and Metraux 1999.

job loss, divorce, health shocks, etc.[4] We believe that economic theory can help us design mechanisms that will reduce the flow to homelessness for the fraction of the population who become homeless due to bad shocks. If such polices could be useful, then the burden on the system to deal with the rest of the homeless population would be more manageable.

The model we build is composed of infinitely lived agents with heterogeneous skills who face idiosyncratic shocks to their incomes. Individuals can be either renters or homeowners, and they value consumption and housing services. Renters are unable to borrow, but homeowners have access to mortgage loans subject to a loan-to-value constraint. Both types of individuals have no access to private insurance markets. Housing supply is endogenous and a function of the housing price. Labor income is private information, but the distribution of income shocks are known by everyone. In each period, rents are due after the realization of the labor income shock. If the total resource available is lower than a minimum amount of consumption needed for survival, the individual defaults on rent and becomes homeless, consuming a minimum amount of shelter provided by the government. Individuals may also choose to be homeless voluntarily when they are in bad economic conditions and find going to the government-provided free shelter is their optimal choice. We calibrate this economy to generate a distribution of income that resembles the U.S. data, paying particular attention to the lower income groups and the challenges they face. In particular, we make sure that the fraction of people whose rent is more than 30-50% of their income matches the data. This population presents the most at-risk group in the U.S. in becoming homeless. These individuals, when faced with successive bad shocks, may find themselves not able to afford even the smallest rental unit and end up being homeless and in need of a shelter.

In our benchmark economy, 0.4% of the population are homeless at the initial steady state, which is meant to characterize the current U.S. economy. We experiment with several revenue neutral policies and examine their impact on homelessness and welfare at the steady state. We find interesting differences between the impact of policies such as rent subsidies to certain sized rental units, housing vouchers based on income, universal basic income, and relaxation of borrowing constraints on the homeless population. For example, rent subsidies directed at particular sized units results in the lowest rate of homelessness while relaxing borrowing constraints results in the highest rate of homelessness. In fact, rental subsidies given to those residing in the smallest unit result in a tenfold decline in homelessness. The main factor that results in lower homelessness in this case is the amount of income subsidies received by those in most need. Targeting the smallest rental units help identify those individuals more accurately. Voluntary homelessness disappears as these small units become more affordable, and defaults on rents declines as more of the vulnerable population are incentivized to reside in small rental units. Such a policy is in direct contrast to some of the housing policies of the U.S. Department of Housing and Urban Development (HUD). For example, among the stated purposes of the Section 8 Housing Choice Voucher Program, the largest housing assistance program, is the desire to promote freedom to choose the kind and location of housing people desire. While there is no

[4]Using data on gentrifications, O'Flaherty (2009) examines the role of rent shocks versus income shocks in causing homelessness. He concludes that income shock appears to be the main shock that leads to homelessness indicating that policies that stabilize income or increase access to borrowing may end up being more useful than policies such as rent control.

question such a goal has very important benefits, it may not be very effective in reducing homelessness. In general, most housing subsidies through HUD are not designed to combat the homelessness problems in the U.S. because of their scarcity. According to Ellen, (2018) in most places, families wait for years to receive a voucher, and only one in four households eligible for a voucher nationally receives any federal housing assistance.[5]

We also find the standard assistance programs for low income individuals, such as food stamps, to be less effective in reducing homelessness compared to subsidies targeted for housing. In particular, subsidies given as housing vouchers result in approximately 19-39% lower homelessness rates compared to the same sized subsidies given as food stamps. Despite this, we find that standard assistance programs such as food stamps are slightly more welfare-improving than housing voucher policy, especially for individuals with low levels of education. Using a model economy calibrated to mimic the general features of the U.S economy also allows us to examine the impact of the macroeconomic changes on homelessness, such as the implications of changes in housing market conditions. For example, we find that an exogenous 5% decline in the housing supply leads to a 13% increase in the rental rate and a 30% increase in homelessness.

The existing literature on homelessness contains important information about the characteristics of the homeless population, investigation of its causes, and policies to combat homelessness, with many focusing on prevention programs offered by different cities or states.[6] O'Flaherty (2009, 2012a, 2012b) provides excellent theoretical frameworks and considers a range of issues such as investigating the type of shocks that lead to homelessness, introducing dynamic elements to the homelessness question, and examining the potential impact of housing subsidies. We contribute to this literature by providing, perhaps the first model of homelessness embedded in a fully calibrated general equilibrium model of the U.S. This framework not only allows us to quantitatively examine the impact of a number of potential measures that may help reduce the flow to homelessness but also provides a framework for examining the impact of macroeconomic changes on homelessness, and for decomposing the relative importance of potential factors contributing to homelessness.

2 The Model

In this section, we develop a dynamic general equilibrium model of homelessness. Our model consists of heterogeneous individuals, competitive financial institutions, and the government. Individuals face idiosyncratic income shocks and make consumption, saving, and housing choices. Financial institutions take deposits from individuals, provide mortgage loans to homeowners, and hold rental housing units as in Gervais (2002) and İmrohoroğlu et al. (2018). The government collects income taxes and uses the tax revenues to finance government expenditures, and public insurance programs.

[5]See also Aurand et al. (2017) and Bieri and Dawkins (2019).

[6]The following represents a small sample of relevant papers: Culhane (2020); Glomm and John (2001); Rossi (1990); Link et al. (1994); Goodman, Messeri, and O'Flaherty (2016).

2.1 Households

The economy consists of a continuum of infinitely-lived individuals who value consumption, c_t, and housing services, h_t. They maximize:

$$E\sum_{t=0}^{\infty}\beta^t(c_t,h_t)$$

where β is the subjective discount factor. Individuals are heterogeneous with respect to their permanent lifetime labor ability ϵ. Labor income of an individual consists of an economy wide wage rate w, a permanent lifetime labor ability ϵ, and a stochastic component, μ_t, which is governed by a first order Markov Chain with the transition probability matrix $\Omega(\mu,\mu')$. Individuals have no access to private insurance markets, but they are able to accumulate assets.

Individuals can either own or rent houses. Renters are unable to borrow, but homeowners get access to the mortgage market with a loan-to-value constraint η. Let m_t represent the financial assets held by an individual. Positive financial assets are deposited in financial institutions earning the deposit rate of r_t^d, and negative financial assets represent mortgages (for homeowners) that charge a mortgage rate of r_t^m. Let r_t represent the interest rate on financial assets m_t and thus is given by:

$$r_t=\begin{cases} r_t^d, & if \quad m_t\geq 0;\\ r_t^m, & if \quad m_t<0.\end{cases}$$

Houses come in discrete sizes, that is, $h\in(\{h_i\}_{i=1}^n,\underline{h}\})$, where $\{h_i\}_{i=1}^n$ are the housing units in the market available for either rent or sale, and $\underline{h}$ represents the government-provided shelter for the homeless population. For technical convenience, we assume that the sizes of all rental units are smaller than the smallest owner-occupied unit, which is denoted by $\underline{h}^o$, that is, any unit $h_i<\underline{h}^o$ is a rental unit and the rest of the housing units are owner-occupied houses.

Housing capital can be turned into housing services via a linear technology, and it depreciates at the rate of δ in each period. The value of house h_t is given by $p_t h_t$ where p_t represents the housing price at period t. The total amount of housing supply is assumed to be determined by the housing supply function, $H(p_t)$, which is an increasing function of the housing price. We follow the urban economics literature and assume that $H(p_t)$ takes the following functional form, $H(p_t)=\zeta_2 p_t^{\zeta_1}$, and discipline the key parameter ζ_1 using empirical estimates of the housing supply elasticity provided in the literature (see Baum-Snow and Han (2020)). We further discuss the functional form and other properties of the housing supply function in the calibration section.

