

# Endogenous Labor Supply, International Trade and Macroeconomic Dynamics (September 2008)

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## Abstract

Models featuring endogenously determined trade patterns have had mild successes in matching the empirical properties of international business cycles. Although these models are "Ricardian" in the sense that countries trade labor services, changes in employment are omitted (labor is supplied inelastically). The effect of foreign booms and recessions on domestic labor markets can therefore not be analyzed. Also, ignoring how labor market outcomes affect the pattern of trade overlooks an important mechanism of business cycle transmission across countries. This paper develops a two-country dynamic stochastic general equilibrium model featuring an endogenously determined trade pattern. Households are permitted to choose how much labor is supplied to the market, allowing for changes in employment over time. The model is capable of generating procyclical employment and positive correlation of employment across countries. The model shows that it is possible to have an increase in employment even during economic downturns due to expansions abroad. Other stylized facts that past models have had difficulty generating (positive correlation of output, consumption and investment across countries and procyclical exports) are also achieved by imposing strong market frictions in international asset markets. Keywords: International Business Cycles, Labor Supply, Imperfect Competition, International Trade. JEL: F12, F16, F21, F41.

## 1 Introduction

Increasing concerns over the business cycle fluctuations of our trading partners has highlighted the modern era of globalization and trade liberalization. Failure to understand how business cycles are transmitted from one country to another leads to problems in making effective policy recommendations. In the mid-1980's, many researchers<sup>1</sup> began extending the newly developed real business cycle models of Kydland and Prescott (1982) and Long

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<sup>1</sup>Dellas (1986), Stockman and Svensson (1987), and Cantor and Mark (1988) in particular.

and Plosser (1983) to multiple countries. The general consensus of this early work was that there are two potential sources of business cycle synchronization: trade and financial linkages (particularly in inputs) and correlated productivity shocks<sup>2</sup>.

Problems arose in simulating realistic business cycles using models that incorporated trade and financial linkages. Mendoza (1991) and Backus, Kehoe and Kydland (1992) proposed two-country dynamic stochastic general equilibrium (DSGE) models featuring complete financial markets (which facilitate international risk sharing), correlated productivity shocks, frictionless trade and capital adjustment costs. Unlike the real economy, consumption among countries in model simulations had high correlation while output had negative or low correlation (known as the consumption-output anomaly)<sup>3</sup>. To a somewhat lesser extent, international correlations between investment, savings and employment were also difficult the match.

Resolving the consumption-output anomaly became the target for the next wave of research. Many authors targeted open financial markets as the source of the problem. Positive productivity shocks in one country tended to increase the returns to capital in that country. Foreign agents would transfer investment dollars to the country where capital earned the highest return, causing a decline in capital and thus output in their own country. The country that was already experiencing an expansion would accept foreign investment dollars and expand even more. The model then predicts negative correlation of output across countries. Baxter and Crucini (1995), Kollmann (1996), Heathcote and Perri (2000), Kose and Yi (2001, 2002, 2005) and Olivero (2004) consider incomplete asset markets to try to retard the effect of capital exodus<sup>4</sup>. Improvements in matching the stylized facts remained modest.

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<sup>2</sup>Canova and Dellas (1993) develop a DSGE including domestic and foreign consumption as well as trade in intermediate goods to explicitly analyze the role of international trade in transmitting and synchronizing cycles. Their model's results suggest that business cycles could become correlated through input trade, depending on the magnitude of trade linkages.

<sup>3</sup>This result occurs essentially from using financial markets to pool risk.

<sup>4</sup>Baxter and Crucini (1995) and Kollmann (1996) included incomplete asset markets. Heathcote and Perri (2000) closed financial markets completely and compared the results to those that occur in complete markets. Kose and Yi (2001, 2002, 2005) develop a three-country model to analyze the impact of trade linkages on cyclical synchronization under both open and closed financial markets. Kehoe and Perri (2002) and Olivero (2004) considered endogenously constrained financial markets by including imperfectly enforceable lending agreements and non-competitive lenders respectively.

Relatively fewer analyses explicitly considered the behavior of factors related to trade linkages (imports, exports, terms of trade and exchange rates). Backus, Kehoe and Kydland (1994) introduced the "j-curve" which describes the shape of the cross-correlation function between the trade balance and the terms of trade<sup>5</sup>. Mendoza (1995), Zimmerman (1999) and Cuñat and Maffezzoli (2002) also focused on matching moments of exchange rates, the trade balance and terms of trade within two-country DSGE models<sup>6</sup>. These models continued to have difficulty in matching stylized facts related to the international economy.

The models described so far were heavily influenced by achievements in macroeconomics. The evolution and determination of the trade pattern had been exogenous. It may have been difficult to achieve realistic results in these frameworks if changes in the trade pattern over time have had an effect on business cycle transmission across countries. A small, but separate sect of the international real business cycle (IRBC) literature developed drawing upon ideas from the international trade literature, allowing the trade pattern to be endogenously determined. In a broad sense, this is done by first endogenizing firm entry, then allowing firms to choose to be exporters. The effect of changing trade patterns on business cycles becomes two-fold: one effect coming from new firms entering the market while the other comes from changes in the composition of exports. Head (2002), Cook (2002) and Ghironi and Melitz (2005), for example, incorporated endogenous firm entry in traded goods markets then analyzed the effect on business cycles across countries.

The Ghironi-Melitz model, in particular, has recently become quite popular. While focusing on deviations from purchasing power parity and the Harrod-Balassa-Samuelson effect, the Ghironi and Melitz framework made progress in reducing the consumption-output anomaly. In addition to being fairly general, the model serves as a bridge between traditional macroeconomic theory and new models of international trade<sup>7</sup>. Although the Ghironi-Melitz framework is able to match many stylized facts, their model oversimplifies the labor market

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<sup>5</sup>The trade balance tends to be negatively correlated with current and future movements in the terms of trade while positively correlated with past movements of terms of trade. Backus, Kehoe and Kydland (1994) produced a DSGE model that replicates this property.

<sup>6</sup>Mendoza (1995) and Zimmerman (1999) included exogenous terms of trade shocks and exchange rate shocks respectively. These models differ from earlier research in the source of shocks, but relate to Backus, Kehoe and Kydland's interest in the properties of the international market.

<sup>7</sup>The model is based in new trade theory, featuring non-competitive markets incorporating intraindustry trade.

and bypasses a discussion of international employment. With this feature omitted, we cannot predict how downturns in the economies of our trading partners affect our employment rate. Further, since the Ghironi-Melitz model is "Ricardian" in spirit<sup>8</sup>, the impact of changes in employment on the trade pattern can change our understanding of international comovement.

In this paper, I construct a more developed labor market within the Ghironi-Melitz framework. I allow households to endogenously determine the amount of labor hours to supply to the market by incorporating leisure into the agent's instantaneous utility function. Households choose how much to consume, how much to work, how many bonds (both domestic and foreign bonds) to purchase and how many shares of a mutual fund<sup>9</sup> to buy. Bond purchases are subject to quadratic adjustment costs which are rebated to consumers in lump sum. A labor supply equation along with three Euler equations (one for domestic bonds, one for foreign bonds and one for mutual fund shares) are constructed from the first order conditions of the household's problem.

On the producer's side of the model, monopolistically competitive producers manufacture unique intermediate goods used in the production of aggregate consumption. In each period, an unbounded mass of potential producers exist. After paying a fixed entry cost, firms that wish to enter the market receive a firm-specific productivity draw. Based on the productivity draw, firms choose how much to produce for the domestic market, how much to produce for export, and the prices they charge in each market<sup>10</sup>. Exporters are subject to iceberg transportation costs and a fixed export fee paid each period. Firms remain in the market until hit with an exogenous "death shock". Firms do not know their productivity draw prior to entering the market, so the decision to enter is based on expectations of their future profit stream constructed using average prices (Melitz (2003)).

A series of non-linear market clearing conditions and definitions characterize the model which is then linearized around a symmetric steady state. Business cycles in the model are driven by aggregate productivity shocks. After specifying the shock process, the model is calibrated with reasonable parameter values and solved using the brute force method of

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<sup>8</sup>In the Ghironi-Melitz model, countries trade intermediate products for which labor is the only input. Effectively, the countries are trading labor services.

<sup>9</sup>The mutual fund distributes any firm profits to households in the form of dividends.

<sup>10</sup>The existence of fixed costs supports the assumption that each firm produces a unique variety.

undetermined coefficients developed by Uhlig (1999). The model is simulated first under the assumption of inelastic labor supply. Both low and high bond adjustment costs are considered to capture the benefits that increased asset market frictions have on international business cycle correlation. Next, the model is simulated under the assumption of endogenously determined labor supply.

The model succeeds in simulating more realistic output volatility, procyclical employment and positive international correlation of employment when labor supply is endogenous. When there is a positive shock to aggregate productivity, output and consumption increase as does the demand for labor as new firms enter the market. Increases in consumption, however, reduce the amount of labor services the households are willing to supply. The reduction in labor supply, however, does not outweigh the increase in labor demand, leading to higher wages and more employment in equilibrium (procyclical employment). Output increases further as employment expands (high output volatility). When bond adjustments induce an exodus of investment and an economic contraction in the other country, foreign workers end up supplying more labor in equilibrium (positive international correlation of employment). Domestic expansions result in an increase in employment as do foreign upswings (although the economy at home may be contracting).

The model also produces business cycle comovement and procyclical exports when stronger asset market frictions are imposed. High bond adjustment costs prevent a positive productivity shock in one country from causing an exodus of investment in the other, since foreign households become less willing to purchase domestic bonds. The international correlations of output, consumption and investment are driven by underlying correlation of the technological innovations. Stronger asset market frictions do not distort the improvements made by incorporating endogenous labor supply.

