An Extended Class of Instrumental Variables for the Estimation of Causal Effects

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Abstract: This paper builds on the structural equations, treatment effect, and machine learning literatures to provide a causal framework that permits the identification and estimation of causal effects from observational studies. We begin by providing a causal interpretation for standard exogenous regressors and standard "valid" and "relevant" instrumental variables. We then build on this interpretation to characterize *extended instrumental variables* (EIV) methods, that is methods that make use of variables that need not be valid instruments in the standard sense, but that are nevertheless instrumental in the recovery of causal effects of interest. After examining special cases of *single* and *double* EIV methods, we provide necessary and sufficient conditions for the identification of causal effects by means of EIV and provide consistent and asymptotically normal estimators for the effects of interest.

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1. Introduction

The structural equations framework is predominant in economics for the purposes of modeling, identifying, and estimating causal effects of interest. The early work of the Cowles Commission by Tinbergen, Frisch, Koopmans, Haavelmo, Marshack, Simon, Wold, Strotz, and others studied the identification and estimation of causal effects (e.g. Haavelmo, 1943, 1944; Simon, 1953, 1954; Strotz and Wold, 1960; Fisher 1966.) This literature also introduced notions of "endogeneity" and "exogeneity." For the most part, standard textbooks currently define these concepts to mean respectively the correlation or lack thereof between a structural equation's observed explanatory variables and its unobserved variables. With the introduction of these concepts, it became evident that standard methods of estimation such as least squares regression fail to provide a consistent estimator for the effect of interest in the "endogeneous" regressor case.

Reiersøl (1945) formalized the method of "instrumental variables", originally introduced by Philip Wright (1928) building on Sewall Wright's (1921, 1923) work on "path analysis", within the structural equations framework. Ever since, the method of instrumental variables has played a central role in econometrics to overcome the problem of endogeneity (e.g. Heckman, 1997; Angrist and Krueger, 2001; Heckman, Urzua, and Vytlacil, 2005.) The definition may vary somewhat depending on the context, but in the familiar context of a linear structural equations system, instrumental variables are defined as variables that are uncorrelated with the equation's unobserved variables, i.e. "valid", and correlated with the included explanatory variables, i.e "relevant."

Recently, advances across a variety of disciplines have resulted in alternative frameworks and methods to identify and estimate causal effects from observational studies in the presence of endogeneity.

In particular, developments in labor economics (Roy 1951; Heckman and Robb, 1985; Hahn, 1998; Heckman, Ichimura, and Todd, 1998; Heckman, LaLonde, and Smith, 1999; Hirano, Imbens, and Ridder, 2003; Hirano and Imbens, 2004; Heckman and Vytlacil, 2005, etc.) have put forward a variety of methods, such as those based on matching and the propensity score, that permit this identification and estimation.

An extensive statistical literature on observational studies (e.g. Rubin, 1974; Rosenbaum, 2002) also emerged, building on the experimental design work of R.A. Fisher, Cox, Neymann, Kempthorne, and others. This "treatment effect" literature introduced the "potential outcome model" as well as the notions of "ignorability" and "propensity score" for measuring causal effects (e.g., Rosenbaum and Rubin, 1983; Holland, 1986.) Angrist, Imbens, and Rubin (1996) have related this framework to the method of instrumental variables.

Another line of research into the identification of causal effects has emerged in the machine learning literature in the work of Pearl (1988, 1995, 2000), Spirtes, Glymour, and Scheines (1993), and Dawid (2002) among others. In particular, Pearl (1995) introduced two methods related to the labor economics and treatment effect literatures,

the "back door" and the "front door" methods. A distinctive feature of this literature is the use of "directed acyclic graphs" (DAGs) to represent causal relationships and the use of graphical criteria to determine if particular causal effects are identifiable without particular attention to the estimation of these causal effects.

White (2006) and White and Chalak (2006a) propose the "settable system" framework as a means of unifying these distinct approaches to the study of causality. In those papers, particular attention is paid to the identification and estimation of causal effects in a setting most analogous to the classical case of exogenous regressors. Here we broaden our focus to apply this framework to analyze identification and estimation of causal effects in the presence of endogenous regressors generally. Consistent with Dawid (1969, 2000), we show that all of the methods that emerge, including those above, require one or more independence or conditional independence relationships to hold between observed variables and corresponding unobserved variables of the system under study.

Specifically, the contribution of this paper is to provide a novel and detailed examination of the ways in which causal structures can give rise to observed variables other than the cause or treatment of interest that can play an *instrumental* role in permitting the identification and estimation of causal effects of interest. In this sense, this paper extends the standard notion of instrumental variables to accommodate variables that are not necessarily uncorrelated with the unobserved causes of a response variable of interest but that can nevertheless be instrumental in permitting the recovery of useful estimates of causal effects of interest.

Consider, for example, the following simple structural equations system, where X, Y, and Z are variables with observed realizations, U_x, U_y and U_z are unobserved causes of X, Y, and Z respectively, α_o is an unknown real vector, and γ_o and δ_o are unknown real scalars such that:

$$(1) \qquad X \overset{c}{=} U_x' \alpha_o$$

$$(2) \qquad Z \overset{c}{=} \gamma_o X + U_z$$

$$(3) \qquad Y \overset{c}{=} \delta_o Z + U_y,$$

where we use $\overset{c}{=}$ instead of the usual equality sign to emphasize the causal nature of the structural equations. Following Dawid (1979), we use $\perp$ to denote independence between two random variables or vectors and $\not\perp$ to denote otherwise. For this example, we assume that $U_x \not\perp U_y$, $U_x \perp U_z$, and $U_y \perp U_z$.

Consider measuring the total causal effect of X on Y. Substituting structural equation (2) into structural equation (3) we get:

$$(3') \qquad Y \overset{c}{=} \beta_o X + \delta_o U_z + U_y,$$

where $\beta_o \equiv \gamma_o \delta_o$ is the causal effect of X on Y.

Clearly, since $U_x \perp U_y$, the least squares estimator for β_o, say $\hat{\beta}$, is inconsistent, as X is endogenous in the standard sense. Further, Z is an "invalid" instrumental variable, in the standard sense, as it is correlated with the unobserved term in (3′): from (2) we have that Z is correlated with U_z. Also, since X causes Z from (2) and X and U_y are correlated, we have that Z is correlated with U_y.

This system presents a situation where it seems that the causal effect of X on Y cannot be consistently estimated. Nevertheless, results of Section 4.1.2 demonstrate that, under mild conditions, a consistent estimator for the total causal effect of X on Y is given by:

$$\tilde{\beta} = \{(\boldsymbol{X}'\boldsymbol{X})^{-1}(\boldsymbol{X}'\boldsymbol{Z})\} \times \{[\boldsymbol{Z}'(\boldsymbol{I} - \boldsymbol{X}(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}')\boldsymbol{Z}]^{-1}[\boldsymbol{Z}'(\boldsymbol{I} - \boldsymbol{X}(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}')\boldsymbol{Y}]\},$$

where $\boldsymbol{X}$, $\boldsymbol{Y}$, and $\boldsymbol{Z}$ each denote $n \times 1$ data vectors.

This structural equation system permits the use of Pearl's (1995) "front-door" method, providing an example of a situation where a variable Z is instrumental in the recovery of the causal effect of X on Y even though it is an "invalid" instrument as currently defined in the literature. The challenge here is not that β_o is unidentified, but that it is *not identified by exogenous regressors or standard exogenous instruments*. Nor is this example the only possibility of this sort.

In a nutshell, the standard method of instrumental variables is not the whole story. There are other *extended instrumental variables* (EIV) methods that we can employ to identify and estimate causal effects of interest in the endogenous regressor case. These methods are characterized by alternative moment conditions and exclusion restrictions that parallel those in the standard instrumental variable case. A main goal of this paper is to begin a systematic exploration of these methods and their interrelations.

This paper is organized as follows. In Section 2, we state our assumptions and discuss the data generating structural equations systems of interest to us here. In Section 3, we employ the framework of Section 2 to provide a fully explicit causal interpretation of standard regression and IV methods, extending previous work and setting the stage for subsequent developments. Section 3.1 examines the case of *exogenous regressors* (XR) where regressors X act as their own instruments to identify the causal effects of interest. In Section 3.2, we study causal identification via standard *exogenous instruments* (XI) Z. There, we examine the standard "validity" and "relevancy" conditions, and we employ our framework to relax conditions presented in previous studies, such as that of Angrist, Imbens, and Rubin (1996), supporting a causal interpretation of standard instrumental variables methods. Section 3.3 shows how causal identification breaks down in situations where standard IV methods fail.

Section 4 begins our study of EIV methods, where the use of *conditional* extended instrumental variables Z, *conditioning* extended instrumental variables W, or both together permit the identification of the causal effect of a potentially endogenous X on the

response of interest *Y*. In Section 4.1 we discuss *single* EIV methods, that is, methods that use either conditioning EIV or conditional EIV but not both to identify causal effects. Section 4.1.1 defines and discusses conditioning instruments and the method of *conditionally exogenous regressors given conditioning instruments* (CXR|I), relating these to the method of matching, Rosenbaum and Rubin's (1983) ignorability condition, Pearl's (1995) back door method, and White's (2006) predictive proxies. In Section 4.1.2, we examine conditional instruments and the method of *conditionally exogenous instruments given regressors* (CXI|R). We relate these to the standard method of instrumental variables and to Pearl's (1995) front door method.

In Section 4.2, we discuss special cases of *double* EIV methods where the joint use of conditional and conditioning EIV is needed for the identification of effects of interest. In particular, we discuss the methods of *conditionally exogenous instruments given conditioning instruments* (CXI|I), *conditionally exogenous instruments and regressors given conditioning instruments* (CXIR|I), and *conditionally exogenous instruments given regressors and conditioning instruments* (CXI|RI.)

Section 5 provides a "master theorem" that contains the various EIV identification results as special cases or delivers them as immediate corollaries, stating both necessary and sufficient conditions for identification of causal effects.

Section 6 discusses how causal matrices can be used to characterize the cases where the identification of causal effects via EIV methods obtains. We illustrate by showing that our single EIV methods exhaust the possibilities for identification of causal effects using a single EIV. Section 7 presents straightforward conditions that ensure the consistency and asymptotic normality of the extended instrumental variables estimators considered here. Section 8 concludes, with final remarks and a discussion of directions for future research. Proofs of formal results are gathered into the Mathematical Appendix.

2. Causal Data Generating Systems

Economists and econometricians interested in measuring causal effects have long understood the distinction between predictive and causal inquiries and in particular the dictum that correlation need not imply causation. Goldberger (1991, p. 337) states that "the causal requirement that in regression the *x*'s have to be the variables that actually determine *y* does not appear in the specification of the [classical regression] model: nothing in the [classical regression] model requires that the *x*'s cause *y*." Thus, economists have been concerned with developing methods to measure causal effects beyond the linear regressions that they perceive as convenient carriers of predictive relationships in the form of conditional correlations between variables (see, e.g., Angrist and Krueger, 1999; Heckman, LaLonde, and Smith, 1999; Heckman, 2000; and Hoover, 2001.)

We employ a familiar structural equations system to represent a causal structure, *S*. In particular, we consider data generated as a special case of the recursive system

$$X_1 \overset{c}{=} r_1(X_0)$$
$$X_2 \overset{c}{=} r_2(X_1, X_0)$$
$$\vdots$$
$$X_J \overset{c}{=} r_J(X_{J-1}, \ldots, X_1, X_0),$$

where X_0 is a random vector, and for $j = 1, \ldots, J$, X_j is a random variable and r_j is an unknown scalar-valued response function.

In writing this system, we use the notation $\overset{c}{=}$ to emphasize that the J structural equations appearing in the system S are neither equations nor regressions. Instead, they represent "causal links" (Goldberger, 1972, p.979) that embody directionality from cause to effect. In particular, the right hand side variables of every structural equation determine the value of the corresponding left hand side variable but the converse is not necessarily true. The structural equations are thus directional "autonomous" mechanisms describing how every variable in the model is generated (Haavelmo 1943, 1944; Strotz and Wold, 1960; Pearl, 2000; White and Chalak, 2006a.) Conceptually, these mechanisms can be independently manipulated without necessarily modifying any of the other generating structural equations in the system. The autonomy of these equations is a vital requirement, for it enables the researcher to evaluate causal relationships by means of hypothetical interventions where X_j is set to some different value, X_j^*. Strotz and Wold (1960) describe such a manipulation as "wiping out" the structural equation that generates X_j, thus suspending the mechanism by which X_j was naturally generated, and setting X_j to X_j^*, whenever it appears as a right hand side variable in the other structural equations of the system, thus enabling the manipulation to manifest its effect on the rest of the variables in the system. White and Chalak (2006a) provide a rigorous formalization of this notion.

Observe that the vector X_0 does not appear on the left hand side of any causal relation in this system. If the system provides a complete description of the causal relations of the structure, then X_0 is not caused by any of the other variables of the system. Following White and Chalak (2006a), we refer to such variables as *fundamental*.

Our first formal assumption modifies the notation above somewhat to accommodate the structures of interest to us here.

Assumption A.1(a): Data Generating Structural Equations System: For $j = 1, \ldots, J$, let U_j be random vectors with unobserved realizations, and let the response functions r_j be unknown real-valued measurable functions such that observable random variables $X_1, \ldots, X_J$ are generated as:

$$X_1 \overset{c}{=} r_1(U_1)$$
$$X_2 \overset{c}{=} r_2(X_1, U_2)$$
$$\vdots$$

.
.

$$X_J \overset{c}{=} r_J(X_{J-1}, \ldots, X_1, U_J). \qquad \blacksquare$$

The U_j's may have differing dimensions. We collect them together into the random vector $X_0 \equiv (U_1', \ldots, U_J')'$. The data are thus generated by the system $S \equiv (X_0, r_1, \ldots, r_J)$ associated with the above collection of J structural equations.

In A.1(a) we do not specify that X_0 is fundamental, implying that A.1(a) does not provide a complete specification of the causal structure. This provides flexibility in handling dependence, and in particular causal relationships, that may hold among the U_j's. For the moment, we defer specifying these.

For notational convenience, we write $\mathbb{U} \equiv X_0$ and $\mathbb{X} \equiv (X_1, \ldots, X_J)$, and we let $\mathbb{V} \equiv (\mathbb{X}, \mathbb{U}) \equiv (V_1, \ldots, V_G)$ denote the vector of all observed and unobserved variables in the system. To maintain a tight focus for our analysis, we study the identification and estimation of causal effects in the structural equation system given in A.1(a) for the rest of this paper, leaving the task of suggesting and testing for causal models for other work (in progress).

We formally view the structure of A.1(a) as a settable system as defined by White and Chalak (2006a), so that references here to notions of setting, cause, and effect are as formally defined there. In particular, we view settings as the means by which manipulations of the mechanism that naturally generates a variable can be effected (Holland, 1986; Spirtes, Glymour, and Scheines, 1993; Pearl, 2000; and White and Chalak, 2006a.) Given A.1(a), the following working definition of causality suffices. Specifically, X_j does not cause X_k when $j \geq k$ (including $k = 0$), whereas X_j may cause X_k when $j < k$. For $j < k$, we say that X_j does not cause X_k relative to S if $r_k(X_{k-1}, \ldots, X_1, U_k)$ defines a function constant in X_j. In this case, we say that the response function r_k depends trivially on X_j. Otherwise, we say that X_j causes X_k relative to S. This notion corresponds to "direct" or "immediate" causality as defined by Pearl (2000.) As we have written this system, we view U_k as a cause of X_k, whereas U_j does not cause X_k for $j \neq k$.

The object of interest in this paper is the *average total causal effect* of an observable[2] X_j on another observable X_k. Thus, we are interested in the full effect of X_j on the dependent variable X_k, channeled via all routes in the system and averaged over the unobserved causes U_k of X_k. This is in contrast to the direct or immediate average effect on a variable where the effects due to intermediate causes are not accounted for. White and Chalak (2006a) discuss the identification of causal effects more generally, including covariate-conditioned effects on other features of the distribution of the response such as the variance and the quantiles.

[2] As we implicitly rely on White and Chalak's (2006a) settable system framework, it is more appropriate to refer to causal relationships as holding between *settable variables*, as defined there. Our present usage is intended to be a convenient shorthand.

The unobserved causes U are included in the structural equations to accommodate either the unobservability of known variables key to determining the target variable, the researcher's ignorance of the full causal mechanism that generates the target, or both. We permit dependence among the elements of U to accommodate dependence between the observed and unobserved variables of the same structural equation, resulting in endogeneity. This dependence may arise from causal relations among the elements of U.

In the structure provided by A.1(a), all variables have a causal status. For simplicity, we have not introduced attributes, that is, non-causal response modifiers (see White and Chalak, 2006a). A key role played by attributes is to introduce heterogeneity into the structural system, an essential aspect of economic reality (see, e.g., Heckman, 1997; Heckman, Urzua, and Vytlacil, 2005; and Heckman and Vytlacil, 2005). The structures we consider can be straightforwardly generalized to handle this heterogeneity, but we refrain from doing so here in order to maintain a sharp focus for our analysis.

For what follows we make use of the fact that every settable system S has an associated "causal matrix," $C_S = [c_{gh}]$. This is an adjacency matrix in which every observed and unobserved variable of system S has a corresponding row and a corresponding column. Thus C_S is a $G \times G$ matrix. An entry $c_{gh} = 1$ indicates that V_g is an immediate cause of V_h. An entry $c_{gh} = 0$ indicates that V_g does not immediately cause V_h. We impose the convention that a variable does not cause itself, so $c_{gg} = 0$ for $g = 1, \ldots, G$.

For example, when Assumption A.1(a) holds and the unobservables are scalar, C_S has the following form:

$$C_S = \left[\begin{array}{c:c} C_{S_1} & C_{S_2} \\ \hdashline C_{S_3} & C_{S_4} \end{array}\right] = \left[\begin{array}{c|ccc:ccc} & X_1 & \cdots & X_J & U_1 & \cdots & U_J \\ \hline X_1 & 0 & & & 0 & & 0 \\ \vdots & \vdots & \ddots & & \vdots & & \\ X_J & 0 & \cdots & 0 & 0 & & 0 \\ \hline U_1 & 1 & 0 \cdots & 0 & 0 & & \\ \vdots & 0 & \ddots & \vdots & & \ddots & \\ \vdots & \vdots & & \vdots & & & \\ U_J & 0 & \cdots 0 & 1 & & & 0 \end{array}\right]$$

The triangularity of the system assumed in A.1(a) ensures that C_{S_1} is upper triangular with zeros along the diagonal. Blank entries in C_S above indicate elements that can take either the values 0 or 1, reflecting the fact that X_j may or may not cause X_k when $j < k$. Assumption A.1(a) further specifies that none of the X's can cause any of the U's. Thus C_{S_2} is the $J \times J$ zero matrix. We also have that U_k does not cause X_j for $j \neq k$. As a result, C_{S_3} is the $J \times J$ identity matrix. We do not rule out the possibility that U_k can cause U_j. This leaves the elements of C_{S_4} unspecified, apart from the zero diagonal. Consequently, C_{S_1} and C_{S_4} determine C_S.

We note that there corresponds a unique structural equations system S given by A.1(a) for a given causal matrix C_S but that the converse is not true. This is so because the causal matrix explicitly specifies all causal relationships, including those holding among the unobserved variables, but these are unspecified in the structural equations system S of A.1(a). It follows that different causal matrices can potentially generate the same statistical dependence relationships among the unobserved variables in S. We discuss this further below.

In addition to the recursive structure among the observed variables imposed in A.1(a), we assume the following:

Assumption A.1(b): Acyclicality: For each $h \leq G$ and each set of h distinct elements, say $\{g_1, \ldots, g_h\}$, of $\{1, \ldots, G\}$, we have:

$$c_{g_1 g_2} \times c_{g_2 g_3} \ldots \times c_{g_h g_1} = 0. \qquad \blacksquare$$

The recursive structure of A.1(a, b) rules out mutual causality or cycles in S. Mutual causality occurs when V_g causes V_h and V_h causes V_g. Cycles occur when, for example, V_g causes V_h, V_h causes V_k, and V_k causes V_g. We impose this particular recursive structure for simplicity; the general settable system framework does not require this.