We assume that all rental housing units are held by financial institutions and housing capital earns the same net rate of return as mortgage assets. Thus, the rental rate per unit of housing capital, $rent_t$, is simply the sum of the mortgage rate and the housing capital depreciation rate δ, that is:

$$rent_t = (r_t^m + \delta)p_t, \tag{1}$$

and the rental payment for house h_t is $rent_t h_t$.

Individuals pay a transaction cost when they decide to move to a new housing unit for the next period. This transaction cost is supposed to capture a variety of related costs such as search cost for finding a new housing unit, and fees paid to real estate agents for selling the current house. If the remain in the same house, individuals do not pay the transaction cost. Let $s_h(h_t, h_{t+1})$ represent the transaction cost, which is specified as follows:

$$s_h(h_t, h_{t+1}) = \left\{ \begin{array}{llll} s_1 & if & h_t \neq h_{t+1}, & h_t \geq \underline{h}^o \\ s_2 & if & h_t \neq h_{t+1}, & \underline{h} < h_t < \underline{h}^o \\ s_3 & if & h_t \neq h_{t+1}, & h_t = \underline{h} \\ 0 & if & otherwise & \end{array} \right\}, \tag{2}$$

where s_1 is the cost of moving to a different housing unit for current homeowners, s_2 is that for renters, and s_3 represents the search cost for a current homeless individual residing in a shelter. Here, we assume that the cost for moving to a new housing unit is different for homeowners, renters, and and homeless individuals because they are facing different types of transaction costs in the housing market. In particular, we highlight the additional transaction cost a homeless individual may need to pay in the housing market and how it may affect the entry and exit of homelessness.

There are two channels via which individuals become homeless. In addition to choosing to be homeless (that is, $h_t = \underline{h}$), renters can also become homeless by defaulting on the rental agreement. Note that labor income shocks happen at the beginning of each period. As the housing choice is made before the next period's income shock is realized, a bad enough income shock may force the individual to default on their rental agreement. Specifically, renters would default if after-tax labor income plus financial assets (with interests) is lower than a minimum amount of consumption needed ($\underline{c}$) plus the rental expenditure, $rent_t h_t$, that is, for $\forall h_t \in (\underline{h}, \underline{h}^o)$ an individual (a renter) defaults if:

$$y_t - T(y_t) + (1 + r_t)m_t - rent_t h_t < \underline{c}. \tag{3}$$

Here, y_t is before-tax labor income, which is determined as $y_t = w\epsilon\mu_t$, and $T(y_t)$ is the income tax function. When individuals default, they become homeless in that period and consume a minimum amount of shelter provided by the government, $\underline{h}$. For simplicity, we assume that homeowners do not default on mortgages, and they can only become homeless voluntarily.[7]

The budget constraint of an individual depends on his/her current housing status. Homeless individuals do not pay for the government-provided shelter, and they are allowed to remain homeless in the next period or leave the shelter by either renting or purchasing a house. Their budget constraint is given as follows:

[7] In an transitional economy featuring significant housing price changes, mortgage defaults may become more relevant for homeowners. We leave this for future research.

$$c_t + a_{t+1} + s_h(h_t, h_{t+1}) = \quad y_t - T(y_t) + (1 + r_t)m_t. \tag{4}$$

As for current renters, the budget constraint facing them is given as:

$$c_t + a_{t+1} + s_h(h_t, h_{t+1}) = \quad y_t - T(y_t) + (1 + r_t)m_t - rent_t h_t. \tag{5}$$

The renter has to pay rents on his/her housing unit, $rent_t h_t$, but he/she has only financial assets. As the renter does not own any housing capital, he/she does not suffer housing capital depreciation or maintenance cost. The choices facing the renter are similar to the homeowner, and the transaction cost for the renter also depends on his/her housing choice for the next period.

Similarly, a current homeowner (that is, $h_t \geq \underline{h}^o$) faces the following budget constraint:

$$c_t + a_{t+1} + s_h(h_t, h_{t+1}) = \quad y_t - T(y_t) + (1 + r_t)m_t + (1 - \delta)p_t h_t. \tag{6}$$

The homeowner has both financial assets and housing capital. While financial assets earn the interest rate r_t, housing capital depreciates at the rate of δ every period. Therefore, the current house, h_t, is worth $(1 - \delta)p_t h_t$ after depreciation.[8] The homeowner chooses current consumption, decides on housing choice for the next period, and carries assets to the next period. Here, a_{t+1} represents the net assets an individual carries to the next period, that consist of both financial assets and housing capital if the individual decides to continue to be a homeowner in the next period, that is $h_{t+1} \geq \underline{h}^o$, and consist of only financial assets if otherwise. That is, a_{t+1} can be specified as follows:

$$a_{t+1} = \begin{cases} m_{t+1} & , \qquad if \; h_{t+1} < \underline{h}^o; \\ m_{t+1} + p_t h_{t+1} & , \qquad if \; h_{t+1} \geq \underline{h}^o. \end{cases} \tag{7}$$

Note that individuals get access to mortgage loans if they decide to purchase a house, but they are subject to the loan-to-value constraint η. If they are renters or homeless, they have no access to borrowing. In other words, m_{t+1} and h_{t+1} have to satisfy the following constraints:

$$m_{t+1} \geq \begin{cases} 0, & \forall h_{t+1} < \underline{h}^o. \\ -\eta p_t h_{t+1}, & \forall h_{t+1} \geq \underline{h}^o; \end{cases} \tag{8}$$

[8]The underlying assumption here is that homeowners pay the maintenance cost to reverse the depreciation of the house occurred during the current period. This assumption is made for technical convenience. It can also be interpreted as the homeowner sells his/her depreciated current house and buys a new house of the same type (but without paying any search costs).

The state faced by an individual at the beginning of period t can be summarized by the vector $\Gamma = (m_t, h_t, \mu_t, \epsilon)$, that is, financial assets, the housing choice determined in the previous period, income shock and permanent labor ability. Let $W(m_t, h_t, \mu_t, \epsilon)$ represent the value function of the individual before the housing default decision and $V(m_t, h_t, \mu_t, \epsilon)$ represent the value function after the default decision, either staying home or becoming homeless. That is:

$$W_t(m_t, h_t, \mu_t, \epsilon) = \begin{cases} V_t(m_t, \underline{h}, \mu_t, \epsilon), & if \quad default; \\ V_t(m_t, h_t, \mu_t, \epsilon), & if \quad otherwise. \end{cases}$$

We denote the default status of each individual by $D(\Gamma)$ with $D = 1$ representing defaulting on the rental agreement.[9]

The value function of an individual after the default decision, $V_t(\Gamma)$, can be specified as follows:

$$V_t(\Gamma) = \max_{c_t, h_{t+1}, a_{t+1}} u(c_t, h_t) + \beta E_t W_{t+1}(\Gamma')$$

subject to the constraints: equation 6, 5, 4, 7, and 8.

2.2 Financial Institutions

There exist a large number of financial institutions running in the background. These financial institutions collect deposits from individuals, provide mortgage loans to homeowners, and invest in rental housing capital that are rented by renter individuals.

In the benchmark economy, we set the mortgage rate and the deposit rate equal to each other. Financial institutions act competitively, and they generate zero profits.[10]

2.3 Government

The government collects labor income taxes from individuals; incurs government expenditures, G; and also runs public insurance programs that provide assistance to segments of the population that need help. In the benchmark model, we only consider public insurance that provides free shelter services, $\underline{h}$, to the homeless population. In additional computational experiments, we also consider other public programs such as one that provides in-kind transfers (in terms of consumption goods) like food stamps, and examine their effectiveness in reducing the flow to homelessness.

[9] Note that for individuals who voluntarily became homeless (i.e, $h_t = \underline{h}$), the default choice is irrelevant and we set it to 1 for convenience.

[10] Alternatively, we can allow the mortgage rate to be different from the deposit rate, and the gap between the two rates can be understood as the financial cost such as administrative costs financial institutions incur during the process of financial transactions. Under the assumption of perfect competition, financial institutions still generate zero profits. We consider this alternative case in our sensitivity analysis.