The remainder of the paper is as follows. First, the data of international business cycles will be presented. The model will then be discussed and a series of numerical experiments will be suggested. Then the results will be analyzed. Finally, the paper concludes.

## 2 Stylized Facts

We can organize the data on international comovement into three categories. First, we want to look at internal business cycles ("intranational" business cycle statistics). Quarterly data for the United States (1957.1-2007.1) is extracted from the International Financial Statistics database maintained by the World Bank. Logs for deflated measures of output (Y), consumption (C), investment (I)<sup>11</sup>, and labor (L)<sup>12</sup> are detrended using the HP filter. These business cycle statistics are reported in table 1. As is commonly found, consumption, investment and labor hours are procyclical. Consumption and labor hours are less volatile than output while investment is more volatile.

Next, we consider data on international business cycle comovement. In order to get an accurate picture, we must use comparable data. To do this, I rely on annual data from the Penn World Tables which is adjusted for changes in purchasing power parity. I construct a "European Aggregate" following Backus, Kehoe and Kydland (1992)<sup>13</sup> from 1970-2004. The statistics that are reported in table 2 suggest positive comovement.

Business cycle comovement, however, proves to be time-dependent. Figure 1 shows how the correlation coefficients described in table 2 change over time by looking at 5-year and 10-year rolling window estimates<sup>14</sup>. Data from the entire sample suggests output is more correlated across countries than consumption (the data used to isolate the consumption-output anomaly). Looking at the 10-year window, we see that output is more correlated across countries than consumption before 1985, but consumption is more correlated than output after 1985. In general, we can say that output correlation and consumption correlation are "close" and generally positive. Investment correlation, however, is low and occasionally negative.

If we believe that trade and financial linkages serve as a medium for business cycle transmission, it is also important to match data from the international market. Table 3 shows

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<sup>11</sup>Data for investment includes gross fixed capital formation plus changes in inventories.

<sup>12</sup>Labor data is generated from civilian employment measures from the OECD database.

<sup>13</sup>The European aggregate consists of Austria, Finland, France, Germany, Italy, Switzerland and the United Kingdom. Annual labor data is taken from the OECD database and omits Finland due to missing data.

<sup>14</sup>Figure 1: At any time,  $t$ , the figure plots the international correlations of output, consumption and investment using data from the preceding  $j$  years, where  $j = 5$  and 10 respectively.

quarterly volatility and cyclicity measures for the U.S. participation in the international market <sup>15</sup>. These measures include imports (IM), exports (EX), the trade balance as a fraction of output (Net Exports/Output = NX/Y), the terms of trade (TOT, measured as the ratio of import prices to export prices) and the real exchange rate index (Q). The data suggests high volatility of quantities (imports and exports) and prices (terms of trade and the real exchange rate). Exports and imports are procyclical, net exports are counter-cyclical and prices are approximately acyclical.

Figure 2 plots various cross-correlation functions for output, imports, exports, terms of trade and the trade balance as a fraction of output. Graph b in figure 2 describes the "j-curve" property put forth by Backus, Kehoe and Kydland (1994). An increase in the terms of trade today is correlated with an increase in the trade balance in the future. The cross-correlation function describes how long it takes the trade balance to adjust to changes in the terms of trade. A worsening of the terms of trade results in an contemporaneous trade deficit as imports become relatively more expensive than exports. However, the trade balance improves over time as exports rise and imports fall in response to the price changes. Graphs d and f show the dynamic correlations of terms of trade and exports and imports respectively (the components of the trade balance). Together, these graphs describe the story behind the "j-curve" property. An increase in the terms of trade today is correlated with a rise in exports in the future. This correlation remains positive for as long as 5 periods. An increase in the terms of trade today is correlated with an initial increase in imports. This positive correlation falls quickly and becomes negative after 3 periods. Thus, we expect the terms of trade today to be positively correlated with the trade balance (EX-IM) in the future as the impact of exports outweigh the impact of imports. Graphs a, c, e and g show cross-correlation functions between output and the variables of the international market. If trade linkages are the assumed medium for business cycle transmission, these should also be replicated by the model.

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<sup>15</sup>The quarterly data is extracted from 1957.1-2007.1 from the International Financial Statistics Database for imports, exports, the terms of trade (index) and the real exchange rate (index).

### 3 The Model

The theoretical model is heavily influenced by Ghironi and Melitz (2005). I diverge, however, from their framework by including endogenously determined labor supply in the household's decision. The model described here considers two countries, home and foreign (denoted by \*). Both countries are large and assumed to be structurally identical. That in mind, I construct the framework for one country (the home country) knowing that a symmetric framework exists for the foreign counterpart.

#### 3.1 The Consumer's Problem

The most relevant changes I make to the original framework are in the household's problem. It is assumed that there exists a representative household that chooses to work, consume and save. A variety of products are available to the consumers. Denote the universe of intermediate input varieties available to both countries as  $\Omega$ . At any time,  $t$ , a subset  $\Omega_t \subseteq \Omega$  are actually available to home consumers at  $t$ .  $\Omega_t$  contains both domestically produced and imported goods. It need not be the case that  $\Omega_t = \Omega_s$  for  $t \neq s$  nor  $\Omega_t = \Omega_t^*$ . Composite consumption,  $C_t$ , is produced using Dixit-Stiglitz technology:

$$C_t = \left( \int_{\omega \in \Omega_t} c_t(\omega)^{(a-1)/a} d\omega \right)^{a/(a-1)} \quad (1)$$

where  $c_t(\omega)$  denotes the quantity of variety  $\omega$  used in the production of the composite, and  $a > 1$  denotes the elasticity of substitution across varieties. The framework suggests that, ceteris paribus, the amount of composite commodity produced is increasing in the number of available varieties (referred to in the literature as "love of variety" (Dixit and Stiglitz (1977))). This set-up allows us to clearly incorporate imperfect competition on the producer's side of the model. Recent advancements in the trade literature with respect to imperfect competition and trade patterns make this formulation desirable<sup>16</sup>.

Denote  $P_t(\omega)$  as the price for variety,  $\omega$ , and  $P_{C_t}$  as the price index for consumption. It

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<sup>16</sup>Helpman and Krugman (1985) illustrate the emergence of this strand of literature.



is straightforward to construct conditional demand equations for individual varieties,  $c_t$ <sup>17</sup>.

$$c_t(\omega) = C_t(P_t(\omega)/P_{Ct})^{-a}$$

We also construct the associated price index:

$$P_{Ct} = \left( \int_{\omega \in \Omega_t} P_t(\omega)^{1-a} d\omega \right)^{1/(1-a)} \quad (2)$$

The household is endowed with 1 unit of time that it can divide between labor,  $l_t$ , and leisure. Labor earns the real wage,  $w_t$ . The household can choose to save by purchasing bonds,  $B_{t+1}$ , which each cost one unit of consumption, but yield  $(1 + r_{t+1})$  units in the next period. Both domestic and foreign bonds ( $B_*$ ) are available to the household and are subject to a quadratic transaction cost which is rebated, lump-sum ( $\Gamma_t$ ), to the household. The household can also purchase shares of a mutual fund,  $x_{t+1}$ , which entitle the owner to a fraction of the profits of the producing firms. Arbitrage prices the shares at the firms expected discounted value ( $N_{ht}\tilde{v}_t$  for new shares,  $N_{dt}\tilde{v}_t$  for old shares). Households are obligated to hold bonds and stocks for only one period before they are resold in asset markets<sup>18</sup>. The consumer's real period budget constraint is then given by:

$$\begin{aligned} C_t + B_{t+1} + Q_t B_{*t+1} - \frac{n}{2}(B_{t+1}^2 + Q_t B_{*t+1}^2) + (N_{ht}\tilde{v}_t)x_{t+1} = \\ w_t l_t + (1 + r_t)B_t + Q_t(1 + r_t^*)B_{*t} + \Gamma_t + x_t(N_{dt}\tilde{v}_t + N_{dt}\tilde{d}_t) \end{aligned}$$

where  $n > 0$  is the scale parameter on bond adjustment costs and the lump-sum rebate  $\Gamma_t = \frac{n}{2}(B_{t+1}^2 + Q_t B_{*t+1}^2)$  in equilibrium.

I will construct two different formulations for the agent's instantaneous utility function. When considering perfectly inelastic labor supply, as in the original Ghironi-Melitz framework, I assume the period utility function is of the form:

$$U_t(C_t, l_t) = \log C_t$$

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<sup>17</sup>We solve a series of cost-minimization problems to construct demand equations and price indices.

<sup>18</sup>All savings instruments are denoted with the time subscript in which they yield a return.