The acyclicality imposed in A.1(b) ensures that there exists at least one fundamental variable among the variables of the system, a consequence of proposition 1.4.2 of Bang-Jensen and Gutin (2001.) Given A.1(a), none of the X_j's ($j > 0$) can be fundamental, so it must be that at least one element of U is. For ease of reference, we denote the vector of fundamental variables U_0. This may contain some or all of the elements of the U_j's. We could alternatively specify an additional vector of unobserved fundamental variables U_0 to which the other U_j's may be causally related. We forgo this, however, to avoid elaborating our structure beyond what is strictly necessary to deliver our desired results. Note that whenever a variable is fundamental, its corresponding causal matrix column contains a vector of zeroes.

A convenient device for representing causal relations in simple situations that we employ repeatedly below is the "directed causal graph." For each causal matrix C_S, there is a corresponding causal graph G_S. These are variants of the graphs used in Wright's path analysis (Wright, 1921, 1923) and that are lately revived in the machine learning literature as "semi-markovian directed acyclic graphs" (DAGs.) See, for example, Pearl (1988, 1993a, 1993b, 2000) and Spirtes, Glymour, and Scheines (1993). In that literature, the unobserved components of the system are typically not explicitly represented. In contrast, we explicitly represent these due to the central role that they play in econometrics.

The graph G_S consists of a set of nodes (also referred to as vertices), one for each element of V, and a set of arrows A, corresponding to ordered pairs of distinct vertices. An arrow a_{gh} denotes that variable V_g is an immediate cause of V_h. We use solid arrows to denote direct causal relationships between variables with observed realizations. Thus, a solid

arrow from X_j to X_k denotes that X_j is an immediate cause of X_k. For variables with unobserved realizations, we use a dashed arrow from U_j to U_k to denote that U_j causes U_k. We also use a dashed arrow to denote that U_j is a cause of an observed variable X_j. As a convenient shorthand, we use a dashed line connecting U_j and U_k to indicate that either U_j is an immediate cause of U_k or that U_k is an immediate cause of U_j, or to indicate that there is an unobserved cause (e.g., U_0) that causes both U_j and U_k. (In the latter case we omit depiction of the unobserved common cause.) The lack of dashed arrows or dashed lines between the unobserved variables U_j and U_k indicates that $U_j \perp U_k$ as discussed further below. The convention that variables do not cause themselves corresponds to the absence of self-directed arrows in the causal graph G_S.

We next impose some significant simplifying structure:

Assumption A.2: Linearity and Separability: For $j = 1, \ldots, J$, assume that r_j is linear and separable so that the data generating structural equations system S is given by:

$$
\begin{aligned}
X_1 &\overset{c}{=} U_1' \alpha_1 \\
X_2 &\overset{c}{=} \beta_{2,1} X_1 + U_2' \alpha_2 \\
&\vdots \\
X_J &\overset{c}{=} \beta_{J,J-1} X_{J-1} + \ldots + \beta_{J,1} X_1 + U_J' \alpha_J,
\end{aligned}
$$

where, for $j = 1, \ldots, J$, $E(U_j) = 0$, α_j is an unknown real vector conforming to U_j, and for $j = 2, \ldots, J$, $\beta_{j,1}, \ldots, \beta_{j,j-1}$ are unknown real scalars. We put $\beta_j \equiv (\beta_{j,j-1}, \ldots, \beta_{j,1})'$. ■

The compelling motivation for imposing the strong structure of linearity and separability is to provide clear insight into the nature of EIV methods in a simple and familiar context, permitting us to make our main points without being distracted by further complications that otherwise arise. Nevertheless, the key insights of this paper extend to the modern "nonparametric" (more accurately, non-separable) setting in which A.2 is replaced by much milder conditions (see, e.g., Matzkin, 2003, 2004, and 2005; Imbens and Newey, 2003; White and Chalak, 2006a). We take this up elsewhere.

In what follows, we specify that certain independence or conditional independence conditions hold between the variables of interest. Following Dawid (1979), we write $X \perp U \mid W$ to denote that X is independent of U conditional on W. Just as independence $X \perp U$ entails $f_{U|X}(u|x) = f_U(u)$ (using the obvious shorthand notation for density or conditional density functions), conditional independence $X \perp U \mid W$ entails $f_{U|X,W}(u \mid x, w) = f_{U|W}(u \mid w)$. Given the linear separable structure assumed in A.2, these assumptions are stronger than is strictly necessary to obtain identification results. Weaker conditions can suffice given A.2, such as suitable conditional mean independence ($E(U \mid X, W) = E(U \mid W)$) or conditional non-correlation ($E(X U \mid W) = E(X \mid W) E(U \mid W)$) assumptions. Nevertheless, we work primarily with independence or conditional independence, first for simplicity

and second because these conditions are required for identification of causal effects in general, such as for the non-separable case or when interest attaches to causal effects on aspects of the distribution of the response other than average effects, such as effects on the quantiles or distribution of the response (see White and Chalak, 2006a.)

Observe that conditional independence implies conditional mean independence and conditional non-correlation. As a convenient convention to accommodate the stronger than necessary independence or conditional independence assumptions adopted here, when we speak of dependence or conditional dependence, we may understand this to result from unconditional or conditional correlation, also implying unconditional or conditional mean dependence.

3. Causal Identification with Exogenous Regressors or Instruments

We first employ the framework of Section 2 to provide a fully explicit causal interpretation of standard regression and IV methods. In placing the standard methods in this context, we will cover some very familiar ground. Nevertheless, by doing so we discover certain aspects of these familiar cases that have previously been overlooked, permitting us to extend previous work and to set the stage for subsequent developments.

Just as Goldberger (1991, p. 337) notes that there is no necessary causal structure embodied in the variables appearing in the standard regression model, there is also no necessary causal structure embodied in the standard treatments of instrumental variables. Although causal relationships were clearly of concern to the Cowles Commission pioneers, an explicit causal focus has disappeared from much of the subsequent literature on instrumental variables methods. For example, White (2001) treats instrumental variables estimation extensively, but nowhere is there any reference to causal structure. The statistical properties of the estimators studied are driven solely by stochastic properties of the variables involved, and in particular certain key moment conditions.

Exceptions to this agnosticism about causal structure in the instrumental variables context are provided by the recent articles of Angrist, Imbens, and Rubin (1996) (AIR), Heckman (1997), and Heckman, Urzua, and Vytlacil, (2005), for example. A main goal of AIR is explicitly to provide a causal account of the method of instrumental variables. Here we provide a causal account of instrumental variables methods complementary to and extending that of AIR. Our account is designed to accord with the philosophy literature on causality, which requires causal inputs in order to derive causal conclusions, as expressed in Cartwright's (1989) dictum "no causes in, no causes out." After all, why would we rely on statistical methods that allege to have identified causal effects without providing a causal account to support that claim?

To facilitate our analysis, we further elaborate our notation. We now let Y denote the scalar response of interest, let the k random variables $X_1, \ldots, X_k$ denote the observed causes of interest, and let the ℓ random variables $Z_1, \ldots, Z_\ell$ denote variables potentially instrumental to identifying the causal effects of interest, all with observed realizations specified in a manner consistent with A.1 and A.2. We put $X \equiv [X_1, \ldots, X_k]'$ and $Z \equiv [Z_1,$

..., Z_ℓ]′. (The X_j's of A.1 and A.2 now correspond to the elements of X, Y, and Z; this new notation helps us to keep track of the various roles played by the different variables of the system.) We denote by U_y, U_{x_1}, ..., U_{x_k}, and U_{z_1}, ..., U_{z_ℓ} the unobserved causes corresponding to the responses, causes, and instruments, respectively, and we write $U_x \equiv [U_{x_1}', \ldots, U_{x_k}']'$, and $U_z \equiv [U_{z_1}', \ldots, U_{z_\ell}']'$. We continue to denote the fundamental unobservables as U_0. We also let $\boldsymbol{X}$, $\boldsymbol{Y}$, and $\boldsymbol{Z}$ denote $n\times k$, $n\times 1$, and $n\times \ell$ matrices containing n identically distributed random observations on X, Y, and Z respectively.

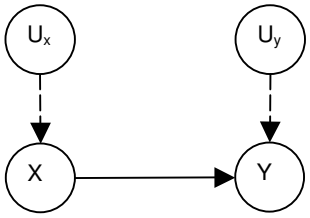
3.1 Exogenous Regressors

The first case we consider is that of exogenous regressors. Under our assumptions, this is the familiar case where simple regression identifies the effect of X on Y. Consider structural equations system S_1 and its corresponding causal graph G_1:

$$(1)\ X \overset{c}{=} \alpha_x U_x$$

$$(2)\ Y \overset{c}{=} X'\beta_o + U_y,$$

where $U_x \perp U_y$.



Graph 1 (G_1)
Exogenous Regressors

In *(1)* above, α_x is a matrix of unknown coefficients that maps the unobserved causes U_x to the observed causes X. The coefficients β_o have causal meaning by virtue of *(2)*.

A main feature of S_1 is that X and Y do not share a common cause. This structure ensures the following unconditional independence relationship:

(XR) *Exogenous Regressors*: $X \perp U_y$

In conformity with standard nomenclature, we refer to X as *exogenous regressors*. Together, A.1 and XR ensure the key moment condition

$$E(XU_y) = 0. \qquad (M1)$$

From *(2)* we have $U_y = Y - X'\beta_o$ (note that this is an equality, not a causal link). Substituting this into $M1$ gives

$$E(XY) - E(XX')\,\beta_o = 0$$

This condition *structurally identifies* the causal coefficients β_o by relating them solely to moments of observable variables. When *stochastic identification* also holds, that is, $E(X X')$ is non-singular, we have *full identification* of β_o. In this case, we have

$$\beta_o = [E(XX')]^{-1}[E(XY)].$$

We formalize this as follows:

Proposition 3.1.1 Suppose A.1 and A.2 hold such that: (i) $Y \overset{c}{=} X'\beta_o + U_y$, where X is $k \times 1$, $k > 0$, β_o is an unknown finite $k \times 1$ vector, and $E(XX')$ and $E(XY)$ exist and are finite. Suppose further that (ii) $E(XX')$ is non-singular; and (iii) XR: $X \perp U_y$ holds.

Then β_o, the average total causal effect of X on Y, is fully identified as:

$$\beta_o = [E(XX')]^{-1}[E(XY)]. \quad \blacksquare$$

Thus, Proposition 3.1.1 identifies the causal coefficient β_o with the statistical association between X and Y, $[E(XX')]^{-1}[E(XY)]$. We refer the use of exogenous regressors to identify the causal effect β_o in this way as the XR method.

The plug-in estimator for β_o is the familiar OLS estimator for a simple linear regression of $\boldsymbol{Y}$ on $\boldsymbol{X}$, $\hat{\beta}_n^{XR} \equiv (\boldsymbol{X}'\boldsymbol{X})^{-1}(\boldsymbol{X}'\boldsymbol{Y})$. In Section 7, we state straightforward conditions ensuring that this estimator and the others we introduce are consistent and asymptotically normal for the causal effect β_o.

Reichenbach's (1956) principle of common cause, applicable here, states that two random variables can exhibit correlation only if one causes the other or if they share a common cause. Here we have that X and Y are correlated. We know that Y does not cause X. The XR condition rules out the possibility that X and Y share a common cause. Given that $Y \overset{c}{=} X'\beta_o + U_y$, it follows that the association between X and Y can only be explained as the effect of X on Y.

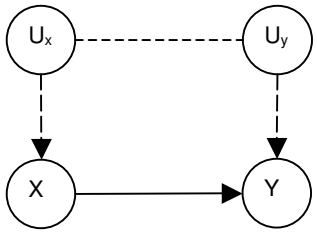
When control over X is possible, XR can be ensured by randomization, for example. Nevertheless, in observational studies, where control is not possible, it is often hard to argue that XR holds. We now examine the case where XR fails from a causal standpoint.

Specifically, consider the structural equations system S_2 and its corresponding causal graph G_2:

(1) $X \overset{c}{=} \alpha_x U_x$

(2) $Y \overset{c}{=} X'\beta_o + U_y$

where $U_x \perp U_y$.



Graph 2 (G_2)
Endogenous Regressors

In S_2, XR does not hold since $X \not\perp U_y$. When this results from $E(XU_y) \neq 0$, then β_o is no longer structurally identified. Instead, we have $E(XY) = E(XX')\,\beta_o + E(XU_y)$, in which unknown moments involving unobservables appear. We thus have the familiar result that regression fails to structurally identify β_o. In particular, the OLS estimator from a linear regression of $\boldsymbol{Y}$ on $\boldsymbol{X}$ is inconsistent for β_o.

In G_2, we cannot necessarily explain the association between X and Y as due to X causing Y, as this could equally be due to the joint response of X and Y to U_y, to U_x, or to an unobserved common cause of U_y and U_x, U_0. In accord with standard parlance, we refer to the failure of XR as *regressor endogeneity* and refer to X as *endogenous regressors* when $X \not\perp U_y$. We also refer to failure of XR as *confoundedness* of causes. In S_2, either U_y or U_x (or U_0) is a *confounding variable* for X and Y. Thus, under A.1, an endogenous regressor is one that shares at least one unobserved common cause with the response variable. Observe that simultaneity is absent from this system and is therefore not responsible for the endogeneity.

3.2 Exogenous Instruments

The presence of endogenous regressors means that β_o cannot be identified by the method of exogenous regressors. Nevertheless, identification is possible when one has available a vector of "proper" instrumental variables, Z. The standard textbook definition is that variables Z are "proper" instrumental variables if they are "valid," i.e. uncorrelated with the "error term," and "relevant," i.e. correlated with the endogenous regressors (e.g., Hamilton, 1994 p.238; Hayashi, 2000 p.191; Wooldridge, 2002 p.83-84):

(i) Z is "valid" if and only if $Corr\,(Z, U_y) = 0$
(ii) Z is "relevant" if and only if $Corr\,(X, Z) \neq 0$

where $Corr\,(\cdot\,,\cdot)$ denotes correlation.

P.G. Wright (1928) first used instrumental variables, which he referred to as "curve shifters," to identify supply and demand elasticities (see Morgan, 1990; Angrist and Krueger, 2001; Stock and Trebbi, 2003.) In describing these variables, P.G. Wright states: "Such additional factors may be factors which (A) *affect* demand conditions without affecting cost conditions or (B) *affect* cost conditions without affecting demand conditions" (P.G. Wright, 1928 p.312; our italics.) The use of the term "affect" suggests that Wright was thinking about causality and not only about statistical correlation. The first thoughts on instrumental variables appear, as expected, to have been driven by causal reasoning and not by statistical or algebraic study.

As we discuss next, standard instrumental variables methods fall into one of two subcategories. In both cases, we refer to these standard instruments as *exogenous instruments* (XI) and refer to this as the XI method.

3.2.1 Observed Exogenous Instruments

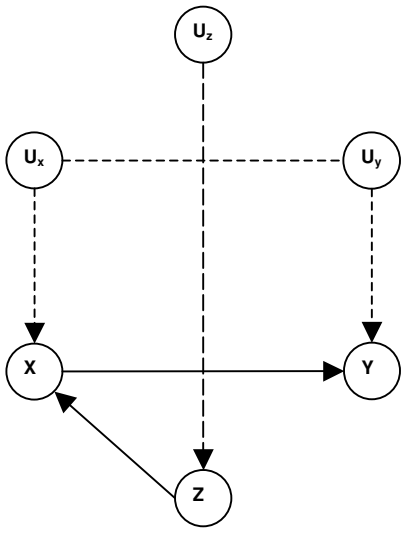
Consider the following structural equations system S_3 and its associated causal graph G_3:

$$(1)\ Z \overset{c}{=} \alpha_z U_z$$
$$(2)\ X \overset{c}{=} \gamma_x Z + \alpha_x U_x$$
$$(3)\ Y \overset{c}{=} X'\beta_o + U_y$$

where γ_x is a $k \times k$ matrix (so $\ell = k$), $U_x \perp U_y$, $U_x \perp U_z$, and $U_y \perp U_z$.

Substituting structural equation *(2)* into structural equation *(3)* and setting $\pi_o \equiv \gamma_x' \beta_o$, we have:

$$(3')\ Y \overset{c}{=} Z'\pi_o + U_x'\beta_o + U_y.$$



Graph 3 (G_3)
Observed Exogenous Instruments (OXI)

In S_3, X is endogenous since XR does not hold. Nevertheless, structural identification of the effect of X on Y is ensured by:

(XI) *Exogenous Instruments*: $Z \perp U_y$

Together with A.1 and A.2, this implies

$$E(ZU_y) = 0. \qquad (M2)$$

Using *(3)* then gives the structural identifying condition $E(Z\,Y) - E(Z\,X')\,\beta_o = 0$.

Identification is complete provided stochastic identification holds; for this, we now require that $E(Z\,X')$ is non-singular. This directly embodies the standard rank and order conditions. Parallel to Proposition 3.1.1, we have

Proposition 3.2.1 Suppose A.1 and A.2 hold such that: (i) $Y \overset{c}{=} X'\beta_o + U_y$, $X \overset{c}{=} \gamma_x Z + \alpha_x U_x$ (with $\ell = k$), and $E(ZX')$ and $E(ZY)$ exist and are finite. Suppose further that (ii) $E(ZX')$ is non-singular; and (iii) XI: $Z \perp U_y$ hold.

Then β_o, the average total causal effect of X on Y, is fully identified as:

$$\beta_o = [E(ZX')]^{-1}[E(ZY)]. \quad \blacksquare$$

The familiar result that the plug-in estimator $\hat{\beta}_n^{XI} \equiv (\boldsymbol{Z}'\boldsymbol{X})^{-1}(\boldsymbol{Z}'\boldsymbol{Y})$ is consistent and asymptotically normal for β_o then holds under mild conditions, provided in Section 7.

In S_3, Z satisfies the following three causal properties that accord with XI and that make Z instrumental for identifying β_o when X and Y are confounded:

(CP:OXI): **Causal Properties of Observed Exogenous Instruments**

(i) Z directly causes X, and the effect of Z on X is identified
(ii) Z indirectly causes Y, and the effect of Z on Y is identified
(iii) Z causes Y only via X

We refer to Z as *observed exogenous instruments* (OXI), as it is the observable vector Z that satisfies the specified causal properties.

These properties justify the indirect least squares (ILS) interpretation of instrumental variables (Haavelmo, 1943, 1944). Specifically, in S_3, since $Z \perp U_x$ and assuming that $E(XZ')$ and $E(ZZ')$ exist and are finite with $E(ZZ')$ non-singular, Proposition 3.1.1 establishes that γ_x , the effect of Z on X, is identified as $E(XZ')[E(ZZ')]^{-1}$. Under mild assumptions, the OLS estimator from a regression on structural equation *(2)*, $(\mathbf{Z'Z})^{-1}(\mathbf{Z'X})$, is consistent for γ_x'. Similarly, since $Z \perp U_x$ and $Z \perp U_y$ and assuming that $E(ZY)$ exists and is finite, Proposition 3.1.1 establishes that π_o, the effect of Z on Y, is identified as $[E(ZZ')]^{-1}E(ZY)$ and can be consistently estimated by $(\mathbf{Z'Z})^{-1}(\mathbf{Z'Y})$, the OLS estimator from a regression on structural equation *(3')*. By CP:OXI (iii) the only way in which Z can affect Y is via X. The effect of X on Y, β_o, thus equals the "ratio" of the effect of Z on Y to that of Z on X, so that $\beta_o = \gamma_x'^{-1} \pi_o$ for γ_x non-singular. β_o is then identified as:

$$\beta_o = \gamma_x'^{-1} \pi_o = \{[E(ZZ')]^{-1}E(ZX')\}^{-1}\{[E(ZZ')]^{-1}E(ZY)\} = [E(ZX')]^{-1}E(ZY).$$

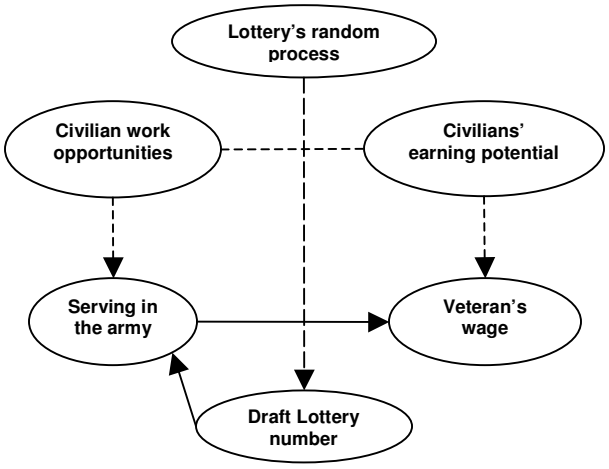
This can be consistently estimated by indirect least squares, that is, as the "ratio" of the two consistent estimators of the effects of Z on Y and the effect of Z on Y:

$$\{(\mathbf{Z'Z})^{-1}(\mathbf{Z'X})\}^{-1}\{(\mathbf{Z'Z})^{-1}(\mathbf{Z'Y})\} = (\mathbf{Z'X})^{-1}(\mathbf{Z'Y}) = \hat{\beta}_n^{XI}.$$

This is essentially the classical account of IV estimation of the coefficients of one of a system of structural equations. It bears explicit statement, however, for two reasons. First, it makes fully explicit all the causal components; second, it provides a "base case" against which variations on this account, provided below, can be compared.