Following Benabou (2002) and Heathcote, Storesletten, and Violante (2017), we model the income tax function as follows:

$$T(y_t) = y_t - \tau_2 y_t^{1-\tau_1},$$

where τ_1 controls the progressivity of the tax function, and the value of τ_2 controls the level of income taxation.

2.4 Equilibrium

Individuals are heterogeneous with respect to financial assets, m_t, rental housing, h_t, idiosyncratic labor income shock μ_t, and permanent labor ability ϵ. Let Γ_t be the vector summarizing the state $(m_t, h_t, \mu_t, \epsilon)$ faced by an individual, and let $\lambda_t^b(\Gamma)$ represent the measure of the distribution (before the default decision) and $\lambda_t(\Gamma)$ represent the measure of the population distribution (after the default decision). The concept of a competitive equilibrium is given as follows:

Given tax function $T(.)$, the prices $\{w, r_t^d, r_t^m\}$, and the housing price p_t, a competitive equilibrium is a sequence of value functions; individual decision rules for consumption of goods, housing, and asset holdings; a measure of individual types $\lambda_t(\Gamma)$ (after the default decision); a measure of individual types $\lambda_t^b(\Gamma)$ (before the default decision); and a housing supply function $H(.)$, such that, for all t:

1. Given the interest rates, the housing price, and the government policy, the individual decision rules solve the individual's dynamic programming problems.

2. The budget constraint for financial institutions holds (or financial assets market clears),

$$\sum_{\underline{h}<h<\underline{h}^o} \lambda_t(\Gamma) p_t h_t = \sum_{\Gamma} m_t \lambda_t(\Gamma), \tag{9}$$

3. π_t fraction of individuals that are homeless

$$\pi_t = \sum_{\Gamma, h_t > \underline{h}} \lambda_t^b(\Gamma) D(\Gamma) + \sum_{\Gamma, h_t = \underline{h}} \lambda_t^b(\Gamma), \tag{10}$$

or

$$\pi_t = \sum_{m_t, \mu_t, \epsilon} \lambda_t(m_t, \underline{h}, \mu_t, \epsilon). \tag{11}$$

Here, λ_t^b and λ_t represent the measures before and after the default decisions in period t. Note that the right-hand side of equation 10 highlights the shares of homeless population via each channel, with the first term representing those who become homeless from defaulting and the second term representing the voluntary homeless population.

4. Housing market clears:

$$\sum_{\Gamma, h_t \neq \underline{h}} h_t(\Gamma)\lambda_t^b(\Gamma) = H(p_t). \tag{12}$$

5. Government tax revenues are used to pay for government expenditure G and to supply shelters where the cost of a unit of shelter housing services is assumed to be the same as that of market housing. In addition, for simplicity we assume that the government bears the financial loss from defaults on the rental agreement, so that financial institutions do not need to adjust their pricing behaviors.[11] Therefore, the government budget constraint can be specified as follows:

$$\sum_{\Gamma} T(y_t)\lambda_t(\Gamma) = rent_t \underline{h}\pi_t + \sum_{\Gamma, h_t \neq \underline{h}} \lambda_t^b(\Gamma)D(\Gamma)rent_t h_t + G_t. \tag{13}$$

Here, the LHS of the equation is the total tax revenues. On the RHS of the budget constraint, the first term represents the total cost of shelter services, and the second term is the financial loss from defaults on the rental agreement. Government expenditure, G_t, is determined in the equilibrium to balance the government budget in each period, and it is assumed to be thrown away.

6. The population distributions evolve according to:

$$\lambda_{t+1}^b(m_{t+1}, h_{t+1}, \mu_{t+1}, \epsilon) = \sum_{\Gamma} I_{m_{t+1}=m'(\Gamma)} I_{h_{t+1}=h'(\Gamma)} \Omega(\mu_t, \mu_{t+1})\lambda_t(\Gamma). \tag{14}$$

$$\lambda_t(m_t, h_t, \mu_t, \epsilon) = \begin{cases} \sum_h \lambda_t^b(m_t, h, \mu_t, \epsilon)D(m_t, h, \mu_t, \epsilon), if\ \ h_t = \underline{h} \\ \lambda_t^b(m_t, h_t, \mu_t, \epsilon)(1 - D(m_t, h_t, \mu_t, \epsilon)), if\ \ h_t \neq \underline{h}. \end{cases} \tag{15}$$

Here, equation 14 describes how the population distribution at the beginning of next period (before the default decision) is determined, with $I_{m_{t+1}=m'(\Gamma)}$ and $I_{h_{t+1}=h'(\Gamma)}$ being the indicator functions for that the policy functions for financial assets and housing are equal to m_{t+1} and h_{t+1}. Equation 15 describes how the population distribution evolves before and after the default decision within period t.

3 Calibration

We calibrate the benchmark model so that the steady state of the economy matches some key moments of the current U.S. economy. Our calibration strategy consists of two stages. In the first stage, we predetermine the values of some standard parameters based on the existing literature or independent estimations. In the the second stage, we calibrate the rest of the parameters by minimizing the distance between key empirical moments constructed from the data and their model counterparts.

[11] Quantitatively, the total amount of financial losses is small due to the low default rate, and the defaults are among the poorest individuals. Therefore, we believe that it shouldn't have a significant effect on market interest rate and rent, and this assumption shouldn't affect our quantitative results significantly.

3.1 Demographics, Preferences, and Labor Income

We assume a model period to be three months, that is, a quarter. We set the subjective time discount factor β to 0.982 to match an annualized wealth-earnings ratio of 3.2 as in the literature.[12] The utility function is assumed to take the following form:

$$u(c,h) = \frac{c^{1-\sigma}}{1-\sigma} + \theta\frac{h^{1-\sigma_h}}{1-\sigma_h},$$

where σ is set to 2, a commonly used value in the quantitative macro literature, and the value of σ_h is set to the same as σ in the benchmark calibration. The remaining parameter, θ, governs the relative importance of utility flow from housing services in utility. As the value of θ affects the demand for housing and thus implicitly affects the housing rent in equilibrium, we calibrate its value to match the fraction of low income population that are rent-burdened in the data. Specifically, Dumont (2019) documents that more than 75% of the low income households (income less than $20,000) pay more than 30% of their income to rent, which we use to pin down the value of θ. The resulting value from this strategy is 2.0 for θ.

All individuals face the same labor income process where the idiosyncratic labor productivity is modeled as a combination of a persistent shock, a transitory shock, and a fixed component following the literature studying heterogeneous agents models.[13] That is, the logarithm of idiosyncratic labor productivity, $log(\mu_j)$, is determined by:

$$log(\mu_t) = \alpha + \kappa_t + \nu_t$$

$$\kappa_t = \rho\kappa_{t-1} + \omega_t.$$

Here, α is the time-invariant fixed component of labor income, which is normally distributed with the variance of σ^2_α . The stochastic part of labor income consists of κ_j, the persistent component, and ν_j, the transitory shock. The persistent component follows an AR(1) process with ρ being the persistence of the income shock and ω_j representing the persistent shock. Both persistent and transitory shocks follow a normal distribution, and their variances are denoted by σ^2_ω and σ^2_υ.

In the benchmark calibration, we set the parameter values for the labor income process based on the estimates provided in Guvenen (2009).[14] Specifically, we set the value of ρ to 0.988, and set the variance of the fixed effect and the variances of the two shocks as follows, that is, $\sigma^2_\alpha = 0.058$, $\sigma^2_\omega = 0.015$ and $\sigma^2_\upsilon = 0.061$. We discretize the labor productivity shock to a 20-state Markov Chain with 5 states for the persistent shock, and 4 states for the transitory shock, and discretize the fixed component into 3 states.[15]

[12] This empirical moment is chosen following Hong and Rios-Rull (2012) and Hosseini, Kopecky, and Zhao (2020), who only target the wealth-earnings ratio of the bottom 95%.