When considering endogenously determined labor supply, the instantaneous utility function takes the form<sup>19</sup>:

$$U_t(C_t, l_t) = \log C_t - \frac{l_t^{1+\lambda}}{1+\lambda}$$

with  $\lambda > 0$ . The agent's maximization problem can then be written as:

$$\begin{aligned} & \max_{\{C_s, l_s, B_{s+1}, B_{*s+1}, x_{s+1}\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \beta^{s-t} [U_t(C_t, l_t)] \quad s.t. \\ & C_s + B_{s+1} + Q_s B_{*s+1} - \frac{n}{2}(B_{s+1}^2 + Q_t B_{*s+1}^2) + (N_{hs} \tilde{v}_s) x_{s+1} = \\ & w_s l_s + (1 + r_s) B_s + Q_s (1 + r_s^*) B_{*s} + \Gamma_s + x_s (N_{ds} \tilde{v}_s + N_{ds} \tilde{d}_s) \end{aligned}$$

The first-order conditions for the consumer's problem generate an equation that guides labor supply in addition to three Euler equations. The labor supply equation is:

$$l_t = C_t^{-1/\lambda} w_t^{1/\lambda} \quad (3)$$

with elasticity of labor supply equal to  $1/\lambda$  when labor supply is endogenous and

$$l_t = 1 \quad (4)$$

when labor is supplied inelastically. The Euler equation for domestic bonds is:

$$C_t^{-1}(1 + nB_{t+1}) = (1 + r_{t+1})\beta E_t C_{t+1}^{-1} \quad (5)$$

Similarly, the Euler equation for foreign bonds is:

$$Q_t C_t^{-1}(1 + nB_{*t+1}) = (1 + r_{*t+1}^*)\beta E_t Q_{t+1} C_{t+1}^{-1} \quad (6)$$

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<sup>19</sup>In their original paper, Ghironi and Melitz use a CRRA utility function:  $U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$  with  $\gamma = 2$ . I have chosen log consumption ( $\gamma = 1$ ) for both cases to ensure balanced growth properties of the model when endogenous labor supply is considered.

The Euler equation for stocks is:

$$\tilde{v}_t = \beta(1 - \delta)E_t\left(\frac{C_{t+1}}{C_t}\right)^{-1}(\tilde{d}_{t+1} + \tilde{v}_{t+1}) \quad (7)$$

### 3.2 The Firm's Problem

We now consider the problem faced by a typical intermediate good producer in the home country. It is assumed that there exists an unbounded mass of firms in the economy that may begin production at any time. As in Ghironi and Melitz (2005), it is assumed that these firms are monopolistically competitive and produce for domestic and foreign markets separately (i.e. markets are segmented). Given the demand for their products derived in the previous section, the firm's problem is as follows:

Step 1: Decide whether or not to enter.

Step 2: Upon entry, choose how much output to produce and what price to set.

Step 3: Death or exit.

In practice, firms will first derive the solutions to step 2 and step 3 before choosing whether or not it's worth it to enter the industry.

Consider a potential domestic producer,  $j$ . If firm  $j$  decides to enter the market, they must pay a one-time entry fee in terms of effective units of labor. Once the entry cost is paid, they receive a firm-specific productivity draw,  $z_{jt}$ . The firm then produces output according to the production function:

$$y_{jt} = Z_t z_j L_{jt}$$

where  $y_{jt}$  denotes the quantity of output firm  $j$  produces,  $Z_t$  denotes an economy-wide technology variable, and  $L_{jt}$  is the quantity of labor firm  $j$  hires in the production of output. The firm will choose how much to produce for the domestic market ( $c_{djt}$ ) and how much to produce for export ( $c_{xjt} = c_{mjt}^*$ ). Firms that choose to export must pay a per-period export fee in terms of effective units of labor. Exports are subject to an iceberg cost,  $\tau \geq 1$ .

Thus, total output for the firm is given by  $y_{jt} = c_{djt} + \tau c_{xjt}$ .

The firm's expenses include a wage bill, a one-time start-up cost, and exporter fees. The wage bill is simply  $w_t L_{jt}$ . Start-up costs are a fixed cost paid in terms of labor once during the period the firm begins production. The "production" of entry is given by  $F_{Et} = Z_t L_{Ejt}$  which suggests a total fixed cost of  $w_t F_{Et} / Z_t$ <sup>20</sup>. Exporter fees are paid each period the firm chooses to export ( $c_{xjt} > 0$ ). The production of the exporter fee is given as  $F_{Xt} = Z_t L_{Xjt}$ . Thus, the per-period exporter cost is  $w_t F_{Xt} / Z_t$ . Dropping the start-up cost, we form the firm's real instantaneous profit maximization problem:

$$\max_{c_{xjt}, c_{djt}} \frac{\Pi_{jt}}{P_{Ct}} = \frac{P_{djt}}{P_{Ct}} c_{djt} + \epsilon_t \frac{P_{xjt}}{P_{Ct}} c_{xjt} - \frac{w_t}{Z_t z_j} (c_{djt} + \tau c_{xjt}) - I_{xt} \frac{w_t F_{Xt}}{Z_t}$$

where  $I_{xt} = 1$  if  $c_{xjt} > 0$ , else  $I_{xt} = 0$ .  $\epsilon_t$  denotes the nominal exchange rate. It is important to note that  $P_{xjt}$  is measured in terms of foreign currency. For any variable, X, we denote *real* prices as  $\rho_X = P_X / P_C$  and substitute the demand functions from the previous section into the firm's problem (as is standard with monopolistic competition) to generate the first order conditions for the firm<sup>21</sup>:

$$c_{djt} = C_t \left( \frac{a}{a-1} \frac{w_t}{Z_t z_j} \right)^{-a}$$

$$c_{xjt} = C_t^* \left( \frac{a}{a-1} \frac{w_t}{Z_t z_j} \frac{\tau}{Q_t} \right)^{-a}$$

where  $Q_t = \epsilon_t P_{Ct}^* / P_{Ct}$  is the real exchange rate. For simplicity, it is assumed that  $\epsilon_t = 1$ .

The demand equations are used to find the optimal real prices charged by the firm:

$$\rho_{djt} = \frac{a}{a-1} \frac{w_t}{Z_t z_j} \tag{8}$$

$$\rho_{xjt} = \frac{a}{a-1} \frac{w_t}{Z_t z_j} \frac{\tau}{Q_t} \tag{9}$$

The prices suggest a constant markup over marginal cost ( $w_t / Z_t z_j$ ).

<sup>20</sup>The presence of a fixed entry fee induces constantly falling average cost, ensuring each firm produces a unique variety.

<sup>21</sup>It is assumed that firms do not observe the impact of price setting on the aggregate price level when solving the firm's maximization problem.

Profits for the firm can be divided into "domestic production profits" and "export profits".

They are given as:

$$d_{dj t} = \frac{1}{a} C_t \rho_{dj t}^{1-a} \quad (10)$$

$$d_{xj t} = \left\{ \begin{array}{l} \frac{1}{a} C_t^* Q_t \rho_{xj t}^{1-a} - \frac{w_t F_{Xt}}{Z_t} \text{ if } c_{xj t} > 0 \\ 0 \text{ if } c_{xj t} = 0 \end{array} \right\} \quad (11)$$

For a continuum of potential firms, there exists some productivity draw,  $z_{xt}$ , such that the firm with that draw earns zero profits whether it exports or not (the exporter cutoff). For this firm,  $\frac{1}{a} C_t^* Q_t \rho_{xj t}^{1-a} = \frac{w_t F_{Xt}}{Z_t}$ .

Without knowing their productivity draw prior to entry, firms form an expectation of their profits before deciding to enter the market. To do this, we employ the "special averages" developed by Melitz (2003). First, it is assumed that the productivity draws follow a Pareto distribution with CDF  $G(z) = 1 - (z_{\min}/z_j)^k$  and PDF  $g(z) = k z_{\min}^k z_j^{-k-1}$  where  $k$  denotes the shape parameter of the distribution. Next, define the special productivity averages as:

$$\begin{aligned} \tilde{z}_d^{1-a} &= \int_{z_{\min}}^{\infty} z_j^{1-a} g(z) dz \\ \tilde{z}_{xt}^{1-a} &= \int_{z_{xt}}^{\infty} z_j^{1-a} \frac{g(z)}{1-G(z_{xt})} dz \end{aligned}$$

Finally, substitute these definitions in constructing average prices:  $\tilde{\rho}_{dt}^{1-a} = \int_{z_{\min}}^{\infty} \rho_{dj t}^{1-a} g(z) dz = \left(\frac{a}{a-1} \frac{w_t}{Z_t \tilde{z}_d}\right)^{1-a}$  and  $\tilde{\rho}_{xt}^{1-a} = \int_{z_{xt}}^{\infty} \rho_{xj t}^{1-a} \frac{g(z)}{1-G(z_{xt})} dz = \left(\frac{a}{a-1} \frac{w_t}{Z_t \tilde{z}_{xt} Q_t} \tau\right)^{1-a}$ . We define  $N_{dt}$  as the number of firms that produce for the domestic market and  $N_{xt}$  as the number of producing firms that produce for export. The model is recast in terms of average prices:

$$\tilde{\rho}_{dt} = \left(\frac{a}{a-1} \frac{w_t}{Z_t \tilde{z}_d}\right) \quad (12)$$

$$\tilde{\rho}_{xt} = \left(\frac{a}{a-1} \frac{w_t}{Z_t \tilde{z}_{xt} Q_t} \tau\right) \quad (13)$$

$$\tilde{d}_{dt} = \frac{1}{a} C_t \tilde{\rho}_{dt}^{1-a} \quad (14)$$

$$\tilde{d}_{xt} = \frac{1}{a} C_t^* Q_t \tilde{\rho}_{xt}^{1-a} - \frac{w_t F_{Xt}}{Z_t} \quad (15)$$

Completing the integral for the special productivity averages suggests that  $\tilde{z}_D = \left(\frac{k}{k+1-a}\right)^{1/(a-1)} z_{\min}$  and  $\tilde{z}_{xt} = \left(\frac{k}{k+1-a}\right)^{1/(a-1)} z_{xt}$  where  $k > a - 1$  for boundedness. Knowing that all existing firms produce for the domestic market, and a fraction of those become exporters, the expected per-period profit for a potential firm, on average, is:

$$\tilde{d}_t = \tilde{d}_{dt} + (1 - G(z_{xt}))\tilde{d}_{xt} \quad (16)$$

There are two important features guiding the expected value of the firm's lifetime profits. The first is a lag in production. A firm that enters in period  $t$  starts producing at period  $t + 1$ . The entering firm, however, is still *counted* as a firm in period  $t$ . The total number of firms that exist at period  $t$ ,  $N_{ht}$ , is given by the number of producing firms that already exist plus the number of new firms,  $N_{et}$ .