The work of Angrist (1990) provides an example of the use of OXI in which all the causal elements are clear. Angrist is interested in measuring the effect of serving in the military during the Vietnam War on the civilian wage of a veteran after the war. Serving in the Vietnam War and the veteran's civilian wage could possibly be confounded by variables such as the individual's ability and education level, since these variables might jointly affect whether an individual joins the military and his/her civilian wages. Since serving in the military could thus potentially be an endogenous regressor, Angrist employs the Vietnam draft lottery number as an observed exogenous instrument in our

terminology. The lottery number was randomly assigned to individuals based on their date of birth and dictated that individuals whose date of birth corresponds to a low number (a one below a certain threshold) have to serve in the army, whereas those whose date of birth corresponds to a high number do not.



Graph 4 (G_4)
OXI for the Effect of Serving in the Army on Veteran's Wage

We now show that Angrist's use of the draft lottery number as an instrument satisfies CP:OXI. Implicitly, Angrist assumes that the randomness of the draft lottery number makes it statistically independent of unobserved factors that affect whether an individual joins the military or his/her civilian wages, and that the draft lottery number does not affect the veteran's wages except via serving in the army (see Graph 4.) If the data are indeed generated as in G_4, then the draft lottery number is a proper observed exogenous instrument.

A main goal of AIR is to provide an explicit causal account of the operation of the method of instrumental variables. To this end, AIR employ the "potential outcome" framework. We now compare the present approach with that of AIR. We maintain AIR's notation with the only change being our use of X_i and $\boldsymbol{X}$ instead of their D_i and $\boldsymbol{D}$ to denote the receipt of treatment. AIR let $i = 1, \ldots, n$ denote individuals in the population of interest and assume that the assignment Z_i to a binary treatment is "ignorable" but that the receipt of the treatment X_i is "non-ignorable." AIR list the following sufficient assumptions for the IV estimator to have a "causal interpretation", namely that of "an average causal effect for a subgroup of units, the compliers":

(a) Single Unit Treatment Value Assumption (SUTVA):
if $Z_i = Z_i'$ then $X_i(\boldsymbol{Z}) = X_i(\boldsymbol{Z}')$;
if $Z_i = Z_i'$ and $X_i = X_i'$ then $Y_i(\boldsymbol{Z}, \boldsymbol{X}) = Y_i(\boldsymbol{Z}', \boldsymbol{X}')$
(b) The treatment assignment Z_i is random
(c) Exclusion restriction: $\boldsymbol{Y}(\boldsymbol{Z}, \boldsymbol{X}) = \boldsymbol{Y}(\boldsymbol{Z}', \boldsymbol{X})$ for all $\boldsymbol{Z}$ and $\boldsymbol{Z}'$ and for all $\boldsymbol{X}$
(d) Nonzero average causal effect of Z on X
(e) Monotonicity: $X_i(1) \geq X_i(0)$ for all $i = 1, \ldots, n$.

If we let the variables *X, Y, Z,* U_x, U_y, and U_z in S_3 pertain to a given individual in the population, the OXI case satisfies AIR's assumptions. In that case, assumption (a) is satisfied by A.1, so that the left- and right-hand side variables in every structural equation in S_3 pertain only to a given individual. Assumption (b) in AIR (or more weakly the ignorability of the assignment of *Z*) and the OXI case share the same statistical implications, as $U_x \perp U_z$ and $U_y \perp U_z$. Assumption (c) states that any effect of *Z* on *Y* must go through *X*; this is ensured by structural equations *(2)* and *(3)* in S_3. Assumption (d) states that *Z* has an effect on the treatment *X*, which is ensured by structural equation *(2)* in S_3 with γ_x non-singular. Finally, assumption (e) assumes that the causal relationship between *Z* and *X* is monotonic in the sense that the direction of the effect of the assignment to treatment on the actual treatment is the same for all individuals. This is implicitly ensured in the structural equations of S_3 as γ_x is the same for each individual, given our assumed absence of heterogeneity.

3.2.2 Proxies for Unobserved Exogenous Instruments

In satisfying CP:OXI, S_3 provides a causal account of the standard method of instrumental variables, but it imposes the strong requirements that $U_x \perp U_z$ and $U_y \perp U_z$, or that *Z* is random (or ignorable) in AIR's language. Nevertheless, random instruments are infrequent and hard to argue for in economics generally, as it is largely an observational science. Further, neither the standard relevancy and validity conditions nor Proposition 3.2.1 necessarily require *Z* to be ignorable or random. It suffices that *Z* is correlated with *X* and uncorrelated with U_y. In particular, Proposition 3.2.1, as is standard in the econometrics literature (e.g. Heckman, 1996), does not require $Z \perp U_x$: the effect of *Z* on *X*, usually estimated from a "first stage" regression, need not be identified.

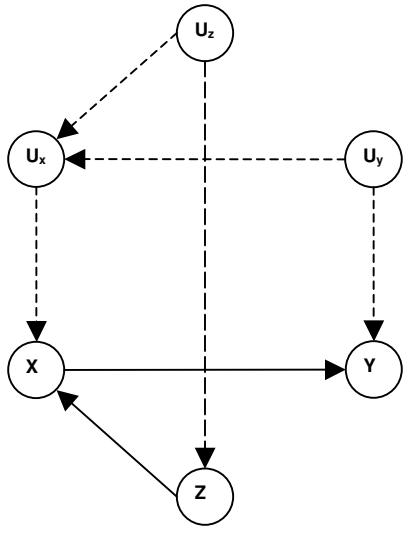
In this section, we present a causal account of the method of instrumental variables when *Z* is relevant and valid but is not random or ignorable. We thus present a causal explanation supporting the standard relevancy and validity moment conditions without having to impose further assumptions. In so doing, we relax AIR's conditions for the standard method of instrumental variables to consistently estimate a causal effect.

For this, consider the structural equations system S_5 and its associated causal graph G_5:

$$(1)\ Z \overset{c}{=} \alpha_z U_z$$
$$(2)\ X \overset{c}{=} \gamma_x Z + \alpha_x U_x$$
$$(3)\ Y \overset{c}{=} X' \beta_o + U_y$$

where γ_x is a $k \times k$ matrix (so $\ell = k$),
$U_x \perp U_y$, $U_x \perp U_z$ and $U_y \perp U_z$.

Substituting structural equation *(2)* into structural equation *(3)* with $\pi_o \equiv \gamma_x' \beta_o$ we have:



Graph 5 (G_5)
Proxies for Unobserved Exogenous Instruments (PXI)

$$(3') \; Y \overset{c}{=} Z'\pi_o + U_x'\alpha_x'\beta_o + U_y$$

Note that here $U_x \not\perp U_z$, where in S_3 we have $U_x \perp U_z$.

Since XR does not hold, Proposition 3.1.1 need not hold, and the usual OLS estimator is generally inconsistent for β_o. However, XI is satisfied as $Z \perp U_y$. Since we further have that $Z \not\perp X$, Z is a relevant and valid standard instrumental variable. As a result, Proposition 3.2.1 applies to S_5, so β_o is identified as $\beta_o = [E(ZX')]^{-1} E(ZY)$.

Clearly, Z in S_5 satisfies XI. Nevertheless, S_5 differs fundamentally from S_3, in that the causal properties making Z in S_5 instrumental for identifying β_o are satisfied not by Z but instead by the unobservable causes U_z. If these were observable, they could act as standard instruments. In their absence, the observable vector Z turns out to act as a proxy for the unobservables U_z. Accordingly, we refer to Z in S_5 as *proxies for (unobserved) exogenous instruments* (PXI) to distinguish this case from OXI. The causal properties for this case are:

(CP:PXI) Causal Properties for Proxies for Unobserved Exogenous Instruments

(i) U_z indirectly causes X, and the full effect of U_z on X could be identified via XR had U_z been observed
(ii) U_z indirectly causes Y, and the full effect of U_z on Y could be identified via XR had U_z been observed
(iii) U_z causes Y only via X
(iv) if Z causes Y, it does so only via X

Conditions (i), (ii) and (iii) of CP:PXI are essentially identical to their analogues of CP:OXI with the unobservable U_z appearing instead of the observable Z. Note that in (i) and (ii) we refer to the *full* effect of U_z on X and Y respectively. In (i), this includes not only the effect of U_z on X through Z, but also its effect through U_x, and similarly for the effect on Y in (ii) (see *(3')*). Condition (iv) is analogous to the exclusion restriction (iii) of CP:OXI, but here we do not require that Z causes Y.

In the PXI case, the effect of X on Y can be represented as the "ratio" of the full effect of U_z on Y to the full effect of U_z on X; however, the unobservability of U_z prohibits a direct computation. Moreover, in S_5 the effects of Z on Y and of Z on X are *not* identified as they are in the OXI case. The classical account of instrumental variables as indirect least squares does not work here. It thus might seem that the identification of the causal effect of interest is precluded in this case. Fortunately, however, Z plays the role of a *proxy* for U_z that, given the causal structure of S_5, enables it to identify β_o, the effect of X on Y.

Specifically, this structure ensures that Z and X as well as Z and Y are confounded by the same variables, U_z. When U_z renders the regression estimator of the effect of Z on X inconsistent, it simultaneously and systematically renders the estimator of the effect of Z

on Y inconsistent in just the right way to leave the ratio of these confounded effects informative for the effect of interest.

To demonstrate, suppose that $E(XZ')$, $E(ZZ')$, $E(ZY)$, and $E(U_xZ')$ exist and are finite and that the needed inverses exist. The effect of Z on X, γ_x, is not identified as $E(XZ')[E(ZZ')]^{-1}$ from *(2)* since $Z \perp U_x$. Instead, we have

$$\gamma_x = E(XZ')[E(ZZ')]^{-1} - \alpha_x E(U_x Z')[E(ZZ')]^{-1} .$$

Similarly, since $Z \perp U_x$, the effect of Z on Y, π_o, is not identified as $[E(ZZ')]^{-1}E(ZY)$ from *(3′)*. Instead we have

$$\pi_o = [E(ZZ')]^{-1}E(ZY) - [E(ZZ')]^{-1}E(ZU_x')\ \alpha_x'\ \beta_o.$$

Nevertheless, β_o, the effect of X on Y, is identified from *(3)* as $\beta_o = E(ZX')^{-1}E(ZY)$. To verify this from the expressions above, we write

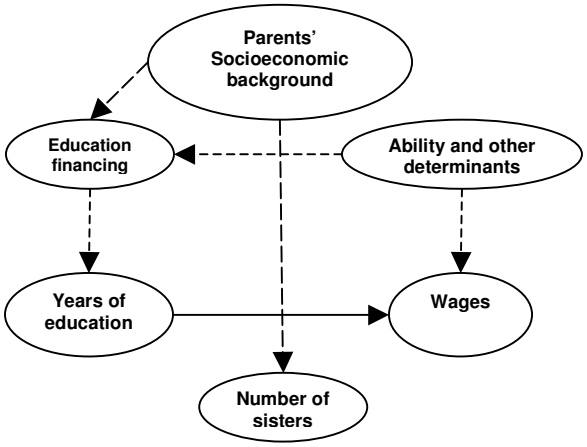
$$\begin{aligned} E(ZX')^{-1}E(ZY) &= \{[E(ZZ')]^{-1}E(ZX')\}^{-1}\ [E(ZZ')]^{-1}E(ZY) \\ &= \{\ \gamma_x' + [E(ZZ')]^{-1}E(ZU_x')\ \alpha_x'\}^{-1}\ [\pi_o + [E(ZZ')]^{-1}E(ZU_x')\ \alpha_x'\ \beta_o] \\ &= \{\ \gamma_x' + [E(ZZ')]^{-1}E(ZU_x')\ \alpha_x'\}^{-1}\ \{\gamma_x' + [E(ZZ')]^{-1}E(ZU_x')\ \alpha_x'\}\beta_o \\ &= \beta_o. \end{aligned}$$

The above expression also clearly shows that we may have γ_x equal to *zero*, so that in S_5, the PXI Z do *not* have to cause X, whereas γ_x had to be invertible in S_3 to support ILS. As long as Z and X share a common unobserved cause (U_z), they possess the statistical association required to identify the effect of interest. We provide an example below. When $\gamma_x = 0$, Z can be thought of as a "pure predictive proxy" for U_z, the true causal instrumental variable that allows the recovery of the effect of X on Y. (We further discuss predictive proxies below.) The PXI case thus makes use of a causally meaningful instrument that satisfies the relevancy and validity moment conditions but that does not satisfy the conditions of Angrist, Imbens and Rubin (1996.) In particular, PXI provides a causal account of the method of instrumental variables that relaxes two of AIR's assumptions, namely, random assignment (or more weakly the ignorability) of Z (assumption (b)) and nonzero average causal effect of Z on X (assumption (d)).

We note that in the PXI case, a function of two inconsistent estimators, the OLS estimators of γ_x' and π_o for structural equations *(2)* and *(3′)*, is itself a consistent estimator for the effect of interest, β_o. Thus, identification strategies that advocate the recovery of causal effects as functions only of *identifiable* effects, as in Pearl (2000, p.153-154), miss recovering certain identifiable causal effects.

A number of applied papers in economics that use the standard method of instrumental variables to estimate the effect of a potentially endogenous X on Y implicitly employ the PXI method to justify the validity and relevancy of their instruments. Consider, for example, measuring the effect of the number of years of education an individual receives on his/her future wages, as in Butcher and Case (1994) (BC).

As BC note, an individual's years of education and wages can be confounded by unobserved variables such as that individual's ability, making the variable "years of education" potentially endogenous. To avoid this problem, BC employ the numbers of sisters in a family as an instrument. They argue that daughters in families with a larger number of sisters tend to have lower levels of education and that this association is unlikely to be related to their future wages by means other than their educational attainment. In our framework, BC attempt to use a statistical association between the number of sisters and the level of education that the daughters attain without necessarily having that one causes the other. For instance, we can postulate that aspects of the parents' socioeconomic background and capacity to help finance the daughters' college education is what is generating the statistical association between the number of sisters and the education level (see G_6.) If the data are indeed generated as in G_6, then the number of sisters is a proxy for the unobserved exogenous instrument(s) "parents' socioeconomic background."



Graph 6 (G_6)
PXI for the effect of Education of wages

The OXI and PXI methods allow the identification of the effect of an endogenous X on the response of interest Y, as they employ instruments Z that satisfy the standard validity and relevance conditions. We thus refer to such variables Z as "proper standard instruments." Of equal importance is that Z provides a source of variation that precedes X and that affects Y only via X if at all. We thus also refer to OXI or PXI Z as *pre-cause instrumental variables*, as they causally precede the cause of interest X. We also refer to any Z satisfying XI as an *unconditional* instrumental variable, to distinguish it from the *conditional* instrumental variables discussed below.

3.3 Failures of Identification

In this section, we examine how structural identification of β_o via the XI method fails in the standard "irrelevant," "invalid," and "under-identified" cases. We thus demonstrate how, under A.1 and A.2, our causal framework accounts for not only the successes of the standard method of instrumental variables but also its failures.

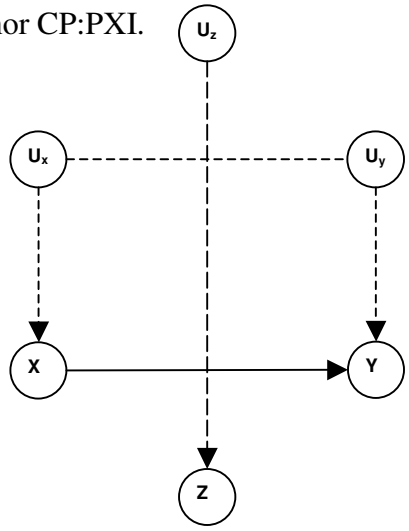
3.3.1 Irrelevant Exogenous Instruments

Not only must proper instruments Z be valid, they must also be relevant, i.e. correlated with X, in order to ensure the identification of the effect of X on Y. Structural equation system S_7 and its associated causal graph G_7 depict the irrelevant XI case and demonstrate how an irrelevant XI satisfies neither CP:OXI nor CP:PXI.

Let S_7 be given by:

$$(1)\ Z \overset{c}{=} \alpha_z U_z$$
$$(2)\ X \overset{c}{=} \alpha_x U_x$$
$$(3)\ Y \overset{c}{=} X'\beta_o + U_y$$

where $U_x \perp\!\!\!\perp U_y$, $U_x \perp\!\!\!\perp U_z$ and $U_y \perp\!\!\!\perp U_z$.



Graph 7 (G_7)
Irrelevant Exogenous Instruments

Even though Z is a valid standard instrument satisfying XI, it cannot be used to identify the effect of X on Y, because it is no longer true that the effect of X on Y can be represented as the ratio of the effect of Z (resp. U_z) on Y and the effect of Z (resp. U_z) on X. Both of these effects are zero, and their ratio is hence undetermined. In S_7, neither CP:OXI(i) nor CP:PXI(i) hold, since neither Z nor U_z cause X. We thus call these irrelevant exogenous variables. Observe that when $\ell = k$ (as assumed here), the presence of irrelevant exogenous variables causes condition (ii) of Proposition 3.2.1 (stochastic identification) to fail. The causal effect β_o fails to be identified in this case.

3.3.2 Endogenous Instruments

We next examine the failure of XI, condition (ii) of Proposition 3.2.1. In this case $Z \perp\!\!\!\perp U_y$; in accord with standard terminology we call such Z *endogenous instruments*. There are a number of ways that this can occur, which we now consider in some detail.