[13] For example, see Storesletten, Telmer, and Yaron (2004) and Guvenen (2009).

[14] Specifically, we use the estimates reported in Panel A of Table 1 in Guvenen (2009). These estimates are for the RIP labor income process, which is consistent with the one specified here.

[15] Following Kopecky and Suen (2010), we discretize the persistent shock using the Rouwhorst (1995) method . The transitory

Labor efficiencies for high and low-ability individuals are calibrated to match those of college and non-college graduates as reported in Heathcote, Storesletten, and Violante (2013) who estimate, using the CPS data, that the college premium was approximately 68% and the fraction of college graduates was 28.7% among the cohort of age 25-29 in 2001-2005. Therefore, we set $\epsilon_l = 1$ and $\epsilon_h = 1.68$ with the corresponding population shares being 71.3% and 28.7%, respectively.

The economy wide wage rate, w, is normalized so that the average earnings in the benchmark economy is equal to 1. This exercise gives $w = 0.58$.

Table 1: Calibration

Parameter	Description	Value
α	capital income share	0.33
δ	capital depreciation rate	0.07
σ and σ_h	CRRA risk aversion parameters	1.5
r	interest rate	0.985%
w	wage rate	0.58
β	time discount factor	0.982
ζ_1	housing supply function	3
ζ_2	housing supply function	0.0065
η	loan-to-value constraint	0.8
$s_1(h)$	housing transaction cost	see text
s_2	housing transaction cost	0
s_3	housing transaction cost	0.0025
θ	utility weight on housing services	2.0
h_t	housing unit sizes	see text
$\underline{h}$	minimum amount of shelter	0.082
ρ	income shock persistence	0.988
σ^2_ω	variances of income shocks	0.015
σ^2_υ	variances of income shocks	0.061
σ^2_α	variances of income shocks	0.058
ϵ_l and ϵ_h	education-specific productivity	1.0 and 1.68
	fraction of college graduates	28.7%
τ_1	income tax function parameter	0.036
τ_2	income tax function parameter	0.902

3.2 Housing Market and Financial Institutions

We assume that housing capital depreciates at the same rate as physical capital, and set the depreciation rate to 7% according to Gomme and Rupert (2007). Housing price, p, is endogenously determined in the equilibrium to clear the housing market. We set the housing grids based on data on the square footage of houses for renters and homeowners from the U.S. Census Bureau, American Housing Survey 2017. This

shock and the fixed component are discretized using the standard Tauchen method (1986). The resulting values of μ and its corresponding transition matrix $\Omega(\mu, \mu')$ are not reported to save space and are available upon request form the authors.

results in the following set of housing unit sizes: 0.22, 0.35, 0.49, 0.7, 0.75, 1.26, and 2.79, with the first four unit sizes being rental units and the last three unit sizes being the owner-occupied units.[16] Here, we use finer grids for the lower tail of the housing size distribution due to its increased relevance for the issue of homelessness. That is, the four rental unit grids correspond to the 5th, 25th, 50th, and the 75th percentiles of the rental unit distribution in the data, respectively. The housing grids for owner-occupied houses correspond to the 25th, 75th, and the 95th percentiles of owner-occupied houses in the data. We calibrate the size of a shelter unit $\underline{h}$ to match the fraction of homeless population in the U.S. data. This calibration exercise results in a value of 0.098 for $\underline{h}$.

The housing supply function, $H(p_t)$, is assumed to take the following functional form:

$$H(p_t) = \zeta_2 p_t^{\zeta_1},$$

where ζ_1 determines the elasticity of housing supply. In our benchmark calibration, we set the value of ζ_1 based on the empirical estimates of the housing supply elasticity provided in the urban literature. According to Baum-Snow and Han (2020), the elasticity of housing supply is approximately 3, which is assigned as the value of ζ_1 in the benchmark calibration. We calibrate the value of ζ_2 to match a moment related to the distribution of housing sizes. Specifically, as the state of homelessness is more relevant for individuals living in small housing units, we choose the value of ζ_2 so that the fraction of population living in the smallest rental unit matches its data counterpart, that is, 3%. The resulting value of ζ_2 is 0.0065.

The transaction cost in the housing market captures two types of costs: the real estate agent fees for selling an owned housing unit, and the transaction cost of finding a new housing unit. If remaining in the same house, individuals do not pay the transaction cost, but homeowners need to pay the maintenance cost, which is equal to the housing capital depreciation.[17] The transaction cost for homeowners captures real estate agent fees for selling a house. In the benchmark calibration, we assume that the real estate agent fees are 6% of the housing value, and thus the value of $s_1(h_t)$ is determined by $s_1(h_t) = 0.06 p_t h_t$. We follow the tradition in the literature and assume no search cost for finding a new housing unit; thus, s_2 is set to 0. The transaction cost facing the homeless affects their exit rates from homelessness and thus the homeless duration spell. Thus, we calibrate s_3 to match the homelessness duration in the data. According to Allgood and Warren (2003), the median spell of homelessness is 270 days in data from the National Survey of Homelessness Assistance Providers and Clients (NSHAPC). We calibrate s_3 to match this moment, and the resulting value is 0.0025.[18]

Financial institutions act competitively. Thus, the no-arbitrage condition implies that the deposit interest rate, the mortgage rate, and the rate of return from housing capital should follow equation 1. In the

[16] These housing unit sizes are based on our calculations from the data.

[17] This assumption is made for technical simplicity. It can also be interpreted as this type of homeowner selling his/her current home and buying a new housing unit of the same type without paying the transaction cost.

[18] There exist few empirical estimates on the duration of homelessness in the literature, with a wide range of values. This is partly due to data limitation. In sensitivity analysis, we explore alternative calibration strategies that imply different duration of homelessness.

benchmark calibration, we set the deposit rate and the mortgage rate to be 4% annually, that is, 0.985% quarterly. We set the loan-to-value constraint to be 80% based on the actual mortgage policies in the U.S., that is, $\eta = 0.8$.

3.3 Government Parameters

The government collects labor income taxes from individuals, and it also runs public insurance programs that provide various assistance to the population that needs help. In the benchmark model, we only consider public insurance that provides free shelter services, $\underline{h}$, to the homeless population. In additional computational experiments, we also consider other public programs such as one that provides in-kind transfers (in terms of consumption goods) like food stamps and Universal Basic Income (UBI) and examine their effectiveness in reducing the flow to homelessness.

Following Benabou (2002) and Heathcote et al. (2017), we model the income tax function as follows:

$$T(y_t) = y_t - \tau_2 y_t^{1-\tau_1},$$

where τ_1 controls the progressivity of the tax function, and the value of τ_2 controls the level of income taxation. The value of τ_1 is set to 0.036 and the value of τ_2 is chosen to be 0.902 based on the estimates provided by Guner, Kaygusuz, and Ventura (2014).

The government also incurs government expenditure, G. We assume that the government budget holds in each period. That is, the government expenditure, G, is endogenously determined in equilibrium.

Table 1 summarizes the main results of the benchmark calibration. This calibration generates an average housing size of 1.63, and a market-clearing housing price of 6.3.[19] Overall, our benchmark calibration is successful in matching the empirical targets. In the next section, we will further evaluate the model fit by examining the key properties of the benchmark economy.

4 Quantitative Results

We start this section by examining the properties of the benchmark economy and compare them to the economic conditions in the U.S. Next, we run counterfactual experiments to analyze the effectiveness of a series of relevant policies including rent subsidies, housing vouchers, and relaxation of borrowing constraints in reducing the flow to homelessness.

Table 2 reports the key statistics of the benchmark economy together with the corresponding data counterparts. The model economy is calibrated to match the U.S. data in several relevant dimensions. The share of the population living in the smallest rental units and the percent of low income individuals paying more than 30% of their income to rent are potentially important for capturing the rent burdened individuals.