$$N_{ht} = N_{dt} + N_{et}$$

Second, firms are subject to an exogenous exit shock. The number of firms that "survive" to produce in period  $t + 1$  is given by:

$$N_{dt+1} = (1 - \delta)N_{ht} = (1 - \delta)(N_{dt} + N_{et}) \quad (17)$$

Therefore, the expected value of the firm's lifetime profit stream,  $\tilde{v}_t$ , is given by:

$$\tilde{v}_t = E_t \sum_{s=t+1}^{\infty} [1 - \delta]^{s-t} \left[ \beta^{s-t} \left(\frac{C_s}{C_t}\right)^{-\gamma} \right] \tilde{d}_s$$

where  $\left[ \beta^{s-t} \left(\frac{C_s}{C_t}\right)^{-\gamma} \right]$  is the stochastic discount factor of the household and  $(1 - \delta)$  is the firm's survival probability. Notice that repetitive forward substitution of the Euler equation for stocks generates the expected value of the firm's lifetime profit stream. Firms continue to enter as long as the discounted value of their profit stream exceeds the cost of entry. Therefore, the entry cutoff is determined by:

$$\tilde{v}_t = w_t F_{Et} / Z_t \quad (18)$$

### 3.3 Market Clearing and Equilibrium

I now turn to important market clearing conditions implied by the model. The most important of these conditions for this discussion is labor market clearing. As determined in the consumer's problem, labor supply is given as  $l_t = C_t^{-1/\lambda} w_t^{1/\lambda}$  when labor supply is endogenous and  $l_t = 1$  when labor is supplied inelastically. Labor demand comes from three sources: production of intermediate inputs, exporter costs and start-up costs. Labor demand for intermediate goods production is derived from the labor requirement curve, the demand equations and average prices. We can calculate total labor demand for production of goods as  $N_{dt} \tilde{d}_{dt} \left(\frac{a-1}{w_t}\right) + N_{xt} \frac{a-1}{w_t} \tilde{d}_{xt} + N_{xt}(a-1)F_{Xt}/Z_t$ . Labor demand from the payment of exporter fees is simply  $N_{xt}F_{Xt}/Z_t$ . Labor demand for the payment of start-up fees is  $N_{Et}F_{Et}/Z_t$ . Total labor demand is then:

$$L_t = \frac{a-1}{w_t} (N_{Dt} \tilde{d}_{dt} + N_{xt} \tilde{d}_{xt}) + (aN_{xt}F_{Xt} + N_{Et}F_{Et})/Z_t \quad (19)$$

The equation that governs equilibrium in the labor market when labor supply is endogenously determined is:

$$C_t^{-1/\lambda} w_t^{1/\lambda} = \frac{a-1}{w_t} (N_{Dt} \tilde{d}_{dt} + N_{xt} \tilde{d}_{xt}) + (aN_{xt}F_{Xt} + N_{Et}F_{Et})/Z_t \quad (20)$$

When labor is supplied inelastically, the labor market clearing condition is:

$$1 = \frac{a-1}{w_t} (N_{Dt} \tilde{d}_{dt} + N_{xt} \tilde{d}_{xt}) + (aN_{xt}F_{Xt} + N_{Et}F_{Et})/Z_t \quad (21)$$

The second market clearing condition ensures market clearing in the mutual fund market (shares of the portfolio sum to unity):

$$x_{t+1} = x_t = 1 \quad (22)$$

The third market clearing condition ensures zero net supply of bonds. The representative

agent in each country is either a borrower or a lender, but not both. If the agent produces bonds to sell, they must be purchased by agent in the foreign country:  $B_{t+1} = -B_{t+1}^*$ . Thus, we generate the zero net supply conditions:

$$\begin{aligned} B_{t+1} + B_{t+1}^* &= 0 \\ B_{*t+1} + B_{*t+1}^* &= 0 \end{aligned} \tag{23}$$

The presence of adjustment costs ensure that the unique steady state bond issuance is zero ( $B = 0$ ).

The final market clearing condition ensures balance of payments. We impose mutual fund market clearing and lump-sum transfers of bond adjustment costs<sup>22</sup>:

$$\begin{aligned} 2(1 + r_{t+1})B_t + 2(1 + r_{t+1}^*)Q_t B_{*t} &= [C_t - Q_t C_t^*] + [N_{et}\tilde{v}_t - Q_t N_{et}^*\tilde{v}_t^*] + 2[B_{t+1} + Q_t B_{*t+1}] \\ &\quad - [w_t l_t - Q_t w_t^* l_t^*] - [N_{dt}\tilde{d}_{dt} - Q_t N_{dt}^*\tilde{d}_{dt}^*] \\ &\quad - [N_{xt}\tilde{d}_{xt} - Q_t N_{xt}^*\tilde{d}_{xt}^*] \end{aligned} \tag{24}$$

The full equation system that characterizes the model is described in the appendix. 29 non-linear equations are generated and then log-linearized around a symmetric steady state (denoted by  $\hat{\cdot}$ )<sup>23</sup>. In addition to these equations, the stochastic processes for the technology innovations must be specified. Productivity shocks are assumed to take the following VAR form:

$$\begin{aligned} \begin{bmatrix} \hat{Z}_t \\ \hat{Z}_t^* \end{bmatrix} &= \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} \hat{Z}_{t-1} \\ \hat{Z}_{t-1}^* \end{bmatrix} + \begin{bmatrix} \xi_{Zt} \\ \xi_{Zt}^* \end{bmatrix} \\ E_t(Z_t) &= E_t^*(Z_t^*) = \bar{Z} \\ \sigma_{\xi_{Zt}} &= \sigma_{\xi_{Zt}^*} = \sigma_{\xi} \\ \rho_{\xi_{Zt}\xi_{Zt}^*} &= \rho_{\xi_Z\xi_Z^*} \end{aligned} \tag{25}$$

<sup>22</sup>A description of how the balance of payments equation is generated is in the appendix.

<sup>23</sup>Since the variable,  $B_t$ , has an expected value of zero in equilibrium, it is linearized with respect to steady state consumption:  $\hat{B}_t = \frac{B_t - 0}{C} = \frac{B_t}{C}$ .



Additional definitions are also required. Following Ghironi and Melitz (2005), income (GDP) is defined as:

$$Y_t = w_t l_t + N_{dt} \tilde{d}_{dt} \quad (26)$$

Total imports is constructed using the average revenue foreign exporters earn from their sales abroad:

$$IM_t = N_{xt}^* C_t \tilde{\rho}_{xt}^{*1-a} \quad (27)$$

Total exports is constructed using the average revenue domestic exporters earn from their sales abroad:

$$EX_t = Q_t N_{xt} C_t^* \tilde{\rho}_{xt}^{*1-a} \quad (28)$$

Terms of trade is defines as the ratio of import prices to export prices:

$$TOT_t = \frac{\tilde{\rho}_{xt}^*}{Q_t \tilde{\rho}_{xt}} \quad (29)$$

Investment is defined as expenditures on new firm entry:

$$I_t = N_{et} \tilde{v}_t \quad (30)$$

In the model, the price of consumption ( $P_{Ct}$ ) is measured as a welfare-based price index following Feenstra(2003). It is thus important to transform this welfare-based index into one which closer matches the price index calculated in the data. To do so, average prices are substituted into the definition of the consumption price index:

$$1 = [N_{dt} \tilde{\rho}_{dt}^{1-a} + N_{xt}^* \tilde{\rho}_{xt}^{*1-a}]^{1/(1-a)}$$

Note that the price index is sensitive to the number of varieties. If we assume that all prices are the same on average,  $\tilde{\rho} = \tilde{P}_t / P_{Ct}$ , we construct:

$$\frac{P_{Ct}}{\tilde{P}_t} = [N_{dt} + N_{xt}^*]^{1/(1-a)} \quad (31)$$

Any variable measured in terms of real consumption,  $X_t$ , is adjusted to this index:  $\tilde{X}_t = \frac{P_{Ct}X_t}{\tilde{P}_t}$ . Further, since the real exchange rate is constructed using the welfare-based price indices ( $Q_t = P_{Ct}^*/P_{Ct}$ ), we construct an adjusted real exchange rate:

$$\tilde{Q} = \tilde{P}_t^*/\tilde{P}_t = Q_t \left[ \frac{N_{dt} + N_{xt}^*}{N_{dt}^* + N_{xt}} \right]^{1/(1-a)} \quad (32)$$

## 4 Numerical Experiments

I start by employing a calibration that most closely matches the original Ghironi and Melitz (2005) framework. Many parameter values appear in past literature and are widely accepted (table 4). The values for the discount factor (0.99 for quarterly simulations, 0.96 for annual simulations) is standard in the literature. The values for the probability of death are calibrated to match data on US job destruction (2.5% quarterly, 10% annually). The elasticity of substitution between intermediate inputs, the shape parameter on the Pareto distribution, the lower bound on the Pareto distribution, the iceberg cost on intermediate inputs and the costs of entry and export are set to match Ghironi and Melitz (2005).