We first note that a potential instrument Z does not need to be relevant in order to be endogenous. An example of an irrelevant and endogenous instrument Z is one such that Z doesn't cause X and $U_z \perp\!\!\!\perp U_x$, but both U_x and U_z cause U_y. Turning now to relevant instruments, consider the system S_8:

$$(1)\ Z \overset{c}{=} \alpha_z U_z$$

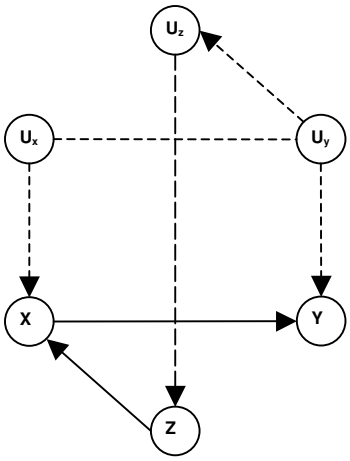
$$(2)\ X \overset{c}{=} \gamma_x Z + \alpha_x U_x$$

$$(3)\ Y \overset{c}{=} X'\beta_o + U_y$$

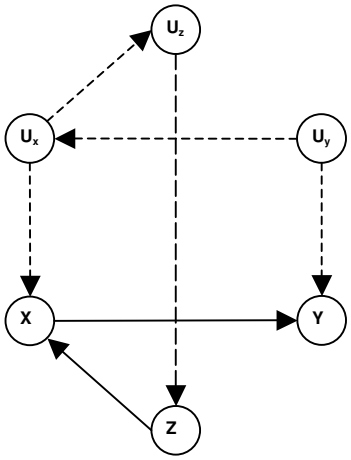
where $U_x \perp U_y$, $U_x \perp U_z$, and $U_y \perp U_z$. Substituting *(2)* into *(3)* with $\pi_o \equiv \gamma_x' \beta_o$ gives

$$(3')\ Y \overset{c}{=} Z'\pi_o + U_x'\alpha_x'\beta_o + U_y.$$

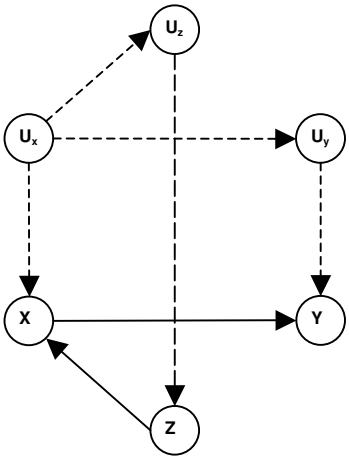
We have endogenous instruments because $U_y \perp U_z$, implying that XI fails. This can occur in several ways. For example, correlation between U_z and U_y can arise because either U_y causes U_z (see G_{8a}, G_{8b}) or U_x causes both U_z and U_y (see G_{8c}). In this case, CP:OXI(ii) does not hold, as Z and Y are confounded; and CP:PXI(ii) does not hold, as U_z and Y are confounded, implying that the full effect of U_z on Y would not be identified had U_z been observed. In fact, CP:PXI(i) also fails here, as U_z and X are also confounded.



Graph 8*a* (G_{8a})
Endogenous Instruments

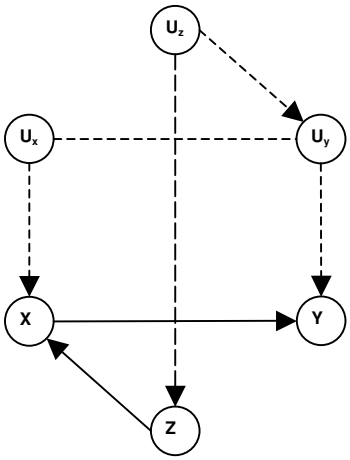


Graph 8*b* (G_{8b})
Endogenous Instruments

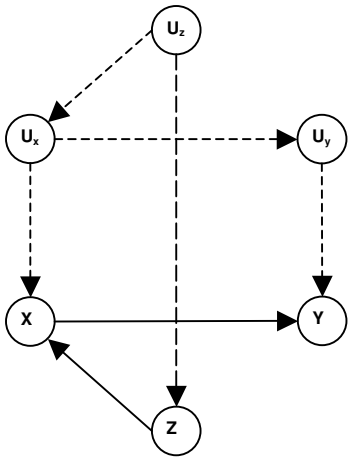


Graph 8*c* (G_{8c})
Endogenous Instruments

Alternatively, an endogenous instrument occurs when U_z affects U_y via a channel other than X. Since by assumption Z can't cause U_y, we need consider only the case where U_z causes Y via an intermediate cause other than X (see G_{8d} and G_{8e}). In this case, CP:OXI(ii) does not hold as Z and Y are confounded; and CP:PXI(iii) does not hold as U_z causes Y via an intermediate cause other than X. The effect of X on Y can thus no longer be expressed as the ratio of the effect of U_z on Y and the effect of U_z on X.



Graph 8*d* (G_{8d})
Endogenous Instruments



Graph 8*e* (G_{8e})
Endogenous Instruments

The conclusion of Proposition 3.2.1 fails in this case because structural identification fails. From structural equation *(3)* we have

$$E(ZY) = E(ZX')\,\beta_o + E(ZU_y),$$

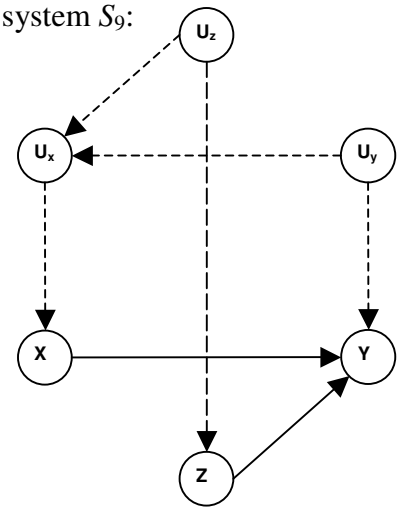
but $E(ZU_y)$ does not vanish, precluding structural identification of β_o as $E(ZX')^{-1}E(ZY)$.

3.3.3 Under-Identified Exogenous Instruments

We now consider what happens when instruments Z are valid and relevant, but condition (i) of Proposition 3.2.1 fails. Specifically, consider the system S_9:

$$(1)\ Z \overset{c}{=} \alpha_z U_z$$
$$(2)\ X \overset{c}{=} \alpha_x U_x$$
$$(3)\ Y \overset{c}{=} X'\beta_o + Z'\gamma_o + U_y$$

where $U_x \perp\!\!\!\perp U_y$, $U_x \perp\!\!\!\perp U_z$, and $U_y \perp\!\!\!\perp U_z$.



Graph 9 (G_9)
Under-identified XI

In this case, the regressors X are endogenous, as $U_x \perp\!\!\!\perp U_y$, but we have that Z is relevant since $X \perp\!\!\!\perp Z$, and valid since $Z \perp\!\!\!\perp U_y$. It follows from *(3)*, however, that

$$\begin{aligned}[E(ZX')]^{-1}E(ZY) &= [E(ZX')]^{-1}E[Z(X'\beta_o + Z'\gamma_o + U_y)] \\ &= \beta_o + [E(ZX')]^{-1}E(ZZ')\,\gamma_o,\end{aligned}$$

Once again, structural identification of β_o fails, this time due to the presence of the unknown (non-zero) γ_o. In terms of CP:OXI and CP:PXI, the problem is that Z enters the structure that determines Y directly, and not solely via X. This violates property (iii) of CP:OXI and property (iv) of CP:PXI, since Z affects Y directly instead of via X.

Viewed in this way, the lack of structural identification appears as a kind of "omitted variables bias," resulting from the failure to include Z in the instrumental variables regression. This difficulty cannot, however, be resolved by including Z in the IV regression as then one is attempting to identify both β_o and γ_o, and there are not enough proper instruments to do this. The standard order condition for identification requires the availability of at least one valid instrument for each right-hand side variable of a structural equation. Attempting to include Z in the IV regression places us in the classical "under-identified" case in which there are more right-hand side variables than valid

instruments. In addition to condition (i) of Proposition 3.2.1 not holding, this causes condition (ii) to fail too for the IV regression that includes both X and Z as regressors and uses only Z as instruments.

4 Extended Instruments

As the example in the introduction demonstrates, it is possible to identify causal effects of interest even in the absence of exogenous regressors or instruments. We now investigate situations in which vectors Z or W are not valid instruments in the standard sense, as they are correlated with the error term U_y, but are nevertheless instrumental in identifying the effect of X on Y. We therefore call such variables *extended* instrumental variables (EIV.) In particular, we introduce the notions of *conditional* or *conditioning* EIV. In each case, we explicitly reference two key conditions that together ensure the full identification of the causal effect of interest: (i) a conditional independence relationship that parallels the validity condition for the standard instrumental variables method, ensuring what White and Chalak (2006a) call *structural identification*; and (ii) the analog of the standard relevance, order, and rank conditions for identification, ensuring what White and Chalak (2006a) call *stochastic identification*.

4.1 Single EIV Methods

We first treat the case in which a single EIV can identify the causal effect of interest.

4.1.1 Conditioning Instruments

The results of Section 3.2 concern identification of causal effects using a single vector of what we have called "unconditional" instruments Z. We now consider single EIV methods that employ a vector of *conditioning* instruments W to proxy for the effects of unobserved confounding variables for X and Y. We extend our notation by writing $W \equiv [W_1, \ldots, W_m]'$, $U_w \equiv [U_{w_1}', \ldots, U_{w_m}']'$, and letting $\boldsymbol{W}$ denote an $n \times m$ matrix of identically distributed observations on W.

The treatment effect literature has introduced two central methods to treat the problem of confoundedness: randomization and matching (see for e.g. Rubin, 1974; Rosenbaum, 2002.) R.A. Fisher (1949, p. 12) argues that randomization is the "reasoned basis" for inference (see Rosenbaum, 2002, p. 21.) If randomization is feasible, randomly assigning agents to treatment and control groups ensures that there aren't systematic confounding variables for the cause and effect of interest. Since the process is random, the distribution of the assignment mechanism is known: it gives equal probability to every possible treatment assignment. We saw above in cases S_1 and S_3 that randomization permits identification of causal effects. Randomization, however, is uncommon in observational studies, where the researcher usually lacks the ability to control the variables of interest.

In non-randomized studies, matching units that share common causes or attributes from the treatment and control groups provides a way forward. By conditioning on the information in the true confounding variables, it is possible to interpret the remaining

conditional association between the putative cause and effect as the causal effect of the first on the second. Developments along these lines include "selection on observables" (Barnow, Cain, and Goldberger, 1980; Heckman and Robb, 1985), the "ignorability condition" and "propensity score" (Rubin, 1974; Rosenbaum and Rubin, 1983), the "back-door" method (Pearl, 1995), and "predictive proxies" (White, 2006; White and Chalak, 2006a.) In labor economics, matching methods are well established and have been discussed in the contexts of the distribution of earnings, policy evaluation, and the return to education and training programs, for example (see Roy, 1951; Heckman and Robb, 1985; Heckman, Ichimura, and Todd, 1998.)

We now investigate causal structures in which conditioning instruments W permit the identification of the effect of an endogenous X on Y. Thus, consider structural equations system S_2, in which X is endogenous because $U_x \not\perp U_y$. Suppose that this dependence arises from the presence of a common cause for both U_x and U_y. It is instructive to start with the extreme case where we actually observe the true common causes or confounding variables W that jointly determine U_x and U_y. Of course, this violates our assumption that observables do not cause unobservables (A.1(a)), so this is only a temporary expedient adopted to provide useful insight. We will remove this shortly. To proceed, consider structural equations system S_{10a} and its associated causal graph G_{10a}:

(1) $W \overset{c}{=} \alpha_w U_w$

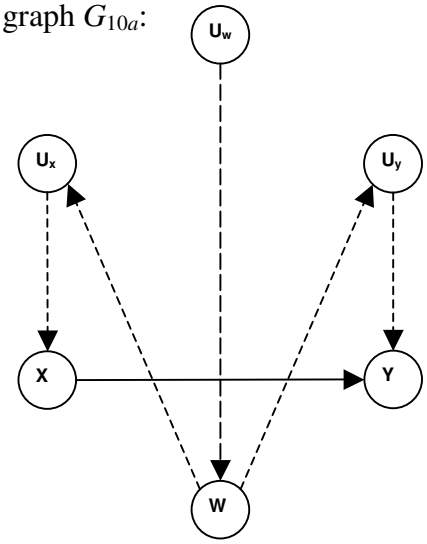
(2) $U_{x_1} \overset{c}{=} \gamma_{x_1} W$

(3) $U_{y_1} \overset{c}{=} \gamma_{y_1} W$

(4) $X \overset{c}{=} \alpha_{x_1} U_{x_1} + \alpha_{x_2} U_{x_2}$

(5) $Y \overset{c}{=} X' \beta_0 + U_{y_1} + U_{y_2}$

so that $U_x \perp U_y$, $U_x \perp U_w$, and $U_y \perp U_w$,
where $U_x \equiv (U_{x_1}', U_{x_2}')'$ and $U_y \equiv (U_{y_1}, U_{y_2})'$,
with $U_w \perp U_{x_2}$, $U_w \perp U_{y_2}$, and $U_{x_2} \perp U_{y_2}$.



Graph 10*a* (G_{10a})
Conditioning Instruments

Regressor endogeneity arises from correlation between U_{x_1} and U_{y_1} resulting from the common cause W. The unobservable causes U_{x_2} and U_{y_2} provide independent sources of variation[3] ensuring that X is not entirely determined by W and that Y is not entirely determined by X and W.

[3] In subsequent structural equations systems of Section 4, we sometimes drop explicit reference to components of vectors of unobserved causes for notational convenience, keeping in mind that these vectors are not entirely determined by other unobserved causes and thus that they include independent sources of variation, such as U_{x_2} and U_{y_2} in S_{10a} (and S_{12} below), necessary for stochastic identification.

In S_{10a}, once we condition on W, we are guaranteed that the remaining association between X and Y can be interpreted only as the causal effect of X on Y. The key conditional independence condition obvious in S_{10a} that parallels XR and XI above is

(CXR|I) *Conditionally Exogenous Regressors given Conditioning Instruments:*
$$X \perp U_y \mid W$$

When this condition holds for some vector W generally, we call W *conditioning instruments* to emphasize their role in ensuring this conditional exogeneity.

The role of S_{10a} is merely to motivate CXR|I. We emphasize that S_{10a} is by no means a necessary structure for CXR|I to hold. As we discuss below, CXR|I can also hold for properly chosen W even when the true confounding variables for X and Y cannot be observed.

Just as XR and XI can deliver structural identification of β_o, so can CXR|I. Specifically, the key moment condition resulting from CXR|I in our linear separable framework is:

$$E(XU_y \mid W) = E(X \mid W) \times E(U_y \mid W). \qquad (M3)$$

To see how this condition structurally identifies β_o, rewrite *(M3)* as

$$E([X - E(X \mid W)]\, U_y \mid W) = 0,$$

replace $E(X|W)$ with its regression representation $E(XW')[E(WW')]^{-1}W$, and take expectations on both sides above to get

$$E(\, [X - E(XW')[E(WW')]^{-1}W\,]\, U_y) = 0.$$

This and structural equation *(5)* imply that

$$E([X - E(XW')[E(WW')]^{-1}W]\, [Y - X'\beta_o]\,) = 0,$$

so that β_o is structurally identified as

$$\{E(X\,X')\, - E(XW')[E(WW')]^{-1}\, E(W\,X')\}\, \beta_o = E(XY) - E(XW')[E(WW')]^{-1}E(WY).$$

Note that this derivation relies only on $Y \overset{c}{=} X'\beta_o + U_{y_1} + U_{y_2}$, the linear regression representation $E(XW')[E(WW')]^{-1}W$ for $E(X \mid W)$, and CXR|I. The specific structure of S_{10a} is not required.

When stochastic identification holds, i.e., $E(X\,X')\, - E(XW')[E(WW')]^{-1}\, E(W\,X')$ is non-singular, β_o, the average total causal effect of X on Y, is identified as:

$$\beta_o = \{E(XX') - E(XW')[E(WW')]^{-1}E(WX')\}^{-1} \times \{E(XY) - E(XW')[E(WW')]^{-1}E(WY)\}.$$

We defer a formal statement of this result until we have further explored the causal structures relevant to this case. From a causal perspective, identification holds because after conditioning on W, the association remaining between X and Y can only be explained as a response in Y due to variation in X.

Under mild conditions, a consistent and asymptotically normal plug-in estimator for β_o is

$$\hat{\beta}_n^{CXR|I} = \{\boldsymbol{X}'(\boldsymbol{I} - \boldsymbol{W}(\boldsymbol{W}'\boldsymbol{W})^{-1}\boldsymbol{W}')\,\boldsymbol{X}\}^{-1}\{\boldsymbol{X}'(\boldsymbol{I} - \boldsymbol{W}(\boldsymbol{W}'\boldsymbol{W})^{-1}\boldsymbol{W}')\,\boldsymbol{Y}\}.$$

Even though W plays an instrumental role in identifying β_o, there is no requirement that W be exogenous. For example, in S_{10a} we clearly have that W is endogenous, as $W \not\perp U_y$. Conditioning instruments are thus not standard instruments, motivating their description as extended instrumental variables (EIV). We call $\hat{\beta}_n^{CXR|I}$ an EIV estimator.

Inspecting $\hat{\beta}_n^{CXR|I}$, we see that it has the form of a standard instrumental variables estimator using as *derived* standard instruments the residuals of the regression of X on W, $X - E(XW')[E(WW')]^{-1}W$. Nevertheless, we do not put these derived instruments on an equal footing with W, as it is W that provides the natural causal explanation enabling the recovery of the effect of X on Y. Of even greater significance, however, is the fact that as White and Chalak (2006a) show, when A.2 is relaxed to permit non-separable structures, these derived instruments no longer play an essential role, whereas W (the vector of "predictive proxies" in White and Chalak's (2006a) terminology) continues to play the instrumental role in identifying the causal effects of interest.

The estimator $\hat{\beta}_n^{CXR|I}$ is easily recognized as the Frisch-Waugh (1933) partial regression estimator, obtained by regressing $\boldsymbol{Y}$ on the residuals $(\boldsymbol{I} - \boldsymbol{W}(\boldsymbol{W}'\boldsymbol{W})^{-1}\boldsymbol{W}')\boldsymbol{X}$ from a regression of $\boldsymbol{X}$ on $\boldsymbol{W}$. This can also be obtained as the coefficient estimator associated with $\boldsymbol{X}$ from a *simple linear regression* of $\boldsymbol{Y}$ on both $\boldsymbol{X}$ and $\boldsymbol{W}$.

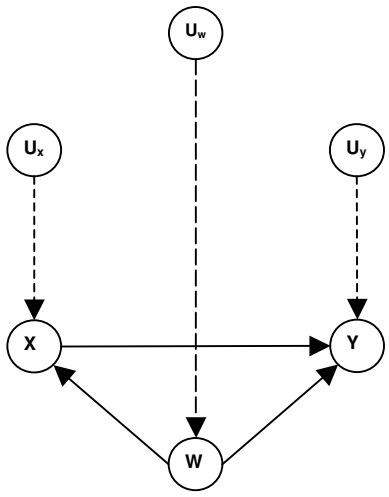
This latter regression emerges naturally from S_{10a}, after performing the simple substitutions required to enforce our convention that observables do not cause unobservables. Substituting *(2)* into *(4)* and *(3)* into *(5)* in S_{10a} gives the structure S_{10b}:

(1) $W \overset{c}{=} \alpha_w U_w$

(2) $X \overset{c}{=} \gamma_x W + \alpha_x U_x$

(3) $Y \overset{c}{=} X'\beta_o + W'\gamma_o + U_y$

with $U_w \perp U_x$, $U_w \perp U_y$, and $U_x \perp U_y$.



Graph 10*b* (G_{10b})
Conditioning Instruments

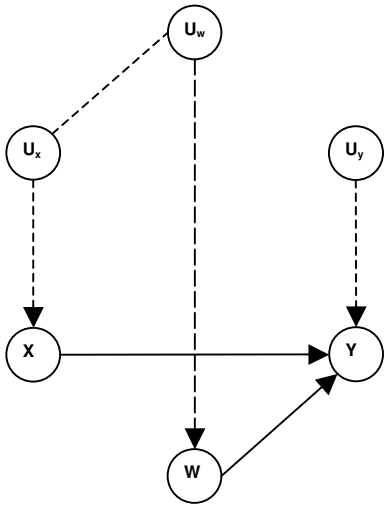
In formulating S_{10b}, we have adjusted the notation in the natural way. With the given independence conditions, we are back to the case treated by Proposition 3.1.1 as X and W jointly satisfy XR. In S_{10b}, both β_o, the causal effect of X on Y, and γ_o, the causal effect of W on Y are identified. Observe that the latter is only the direct causal effect of W on Y. The *full* causal effect of W on Y is given by $\gamma_o + \gamma_x' \beta_o$, which is identified from a regression of Y on W only.