[19] The market-clearing housing price results in a total housing demand equal to housing supply.

Based on data from the American Housing Survey of the U.S. Census Bureau (2017) and our construction of housing grids, the share of the population living in the smallest unit ($h_1 = 0.22$) is approximately 3%. On the other hand, according to Dumont (2019), more than 75% of the low income households (income less than $20,000) pay more than 30% of their income to rent. In our benchmark model, this share of the rent-burdened population is 74% while 3% of individuals live in the smallest rental units. The benchmark model also matches the wealth earnings ratio, and the income distribution, particularly of the bottom tail, fairly well. As documented in the 2013 SCF data by Kuhn and Rios-Rull (2016), the bottom 5% of Americans earn 0.4% of the total income. This number is 0.7% for the bottom 5-10% and 2.3% for the bottom 10-20%. In the benchmark model, the share of total income earned by the bottom 5% is 0.6%, and the shares for the bottom 5%-10% and the 10%-20% are 0.9% and 2.5%, respectively. The benchmark model also matches the rest of the income distribution reasonably well. The homeownership rate in the model is 73%, slightly above its data counterpart of 66%.

With this calibration, the model generates a homeless population share of 0.4%. It is hard to determine the fraction of the homeless in the data for economic reasons. In 2017, the total sheltered homeless population in the U.S was 1,416,908. In the 2019 survey of one-night estimates, which counts both the sheltered and the unsheltered individuals on one night, about 568,000 individuals were homeless. Of these, 1/3 were unsheltered and 2/3 were sheltered. Assuming that the fraction of unsheltered homeless is always 1/3 of the total homeless population would result in 2,125,362 homeless individuals (0.64%) of the population in a year. This may be an overestimation if the unsheltered homeless have a higher duration of homelessness, however. Also, according to the Substance Abuse and Mental Health Services Administration, 20-25% of the homeless population in the United States suffers from some form of severe mental illness. Economic incentives may be less effective for such individuals. Taking into account these complications, we target a homeless population share of 0.4% in our calibration.[20]

[20]According to Allgood and Warren (2003), the median spell of homelessness is 270 days in data from the National Survey of Homelessness Assistance Providers and Clients (NSHAPC). Our model generates an average homeless duration of 3.2 quarters.

Table 2: Key Statistics: Benchmark Model v.s. Data

Statistic	Data	Benchmark
Pop. share living in the smallest rental unit	3%	3%
Low-income share with rents > 30% of income	75%	74%
Homeownership rate	66%	73%
Wealth-earnings ratio (annualized)	3.2	3.2
Housing price	..	6.30
Aggregate housing size	..	1.63
Income shares		
Income share of bottom 5%	0.4%	0.55%
Income share of 5-10%	0.7%	0.89%
Income share of 10-20%	2.3%	2.5%
Income share of 20-40%	6.5%	8.0%
Income share of 40-60%	10.9%	13.2%
Income share of 60%+	79.5%	74.9%
Homeless population share	$0.35 - 0.5\%$	0.4%

Table 3 reports additional statistics along the income distribution. The second column of the table shows that most of the homeless are from the bottom 5% of the income distribution. Among this income group, 8% of them is homeless with 2.5% directly coming from defaults. The homeless shares for the rest of the income groups are negligible. The average housing size is also highly related to household income. As shown in the fourth column of the table, the bottom 5% and the bottom 5-10% income groups have an average housing size of 0.27 and 0.45. This number increases substantially as income increases, and it reaches 2.59 for the top income group.

Table 3: Key Statistics Across the Income Distribution

Income groups	Homeless	Defaults	Housing Size
Bottom 5%	8.0%	2.5%	0.27
5-10%	0.07%	0.06%	0.45
10-20%	0.00%	0.00%	0.57
20-40%	0.00%	0.00%	0.95
40-60%	0.00%	0.00%	1.56
60%+	0.00%	0.00%	2.59

4.1 Policy Experiments

We use the calibrated benchmark economy as a lab to quantitatively examine the effectiveness of policies that may help reduce the flow to homelessness. Specifically, in this section, we consider housing vouchers based on income, rent subsidies to designated housing units, and relaxation of borrowing constraints. In the next section, we then compare the impact of these policies with policies designed to help the poor individuals without targeting housing specifically, such as transfers in the form of consumption goods and Universal

Basic Income (UBI).

4.1.1 Housing Vouchers

In the U.S., the largest low-income housing subsidy program is the Housing Choice Voucher Program managed by the Department of Housing and Urban Development (HUD). The recipients of vouchers use them to rent units in the private market. To be eligible for these vouchers, a household's income has to be below an eligibility threshold of 30%-80% of the area median income. In the experiment below, we conduct a housing voucher policy, providing vouchers to relevant population groups to cover rental payments.[21] We set the payroll tax rate to 0.2%, which is approximately equivalent to the 2019 budget of the HUD for relevant programs.

Housing voucher policy is administered in three different ways: (1) vouchers, v^r, to renters with labor income below one third of the population average; (2) vouchers only to renter individuals with labor income below 50% of the population average; and (3) vouchers to all renters. The budget constraint for the housing voucher policy targeting individuals with labor income below one third of the population average can be written as follows:

$$\sum_{\Gamma} \tau_t y_t \lambda_t(\Gamma) = \sum_{\Gamma, h<\underline{h}^o} min(v^r, rent_t h_t) I_{y_t<0.33} \lambda_t^b(\Gamma). \tag{16}$$

Similarly, the government budget constraint in the second and the third cases are:

$$\sum_{\Gamma} \tau_t y_t \lambda_t(\Gamma) = \sum_{\Gamma, h<\underline{h}^o} min(v^r, rent_t h_t) I_{y_t<0.5} \lambda_t^b(\Gamma), \tag{17}$$

and

$$\sum_{\Gamma} \tau_t y_t \lambda_t(\Gamma) = \sum_{\Gamma, h<\underline{h}^o} min(v^r, rent_t h_t) \lambda_t^b(\Gamma). \tag{18}$$

Housing vouchers can only be used to offset the rental payments for eligible individuals. Thus, the budget constraint of an eligible individual becomes:

$$c_t + a_{t+1} + s_h(h_t, h_{t+1}) = y_t - T(y_t) - \tau_t y_t + (1 + r_t) m_t - max(rent_t * h_t - v^r, 0).$$

Here the last term on the right-hand side of the budget constraint indicates that the housing voucher, v^r, is only valuable up to the level of actual rental payments $rent_t h_t$.

[21] There are many details of the voucher program that we do not model here. For example, typically, recipients pay 30% of their income on rent and the vouchers cover the rest up to the local maximum payment standard set between 90 and 110 percent of the Fair Market Rent, which is defined as either the 40th or 50th percentile of rents in the metropolitan area, depending on market conditions. The units that can be rented have to pass certain quality standards. There are long waiting times for eligibility.

The results from these three experiments are reported in columns 3-5 of Table 4. Overall, housing voucher policies are effective in reducing the homeless population share. When the policy targets only individuals with labor income below one third of the population average, the population share of homelessness declines to 0.14%. As the policy targets more individuals, each eligible individual receives fewer benefits because of the revenue-neutral nature of the experiments. As a result, the policy's impact on the flow to homelessness also becomes smaller. The homeless population share is 0.20% in the second experiment and 0.22% in the third experiment. In the benchmark, 0.13% of the population (about 33% of the homeless) becomes homeless after defaulting on their rent payments. The rest chose to become homeless due to their income prospects. With housing vouchers, the fraction of the homelessness due to defaults increases to 57% for case (1) and to about 45% for cases (2) and (3). We will discuss the reasons for changes in defaults versus voluntary homelessness across different experiments in more detail later.