The productivity process is calibrated as<sup>24</sup>:

$$\begin{aligned} \begin{bmatrix} \hat{Z}_t \\ \hat{Z}_t^* \end{bmatrix} &= \begin{bmatrix} 0.99 & 0 \\ 0 & 0.99 \end{bmatrix} \begin{bmatrix} \hat{Z}_{t-1} \\ \hat{Z}_{t-1}^* \end{bmatrix} + \begin{bmatrix} \xi_{Zt} \\ \xi_{Zt}^* \end{bmatrix} \\ \sigma_{\xi_{Zt}} &= \sigma_{\xi_{Zt}^*} = \sigma_{\xi} = 0.00852 \\ \rho_{\xi_{Zt}\xi_{Zt}^*} &= \rho_{\xi_Z\xi_Z^*} = 0.258 \end{aligned} \quad (33)$$

The other model parameters,  $(n, \bar{Z}, \lambda)$ , will be thought of as "free parameters" and will be calibrated under different experiments discussed shortly<sup>25</sup>.

<sup>24</sup>Backus, Kehoe and Kydland (1992) calibrate technology shocks as a VAR process with spillovers:

$$\begin{aligned} \begin{bmatrix} \hat{Z}_t \\ \hat{Z}_t^* \end{bmatrix} &= \begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix} \begin{bmatrix} \hat{Z}_{t-1} \\ \hat{Z}_{t-1}^* \end{bmatrix} + \begin{bmatrix} \xi_{Zt} \\ \xi_{Zt}^* \end{bmatrix} \\ \sigma_{\xi_{Zt}} &= \sigma_{\xi_{Zt}^*} = \sigma_{\xi} = 0.00852 \\ \rho_{\xi_{Zt}\xi_{Zt}^*} &= \rho_{\xi_Z\xi_Z^*} = 0.258 \end{aligned}$$

Baxter and Crucini (1995) note that this specification is not statistically different from a near-unit-root process without spillovers.

<sup>25</sup> $\bar{Z}$  is typically set to 1.

The model is solved numerically using the brute force algorithm developed by Uhlig (1999). Simulations are performed in order to generate a set of statistics to compare to the data. In each experiment, a 50-period model<sup>26</sup> is simulated 200 times by drawing a random vector of innovations. During each simulation, I make the appropriate price adjustments as described above, then apply the HP filter<sup>27</sup>. I calculate each summary statistic (volatility, cyclicity, etc.) for each simulation and then report the average statistics across simulations in the results tables.

## 4.1 The Benchmark Case

To match the original Ghironi and Melitz (2005) framework as closely as possible, I set  $n = 0.0025$  and omit endogenous labor supply from the model. Simulation results are reported in column (2) of table 5. The model fails to predict positively correlated business cycles across countries, procyclical exports, terms of trade volatility and exchange rate volatility in simulations. Figure 3 plots the cross-correlation functions for output, imports, exports the ratio of the trade balance to GDP and the terms of trade. A j-shaped cross-correlation function for the trade balance to output ratio and terms of trade (graph b in figure 3) is not achieved.

### 4.1.1 High Bond Adjustment Costs

**International Business Cycle Statistics** We might believe that the failure of the model to generate realistic business cycle comovement is due to the openness of financial markets, as past literature has suggested. Suppose, for the following discussion, that the home country experiences a positive shock to aggregate productivity. Output and profits for existing firms rise. This drives up the return to mutual fund shares, spurring investment in new firm construction. With larger output and more varieties of firms becoming available, composite consumption rises. At the aggregate level, there is more demand for labor to pay start-up costs and exporter fees (although each individual firm may demand less labor for production when average productivity is higher) which puts upward pressure on wages.

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<sup>26</sup>Short time periods are preferred to prevent trade from collapsing during the simulations.

<sup>27</sup>Since the model exhibits a high degree of persistence, we take out the low-frequency trend using the HP filter.

As the return to mutual fund shares in the home country rises, the return to home bonds does as well via the Euler equation. Foreign agents adjust their asset portfolios in the favor of home bonds which now yield a higher return, reducing investment in new firm construction in the foreign country. Output, wages and consumption decline in the foreign country due to the firm attrition caused by the exodus of investment. Business cycles across countries becomes negatively correlated.

In the international market, the positive productivity shock in the home country leads to an increase in the demand for imports. The number of firms in the foreign country is now lower, each firm facing stronger demand from abroad. Although the home country ultimately imports more, the price of imported goods is driven up (a rise in the value of imports overall). The foreign country has had a contraction resulting in less demand for exports from the home country. However, increased wages in the home country have led to increased prices for home-produced goods. Ultimately, the increase in the price of products exported from the home country and a reduction in exports result in an overall reduction in the value of exports in the model. Exports become counter-cyclical.

Increasing the scale parameter on bond adjustment costs ( $n = 1$ ) would make agents less willing to change their bond market portfolios. The results from this experiment are shown in column (3) of table 5. Consumption and output become more positively correlated across countries. Investment correlation remains low. Exports become procyclical and the volatility of the trade balance declines.

Why business cycles become more positively correlated when bond market adjustment costs increase is quite straight-forward. Foreign agents are no longer so willing to adjust their holdings of bonds issued by the home country when there is a positive shock to home productivity. There is no longer a dramatic fall in foreign firm construction and foreign labor demand. Consumption, output and investment increase in the home country, as does their demand for imported goods. Prices for imported goods rise due to the increased demand, increasing the profitability of foreign exporters. As foreign exporters expand production to meet the demand for their products, foreign labor demand increases driving up foreign wages. Foreign income rises, along with foreign consumption and foreign demand for home's

exports. The wage increase in the foreign country does result in firm attrition. Even though more firms die than enter, each entering firm becomes more profitable (resulting in increased investment). The positive international correlations of output, consumption and investment are further influenced by the underlying correlation of the productivity innovations.

The model predicts that increased demand for home's exports by foreign households and higher export prices (due to higher domestic wages) result in a larger value of exports in equilibrium (procyclical exports). Intuitively, procyclical exports also result from the increased pressure to maintain balanced trade. With high bond adjustment costs, agents are less willing to finance trade deficits by issuing debt. A positive productivity shock in the home country increases output and consumption, leading to an increase in imports. An increase in exports partially offsets the need to issue costly bonds to cover the resulting trade deficit. This results in less volatility in the international market.

**Cross-correlation Functions** As already described, the positive productivity shock in the home country leads to a rise in the value of imports, an increase in the price of imported goods, a reduction in the value of exports and a rise in the price of exported goods when bond adjustment costs are low. Net exports thus declines, as does the trade balance-to-output ratio (since output is increasing). The effect on the terms of trade depends on which is larger, the change in import prices or the change in export prices. The model predicts the increase in export prices outweighs the increase in import prices, resulting in a decline of the terms of trade. Exports and net exports are contemporaneously positively correlated with the terms of trade while imports are negatively correlated.

Due to the lag in production, new firms that are built in the home country do not begin to produce right away. As new firms begin production, the home country starts to import less (since home goods and foreign goods are generally substitutes) and export more. Import prices fall quickly as demand for imported goods contracts while export prices continue to remain high due to persistently higher wages paid in the home country; the terms of trade continues to fall over time. Exports become negatively correlated with the terms of trade while imports become positively correlated in future periods. The trade balance is then initially positively correlated with the terms of trade but becomes negatively correlated over

time. These results are shown in graphs b, d and f of figure 3. The "j-curve" shape that we expect to see in the cross-correlation function between terms of trade and the trade balance-to-output ratio is not captured when bond adjustment costs are low.

Because exports are counter-cyclical when bond adjustment costs are low, the shape of the cross-correlation function between output and exports exhibits a contemporaneous "dip". As exports increase over time, the cross-correlation function begins to rise. Similarly, since imports are procyclical but fall over time, the cross-correlation function between output and exports exhibits a "tent-shape". Both these cross-correlation functions roughly match the shapes seen in the data. The cross-correlation function between imports and exports, however, clearly does not since these two factors move in opposite directions contemporaneously.

The dynamic properties of exports, imports and terms of trade are more subtle when bond adjustment costs are high. There is less firm attrition in the foreign country and less firm construction in the home country when there is a positive shock to home productivity since foreign agents are less willing to reduce firm construction to purchase home bonds. The increase in future exports that occurs because new firms in the home country begin production is not as large as it was when bond adjustment costs were low. The negative correlation between exports and terms of trade in the future is now muted. The underlying correlations of the productivity innovations drive the dynamic correlations of exports, imports and the terms of trade over time, resulting in the "tent-shaped" cross-correlation function for the trade balance to output ratio and terms of trade seen in graph b of figure 4.

The cross-correlation functions between output and exports now exhibits a "tent-shape" since exports are procyclical. The shape of the cross-correlation function between output and imports is virtually unchanged. Now that exports and imports move in the same direction contemporaneously, the cross-correlation function between imports and exports obtained from the simulations more closely matches the data.

## 4.2 Endogenous Labor Supply

I now turn to the model with endogenous labor supply. Previous macroeconomics literature suggests that endogenously determined labor supply has the potential to increase the volatility of output if employment is procyclical. Since the two countries described in the economy essentially trade labor services, changes in the labor market can also affect the pattern of trade, altering the international correlation of business cycles. In the model I have constructed, supply-side changes in the labor market buffers against negatively correlated business cycles, results in positive international correlation of employment and generates more volatile output.

**International Business Cycle Statistics** To ensure well-behaved simulations, I experiment with unit-elastic labor supply ( $\lambda = 1$ ). I first consider low bond adjustment costs,  $n = 0.0025$ . The results are reported in column (4) of table 5. At first glance, the volatility of output is improved. Employment in the model is procyclical and the international correlation of employment is appropriate. Also, comparing the model to the case when labor is inelastically supplied (column (2) of table 5), the negative correlation of output, investment and consumption has been reduced.