As noted above, S_{10a} or S_{10b} is sufficient but not necessary for CXR|I. Structures satisfying Pearl's (1995; 2000, pp. 79-81) "back-door" criterion, in which an observable (here W) mediates an indirect link between X and Y also ensure CXR|I. In Pearl's framework, W is modeled either as the vector of common causes (G_{10a}, G_{10b}), or as an effect of the unobserved common cause and a cause of either Y or X (G_{11a}, G_{11b}). In G_{11a} and G_{11b} below, CXR|I holds because either U_x causes Y via W or U_y causes X via W. We do not observe the confounding variable, which is U_x in G_{11a} and U_y in G_{11b}. Instead, W acts as an observable proxy for the true confounding variables in each case.

Specifically, let S_{11a} be given by

$$(1)\ W \overset{c}{=} \alpha_w U_w$$
$$(2)\ X \overset{c}{=} \alpha_x U_x$$
$$(3)\ Y \overset{c}{=} X'\beta_o + W'\gamma_o + U_y$$

with $U_w \perp\!\!\!\perp U_x$, $U_w \perp\!\!\!\perp U_y$, and $U_x \perp\!\!\!\perp U_y$.

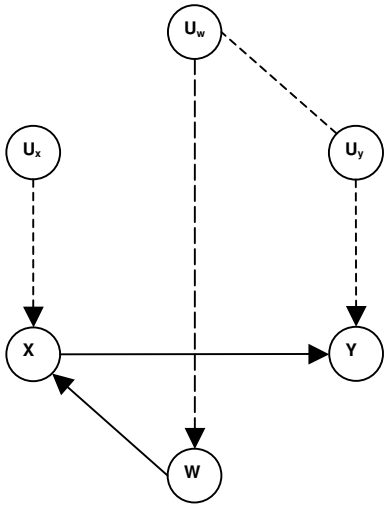


Graph 11*a* (G_{11a})
Conditioning Instruments

Similarly, let S_{11b} be given by

$$(1)\ W \overset{c}{=} \alpha_w U_w$$
$$(2)\ X \overset{c}{=} \gamma_x W + \alpha_x U_x$$
$$(3)\ Y \overset{c}{=} X'\beta_o + U_y$$

with $U_w \perp\!\!\!\perp U_x$, $U_w \perp\!\!\!\perp U_y$, and $U_x \perp\!\!\!\perp U_y$.



Graph 11*b* (G_{11b})
Conditioning Instruments

There are a number of noteworthy features about each of these structures. First consider S_{11a}. Note that the direction of causality between U_x and U_w is not specified in G_{11a}. S_{11a} thus corresponds to three possible back door structures. For all of these structures, X and W jointly satisfy XR in *(3)*, so Proposition 3.1.1 holds; the situation for S_{11a} is completely parallel to that for S_{10b}. In S_{11a}, W is a structurally relevant exogenous variable correlated with X, so omitting W from the identifying regression results in the classical "omitted variable bias." The inclusion of W is thus crucial to identifying β_o.

Now consider G_{11b}. For concreteness, suppose U_y causes U_w. Now X is endogenous because $X \not\perp U_y$. Also, W is endogenous because $W \not\perp U_y$. Nevertheless, given CXR|I, β_o is structurally identified. If stochastic identification also holds, β_o is identified as

$$\beta_o = \{E(XX') - E(XW')[E(WW')]^{-1}E(WX')\}^{-1} \times \{E(XY) - E(XW')[E(WW')]^{-1}E(WY)\}.$$

As we observed above, this is consistently estimated by the coefficients on X from a regression of Y on both X and W. But this is truly a remarkable situation: here we have causally meaningful coefficients β_o identified and consistently estimated using a regression that not only contains endogenous regressors X, but also *structurally irrelevant* and *endogenous* regressors W. (We call W "structurally irrelevant" because W does not appear in *(3)* of S_{11b}.) According to the textbooks, such a regression should yield nonsense. Nevertheless, the causal structure ensures structural identification of β_o.

What about the remaining coefficients in this regression, those associated with W? In the context of S_{11b}, these have no causal interpretation. Instead they have only a predictive interpretation, as discussed in detail by White (2006) and White and Chalak (2006a.) We thus have an interesting situation in which some regression coefficients have a causal meaning (those associated with X), but others do not (those associated with W.) That is to say, not all of the regression coefficients need to have signs and magnitudes that make causal sense. This constitutes an instance of what Heckman (2006) has termed "Marshak's maxim," which holds that we may identify certain economically meaningful components of a given structure (here β_o) without having to identify the entire structure.

Nor does Pearl's back door method exhaust the possibilities for achieving CXR|I. Another possibility, discussed in depth by White (2006) and White and Chalak (2006a) is the case of "predictive proxies." In the present framework, predictive proxies arise from structures such as S_{12}, which violates Pearl's back-door criterion:

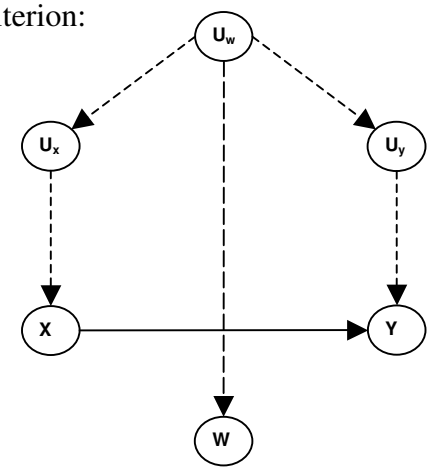
(1) $W \overset{c}{=} \alpha_{w_1} U_{w_1} + \alpha_{w_2} U_{w_2}$

(2) $U_{x_1} \overset{c}{=} \gamma_{x_1} U_{w_1}$

(3) $X \overset{c}{=} \alpha_{x_1} U_{x_1} + \alpha_{x_2} U_{x_2}$

(4) $U_{y_1} \overset{c}{=} \gamma_{y_1} U_{w_1}$

(5) $Y \overset{c}{=} X'\beta_o + U_{y_1} + U_{y_2}$



Graph 12 (G_{12})
Conditioning Instruments

where $U_{w_1} \perp U_{w_2}$, $U_{w_2} \perp U_{x_2}$, $U_{w_2} \perp U_{y_2}$, and $U_{x_2} \perp U_{y_2}$, with $U_w \equiv (U_{w_1}', U_{w_2}')'$, $U_x \equiv (U_{x_1}', U_{x_2}')'$, and $U_y \equiv (U_{y_1}', U_{y_2})'$, so that $W \perp U_y$, and $X \perp U_y$.

In S_{12}, we view U_{w_1} as an unobserved common cause for X and Y; the predictive proxy W can be viewed as a measurement error-laden version of U_{w_1}. We see that both X and W are endogenous; however, Proposition 4.4 of White and Chalak (2006a) applies to establish that CXR|I holds here. The key to this is the ability of W to predict U_w (hence X) sufficiently well that U_y contains no additional information useful in predicting X. Just as in S_{11b}, the causal effect of interest is identified from a regression containing endogenous X and structurally irrelevant endogenous W. Our comments about S_{11b} fully apply to S_{12}.

We may refer to unobserved confounding variables for X and Y as the *common causes* of X and Y. Given its role as an observable proxy for the unobserved common causes, we may also refer to W ensuring CXR|I as a vector of *common cause instruments*.

The CXR|I condition can play a central role in the treatment effects and matching literature. Using Y_x to denote the value that Y would take had X been set to x (the "potential outcome"), it can be shown that when $Y \overset{c}{=} X'\beta_o + U_y$ and CXR|I holds, then the key "ignorability" or "unconfoundedness" condition $Y_x \perp X \mid W$ of Rosenbaum and Rubin (1983) holds. (See White (2006, proposition 3.2).)

We conclude this section with a formal identification result under CXR|I.

Proposition 4.1.1 Suppose A.1 and A.2 hold such that: (i) $Y \overset{c}{=} X'\beta_o + U_y$, and $E(XX')$ and $E(XY)$ exist and are finite. Suppose further that (ii) there exists a random vector W such that $E(XW')$, $E(WW')$, and $E(WY)$ exist and are finite; $E(WW')$ is non-singular and $E(X \mid W) = E(XW')[E(WW')]^{-1}W$; (iii) $E(XX') - E(XW')[E(WW')]^{-1}E(WX')$ is non-singular; and (iv) CXR|I: $X \perp U_y \mid W$ holds.

Then β_o, the average total causal effect of X on Y, is fully identified as:

$$\beta_o = \{E(XX') - E(XW')[E(WW')]^{-1}E(WX')\}^{-1} \{E(XY) - E(XW')[E(WW')]^{-1}E(WY)\}. \blacksquare$$

In contrast to the XI method, we note that the CXR|I method does not require $\ell = k$, either for structural or for stochastic identification.

White and Chalak (2006a) present further substantial analysis for identification of average and other causal effects using predictive proxies for the general nonlinear and non-separable case (where A.2 is removed.) White and Chalak (2006b) discuss related parametric and nonparametric estimation methods and provide several tests for CXR|I.

Because of the straightforward framework provided by CXR|I for identifying causal effects (in particular, because there are no necessary exclusion restrictions involved) there

is no need to provide a list of causal properties for CXR|I parallel to CP:OXI or CP:PXI. Note, however, that because we are interested in the total effect of X on Y, we do not permit X to cause W.

4.1.2 Conditional Instruments

In Section 3.2, we discuss the use of standard exogenous instrumental variables Z to identify the effect of the potentially endogenous X on Y as the ratio of the effect of Z on Y and that of Z on X. In this section, we demonstrate how a single vector of extended instruments Z, that we refer to as *conditional instruments*, permits the identification of the causal effect of the endogenous X on Y as the *product* of the effects of X on Z and that of Z on Y. We also refer to this class of extended instruments as *intermediate cause* instrumental variables as these variables mediate the effects of X on Y.

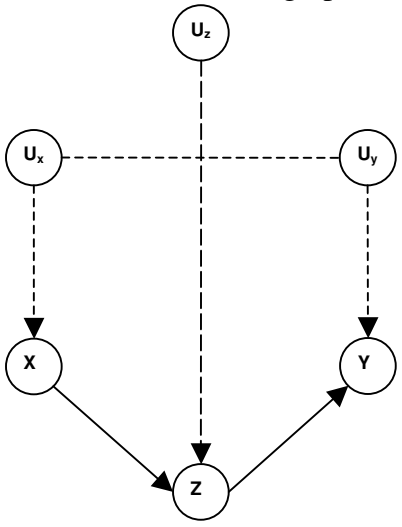
To illustrate, consider structural equation system S_{13} and its associated causal graph G_{13}:

$$(1)\ X \overset{c}{=} \alpha_x U_x$$
$$(2)\ Z \overset{c}{=} \gamma_z X + \alpha_z U_z$$
$$(3)\ Y \overset{c}{=} Z'\delta_o + U_y$$

where $U_x \perp\!\!\!\perp U_y$, $U_x \perp U_z$, and $U_y \perp U_z$.

Substituting structural equation *(2)* into structural equation *(3)* we get:

$$(3')\ Y \overset{c}{=} X'\beta_o + U_z'\alpha_z'\delta_o + U_y;\ \text{with } \beta_o \equiv \gamma_z'\delta_o.$$



Graph 13 (G_{13})
Conditional Instruments

This is the structure described in the introduction. We immediately see that X is endogenous, and it is also clear that Z is endogenous, so neither standard regression nor standard instrumental variables methods can identify β_o, the effect of interest. Nor do we have CXR|I, as $X \perp\!\!\!\perp U_y \mid Z$, so there are no conditioning instruments available. Nevertheless, β_o is structurally identified as a result of our next conditional independence relationship:

(CXI|R) *Conditionally Exogenous Instruments given Regressors:* $Z \perp\!\!\!\perp U_y \mid X$

We refer to such Z as *conditional instruments* and refer to methods that identify β_o using these extended instrumental variables as CXI|R methods.

For our linear separable system, CXI|R implies the key moment condition

$$E(ZU_y \mid X) = E(Z \mid X) \times E(U_y \mid X) \qquad (M4)$$

Parallel to our analysis of CXR|I, it follows from this moment condition that

$$E(\,[Z - E(ZX')[E(XX')]^{-1}X\,]\,U_y\,) = 0.$$

This and structural equation *(3)* imply

$$E(\,[Z - E(ZX')[E(XX')]^{-1}X\,]\,[Y - Z'\delta_o\,]) = 0,$$

so that δ_o is structurally identified as

$$\{E(Z\,Z')\; - E(ZX')[E(XX')]^{-1}\,E(X\,Z')\}\;\delta_o = E(ZY) - E(ZX')[E(XX')]^{-1}E(XY).$$

That is, δ_o is structurally identified by CXR|I with regressors Z and conditioning instruments X. Full identification of δ_o holds given stochastic identification; here, this requires the non-singularity of $\{E(Z\,Z')\; - E(ZX')[E(XX')]^{-1}\,E(X\,Z')\}$.

If γ_z can be also be identified, then identification of β_o follows, as $\beta_o \equiv \gamma_z'\,\delta_o$. In S_{13}, we see that γ_z is structurally identified by XR, as $X \perp U_z$. If γ_z is stochastically identified (i.e., $E(XX')$ is non-singular), Proposition 3.1.1 gives $\gamma_z' = [E(XX')]^{-1}E(XZ')$.

We have the following formal result.

Proposition 4.1.2 Suppose A.1 and A.2 hold such that: (i) $Z \overset{c}{=} \gamma_z X + \alpha_z U_z$, $Y \overset{c}{=} Z'\delta_o + U_y$, where $E(XX')$, $E(XZ')$, $E(ZZ')$, $E(ZY)$, and $E(XY)$ exist and are finite. Suppose further that (ii) (a) $E(XX')$ is non-singular and (b) $\{E(Z\,Z')\; - E(ZX')[E(XX')]^{-1}\,E(X\,Z')\}$ is non-singular; and (iii) (a) XR: $X \perp U_z$ and (b) CXI|R: $Z \perp U_y \mid X$ hold.

Then $\beta_o \equiv \gamma_z'\,\delta_o$, the average total causal effect of X on Y, is identified as:

$$\beta_o = [E(XX')]^{-1}E(XZ') \times \{E(ZZ') - E(ZX')[E(XX')]^{-1}E(XZ')\}^{-1} \times \{E(ZY) - E(ZX')[E(XX')]^{-1}E(XY)\}\ \blacksquare$$

In contrast to the XI method, we note that the CXI|R method does not require $\ell = k$. This condition is required neither for structural nor for stochastic identification.

Section 7 gives straightforward conditions under which the plug-in estimator $\hat{\beta}_n^{CXI|R}$ is a consistent and asymptotically normal estimator for β_o where:

$$\hat{\beta}_n^{CXI|R} \;\equiv\; (\boldsymbol{X'X})^{-1}(\boldsymbol{X'Z}) \times [\boldsymbol{Z'}(\boldsymbol{I} - \boldsymbol{X}(\boldsymbol{X'X})^{-1}\boldsymbol{X'})\boldsymbol{Z}]^{-1}[\boldsymbol{Z'}(\boldsymbol{I} - \boldsymbol{X}(\boldsymbol{X'X})^{-1}\boldsymbol{X'})\boldsymbol{Y}]$$

$$\equiv\; \hat{\gamma}_n^{XR}{}'\;\hat{\delta}_n^{CXR|I}\,.$$

Although this method uses a single vector of extended instrumental variables Z to identify the causal effect of interest, both these instruments and the regressors X play dual roles in the process. The extended instruments play the dual role of a response for X and a cause for Y. The regressors serve as exogenous regressors with respect to U_z in *(2)* and as conditioning instruments with respect to U_y in *(3)*, as is explicit in (iii)(a) and (iii)(b). We reflect these latter roles in our notation $\hat{\gamma}_n^{XR}$ and $\hat{\delta}_n^{CXR|I}$ above.

Analogous to the case of unconditional instruments, we can state a succinct set of causal properties required to ensure that the conditional instruments Z identify the effect of interest when X and Y are confounded:

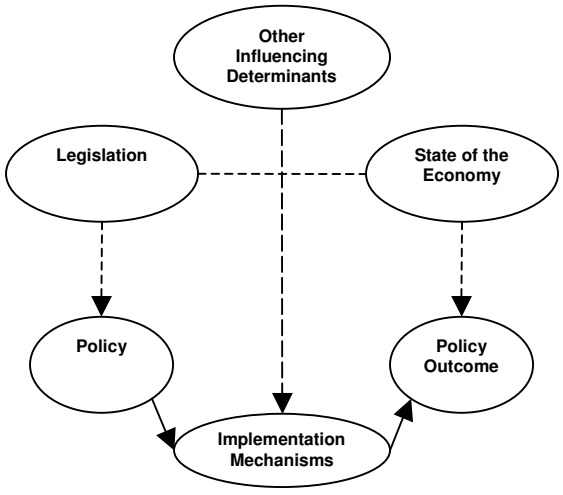
(CP:XI|R): **Causal Properties of Conditionally Exogenous Instruments Given Regressors**

(i) The effect of X on Z is identified
(ii) The effect of Z on Y is identified
(iii) If X causes Y, it does so only via Z

As is readily verified, S_{13} satisifies CP:XI|R.

The CXI|R method corresponds to the "front-door" method introduced by Pearl (1995). Pearl (1995, 2000) discusses the front-door method primarily in relation to the back-door method discussed in Section 4.1.1. In particular, the treatment effect literature typically applies CXR|I (back door) to identify the effect of interest in the presence of confounding by conditioning on a covariate (the conditioning instrument W) that is *not* affected by the treatment. In the CXI|R (front-door) case, we condition on a variable that *is* affected by the treatment (indeed, that mediates the treatment) to allow the identification and consistent estimation of the causal effect of the treatment X on Y.

The CXI|R method can play a particularly useful role in measuring the effects of certain policies as illustrated in G_{14}. In particular, we might be interested in evaluating the outcome of a policy that we think is endogenous since it is determined by legislation that is correlated with the state of the economy, which also determines the policy outcome.



Graph 14 (G_{14})
Policy Evaluation by means of CXI|R

To illustrate, we might be interested in evaluating the effect on students' performance in public schools, as measured by their standardized test scores, of new legislation for education reform (see, for example, Gordon and Vegas, 2005) but suspect that the new education law is endogenous as it is correlated with unobserved causes of the students' performance. For instance, one might argue that the legislation passed due to the poor state of the economy, which itself is a cause of the students' unsatisfactory performance. Under these circumstances, one might recover the effect of the legislation on student performance by employing intermediate causes that are affected by the new policy and that in turn affect student performance. In this case, these intermediate cause instruments should be implementation mechanisms that are responses only to the new policy and are not caused by the unobserved common confounding causes of the policy and the response of interest otherwise. In our example, potential intermediate cause instruments could be funding per student, number of teachers per school, educational attainment of teachers, class size, and so forth.

4.1.3 Other Potential Single Extended Instruments

In Sections 3.1, 3.2, 4.1.1, and 4.1.2, we examine four cases where the identification of the effect of X on Y obtains:

$$\begin{aligned} \text{XR:} &\quad X \perp U_y \\ \text{XI:} &\quad Z \perp U_y \\ \text{CXR|I:} &\quad X \perp U_y \mid W \\ \text{CXI|R:} &\quad Z \perp U_y \mid X \end{aligned}$$

In each case we have either the independence or conditional independence of a single vector of observed variables X or Z from U_y, the unobserved causes of Y. (In stating CXR|I, we could have used the notation Z instead of W, but we keep the notation distinct to make explicit the unique role played by conditioning instruments.) We therefore refer to XR, XI, CXR|I, and CXI|R as *single* EIV methods.

The remaining possibilities of independence or conditional independence from U_y that are so far unexplored when considering only a single vector of unconditional, conditional or conditioning EIV Z or W, are those associated with Y. Clearly, $Y \not\perp U_y$, $Y \not\perp U_y \mid X$, and $Y \not\perp U_y \mid W$ since, by definition, U_y is an immediate cause of Y. Similarly, $X \not\perp U_y \mid Y$ since conditioning on a common effect generally renders the possibly independent X and U_y necessarily dependent. The final possibility to consider is whether identification can be achieved in the case for which instruments Z are conditionally independent of U_y given Y.

A causal structure that generates this final conditional independence relationship is one where Z is a *post-response* instrument, so that X causes Y, which then causes Z. Such a structure is given by structural equations system S_{15} and its associated graph G_{15}. S_{15} perhaps looks promising, as one may consider the possibility of identifying the effect of the endogenous X on Y as the ratio of the effect of X on Z and that of Y on Z, analogous to indirect least squares. Unfortunately, the identification of β_o in this way is not possible, as we now demonstrate.