Table 4: Policy Experiments: Vouchers

Statistic	Benchmark	Housing Vouchers		
		(1)	(2)	(3)
Homeless population share	0.4%	0.14%	0.20%	0.22%
Rental default rate	0.13%	0.08%	0.09%	0.1%
Benefits per eligible individual	..	0.009	0.007	0.006

4.1.2 Rent Subsidy

In this experiment, instead of giving vouchers to eligible individuals, we consider a rent subsidy policy that lowers the effective rental payments on eligible rental units. We conduct three policy experiments: (1) rent subsidy only on the smallest rental unit h_1, (2) rent subsidies on rental units h_1 and h_2, and (3) rent subsidies on rental units h_1, h_2, and h_3. We conduct these policy experiments in a revenue-neutral fashion, that is, in all three experiments rent subsidies are financed by the same payroll tax rate of 0.2%. The rent subsidy is assumed to be proportional to rental payments. The budget constraint of the rent subsidy program targeting only h_1 can be written as follows:

$$\sum_{\Gamma} \tau_t y_t \lambda_t(\Gamma) = \sum_{\Gamma, h=h_1} s_{rent} rent_t h_t \lambda_t^b(\Gamma), \tag{19}$$

where, τ_t is the payroll tax rate, s_{rent} is the rent subsidy rate, and $I_{h=h_1}$ is the indicator function for whether the housing unit is subsidized. In all three experiments, τ_t is set to 0.2%, and s_{rent} is determined in the equilibrium according to equation 19. Similarly, the budget constraints for policies that subsidize the rents on the smallest two or three units are:

$$\sum_{\Gamma} \tau_t y_t \lambda_t(\Gamma) = \sum_{\Gamma, h\in\{h_1,h_2\}} s_{rent} rent_t h_t \lambda_t^b(\Gamma).$$

$$\sum_{\Gamma} \tau_t y_t \lambda_t(\Gamma) = \sum_{\Gamma, h \in \{h_1, h_2, h_3\}} s_{rent} rent_t h_t \lambda_t^b(\Gamma).$$

Note that a direct effect of the rent subsidy policy is to reduce the effective rental rates for eligible rental units. This can be reflected in the budget constraint of a renter as:

$$c_t + a_{t+1} + s_h(h_t, h_{t+1}) = y_t - T(y_t) - \tau_t y_t + (1 + r_t)m_t - (1 - s_{rent}) rent_t * h_t.$$

Table 5: Policy Experiments: Rent Subsidies

Statistic	Benchmark	Rent Subsidy On			Counterfactual
		only h_1	h_1 and h_2	h_1, h_2 and h_3	
Homeless population share	0.4%	0.0045%	0.043%	0.26%	0.0093%
Rental default rate	0.13%	0.0045%	0.04%	0.10%	0.0093%
Voluntary homeless	0.27%	0.0%	0.003%	0.16%	0.0%
Low-income share-rents $>$ 30%	74%	58%	60%	69%	62%
Rental rate	0.175	0.175	0.175	0.175	0.175
Subsidy rate	0%	86%	34%	13%	n.a.

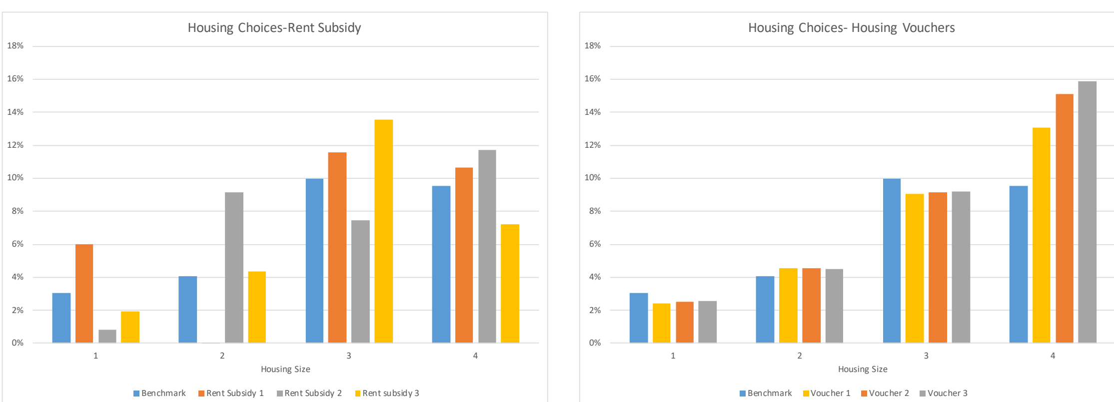
We report the results from the three rent subsidy experiments in Table 5. Overall, all three policies are fairly effective in reducing the homeless population share. In particular, when the policy targets only the smallest rental unit, the population share of the homeless declines almost ninety-fold, to 0.0045%, from its value in the benchmark economy. Voluntary homelessness goes down to zero and the share of low income individuals whose rents are more than 30% of their income declines from 74% at the benchmark to 58%.

As the rent subsidy policy targets more rental units, its effect on the effective rental rates for subsidized units decreases. This is simply because of the revenue-neutral nature of the experiments. When the bottom two and three rental units are subsidized, the corresponding subsidy rates on the rental payments are 34% and 13%, respectively. As a result, the policy's impact on the flow to homelessness also becomes smaller. The homeless population share is 0.043% in the second experiment and 0.26% in the third experiment. Rental subsidies have a noticeable effect on voluntary versus involuntary (due to defaults) homelessness as well. In the benchmark economy, one-third of the homeless (0.13%) are homeless due to defaults and the rest, 0.27%, are homeless voluntarily. When rental subsidies are directed at the smallest rental units, all the homeless are homeless due to defaults. As the subsidy rate declines, the percent of homeless who choose to be homeless voluntarily increases.

While both the housing voucher and rent subsidy policies lower the fraction of the homeless in the economy, there are significant differences in their effectiveness. In particular, the rent subsidy program that targets the smallest units (h_1) lowers the homeless population almost ninety-fold. This is partly due to the large decline in the effective rental rate facing individuals living in subsidized rental units. In this case, the subsidy rate on rental payments for h_1 is 86%, reducing the effective rental rate to less than one fifth of its original value. However, there is one other factor contributing to the decline in homelessness in this case. A rent subsidy program targeting particular units distorts the distribution of housing units in the economy. When the program targets the smallest units, a larger fraction of the low income income individuals choose those units. As a result, these households are less likely to become homeless if they get a bad income shock in a future period.

Figure 1 reports the distribution of housing units in the benchmark economy and in each of the two policy experiments. "Rent Subsidy 1" refers to the experiment where the smallest unit, (h_1), is subsidized, "Rent Subsidy 2" refers to the case where the two smallest units (h_1, h_2), are subsidized, and "Rent Subsidy 3" refers to the case where the smallest three units (h_1, h_2, h_3), are subsidized. In the benchmark economy, only 3% of the population choose to live in the smallest rental unit. With the rent subsidy program that targets the smallest unit, the fraction of the population choosing the smallest unit doubles to 6%. When more than one unit is subsidized, most of the distortion occurs between the largest subsidized rental unit and the rest of the housing units. That is, in the second experiment, the population share choosing h_2 increases the most, and the population share choosing h_3 increases the most in the third experiment. By contrast, in the housing voucher policy, agents choose mostly larger units. Under all the voucher policies, the fraction of the population living in the smallest unit declines, and the fraction living in the fourth unit (i.e., the largest rental unit) increases. For example, in the housing voucher policy that targets those with labor incomes below one third of the population average (voucher 1 in Figure 1), the fraction of the population living in the smallest rental unit is 2.4% (3% in the benchmark), and the fraction living in the largest rental unit is 13% (10% in the benchmark).