We already know the effect of a positive productivity shock on the domestic economy when labor is supplied inelastically: the cost of firm entry and export declines, leading to entry into both the domestic and export market. In the labor market, more labor is demanded to build firms and pay export fees, although individual firms in the economy may require less labor to produce output. In aggregate, the model predicts more labor is demanded in the economy resulting in a higher wage (shown in diagram a of figure 5: labor demand shifts from  $LD_1$  to  $LD_2$ ). Income increases, as does output in equilibrium. More goods are produced, more varieties are produced, and more goods are imported which raises composite consumption.

As previously described, the foreign country enters a contractionary period as investment dollars leave the country. On one hand, foreign demand for labor declines because there is less demand by existing firms for workers (since some of them exit the market) and less demand for workers by start-up firms to pay entry fees (since there is less entry). On the

other hand, a positive productivity shock in the home country increases home's demand for imported goods. Exporter firms in the foreign country demand more labor to expand output to meet demand. The model predicts these two effects nearly offset each other, if not resulting in a modest net decline in labor demand<sup>28</sup>. Foreign wages fall, foreign income and output fall, and foreign consumption falls. Business cycles become negatively correlated across the two countries in the model.

When the labor supply decision is endogenous, the increase in composite consumption from firm entry (more variety) and expansion (more goods produced) in the home country compels households to supply less labor. This is clear from the first order condition governing labor supply:  $l_t = C_t^{-1/\lambda} w_t^{1/\lambda} = w_t/C_t$  for  $\lambda = 1$  (shown in graph b of figure 5: the labor supply curve shifts from  $LS_1$  to  $LS_2$  in response to a positive productivity shock). The boost in employment further increases aggregate output, leading to the higher output volatility seen in the results table.

The positive international correlation of employment is due directly to trade and financial linkages. As previously described, a positive productivity shock in the home country leads to firm attrition in the foreign country due to open asset markets. In foreign labor markets, there is a slight decline in the foreign demand for labor (as seen in graph b of figure 6: labor demand shifts from  $LD_1^*$  to  $LD_2^*$ ). Because the foreign economy has started to contract ( $C_t^*$  is falling), foreign households increase their supply of labor according to the first order condition governing foreign labor supply:  $l_t^* = w_t^*/C_t^*$  (shown in graph b of figure 6: the foreign labor supply curve shifts from  $LS_1^*$  to  $LS_2^*$ ). The slight decline in labor demand is offset by the increase in labor supply, resulting in more employment in the foreign country at lower wages. There is positive international correlation of employment. The model predicts an increase in employment when there is a positive shock to aggregate productivity in either our country or our trading partners. The latter, however, comes with an economic contraction.

Endogenous labor supply also acts as a buffer against the negative business cycle correla-

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<sup>28</sup>This result is deduced from the impulse response function for  $w_t^*$  when there is a positive productivity shock to the home country in the case of inelastic labor supply. Impulse response functions are available upon request.



tion generated by open asset markets. Since the foreign country experiences an increase in employment, there is a less pronounced fall in foreign income and consumption. Comparing the international business cycle comovement with inelastically supplied labor (column (2)) and endogenously determined labor supply (column (4)), we see that business cycles are not as negatively correlated across countries when labor supply is endogenous.

The model exhibits the same poor fit as the benchmark case in terms of international correlations and international market statistics. I increase bond adjustment costs ( $n = 1$ ) to achieve better results (column (5) of table 5). Again, international correlation improves and exports become procyclical. The improvements to output volatility and labor market statistics are not distorted with the increased asset market friction.

**Cross-correlation Functions** I now consider the cross-correlation functions when labor supply is endogenous and bond adjustment costs are low ( $n = 0.0025$ ). When there is a positive productivity shock to the home country, the model with endogenously determined labor supply results in an increase in wages. When labor is supplied inelastically, wages also increased. The price of home's exports rise in both cases. The resulting economic expansion in the home country induces a larger demand for imports and puts upward pressure on import prices while the economic contraction in the foreign country induces a reduction in demand for home's exports regardless of how the labor market is modeled. The cross-correlation functions for the case with endogenous labor supply are nearly identical to those with inelastic labor supply. The same improvements to the cross-correlation functions are made when bond adjustment costs are high ( $n = 1$ )<sup>29</sup>.

## 5 Conclusion

The Ghironi and Melitz (2005) model is capable of capturing many features of international business cycles. It fails, however, to predict positively correlated business cycles across countries, procyclical exports, terms of trade volatility and exchange rate volatility in simulations. By increasing bond adjustment costs, we can generate procyclical exports and positively correlated business cycles across countries at the expense of international market

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<sup>29</sup>The diagrams are omitted from the paper and are available upon request.

volatility. Incorporating endogenous labor supply increases the volatility of output to a realistic level and matches important features of the data regarding employment.

There remains room for additional improvements. The inclusion of additional countries and "common shocks" would likely result in stronger international correlations without having to sacrifice international market volatility. Including capital would provide a more accurate measure of investment, potentially resolving the problem of negatively correlated investment across countries. The inclusion of demand shocks for final goods and a comparison of demand-driven business cycles to the benchmark model may lead to interesting results. These are left for future research.

Table 1 - US Business Cycle Statistics

Volatility (Standard Deviation)	
$\sigma_Y(\%)$	1.54
$\sigma_C/\sigma_Y$	0.75
$\sigma_I/\sigma_Y$	3.41
$\sigma_L/\sigma_Y$	0.61
Cyclicality (Correlation)	
$\rho_{C,Y}$	0.88
$\rho_{I,Y}$	0.90
$\rho_{L,Y}$	0.81

Table 2 - US-European Comovement Statistics

Correlation	
$\rho_{Y_{US},Y_{EU}}$	0.56
$\rho_{C_{US},C_{EU}}$	0.45
$\rho_{I_{US},I_{EU}}$	0.37
$\rho_{L_{UE},L_{EU}}$	0.39

Table 3 - US International Market Statistics

Volatility (Standard Deviation)		Cyclicality (Correlation)	
$\sigma_{IM}/\sigma_Y$	3.31	$\rho_{IM,Y}$	0.60
$\sigma_{EX}/\sigma_Y$	3.74	$\rho_{EX,Y}$	0.28
$\sigma_{NX}/\sigma_Y$	0.40	$\rho_{NX/Y,Y}$	-0.36
$\sigma_{TOT}/\sigma_Y$	1.69	$\rho_{TOT,Y}$	0.07
$\sigma_Q/\sigma_Y$	3.37	$\rho_{Q,Y}$	-0.18

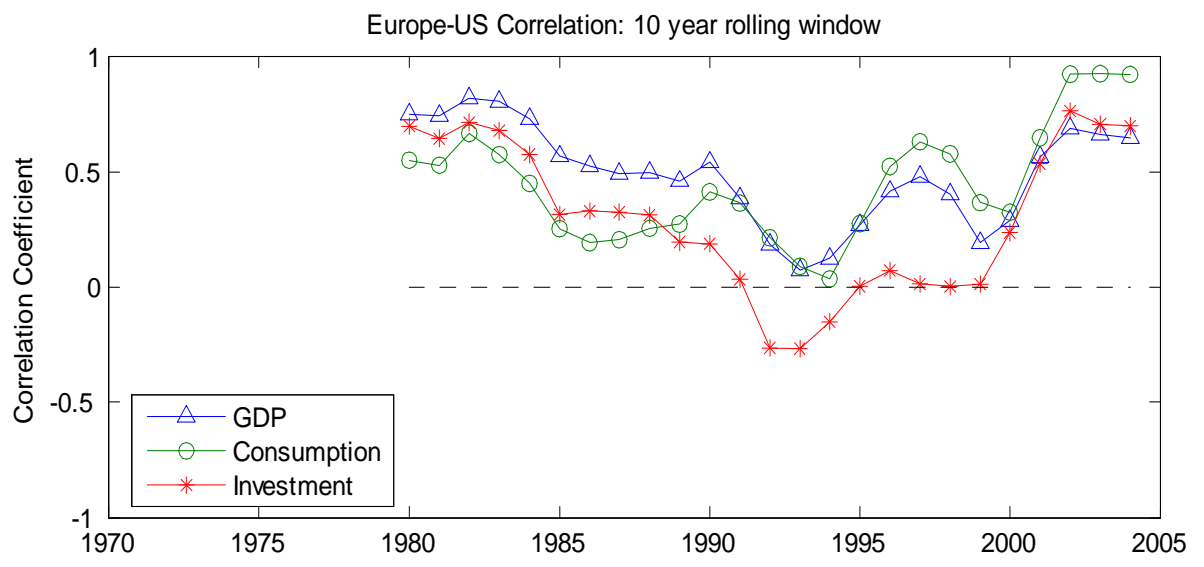
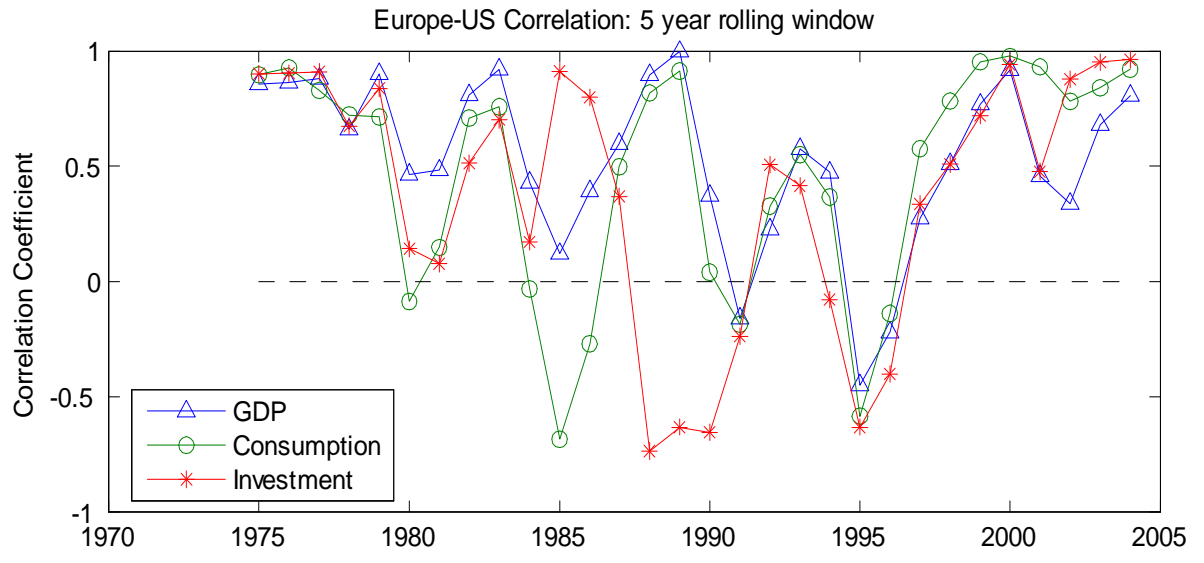