Let S_{15} be given by:

$$(1)\ X \overset{c}{=} \alpha_x U_x$$

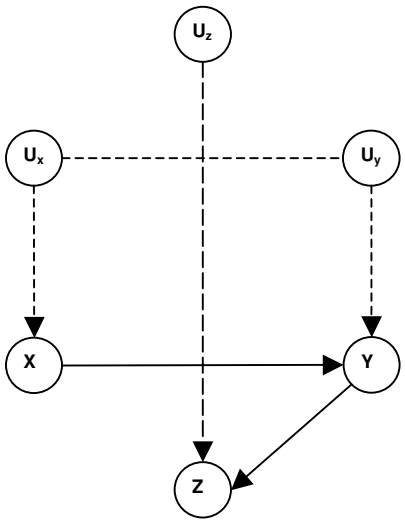
$$(2)\ Y \overset{c}{=} X'\beta_o + U_y$$

$$(3)\ Z \overset{c}{=} \gamma_z Y + \alpha_z U_z$$

where $U_x \perp\!\!\!\perp U_y$, $U_x \perp\!\!\!\perp U_z$, and $U_y \perp\!\!\!\perp U_z$.

Substituting structural equation *(2)* into structural equation *(3)* with $\delta_o \equiv \beta_o\gamma_z$ we get:

$$(3')\ Z \overset{c}{=} X'\delta_o + \gamma_z U_y + \alpha_z U_z.$$



Graph 15 (G_{15})

Unfortunately, the conditional independence condition $Z \perp\!\!\!\perp U_y \mid Y$ is not sufficient to identify β_o in manner analogous to CXR|I or CXI|R. From this condition and algebra analogous to that above, we obtain the moment equations

$$\{E(ZY') - E(ZY')[E(YY')]^{-1}E(YY')\} - \{E(ZX') - E(ZY')[E(YY')]^{-1}E(YX')\}\,\beta_o = 0.$$

The first term above is always zero, however, so even if $\{E(ZX') - E(ZY')[E(YY')]^{-1} E(YX')\}$ is non-singular, the solution for β_o is also always zero.

Another way to see why the above conditional independence is not sufficient for the identification and consistent estimation of β_o is that in S_{15} the effect of Y on Z, γ_z, is identified from Proposition 3.1.1 as $\gamma_z = E(ZY')[E(YY')]^{-1}$ since $Y \perp\!\!\!\perp U_z$. However, the effect of X on Z is not identified as X and Z are confounded by either U_x or U_y. Hence, β_o is itself not identified as the ratio of these two effects.

As this exhausts the possibilities for identification of causal effects using a single vector of instruments, we now consider methods that make use of two vectors of extended instruments.

4.2 Double Extended Instrumental Variables Methods

Sections 3 and 4.1 discuss all the possible ways in which a single vector of unconditional, conditional or conditioning EIVs permits the identification and consistent estimation of the effect of X on Y. We now turn our attention to EIV methods that make joint use of conditional instruments Z, and conditioning instruments W, for the identification of causal effects of interest.

For the remainder of this section, we let the random variable Y be the response of interest, the elements of the $k_1 \times 1, \ldots, k_p \times 1$ random vectors $X_1, \ldots, X_p$ be the causes of interest, and the elements of the $\ell_1 \times 1, \ldots, \ell_q \times 1$ random vectors $Z_1, \ldots, Z_q$ and the $m_1 \times 1, \ldots,$

$m_s \times 1$ random vectors $W_1, \ldots, W_s$ be extended instrumental variables, all with observed realizations as specified in A.1 and A.2. Their corresponding unobserved causes are given by the random variable U_y, and the elements of the random vectors $U_{x_1}, \ldots, U_{x_p}$, $U_{z_1}, \ldots, U_{z_q}$, and $U_{w_1}, \ldots, U_{w_s}$. We put $X \equiv [X_1', \ldots, X_p']'$, $Z \equiv [Z_1', \ldots, Z_q']'$ and $W \equiv [W_1', \ldots, W_s']'$, where X is of dimension $k \times 1$ with $k \equiv k_1 + \ldots + k_p$, Z is of dimension $\ell \times 1$ with $\ell \equiv \ell_1 + \ldots + \ell_q$, and W is of dimension $m \times 1$ with $m \equiv m_1 + \ldots + m_s$. Similarly, we put $U_x \equiv [U_{x_1}', \ldots, U_{x_p}']'$, $U_z \equiv [U_{z_1}', \ldots, U_{z_q}']'$, and $U_w \equiv [U_{w_1}', \ldots, U_{w_s}']'$. Boldface letters denote vectors and matrices of observations of X, Y, Z, and W, as above.

4.2.1 Conditional and Conditioning Instruments: CXI|I

Economic theory can suggest causal models that permit identification of causal effects using more than one type of instrumental variable. In this section, we discuss examples where both conditional and conditioning extended instrumental variables Z and W are needed to ensure structural identification.

4.2.1.a OCXI|I

Our first case is that of *observed conditionally exogenous instruments given conditioning instruments* (OCXI|I). To illustrate, consider structural equations system S_{16a} with associated causal graph G_{16a}.

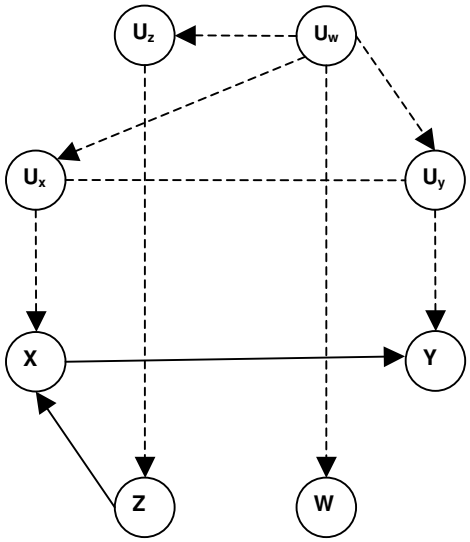
Let S_{16a} be given by:

$$(1)\ W \overset{c}{=} \alpha_w U_w$$
$$(2)\ Z \overset{c}{=} \alpha_z U_z$$
$$(3)\ X \overset{c}{=} \gamma_x Z + \alpha_x U_x$$
$$(4)\ Y \overset{c}{=} X'\beta_o + U_y,$$

where $U_x \perp U_y$, $U_x \perp U_z$, $U_x \perp U_w$, $U_y \perp U_z$, $U_y \perp U_w$, and $U_z \perp U_w$.

Substituting structural equation *(3)* into structural equation *(4)* and setting $\pi_o \equiv \gamma_x' \beta_o$, we have:

$$(4')\ Y \overset{c}{=} Z'\pi_o + U_x'\alpha_x'\beta_o + U_y$$



Graph 16a (G_{16a})
Observed Conditionally Exogenous Instruments Given Conditioning Instruments (OCXI|I)

The key conditional independence relationship that holds in S_{16a} when W is a sufficiently good predictor for U_w (hence Z) is:

(CXI|I) *Conditionally Exogenous Instruments given Conditioning Instruments*:
$Z \perp U_y \mid W$

In contrast to CXR|I, where the regressors are conditionally exogenous, here the conditional instruments Z are those for which the conditioning instruments, W, ensure conditional exogeneity.

Given A.2, the key moment condition for structural identification resulting from CXI|I is:

$$E(ZU_y \mid W) = E(Z \mid W) \times E(U_y \mid W) \qquad (M5)$$

Algebra similar to that for CXR|I delivers the structural identification of β_o under CXI|I.

Proposition 4.2.1 Suppose A.1 and A.2 hold such that: (i) $Y \overset{c}{=} X'\beta_o + U_y$. Suppose further that (ii) there exist random vectors W and Z such that and that $\ell = k$; $E(ZY)$, $E(ZW')$, $E(WW')$, $E(WY)$, and $E(ZX')$ exist and are finite; $E(WW')$ is non-singular and $E(Z \mid W) = E(ZW')[E(WW')]^{-1}W$; (iii) $E(ZX') - E(ZW')[E(WW')]^{-1} E(WX')$ is non-singular; and (iv) CXI|I: $Z \perp U_y \mid W$ holds.

Then, β_o, the average total causal effect of X on Y, is fully identified as

$$\beta_o = \{E(ZX') - E(ZW')[E(WW')]^{-1}E(WX')\}^{-1} \times \{E(ZY) - E(ZW')[E(WW')]^{-1}E(WY)\} \quad \blacksquare$$

Note the requirement that $\ell = k$, analogous to the XI method. None of the methods discussed previously can identify β_o in this case, since none of the other admissible conditional independence relationships hold in S_{16a}.

Section 7 provides straightforward conditions under which the plug-in CXI|I estimator $\hat{\beta}_n^{CXI|I}$ is a consistent and asymptotically normal estimator for β_o, where

$$\hat{\beta}_n^{CXI|I} \equiv [\boldsymbol{Z}'(\boldsymbol{I} - \boldsymbol{W}(\boldsymbol{W}'\boldsymbol{W})^{-1}\boldsymbol{W}')\boldsymbol{X}]^{-1}[\boldsymbol{Z}'(\boldsymbol{I} - \boldsymbol{W}(\boldsymbol{W}'\boldsymbol{W})^{-1}\boldsymbol{W}')\boldsymbol{Y}].$$

This is a standard IV estimator with derived standard instruments $Z - E(ZW')E(WW')^{-1}W$, the residuals from the regression of Z on W. Nevertheless, we do not treat these residuals as the fundamental variables instrumental for the identification of β_o for reasons analogous to those discussed in connection with CXR|I: it is Z and W that provide the natural causal explanation enabling the recovery of the effect of X on Y; further, when A.2 is relaxed to permit non-separable structures, the derived instruments no longer play an essential role, whereas Z and W continue to play the instrumental role in identifying the causal effects of interest.

In S_{16a}, Z satisfies the following causal properties that parallel CP:OXI and permit the identification of the effect of X on Y in a manner analogous to OXI.

(CP:OCXI|I): **Causal Properties of Observed Conditionally Exogenous Instruments given Conditioning Instruments**

(i) Z directly causes X, and the effect of Z on X is identified via CXR|I with conditioning instruments W
(ii) Z indirectly causes Y, and the effect of Z on Y is identified via CXR|I with conditioning instruments W
(iii) Z causes Y only via X

In contrast to CP:OXI, conditioning instruments W are needed in CP:OCXI|I to ensure that the effect of Z on X and that of Z on Y are identified. Since Z is observed, we refer to this case as *observed conditionally exogenous instruments given conditioning instruments* (OCXI|I). Similar to the method of XI, the effect of X on Y is identified here as the "ratio" of the identified effects of Z on X and that of Z on Y.

4.2.1.b PCXI|I

As in the case of XI and CXR|I, we do not need to observe the true underlying cause; it suffices to observe a suitable proxy for it. In fact, this feature applies to all of the EIV methods that we discuss (see Theorem 5.1 and Corollary 5.2 below.) We illustrate this in S_{16b} and associated causal graph G_{16b}.

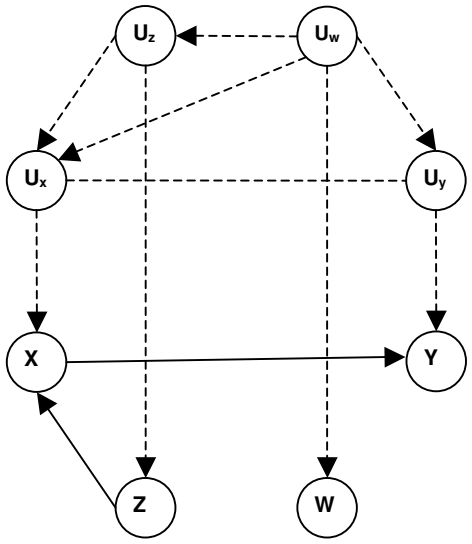
Let S_{16b} be given by:

$$(1)\ W \overset{c}{=} \alpha_w U_w$$
$$(2)\ Z \overset{c}{=} \alpha_z U_z$$
$$(3)\ X \overset{c}{=} \gamma_x Z + \alpha_x U_x$$
$$(4)\ Y \overset{c}{=} X'\beta_o + U_y,$$

where $U_x \perp\!\!\!\perp U_y$, $U_x \perp\!\!\!\perp U_z$, $U_x \perp\!\!\!\perp U_w$, $U_y \perp\!\!\!\perp U_z$, $U_y \perp\!\!\!\perp U_w$, and $U_z \perp\!\!\!\perp U_w$.

Substituting structural equation *(3)* into structural equation *(4)* and setting $\pi_o \equiv \gamma_x' \beta_o$, we have:

$$(4')\ Y \overset{c}{=} Z'\pi_o + U_x'\alpha_x'\beta_o + U_y$$



Graph 16b (G_{16b})
Proxy for Unobserved Conditionally Exogenous Instruments Given Conditioning Instruments (PCXI|I)

Here again CXI|I holds and Proposition 4.2.1 applies to fully identify β_o as for OCXI|I. However, in S_{16b} the effect of Z on X and that of Z on Y are no longer identified and CP:OCXI|I no longer holds. Instead, U_z satisfies these properties and Z plays the role of a proxy for the unobservables U_z. Parallel to PXI, we refer to Z in this case as *proxies for (unobserved) conditionally exogenous instruments given conditioning instruments* (PCXI|I). The causal properties that permit the identification of β_o in this case are:

(CP:PCXI|I) Causal Properties for Proxies for Unobserved Conditionally Exogenous Instruments given Conditioning Instruments

(i) U_z indirectly causes X, and the full effect of U_z on X could be identified via CXR|I with conditioning instruments W had U_z been observed
(ii) U_z indirectly causes Y, and the full effect of U_z on Y could be identified via CXR|I with conditioning instruments W had U_z been observed
(iii) U_z causes Y only via X
(iv) if Z causes Y, it does so only via X

CP:PCXI|I parallels its counterpart CP:PXI, but conditioning instruments W are now needed for the effect of U_z on X and that of U_z on Y to be identified had U_z been observed. Our comments about PXI fully apply here. In particular, the effect of X on Y can be represented as the "ratio" of the full effect of U_z on Y to the full effect of U_z on X. But the effect of Z on Y and that of Z on X are *not* identified in S_{16b} as they are in the OCXI|I case, even after employing conditioning instruments W; the resulting CXR|I estimators are inconsistent. As before, these effects are confounded by the same variables, U_z, in just the right way to leave the ratio of the two CXR|I estimators informative for the effect of interest, β_o. The PCXI|I case is thus another example in which a function of two inconsistent estimators, the CXR|I estimators of γ_x' and π_o from structural equations *(3)* and *(4′)*, is itself a consistent estimator for the effect of interest, β_o. As in the PXI case, we may have γ_x equal to zero, so that in S_{16b}, Z is not required to cause X. When $\gamma_x = 0$, Z acts as a "pure predictive proxy" for U_z.

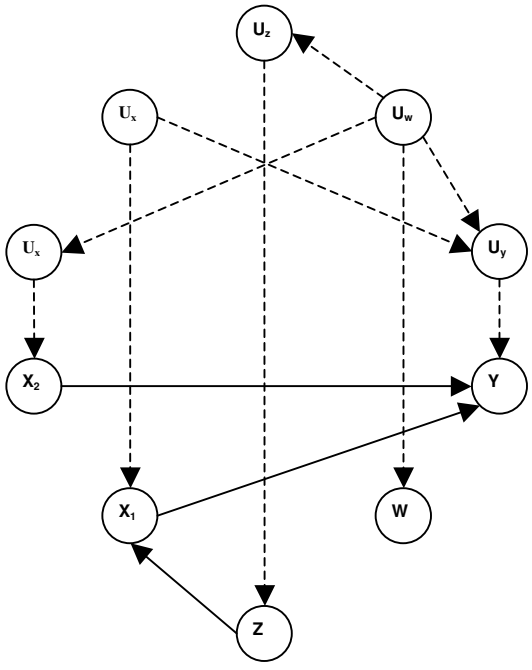
4.2.2 Conditional and Conditioning Instruments: CXIR|I

It can happen that the conditioning instruments W render only a subvector X_2 of $X \equiv [X_1', X_2']'$, conditionally exogenous. In this case, the methods discussed so far cannot structurally identify $\beta_o \equiv [\beta_1', \beta_2']'$. Nevertheless, β_o can be structurally identified if there is another vector of conditional instruments Z for X_1 that is conditionally exogenous given W. This situation is illustrated in S_{17}:

Let S_{17} be given by:

$$(1)\ W \overset{c}{=} \alpha_w U_w$$
$$(2)\ Z \overset{c}{=} \alpha_z U_z$$
$$(3)\ X_1 \overset{c}{=} \gamma_{x_1} Z + \alpha_{x_1} U_{x_1}$$
$$(4)\ X_2 \overset{c}{=} \alpha_{x_2} U_{x_2}$$
$$(5)\ Y \overset{c}{=} X_1'\beta_1 + X_2'\beta_2 + U_y$$

where $U_{x_1} \perp U_{x_2}$, $U_{x_1} \perp U_y$, $U_{x_2} \perp U_y$, $U_{x_1} \perp U_z$, $U_{x_2} \perp U_z$, $U_{x_1} \perp U_w$, $U_{x_2} \perp U_w$, $U_y \perp U_z$, $U_y \perp U_w$, and $U_z \perp U_w$.



Graph 17 (G_{17})
Conditionally Exogenous Instruments and Regressors Given Conditioning Instruments (CXIR|I)

In S_{17}, conditioning on W alone does not permit the identification of β_o as in Proposition 3.3.1 since X_1 is not conditionally exogenous after conditioning on W. Similarly, the use of Z alone does not permit the identification of β_o since Z is endogenous. However, the joint use of Z and W permits the identification and consistent estimation of β_o since conditioning on W when it is a sufficiently good predictor for U_w renders X_2 conditionally exogenous and Z a vector of conditionally exogenous instruments for X_1. The key conditional independence relationship that holds in S_{17} is:

(CXIR|I) *Conditionally Exogenous Instruments and Regressors given Conditioning Instruments:* $(Z, X_2) \perp U_y \mid W$

In fact, CXIR|I is the special case of CXI|I in which X_2 plays the role of a conditionally exogenous instrument for itself. We refer to $\tilde{Z} = [Z', X_2']'$ as a vector of *conditionally exogenous instruments and regressors given conditioning instruments* and refer to the corresponding EIV method as the CXIR|I method. The key moment condition in the linear separable case is given by:

$$E(\tilde{Z}\, U_y \mid W) = E(\tilde{Z} \mid W) \times E(U_y \mid W) \qquad (M6)$$

Proposition 4.2.2 establishes that β_o is fully identified when such a Z and W are available and stochastic identification holds. The condition $\ell = k_1$ is necessary for stochastic identification.

Proposition 4.2.2 Suppose A.1 and A.2 hold such that: (i) $Y \overset{c}{=} X_1'\beta_1 + X_2'\beta_2 + U_y$, with $X \equiv [X_1', X_2']'$ and $\beta_o \equiv [\beta_1', \beta_2']'$. Suppose further that (ii) there exist random vectors W and Z such that and that $\ell = k_1$; with $\tilde{Z} = [Z', X_2']'$, $E(\tilde{Z}\, X')$, $E(\tilde{Z}\, W')$, $E(WW')$, $E(WX')$, $E(\tilde{Z}\, Y)$, and $E(WY)$ exist and are finite; $E(WW')$ is non-singular and $E(\tilde{Z} \mid W) = E(\tilde{Z}\, W')[E(WW')]^{-1}W$; (iii) $E(\tilde{Z}\, X') - E(\tilde{Z}\, W')[E(WW')]^{-1} E(WX')$ is non-singular; and (iv) CXIR|I: $(Z, X_2) \perp U_y \mid W$ holds.