Figure 1: Distribution of Housing Units



To examine the impact of distorting the rental unit choice on the fraction of homeless further, we conduct a counterfactual experiment. We examine what would happen if we were to give the exact amount of the subsidy that was dispersed to different individuals under the rent subsidy program, but give it directly as housing vouchers. In other words, we do not tie the subsidy to certain rental units and let individuals use the benefits for any housing unit. We implement this experiment by giving each earnings group (i.e., each pair of μ_t and ϵ) the same benefits as in the rent subsidy (on h_1 only) policy experiment. We report the results from this counterfactual experiment in the last column of Table 5. In the case, the fraction of the homeless in the economy is 0.0093%, almost twice as high as in the rent subsidy program (on h_1 only). The share of population choosing the smallest unit in this case is 1% as opposed to 6% under the rent subsidy program that targets the smallest unit. More individuals opt for larger units with housing vouchers, which results in higher homelessness.

4.1.3 Role of Borrowing Constraint

Lack of access to credit and bad credit scores are often cited as problems in finding housing. For example, in Glomm and John (2002), agents can become homeless as a result of a temporary negative income shock and due to their inability to borrow. To investigate the role of borrowing constraints on homelessness, we conduct three counterfactual exercises where the borrowing constraints are relaxed gradually. We report the results from these three cases in Table 6 where agents are allowed to borrow up to 10%, 50%, and 100% of the average earnings in the economy.

Table 6: Borrowing Constraints

Statistic	Benchmark	Borrowing constraint % av. inc.		
		10%	50%	100%
Homeless population	0.40%	0.42%	0.50%	0.64%
Rental default rate	0.13%	0.13%	0.11%	0.12%
Voluntary homeless share	0.27%	0.29%	0.39%	0.52%
Aggregate financial wealth	12.71	12.67	12.51	12.33
Aggregate housing wealth	10.27	10.27	10.23	10.20

The results indicate that relaxation of borrowing constraints does not reduce the flow to homelessness.[22] While the fraction of the population who are homeless due to rental defaults declines slightly, there is a significant increase in the fraction of the population who chose to be homeless. Consequently, the homeless population share actually increases to 0.42%, 0.50%, and 0.64%, as the borrowing limit rises from 10% of average earnings to 100% of it. The reason behind this result is that the relaxation of borrowing constraints discourages the accumulation of precautionary wealth. This can be confirmed by the changes in aggregate household financial and housing wealth across these experiments, which are reported in the third and the fourth rows of Table 6.

[22] O'Flaherty (2012a) makes a similar point in a theoretical setting.

4.1.4 Welfare

As discussed so far, while different subsidy programs use the same revenues, they lead to different housing choices, default rates, and homelessness in the economy. To provide a more comprehensive picture, we present the welfare results from these experiments in Table 7. Housing voucher program 1 refers to vouchers to individuals with labor income below one third of the population average, 2 to vouchers only to individuals with labor income below 50% of the population average, and 3 to vouchers given to everyone. The rent subsidy programs starts with subsidies to the smallest unit (1), smallest two units (2), and smallest three units (3). Borrowing constraints 1-3 refer to the cases where the borrowing limit is 10%, 50%, and 100% of average income, respectively.

Among these cases, the policy that maximizes overall welfare is the rent subsidy program to the smallest unit while borrowing constraints results in the lowest welfare. However, there are differences in the welfare implication for college and non-college individuals across different policies. While the welfare of the non-college graduates are maximized in the "rent subsidy 1", welfare of college graduates is not. It is the rent subsidy policy to the two smallest units results that is preferred by the college graduates since they tend to have higher income and are able to afford more than the smallest rental unit. College graduates would also prefer the "housing voucher 1" policy to the "rent subsidy 1" policy since it will allow them to choose a larger rental unit. To examine the "housing vouchers 1" and "rent subsidy 1" cases in more detail, we calculate the consumption compensation that would make the two groups indifferent to being born into an economy with the "rent subsidy 1" policy.[23] In this case, while the agents without a college degree would be willing to give up 1.5% of consumption, those with a college degree would require a 0.3% of consumption compensation to be born into a world with the rent subsidy policy.

Table 7: Welfare Consequences of the Policies (i.e., the values of V)

	Non-College	College	Total
Housing Vouchers			
1	−333.5	−205.6	−296.8
2	−335.0	−206.4	−298.1
3	−335.7	−206.4	−298.6
Rent Subsidy			
1	−328.5	−206.2	−293.4
2	−329.9	−205.0	−294.1
3	−334.5	−205.5	−297.5
Borrowing Constraints			
1	−341.9	−207.5	−303.4
2	−345.9	−209.4	−306.7
3	−351.5	−211.5	−311.3

[23] Consumption compensation is calculated as $(\frac{V_1}{V_2})^{1-\sigma}$ where V_1 is the value with the rent subsidy policy for relevant groups and V_2 is the value under the housing voucher policy.

4.2 Housing Policies v.s. Poverty Reduction Policies

In this section, we compare the impact of policies that target housing with more general policies such as food stamps and Universal Basic Income. The purpose is to enhance our understanding of how the homeless population may be affected by different policies, especially by those that target the poor in general versus those that target housing-related issues directly. We take the housing voucher policy and the revenues spent in that case as the relevant housing policy.

First, we introduce the food stamp program as transfers that are delivered in the form of consumption goods. For comparability, we conduct experiments for this transfer with exactly the same eligibility criteria as for the housing voucher policy (equations 16 to 18). Let tr represent the in-kind transfer in the form of consumption goods, the budget constraints for this transfer policy can be written as follows:

$$\sum_{\Gamma} \tau_t y_t \lambda_t(\Gamma) = \sum_{\Gamma, h<\underline{h}^o} min(tr, c_t) I_{y_t<0.33} \lambda_t^b(\Gamma).$$

$$\sum_{\Gamma} \tau_t y_t \lambda_t(\Gamma) = \sum_{\Gamma, h<\underline{h}^o} min(tr, c_t) I_{y_t<0.5} \lambda_t^b(\Gamma).$$

$$\sum_{\Gamma} \tau_t y_t \lambda_t(\Gamma) = \sum_{\Gamma, h<\underline{h}^o} min(tr, c_t) \lambda_t^b(\Gamma).$$

In contrast to housing vouchers, in-kind transfer tr can only be used to pay for consumption good c. That is, the budget constraint of an eligible individual becomes,

$$max(c_t - tr, 0) + a_{t+1} + s_h(h_t, h_{t+1}) = y_t - T(y_t) - \tau_t y_t + (1 + r_t)m_t - rent_t * h_t.$$

The first term on the left-hand side of the budget constraint indicates that the transfer tr is only valuable up to the level of consumption chosen by the individual. Note that the optimization of the household implies that $c_t \geq tr$. In all three cases, the total amount of subsidies are the same and equal to the total amount distributed under the rent voucher policy.

The results from the consumption good transfer experiments are reported in the last three columns of Table 8. As can be seen, while the consumption good transfers also reduce flow to homelessness, they are less effective than rent vouchers, especially when the policy targets to the lowest income groups. The rent voucher case that targets individuals with the lowest one-third of income results in a homelessness rate of 0.14% while the food stamp policy targeting the same income groups results in 0.23% homelessness.