Figure 1 - Rolling window estimates of international correlations

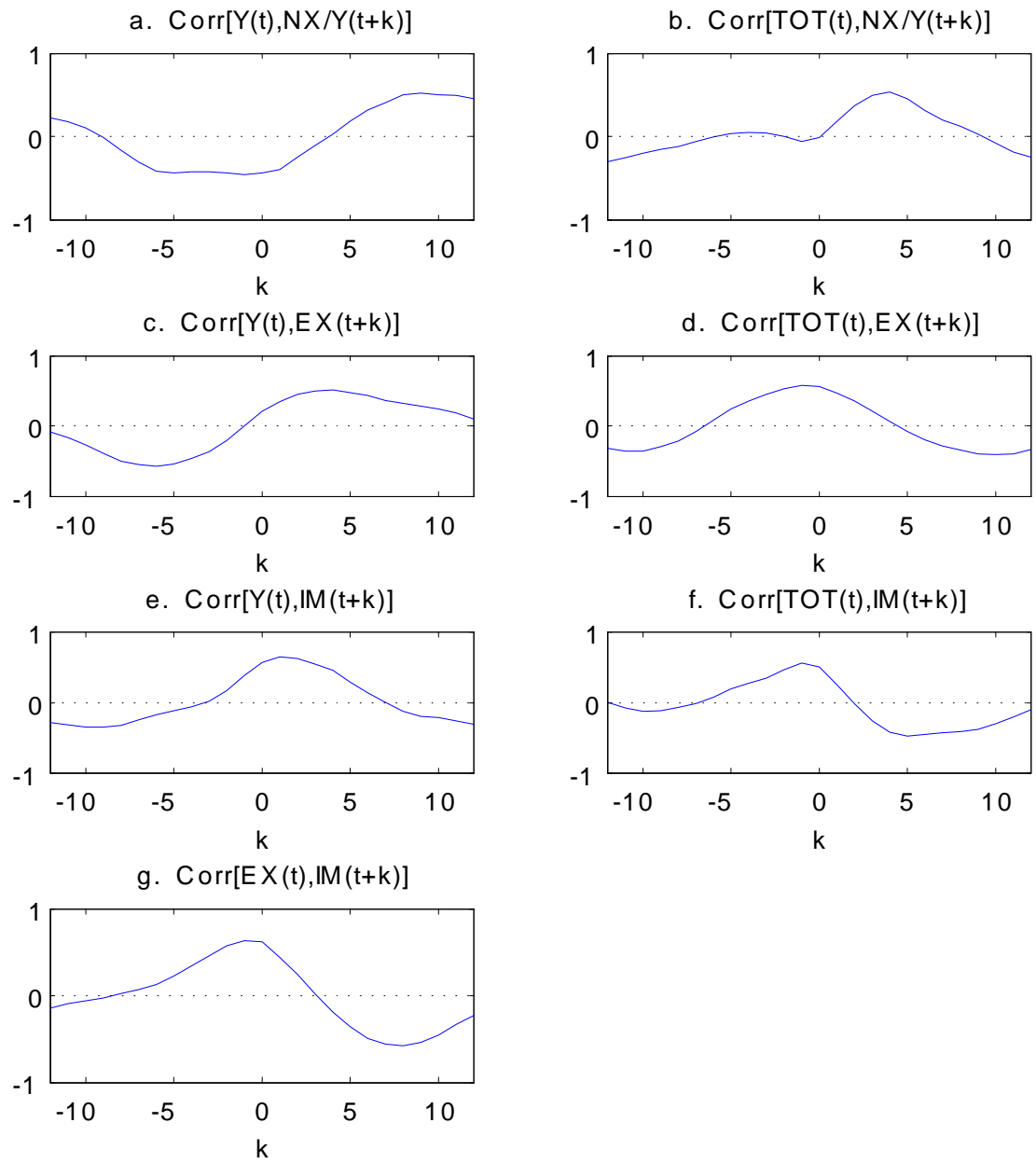


Figure 2 - Cross-correlation functions

Table 4 - Calibrated Parameters

<b>Parameter</b>		<b>Value</b>
$\beta$	Discount factor	0.99, 0.96
$\delta$	Probability of death shock	0.99, 0.96
$\tau$	Iceberg costs associated with intermediate inputs	1.3
$a$	Elasticity of substitution between intermediate inputs	3.8
$k$	Shape parameter on Pareto distribution	3.4
$z_{\min}$	Lower bound on Pareto distribution	1
$\tilde{z}_D$	"Special" average productivity draw	$z_{\min}(\frac{k}{k+1-a})^{1/(a-1)}$
$F_{Et}$	Fixed cost of entry	1
$F_{Xt}$	Fixed exporter costs	$0.235(F_{Et})(\frac{1-\beta(1-\delta)}{\beta(1-\delta)})$

Table 5	(1)	(2)	(3)	(4)	(5)
<b>Intranational Business Cycles</b>					
Volatility (Standard Deviation)					
$\sigma_Y(\%)$	1.54	0.95	0.97	1.23	1.26
$\sigma_C/\sigma_Y$	0.75	0.66	0.63	0.60	0.57
$\sigma_I/\sigma_Y$	3.41	3.85	3.02	4.15	3.43
$\sigma_L/\sigma_Y$	0.61	—	—	0.26	0.27
Cyclicality (Correlation)					
$\rho_{C,Y}$	0.88	0.99	0.99	0.99	0.98
$\rho_{I,Y}$	0.90	0.94	0.98	0.95	0.98
$\rho_{L,Y}$	0.81	—	—	0.98	0.97
<b>International Comovement</b>					
Correlation					
$\rho_{Y,Y^*}$	0.56	-0.01	0.16	0.07	0.15
$\rho_{C,C^*}$	0.45	-0.19	0.12	-0.12	0.09
$\rho_{I,I^*}$	0.37	-0.47	0.10	-0.35	0.11
$\rho_{L,L^*}$	0.39	—	—	0.54	0.25
<b>International Market</b>					
Volatility (Standard Deviation)					
$\sigma_{IM}/\sigma_Y$	3.31	1.10	0.69	0.99	0.62
$\sigma_{EX}/\sigma_Y$	3.74	1.27	0.70	1.11	0.64
$\sigma_{NX}/\sigma_Y$	0.40	0.46	0.18	0.53	0.22
$\sigma_{TOT}/\sigma_Y$	1.69	0.18	0.24	0.17	0.22
$\sigma_Q/\sigma_Y$	3.37	0.14	0.08	0.12	0.08
Cyclicality (Correlation)					
$\rho_{IM,Y}$	0.60	0.89	0.99	0.89	0.98
$\rho_{EX,Y}$	0.28	-0.17	0.27	-0.19	0.24
$\rho_{NX/Y,Y}$	-0.36	-0.58	-0.62	-0.59	-0.60
$\rho_{TOT,Y}$	0.07	-0.48	-0.62	-0.47	-0.60
$\rho_{Q,Y}$	-0.18	-0.61	0.14	-0.62	0.10