Then β_o, the average total causal effect of X on Y, is fully identified as

$$\beta_o = \{E(\tilde{Z}\, X') - E(\tilde{Z}\, W')[E(WW')]^{-1}E(WX')\}^{-1}\{E(\tilde{Z}\, Y) - E(\tilde{Z}\, W')[E(WW')]^{-1}E(WY)\}$$

■

Under mild conditions provided in Section 7, the CXIR|I plug-in estimator

$$\hat{\beta}_n^{CXIR|I} \equiv [\tilde{\boldsymbol{Z}}'(\boldsymbol{I} - \boldsymbol{W}(\boldsymbol{W}'\boldsymbol{W})^{-1}\boldsymbol{W}')\boldsymbol{X}]^{-1}[\tilde{\boldsymbol{Z}}'(\boldsymbol{I} - \boldsymbol{W}(\boldsymbol{W}'\boldsymbol{W})^{-1}\boldsymbol{W}')\boldsymbol{Y}]$$

is a consistent and asymptotically normal estimator for β_o.

4.2.3 Conditional and Conditioning Instruments: CXI|RI

A generalization of the CXI|R method occurs when CXI|R fails but conditioning instruments W, together with regressors X, render the extended instruments Z conditionally exogenous, as in S_{18}.

Let S_{18} be given by:

(1) $W \overset{c}{=} \alpha_w U_w$

(2) $X \overset{c}{=} \alpha_x U_x$

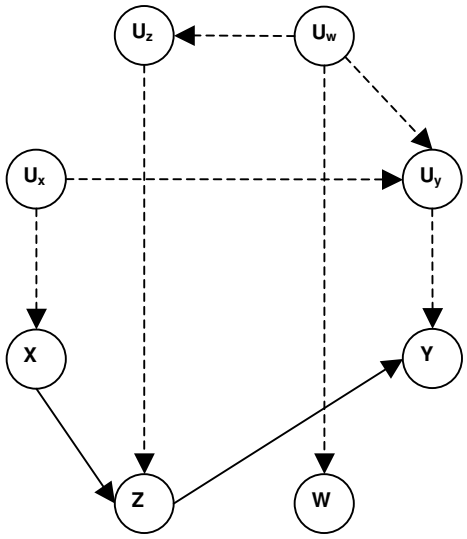
(3) $Z \overset{c}{=} \gamma_z X + \alpha_z U_z$

(4) $Y \overset{c}{=} Z'\delta_o + U_y$

where $U_x \perp U_y$, $U_x \perp U_z$, $U_x \perp U_w$, $U_y \perp U_z$, $U_y \perp U_w$, and $U_z \perp U_w$.

Substituting structural equation *(3)* into structural equation *(4)* with $\beta_o \equiv \gamma_z'\delta_o$, we have:

(4') $Y \overset{c}{=} X'\beta_o + U_z'\alpha_z'\delta_o + U_y$.



Graph 18 (G_{18})
Conditionally Exogenous Instruments given Regressors and Instruments (CXI|RI)

The key conditional independence relationship that holds in S_{18} is:

(CXI|RI) *Conditionally Exogenous Instruments given Regressors and Conditioning Instruments:* $Z \perp U_y \mid (X, W)$

CXI|RI is weaker than CXI|R, its counterpart from Section 4.1. We refer to Z as a vector of *conditionally exogenous instruments given regressors and conditioning instruments* and refer to this EIV method as the CXI|RI method. Now X and W jointly play the role of conditioning instruments for Z. This of course requires that W is a sufficiently good predictor for U_w.

Given linearity and separability, the key moment condition that results from CXR|RI is:

$$E(ZU_y \mid \tilde{W}) = E(Z \mid \tilde{W}) \times E(U_y \mid \tilde{W}), \qquad (M7)$$

where $\tilde{W} = [X', W']'$. Similar to the CXI|R method, the CXI|RI method identifies β_o as the product of the effect of X on Z and that of Z on Y.

Proposition 4.2.3 Suppose A.1 and A.2 hold such that: (i) $Z \overset{c}{=} \gamma_z X + \alpha_z U_z$, $Y \overset{c}{=} Z'\delta_o + U_y$, where $E(XX')$, $E(XZ')$, $E(ZZ')$, and $E(ZY)$ exist and are finite. Suppose further that (ii) there exists a random vector W such that with $\tilde{W} = [X', W']'$, $E(Z\tilde{W}')$, $E(\tilde{W}\tilde{W}')$, and $E(\tilde{W}Y)$ exist and are finite; $E(\tilde{W}\tilde{W}')$ is non-singular and $E(Z \mid \tilde{W}) = E(Z\tilde{W}')[E(\tilde{W}\tilde{W}')]^{-1}\tilde{W}$; (iii) (a) $E(XX')$ and (b) $\{E(ZZ') - E(Z\tilde{W}')[E(\tilde{W}\tilde{W}')]^{-1} E(\tilde{W}Z')\}$ are non-singular; and (iv) (a) XR: $X \perp U_z$ holds and (b) CXI|RI: $Z \perp U_y \mid \tilde{W}$ holds.

Then $\beta_o \equiv \gamma_z'\delta_o$, the average total causal effect of X on Y, is fully identified as:

$$\beta_o = [E(XX')]^{-1}E(XZ') \times$$
$$\{E(ZZ') - E(Z\tilde{W}')[E(\tilde{W}\,\tilde{W}')]^{-1}E(\tilde{W}\,Z')\}^{-1}\{E(ZY) - E(Z\tilde{W}')[\,E(\tilde{W}\,\tilde{W}')]^{-1}E(\tilde{W}\,Y)\}$$

■

Here again, none of the previous EIV methods identify β_o, since their required conditional independence relationships do not hold.

A key feature of S_{18} is that $X \perp U_z$. It should now be clear that this independence relationship could in turn be relaxed to a conditional independence relationship, such as CXR|I: $X \perp U_z \mid W_1$, with W_1 a suitable vector of conditioning instruments. This follows because just as for CXI|R, the effect of X on Y is identified as the product of the effects of X on Z and of Z on Y, given the conditional independence relationships and causal structures ensuring that these two effects are identified, and provided that the effect of X on Y is fully mediated by Z. Pearl (1995, 2000) provides graphical criteria for structural identification of effects of interest to obtain in such a manner via his "front door" method.

Section 7 gives straightforward conditions under which the plug-in estimator $\hat{\beta}^{CXI|RI}$ is a consistent and asymptotically normal estimator for β_o, where:

$$\hat{\beta}_n^{CXI|RI} \equiv [(\boldsymbol{X}'\boldsymbol{X})]^{-1}(\boldsymbol{X}'\boldsymbol{Z}) \times [\boldsymbol{Z}'(\boldsymbol{I} - \tilde{\boldsymbol{W}}\,(\tilde{\boldsymbol{W}}'\,\tilde{\boldsymbol{W}}\,)^{-1}\tilde{\boldsymbol{W}}')\boldsymbol{Z}]^{-1}[\boldsymbol{Z}'(\boldsymbol{I} - \tilde{\boldsymbol{W}}\,(\tilde{\boldsymbol{W}}'\,\tilde{\boldsymbol{W}}\,)^{-1}\tilde{\boldsymbol{W}}')\boldsymbol{Y}]$$

$$\equiv \hat{\gamma}_n^{XR}{}' \hat{\delta}_n^{CXR|I}.$$

In writing this last expression, we engage in a slight abuse of notation, as we do not make explicit the use of $\tilde{\boldsymbol{W}}$ in the CXR|I estimator $\hat{\delta}_n^{CXR|I}$.

4.2.4 Further Comments on Double Extended Instrumental Variables

Like the single EIV case, conditions of the form $Z \perp U_y \mid (Y, W)$ do not permit structural identification of β_o. To see this, let $\tilde{W} \equiv [W', Y']'$, and proceed as in Section 4.1.3. This yields the expression

$$\{E(ZY) - E(Z\tilde{W}')[E(\tilde{W}\,\tilde{W}')]^{-1}E(\tilde{W}\,Y)\} - \{E(ZX') - E(Z\tilde{W}')[E(\tilde{W}\,\tilde{W}')]^{-1}E(\tilde{W}\,X')\}\,\beta_o$$

$$= 0.$$

The first term on the right above is identically zero, however, as it represents the covariance between Y and the residuals of the regression of Z on Y and W. Just as in Section 4.1.3, this provides no useful information for structurally identifying β_o.

The double EIV methods CXI|I, CXIR|I, and CXI|RI, together with the single EIV methods of Sections 3.1, 3.2, and 4.1 thus provide a basis for all EIV methods discussed so far. In fact XR, XI, CXR|I, CXI|I, and CXRI|I constitute an exhaustive set of

"primitive" methods, since other EIV methods, such as CXI|R and CXI|RI, identify causal effects as functions of effects identified by use of one or more of these primitives.

5. A Master Theorem for EIV Identification

The results of Sections 3 and 4 provide a variety of means for identifying the effects of potentially endogenous causes on the response of interest via the use of standard or extended instrumental variables. In this section we summarize these results by stating a "master theorem" that either contains our previous identification results as special cases, as for XI or CXR|I, or delivers them as immediate corollaries, as for CXI|R or CXI|RI. Our master theorem provides not just sufficient conditions for identification, but necessary and sufficient conditions.

Theorem 5.1 Suppose A.1 and A.2 hold for a structural system S such that: (i) $Y \overset{c}{=} X'\beta_o + U_y$, where X is $k\times 1$, $k > 0$, and β_o is finite and $k\times 1$. Suppose further that (ii) Z ($\ell \times 1$, $\ell \geq 0$) and W ($m\times 1$, $m \geq 0$) are random vectors determined by S, and let $\tilde{Z}$ and $\tilde{W}$ be $k \times 1$ and $\tilde{m} \times 1$ vectors respectively such that $[\tilde{Z}', \tilde{W}']' = \boldsymbol{A}\,[X', Z', W']'$, for a given $(k + \tilde{m}) \times (k + \ell + m)$ matrix $\boldsymbol{A}$, and that $E(\tilde{Z}X')$, $E(\tilde{Z}\tilde{W}')$, $E(\tilde{W}\tilde{W}')$, $E(\tilde{W}X')$, $E(\tilde{Z}Y)$, $E(\tilde{W}Y)$ exist and are finite; if $\tilde{m} > 0$, suppose $E(\tilde{W}\tilde{W}')$ is non-singular and that $E(\tilde{Z} \mid \tilde{W}) = E(\tilde{Z}\tilde{W}')[E(\tilde{W}\tilde{W}')]^{-1}\tilde{W}$. If $\tilde{m} = 0$, put $[E(\tilde{W}\tilde{W}')]^{-1} = 0$. Then

(a) $E\{[\tilde{Z} - E(\tilde{Z} \mid \tilde{W})]U_y\}$ exists and is finite.

(b) Stochastic identification holds, that is, there exists a unique β^* such that

$$\{E(\tilde{Z}X') - E(\tilde{Z}\tilde{W}')[E(\tilde{W}\tilde{W}')]^{-1}E(\tilde{W}X')\}\,\beta^* - \{E(\tilde{Z}Y) - E(\tilde{Z}\tilde{W}')[E(\tilde{W}\tilde{W}')]^{-1}E(\tilde{W}Y)\} = E\{[\tilde{Z} - E(\tilde{Z} \mid \tilde{W})]U_y\}$$

if and only if $\{E(\tilde{Z}X') - E(\tilde{Z}\tilde{W}')[E(\tilde{W}\tilde{W}')]^{-1}E(\tilde{W}X')\}$ is non-singular.

(c) Structural identification holds, that is β_o satisfies

$$\{E(\tilde{Z}X') - E(\tilde{Z}\tilde{W}')[E(\tilde{W}\tilde{W}')]^{-1}E(\tilde{W}X')\}\,\beta_o - \{E(\tilde{Z}Y) - E(\tilde{Z}\tilde{W}')[E(\tilde{W}\tilde{W}')]^{-1}E(\tilde{W}Y)\} = 0$$

if and only if $E\{[\tilde{Z} - E(\tilde{Z} \mid \tilde{W})]U_y\} = 0$.

(d) The average causal effect β_o is fully identified as

$$\beta_o = \{E(\tilde{Z}X') - E(\tilde{Z}\tilde{W}')[E(\tilde{W}\tilde{W}')]^{-1}E(\tilde{W}X')\}^{-1} \times \{E(\tilde{Z}Y) - E(\tilde{Z}\tilde{W}')[E(\tilde{W}\tilde{W}')]^{-1}E(\tilde{W}Y)\}$$

if and only if stochastic and structural identification jointly hold. ■

If stochastic identification holds but not structural identification, then we have

$$\beta^* = \beta_o + \{E(\tilde{Z}\,X') - E(\tilde{Z}\,\tilde{W}')[E(\tilde{W}\,\tilde{W}')]^{-1}E(\tilde{W}\,X')\}^{-1}\,E\{[\tilde{Z} - E(\tilde{Z} \mid \tilde{W})]U_y\}.$$

This expresses the probability limit β^* of the plug-in EIV estimator

$$\hat{\beta}_n^{EIV} \equiv [\tilde{\boldsymbol{Z}}'(\boldsymbol{I} - \tilde{\boldsymbol{W}}\,(\tilde{\boldsymbol{W}}'\,\tilde{\boldsymbol{W}}\,)^{-1}\tilde{\boldsymbol{W}}')\boldsymbol{X}]^{-1}[\tilde{\boldsymbol{Z}}'(\boldsymbol{I} - \tilde{\boldsymbol{W}}\,(\tilde{\boldsymbol{W}}'\,\tilde{\boldsymbol{W}}\,)^{-1}\tilde{\boldsymbol{W}}')\boldsymbol{Y}]$$

as the true average causal effect, β_o, plus a "causal discrepancy,"

$$\delta^* = \{E(\tilde{Z}\,X') - E(\tilde{Z}\,\tilde{W}')[E(\tilde{W}\,\tilde{W}')]^{-1}E(\tilde{W}\,X')\}^{-1}\,E\{[\tilde{Z} - E(\tilde{Z} \mid \tilde{W})]U_y\}.$$

If structural identification holds but not stochastic identification, then the estimating equations

$$[\tilde{\boldsymbol{Z}}'(\boldsymbol{I} - \tilde{\boldsymbol{W}}\,(\tilde{\boldsymbol{W}}'\,\tilde{\boldsymbol{W}}\,)^{-1}\tilde{\boldsymbol{W}}')\boldsymbol{X}]\,\beta - [\tilde{\boldsymbol{Z}}'(\boldsymbol{I} - \tilde{\boldsymbol{W}}\,(\tilde{\boldsymbol{W}}'\,\tilde{\boldsymbol{W}}\,)^{-1}\tilde{\boldsymbol{W}}')\boldsymbol{Y}] = 0$$

define a set of solutions converging stochastically to a set that contains β_o, but there is insufficient information to identify which element of the set is the true causal effect.

Theorem 5.1 contains as special cases the XR, CXR|I, XI, CXI|I, and CXRI|I methods. For these cases, an exclusion restriction acts to ensure that when Z is present, Z causes Y only via X. It is straightforward to verify that conditional independence ($\tilde{Z} \perp U_y \mid \tilde{W}$), conditional mean independence ($E(U_y \mid \tilde{Z}, \tilde{W}) = E(U_y \mid \tilde{W})$), and conditional non-correlation ($E(\tilde{Z}\,U_y \mid \tilde{W}) = E(\tilde{Z} \mid \tilde{W})\,E(U_y \mid \tilde{W})$) each imply the necessary structural identification condition $E\{[\tilde{Z} - E(\tilde{Z} \mid \tilde{W})]U_y\} = 0$. We refer to $\tilde{Z} - E(\tilde{Z} \mid \tilde{W})$ as *derived* standard instruments since they satisfy this moment condition. Thus, the causal structure determines the availability of extended instrumental variables. These, in turn dictate the form of derived standard instruments that satisfy moment conditions supporting estimation of identified causal effects.

The next Corollary provides an extension of Theorem 5.1 to cover cases such as CXI|R and CXI|RI, where direct identification of causal effects of interest is not possible but obtains instead as a function of identifiable effects as in Theorem 5.1.

Corollary 5.2 Suppose A.1 and A.2 hold for a structural system S such that $Y \overset{c}{=} X'\beta_o + U_y$, where X is $k \times 1$, $k > 0$, and β_o is finite and $k \times 1$. For $H > 0$, let $\theta_1, \ldots, \theta_H$, be real-valued vectors of structural coefficients of S, and let $b(\cdot)$ be a known measurable real vector-valued function such that $\beta_o = b(\theta_1, \ldots, \theta_H)$. If $\theta_1, \ldots, \theta_H$ are each fully identified as in Theorem 5.1, then, β_o is fully identified as $b(\theta_1, \ldots, \theta_H)$. ■

This covers the intermediate cause instrument case, in which an exclusion restriction acts to ensure that X causes Y only via Z.

6. Characterization of Structural Identification via Causal Matrices: An Illustration with Single EIV

Causal matrices are a powerful way to characterize the causal structures in which the identification of given causal effects of interest obtains. In particular, in other work (in progress), we provide a procedure to generate *conditional independence matrices* from causal matrices. These matrices characterize the conditional independence relationships that hold among the variables of given system S, conditioning on a given subset of variables in system S. (For the empty set, this gives the *independence matrix*.) Thus, given a causal matrix C_S, by inspecting the associated conditional independence matrices it is straightforward to determine whether the necessary exogeneity or conditional exogeneity relationships hold for the identification of given causal effects of interest.

Every causal matrix C_S also has an associated *path matrix* P_S. The k^{th} row and l^{th} column entry of P_S, p_{kl}, takes the value 1 if there is a (V_k, V_l)-path in G_S and equals 0 otherwise. Hence P_S summarizes all direct and indirect causal relationships between the variables of S. The matrices C_S and P_S are related by the operation $P_S = p(C_S)$ where :

$p_{kl} = 1$ if there exists $h > 0$ and a set $\{g_1, \ldots, g_h\}$ with elements in $\{1, \ldots, G\}$ such that $c_{kg_1} \times \ldots \times c_{g_h l} = 1$;

$p_{kl} = c_{kl}$ otherwise.

Thus, p maps an entry $c_{kl} = 1$ in C_S to an entry $p_{kl} = 1$ in P_S, and changes an entry $c_{kl} = 0$ in C_S to an entry $p_{kl} = 1$ in P_S if and only if V_k does not directly cause V_l but there exists a sequence of intermediate variables that mediate an effect of V_k on V_l. These path matrices, in conjunction with their corresponding causal matrices, express concisely the exclusion restrictions necessary for the identification of causal effects.

By examining the conditional independence and path matrices, one can determine whether structural identification of given causal effects of interest obtains. We describe this for the case of the single extended instrumental variable Z and the single cause and response variables X and Y. In work in progress, we discuss the identification of causal effects via causal matrices more generally for single and double EIV methods.

Under A.1 and A.2, the causal matrix for the single EIV case has the form

$$C_S = \left[\begin{array}{c|c} C_{S_1} & C_{S_2} \\ \hline C_{S_3} & C_{S_4} \end{array}\right] =$$

	X	Y	Z	U_x	U_y	U_z
X	0			0	0	0
Y	0	0		0	0	0
Z			0	0	0	0
U_x	1	0	0	0		
U_y	0	1	0		0	
U_z	0	0	1			0

The specified entries in the off-diagonal blocks follow by our conventions, as do the diagonal elements. We also have $c_{21} = 0$ by the acyclicality assumption and the fact the effect of interest is that of *X* on *Y*. Further, the assumed acyclicality of *S* imposes on C_{S_1} three constraints of the form $c_{jk} \times c_{kj} = 0$ and two constraints of the form $c_{jk} \times c_{kl} \times c_{lj} = 0$ for *j, k, l* = 1, 2, 3 as well as three constraints of the form $c_{jk} \times c_{kj} = 0$ and two constraints of the form $c_{jk} \times c_{kl} \times c_{lj} = 0$ on C_{S_4} for *j, k, l* = 4, 5, 6.

Under A1, C_{S_1} admits 9 possible values that we label in relation to *Z* as illustrated by the graphs in Table I.