Table 8: Policy Experiments: Housing Vouchers versus Other Transfers

Statistic	Bench.	Housing Vouchers			Food Stamps			UBI
		(1)	(2)	(3)	(1)	(2)	(3)	
Homeless population share	0.4%	0.14%	0.20%	0.22%	0.23%	0.26%	0.27%	0.36%
Rental default rate	0.13%	0.08%	0.09%	0.1%	0.08%	0.12%	0.12%	0.13%
Benefits/eligible individual	..	0.009	0.007	0.006	0.009	0.007	0.006	0.002
Welfare								
Non-college	−340.9	−333.5	−335.0	−335.7	−333.2	−334.7	−335.4	−339.0
College	−207.5	−205.6	−206.4	−206.7	−205.6	−206.4	−206.7	−206.9

Next, we examine the implications of a universal basic income policy where subsidies are provided without any strings attached to them. Agents are able to use them any way they see fit. We model the UBI policy as a program that uses payroll tax τ_t to finance a lump sump payment b_t to everyone in each period. Therefore, the self-financing budget constraint for the UBI policy can be written as follows:

$$\sum_{\Gamma} \tau_t y_t \lambda_t(\Gamma) = \sum_{\Gamma} b_t \lambda_t(\Gamma).$$

The individual constraints, for homeowners, renters, and homeless individuals, with UBI are given as:

$$c_t + a_{t+1} + s_h(h_t, h_{t+1}) = b_t + y_t - T(y_t) - \tau_t y_t + (1 + r_t)m_t + (1 - \delta)p_t h_t.$$

$$c_t + a_{t+1} + s_h(h_t, h_{t+1}) = b_t + y_t - T(y_t) - \tau_t y_t + (1 + r_t)m_t - rent_t h_t.$$

$$c_t + a_{t+1} + s_h(h_t, h_{t+1}) = b_t + y_t - T(y_t) - \tau_t y_t + (1 + r_t)m_t.$$

As can be seen, UBI policy is much less effective than the rent voucher policy in reducing homelessness in the economy. When the universal basic income policy is in place, the homeless population share is 0.36%, hardly a reduction from the baseline without any policies of 0.4%. The main reason in this case is the insufficient amount of subsidies to the poor individuals. Since the same amount of revenues are dispersed to the entire population instead of only the renters, help provided by UBI is smaller than the help under various voucher programs.

Welfare comparisons between the different policies reveals interesting results. Food stamps, for example, result in slightly higher utility, even with a higher homeless population, for those without a college degree compared to housing vouchers. For the college graduates, there is no visible difference in welfare between these two programs. The UBI policy leads to lower welfare for all individuals compared to either the housing voucher or food stamp policies.

5 Further Discussion

5.1 Housing Supply

Homelessness varies significantly across the U.S. According to the Annual Homeless Assessment Report (2019), of the 567,715 persons experiencing homelessness on a single night in 2019, 151,278 were in California, 92,091 in New York, and 28,328 in Florida. House prices and rents also vary significantly across states. There is extensive literature documenting higher regulations leading to lower housing supplies and higher housing prices. Using data from 44 metro areas between 1985 and 1996, Mayer and Somerville (2000) show that land use regulations lower the steady-state level of new construction and reduce the responsiveness of local supply to price shocks significantly. Ihlanfeldt (2007) shows that greater regulation restrictiveness not only increases house prices but also increases the size of newly constructed homes. Parkhomenko (2020) finds similar results in a model that endogenies regulations.[24] Khater, Kiefer and Yanamandra (2020) examine housing supply shortages in the U.S. states in 2018, compared to historical averages and argue that 29 states have a housing undersupply such as California with a 5.79% and New York with a 1.51% undersupply.

While doing a state level comparison is beyond the scope of our paper, we present results of simple counterfactual exercises to assess the role of housing shortages on homelessness. Specifically, we replace the endogenous housing supply function with exogenous housing supply, and compute counterfactual economies with the following exogenous levels of housing supply:(1) benchmark housing supply ±3%, and (2) benchmark housing supply ±5%. Table 9 summarizes results on the share of the homeless in the population, population defaulting on rents, housing prices, and rental rates for exogenous changes in aggregate housing supply ranging between -5% and 5%. A decline in the housing supply by 5% leads to a 13% increase in the rental rate (from 0.175 in the benchmark to 0.197) and a 30% increase in homelessness (from 0.4% at the benchmark to 0.52%). Most of the increased homeless population due to the decline in housing supply are individuals choosing homelessness as opposed to becoming homeless due to defaults. In the -5% lower housing supply case, for example, the voluntary homeless share in the population (the difference between the total homeless and those from default) is 0.39% while the corresponding number in the benchmark case is only 0.27%.

Table 9: Role of Housing Supply

	Homeless Share	From Default	Housing Price	Rental Rate
Benchmark	0.40%	0.13%	6.30	0.175
Housing Supply				
+3%	0.34%	0.12%	5.93	0.165
+5%	0.28%	0.11%	5.64	0.157
-3%	0.47%	0.13%	6.71	0.187
-5%	0.52%	0.13%	7.09	0.197

[24] See also, Quigley and Rosenthal (2005), Paciorek (2013), and Severen and Plantinga (2018), among others.

5.2 Role of Income Shocks

Income shock is the key factor contributing to homelessness in our model. It is modeled as a combination of a persistent shock (κ) and a transitory shock (ν). In this section, we examine the potential role of each component of income shock in generating homelessness. To examine the contribution of each component, we conduct counterfactual experiments in which we shut down each of them while keeping everything else the same as in the benchmark.[25] Table 10 summarizes results on the share of the homeless in the population, population defaulting on rents, and the income shares for each counterfactual case. As can be seen, the persistent shock is much more important than the transitory shock for generating homelessness. As can be seen in columns 2 and 3 of the table, when the persistent shock is removed, the homeless share of the population goes to 0 while this share slightly increases in the case without the transitory shock. The fixed component of labor productivity (ϵ and α) is also an important factor contributing to homelessness. As shown in column 4, the homeless share declines to 0.02% when the fixed component is shut down.

Table 10: Role of Income Shocks

Statistic	Benchmark	Decomposing Income Shocks		
		Transitory Shock (ν)	Persistent Shock (κ)	Fixed Component (ϵ and α)
Homeless population	0.40%	0.42%	0.0%	0.02%
Rental default rate	0.13%	0.12%	0.0%	0.02%
Voluntary homeless share	0.27%	0.3%	0.0%	0.0%
Income shares				
Income share of bottom 5%	0.55%	0.48%	2.05%	0.66%
Income share of 5-10%	0.89%	1.01%	2.46%	1.06%
Income share of 10-20%	2.5%	2.6%	6.2%	2.8%
Income share of 20-40%	8.0%	6.3%	14.0%	8.9%
Income share of 40-60%	13.2%	15.5%	18.3%	13.9%
Income share of 60%+	74.9%	74.1%	56.9%	72.7%

6 Conclusion

In this paper, we develop a quantitative theory of homelessness and use it to examine the effectiveness of economic policies, such as rental subsidies, housing vouchers, and relaxation of borrowing constraints, in reducing the flow to homelessness. Our findings indicate that rent subsidies directed at particular sized units results in the lowest rate of homelessness while relaxing borrowing constraints results in the highest rate of homelessness. We also find the standard assistance programs for low income individuals, such as food stamps, to be less effective in reducing homelessness compared to subsidies targeted for housing. Despite resulting in higher homelessness, such programs end up generating a slightly higher welfare than the housing

[25]We conduct these counterfactual experiments in the partial equilibrium fashion, that is, the housing price is fixed.

voucher policy, especially for low education individuals.

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7 Appendix

Table 11: Grids for μ and the Associated Transition Matrix

	Income shock grids for μ									
	0.16	0.29	0.55	1.04	1.97	0.51	0.96	1.81	3.40	6.42
	Transition Matrix for μ									
	μ'									
μ	0.493	0.007	0.000	0.000	0.000	0.493	0.007	0.000	0.000	0.000
	0.002	0.493	0.005	0.000	0.000	0.002	0.493	0.005	0.000	0.000
	0.000	0.003	0.493	0.003	0.000	0.000	0.003	0.493	0.003	0.000
	0.000	0.000	0.005	0.493	0.002	0.000	0.000	0.005	0.493	0.002
	0.000	0.000	0.000	0.007	0.493	0.000	0.000	0.000	0.007	0.493
	0.493	0.007	0.000	0.000	0.000	0.493	0.007	0.000	0.000	0.000
	0.002	0.493	0.005	0.000	0.000	0.002	0.493	0.005	0.000	0.000
	0.000	0.003	0.493	0.003	0.000	0.000	0.003	0.493	0.003	0.000
	0.000	0.000	0.005	0.493	0.002	0.000	0.000	0.005	0.493	0.002
	0.000	0.000	0.000	0.007	0.493	0.000	0.000	0.000	0.007	0.493