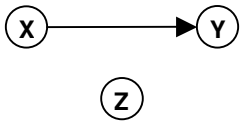
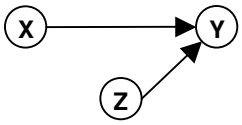
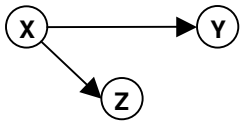
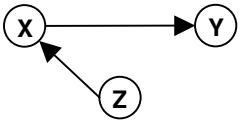
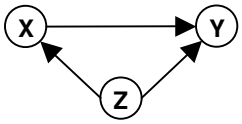
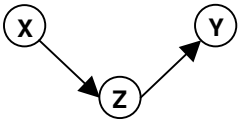
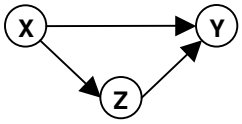
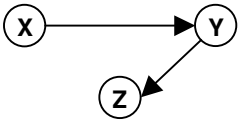
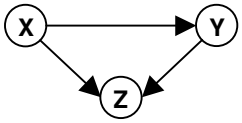
Non Causal, Joint Cause, and Joint Response			
Pre-Cause			
Intermediate Cause			
Post-Response			

Table I
Acyclic Causal Relationships in the Single
Extended Instrumental Variable Case.

Table I displays all possible acyclic causal structures that can relate *X*, *Y*, and a single extended instrument *Z*. These include the pre-cause, intermediate cause, and post-response instrument cases previously discussed. The (1, 1) entry of Table I, the "non-causal" case depicts the potentially valid common cause instrument case, provided that appropriate causal relationships hold among the unobserved variables. Other structures not obeying the exclusion restrictions for identification appear in the second column of the second, third and fourth rows of Table I.

Table I also displays the *joint cause* case where both *X* and *Z* cause *Y* as shown in the (1, 2) entry of Table I and the *joint response* case where *X* causes both *Y* and *Z* as shown in the (1, 3) entry of Table I.

Inspection of C_{S_4} reveals that for every entry of Table I, there are 25 possible acyclic causal structures that can relate the corresponding unobserved variables. Thus in total, C_S

can represent 225 (25×9) potential acyclic causal structures in the single EIV case. The analysis can be simplified by restricting attention to the presence or absence of statistical independence among the unobserved terms, as is standard practice in the structural equations literature. The 25 possible acyclic causal structures among the unobserved variables simplify to 8 possible sets of independence/dependence relationships among U_x, U_y and U_z. Together with the 9 possible entries of Table I, these 8 sets generate 72 possible structural equations systems.

As can be verified, the cases discussed in Sections 3.1, 3.2, and 4.1 are the only ones for which the structural identification of the effect of X on Y obtains in the single EIV case. Specifically, under our assumptions, the values of C_S that have corresponding conditional independence matrices indicating that at least one exogeneity or conditional exogeneity relationship holds, together with corresponding path matrices indicating the appropriate exclusion restrictions, exhaustively characterize all the acyclic causal structures in which the structural identification of the effect of X on Y is possible in the single EIV case. These are precisely the cases presented in Sections 3.1, 3.2, and 4.1.

For example, observe that the second columns of the pre-cause and intermediate-cause categories in Table I violate the exclusion restrictions that Z causes Y only via X in the first case and that X causes Y only via Z in the second case. Hence, the identification of the effect of X on Y is not possible in these cases, even when the appropriate exogeneity or conditional exogeneity conditions hold.

7. Asymptotic Properties of EIV Estimators

Plug-in EIV estimators for causal coefficients identified by Theorem 5.1 have the form

$$\hat{\beta}_n^{EIV} = [\tilde{\boldsymbol{Z}}'(\boldsymbol{I} - \tilde{\boldsymbol{W}}(\tilde{\boldsymbol{W}}'\tilde{\boldsymbol{W}})^{-1}\tilde{\boldsymbol{W}}')\boldsymbol{X}]^{-1}[\tilde{\boldsymbol{Z}}'(\boldsymbol{I} - \tilde{\boldsymbol{W}}(\tilde{\boldsymbol{W}}'\tilde{\boldsymbol{W}})^{-1}\tilde{\boldsymbol{W}}')\boldsymbol{Y}] .$$

Standard arguments easily yield an asymptotic normality result for this estimator.

Theorem 7.1 Suppose the conditions of Theorem 5.1 hold and that

(i) $\tilde{\boldsymbol{Z}}'(\boldsymbol{I} - \tilde{\boldsymbol{W}}(\tilde{\boldsymbol{W}}'\tilde{\boldsymbol{W}})^{-1}\tilde{\boldsymbol{W}}')\boldsymbol{X}/n \xrightarrow{p} Q \equiv E(\tilde{Z}X') - E(\tilde{Z}\tilde{W}')[E(\tilde{W}\tilde{W}')]^{-1}E(\tilde{W}X')$;

(ii) $n^{-1/2}\sum_{i=1}^{n}[\tilde{Z}_i - E(\tilde{Z}_i|\tilde{W}_i)]U_{y,i} \xrightarrow{d} N(0, V)$, where V is finite and positive definite.

Then $n^{1/2}(\hat{\beta}_n^{EIV} - \beta_o) \xrightarrow{d} N(0, Q^{-1}VQ'^{-1})$. ■

Plug-in EIV estimators for average causal effects identified by Corollary 5.2 have the form

$$\hat{\beta}_n^{EIV} = b(\hat{\theta}_n^{EIV}),$$

where $\hat{\theta}_n^{EIV} = (\hat{\theta}_{1,n}^{EIV\prime}, \ldots, \hat{\theta}_{H,n}^{EIV\prime})'$ is a vector of plug-in EIV estimators of the form covered by Theorem 7.1. To state a formal result, let

$$\hat{\theta}_{h,n}^{EIV} \equiv [\tilde{\boldsymbol{Z}}_h'(\boldsymbol{I} - \tilde{\boldsymbol{W}}_h(\tilde{\boldsymbol{W}}_h'\tilde{\boldsymbol{W}}_h)^{-1}\tilde{\boldsymbol{W}}_h')\boldsymbol{X}_h]^{-1}[\tilde{\boldsymbol{Z}}_h'(\boldsymbol{I} - \tilde{\boldsymbol{W}}_h(\tilde{\boldsymbol{W}}_h'\tilde{\boldsymbol{W}}_h)^{-1}\tilde{\boldsymbol{W}}_h')\boldsymbol{Y}_h] \quad h = 1, \ldots, H,$$

$$\zeta_{h,i} \equiv [\tilde{Z}_{h,i} - E(\tilde{Z}_{h,i} | \tilde{W}_{h,i})]\, U_{y_h,i} \qquad i = 1, \ldots, n;\ h = 1, \ldots, H,$$

and put $\zeta_i \equiv (\zeta_{1,i}', \ldots, \zeta_{H,i}')'$.

Theorem 7.2 Suppose the conditions of Corollary 5.2 hold with $\theta_o \equiv (\theta_1', \ldots, \theta_H')'$, and suppose further that

(i) $\tilde{\boldsymbol{Z}}_h'(\boldsymbol{I} - \tilde{\boldsymbol{W}}_h(\tilde{\boldsymbol{W}}_h'\tilde{\boldsymbol{W}}_h)^{-1}\tilde{\boldsymbol{W}}_h')\boldsymbol{X}_h / n \xrightarrow{p} Q_h \equiv E(\tilde{Z}_h X_h') - E(\tilde{Z}_h \tilde{W}_h')[E(\tilde{W}_h \tilde{W}_h')]^{-1} E(\tilde{W}_h X_h'),\ h = 1, \ldots, H$;
(ii) $n^{-1/2}\sum_{i=1}^{n} \zeta_i \xrightarrow{d} N(0, V)$, where V is finite and positive definite.

Then $n^{1/2}(\hat{\theta}_n^{EIV} - \theta_o) \xrightarrow{d} N(0, Q^{-1}V Q'^{-1})$, where $Q = \text{diag}(Q_1, \ldots, Q_H)$.

Suppose further that b is continuously differentiable at θ_o such that $\nabla b(\theta_o)$ (the gradient of b at θ_o) has full column rank. Then with $\hat{\beta}_n^{EIV} \equiv b(\hat{\theta}_n^{EIV})$ and $\beta_o \equiv b(\theta_o)$,

$$n^{1/2}(\hat{\beta}_n^{EIV} - \beta_o) \xrightarrow{d} N(0, \nabla b(\theta_o)' Q^{-1} V Q'^{-1} \nabla b(\theta_o)). \qquad \blacksquare$$

White (2001, ch. 3, 5) gives straightforward primitive conditions ensuring hypotheses (i) (law of large numbers) and (ii) (central limit theorem) of Theorems 7.1 and 7.2.

These plug-in estimators are straightforward to compute, and their asymptotic covariance matrices can be robustly estimated in the usual way under mild conditions (e.g., as in White, 2001, ch. 6). Nevertheless, they are not necessarily asymptotically efficient. Efficiency arises from optimally choosing the extended instruments in a manner somewhat similar to the way in which optimal instruments are chosen in the standard IV framework. Just as GLS-like corrections for conditional heteroskedasticity may be involved in obtaining the optimal instruments in the standard case, such corrections will also play a role in the EIV case. We leave the analysis of the choice of optimal EIV to subsequent research.

8. Conclusion

Building on the structural equations, treatment effects, and machine learning literatures, we utilize the settable system framework of White (2006) and White and Chalak (2006a) to present an explicit and rigorous framework that permits the identification and estimation of causal effects in observational studies with the aid of *extended instrumental*

variables (EIV). EIV methods make use of variables that are not necessarily "valid" instrumental variables in the traditional sense, but that emerge from a given causal structure to enable the recovery of causal effects of interest. In particular, we analyze *single* and *double* extended instrumental variables methods. In the single EIV case, we demonstrate how the use of a single vector of *unconditional*, *conditional*, or *conditioning* EIV permits the identification of causal effects of potentially endogenous causes on the response of interest. In particular, we analyze the *exogenous regressors* (XR), *exogenous instruments* (XI), *conditionally exogenous regressors given conditioning instruments* (CXR|I), and *conditionally exogenous instruments given regressors* (CXI|R) methods. In the XI method, we discuss and provide a causal explanation for two subcategories: the *observed exogenous instruments* (OXI) and the *proxies for unobserved exogenous instruments* (PXI), thereby extending previous causal interpretations of IV methods, such as that of Angrist, Imbens and Rubin (1996.) Our framework also explains the failure of the XI method in the standard irrelevant, invalid, and under-identified cases. In the *double* EIV case, we demonstrate how the joint use of conditional and conditioning EIV permits the identification of causal effects of interest. In particular, we analyze the *conditionally exogenous instruments given conditioning instruments* (CXI|I), the *conditionally exogenous instruments and regressors given conditioning instruments* (CXIR|I), and the *conditionally exogenous instruments given regressors and conditioning instruments* (CXI|RI) methods. We state a master theorem giving necessary and sufficient conditions for the identification of causal effects by means of extended instrumental variables methods and provide straightforward high-level conditions ensuring consistency and asymptotic normality for EIV plug-in estimators of the effects of interest.

By making use of *causal matrices*, *path matrices*, and *conditional independence matrices* it is possible to characterize the cases where the structural identification of causal effects of interest obtains. We illustrate such procedures in the single EIV case, demonstrating that the XR, XI, CXR|I, and CXI|R methods exhaust the single EIV methods capable of structurally identifying causal effects. Work in progress analyzes a procedure for generating conditional independence matrices from causal matrices and establishing identification results for EIV methods more generally.

Here, we consider the identification of causal effects given causal structures specified *a priori*. In that same work in progress, we provide procedures for generating the class of causal matrices that are in agreement with a collection of given (observed) conditional independence matrices. This yields methods for suggesting or ruling out potential causal structures. We propose methods for causal inference based on those results and the identification results provided here.

Future work will analyze the asymptotically efficient choice of EIV in the linear separable case. In other work (White and Chalak, 2006a, 2006b), we analyze nonparametric identification and estimation of general causal effects, relaxing A.2 to the non-separable case. White (2006) and White and Chalak (2006b) provide several tests for conditional exogeneity. Other work in progress extends these tests and proposes new tests for use with EIV methods.

Throughout this paper, we have provided examples of the use of EIV methods relevant to the labor economics and policy evaluation literatures. Our hope is that these methods will prove broadly helpful in empirical applications focused on modeling, understanding, and measuring causal effects of interest.

Mathematical Appendix

Proof of Proposition 3.1.1 From (iii), $E(XU_y) = 0$. From (i), $U_y = Y - X'\beta_o$. Substituting this into $E(XU_y) = 0$ gives $E(XY) - E(XX')\beta_o = 0$. From (ii), $E(XX')$ is non-singular. Thus β_o is fully identified as $\beta_o = [E(XX')]^{-1}\,[E(XY)]$ ■

Proof of Proposition 3.2.1 Analogous to 3.1.1, *mutatis mutandis*. ■

Proof of Proposition 4.1.1 From (iii), $E(XU_y \mid W) = E(X \mid W)\, E(U_y \mid W)$. Equivalently,

$$E([X - E(X \mid W)]\, U_y \mid W) = 0.$$

From (ii), $E(WW')$ is non-singular and $E(X \mid W) = E(XW')[E(WW')]^{-1} W$, so

$$E(\,[X - E(XW')[E(WW')]^{-1}W]\, U_y \mid W) = 0,$$

By the law of iterated expectations

$$E(\,[X - E(XW')[E(WW')]^{-1}W\,]\, U_y) = 0.$$

From (i), $U_y = Y - X'\beta_o$. Substituting this gives:

$$E([X - E(XW')[E(WW')]^{-1}W]\,[Y - X'\beta_o]\,) = 0$$

or

$$\{E(XX') - E(XW')[E(WW')]^{-1}\,E(WX')\}\,\beta_o = E(XY) - E(XW')[E(WW')]^{-1}E(WY).$$

By (iii), $\{E(XX') - E(XW')[E(WW')]^{-1}\,E(WX')\}$ is non-singular. Thus β_o is fully identified as

$$\beta_o = \{E(XX') - E(XW')[E(WW')]^{-1}\,E(WX')\}^{-1}\,\{E(XY) - E(XW')[E(WW')]^{-1}E(WY)\}$$ ■

Proof of Proposition 4.1.2 From (iii)(a), $E(XU_z) = 0$. From (i), $\alpha_z U_z = Z - \gamma_z X$, and from (ii)(a) $E(XX')$ is non-singular. Proposition 3.1.1 thus ensures that γ_z' is fully identified as $\gamma_z' = [E(XX')]^{-1}E(XZ')$. Similarly, from (iii)(b), $E(ZU_y \mid X) = E(Z \mid X) \times E(U_y \mid X)$; from (i), $U_y = Y - Z'\delta_o$; and from (ii)(b), $\{E(ZZ') - E(ZX')[E(XX')]^{-1}\,E(X\,Z')\}$ is non-singular. Since we also have $E(Z \mid X) = E(ZX')[E(XX')]^{-1}X$, δ_o is fully identified by

Proposition 4.1.1 as $\delta_o = \{E(ZZ') - E(ZX')[E(XX')]^{-1}E(XZ')\}^{-1} \times \{E(ZY) - E(ZX')[E(XX')]^{-1}E(XY)\}$. Since $\beta_o \equiv \gamma_z' \delta_o$, β_o is thus fully identified as:

$$\beta_o = [E(XX')]^{-1}E(XZ') \times \{E(ZZ') - E(ZX')[E(XX')]^{-1}E(XZ')\}^{-1} \times \{E(ZY) - E(ZX')[E(XX')]^{-1}E(XY)\} \blacksquare$$

Proof of Proposition 4.2.1 Analogous to 4.1.1, *mutatis mutandis*. ■

Proof of Proposition 4.2.2 Analogous to 4.2.1, replacing Z with $\tilde{Z}$. ■

Proof of Proposition 4.2.3 Analogous to 4.1.2, *mutatis mutandis*. ■

Proof of Theorem 5.1:

(a) From (i) and (ii),

$$E\{[\tilde{Z} - E(\tilde{Z} \mid \tilde{W})]U_y\}$$

$$= E\{[\tilde{Z} - E(\tilde{Z}\tilde{W}')[E(\tilde{W}\tilde{W}')]^{-1}\tilde{W}](Y - X'\beta_o)\}$$

$$= E(\tilde{Z}Y) - E(\tilde{Z}\tilde{W}')[E(\tilde{W}\tilde{W}')]^{-1}E(\tilde{W}Y)$$

$$- E(\tilde{Z}X')\beta_o + E(\tilde{Z}\tilde{W}')[E(\tilde{W}\tilde{W}')]^{-1}E(\tilde{W}X')\beta_o$$

Since β_o is finite and $E(\tilde{Z}Y)$, $E(\tilde{Z}\tilde{W}')$, $[E(\tilde{W}\tilde{W}')]^{-1}$, $E(\tilde{W}Y)$, $E(\tilde{Z}X')$, and $E(\tilde{W}X')$ exist and are finite, it follows that $E\{[\tilde{Z} - E(\tilde{Z} \mid \tilde{W})]U_y\}$ exists and is finite.

(b) Consider the system of equations

$$\{E(\tilde{Z}X') - E(\tilde{Z}\tilde{W}')[E(\tilde{W}\tilde{W}')]^{-1}E(\tilde{W}X')\}\beta - \{E(\tilde{Z}Y) - E(\tilde{Z}\tilde{W}')[E(\tilde{W}\tilde{W}')]^{-1}E(\tilde{W}Y)\} = E\{[\tilde{Z} - E(\tilde{Z} \mid \tilde{W})]U_y\}.$$

It is well known that this system admits a unique solution β^* if and only if $\{E(\tilde{Z}X') - E(\tilde{Z}\tilde{W}')[E(\tilde{W}\tilde{W}')]^{-1}E(\tilde{W}X')\}$ is non-singular.

(c) The result follows immediately from (a).

(d) If stochastic and structural identification hold, we have that $\{E(\tilde{Z}X') - E(\tilde{Z}\tilde{W}')[E(\tilde{W}\tilde{W}')]^{-1}E(\tilde{W}X')\}$ is non-singular and

$$\{E(\tilde{Z}X') - E(\tilde{Z}\tilde{W}')[E(\tilde{W}\tilde{W}')]^{-1}E(\tilde{W}X')\}\beta_o - \{E(\tilde{Z}Y) - E(\tilde{Z}\tilde{W}')[E(\tilde{W}\tilde{W}')]^{-1}E(\tilde{W}Y)\} = 0.$$

It follows that β_o is then fully identified as

$$\beta_o = \{E(\tilde{Z}\,X') - E(\tilde{Z}\,\tilde{W}')[E(\tilde{W}\,\tilde{W}')]^{-1}E(\tilde{W}\,X')\}^{-1} \times \{E(\tilde{Z}\,Y) - E(\tilde{Z}\,\tilde{W}')[E(\tilde{W}\,\tilde{W}')]^{-1}E(\tilde{W}\,Y)\}.$$

To establish the converse, suppose that either stochastic or structural identification fails. If stochastic identification fails, then the inverse of $E(\tilde{Z}\,X') - E(\tilde{Z}\,\tilde{W}')[E(\tilde{W}\,\tilde{W}')]^{-1}E(\tilde{W}\,X')$ does not exist, so β_o cannot have the form given above. If structural identification fails, then $E\{[\tilde{Z} - E(\tilde{Z} \mid \tilde{W})]U_y\}$ is not zero. By (a), β_o satisfies

$$E\{[\tilde{Z} - E(\tilde{Z} \mid \tilde{W})]U_y\}$$

$$= E(\tilde{Z}\,Y) - E(\tilde{Z}\,\tilde{W}')[E(\tilde{W}\,\tilde{W}')]^{-1}E(\tilde{W}\,Y)$$

$$- \{E(\tilde{Z}\,X') - E(\tilde{Z}\,\tilde{W}')[E(\tilde{W}\,\tilde{W}')]^{-1}E(\tilde{W}\,X')\}\,\beta_o.$$

But this is incompatible with the form given above, and the result follows. ■

Proof of Corollary 5.2 Immediate.■

Proof of Theorem 7.1 The proof follows that of theorem 4.26 of White (2001). ■

Proof of Theorem 7.2 The proof of the first result follows that of theorem 4.26 of White (2001). The second result follows from theorem 4.39(i) of White (2001). ■

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