(1) Data

(2) Inelastic labor supply,  $n = 0.0025$

(3) Inelastic labor supply,  $n = 1$

(4) Endogenous labor supply,  $n = 0.0025$

(5) Endogenous labor supply,  $n = 1$

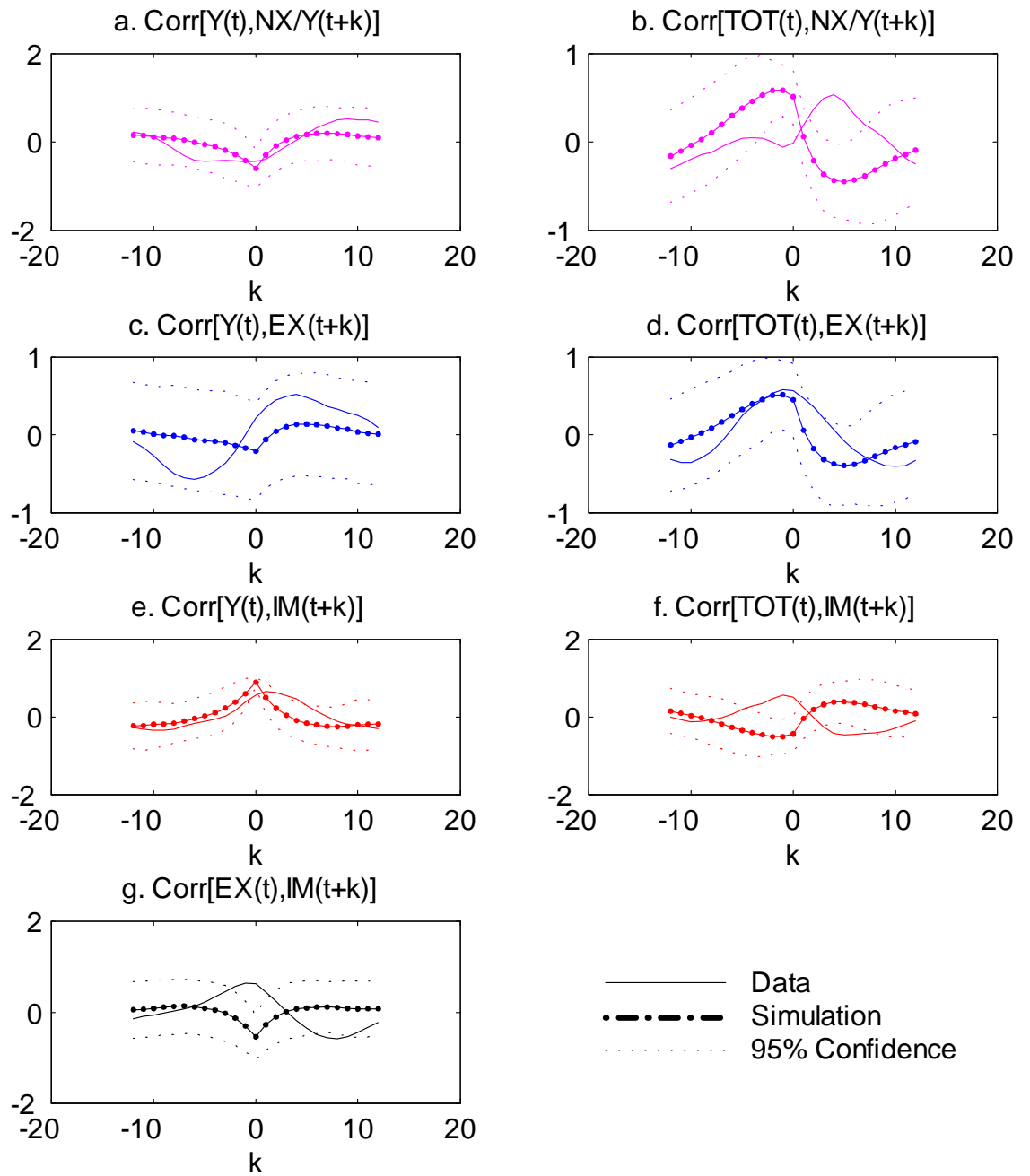


Figure 3 - Cross-correlation functions: Inelastic labor supply,  $n = 0.0025$



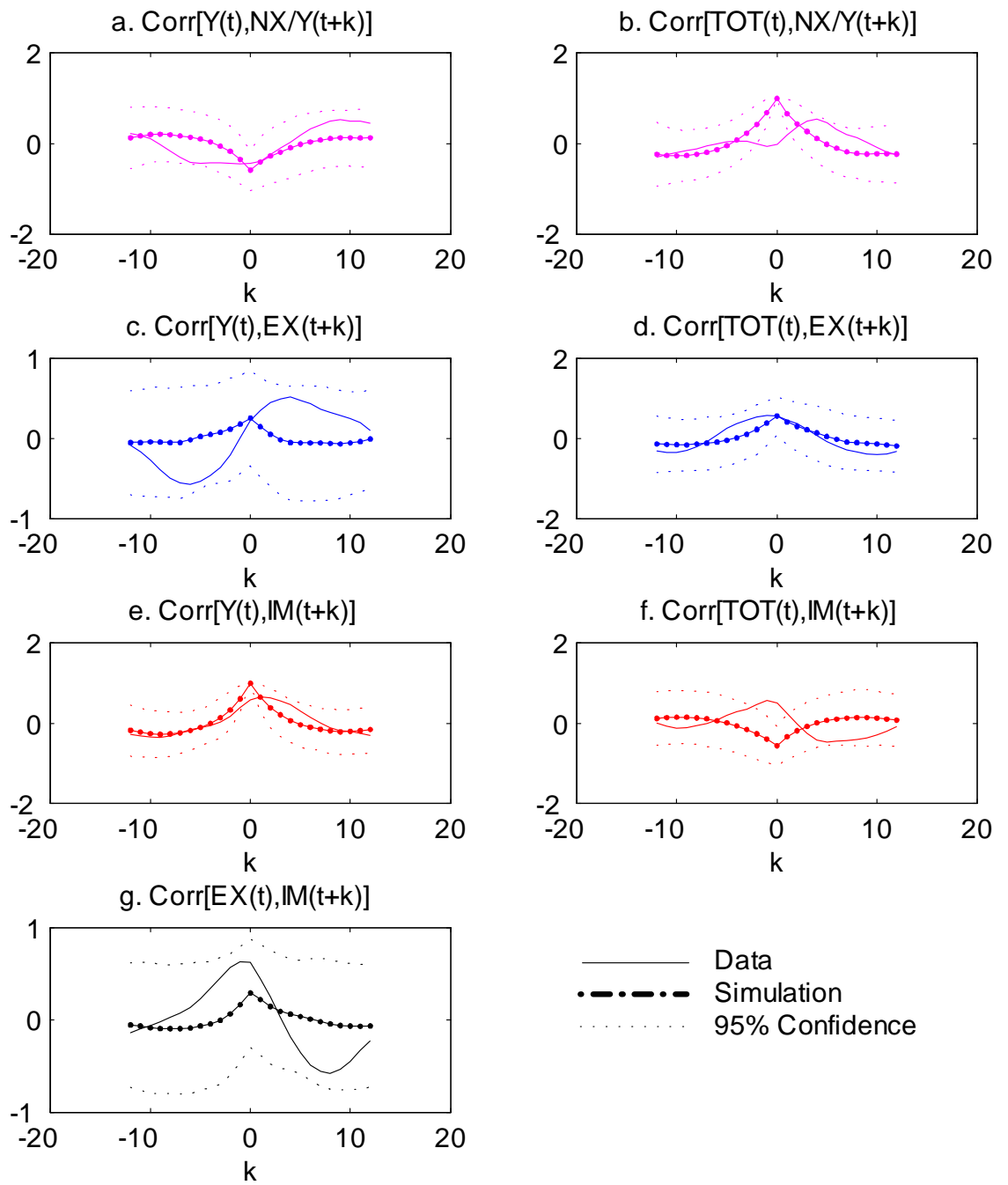


Figure 4 - Cross-correlation functions: Inelastic labor supply,  $n = 1$

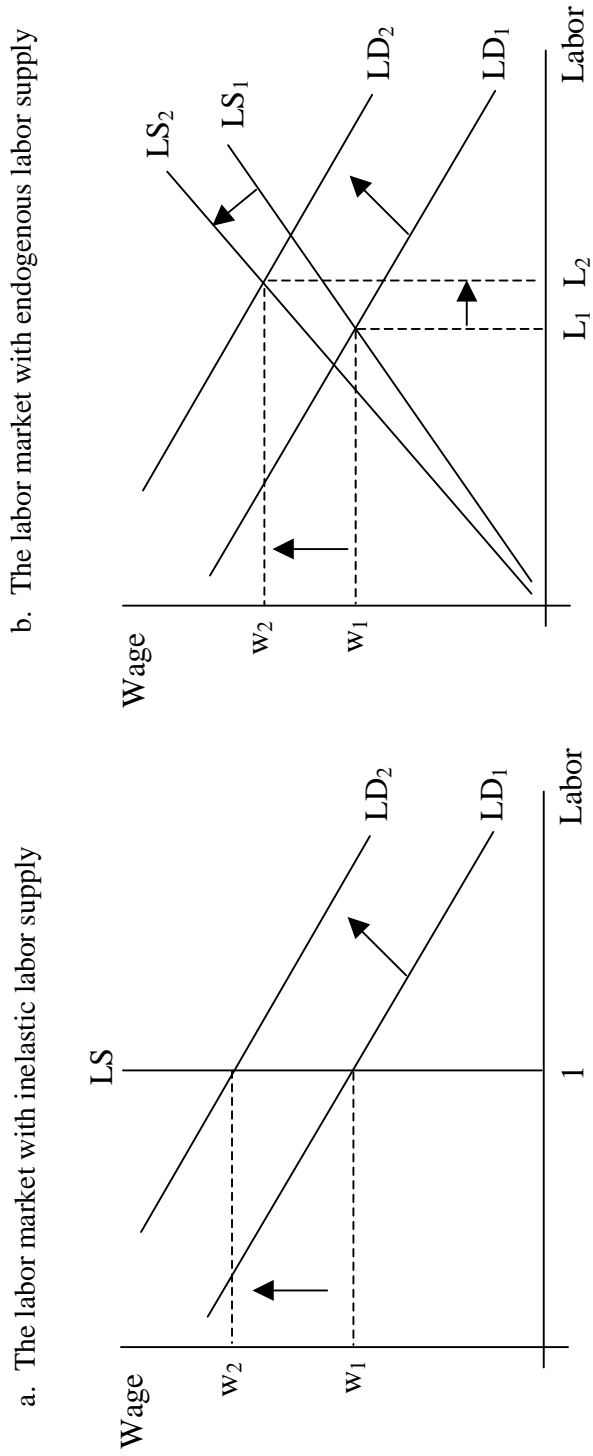
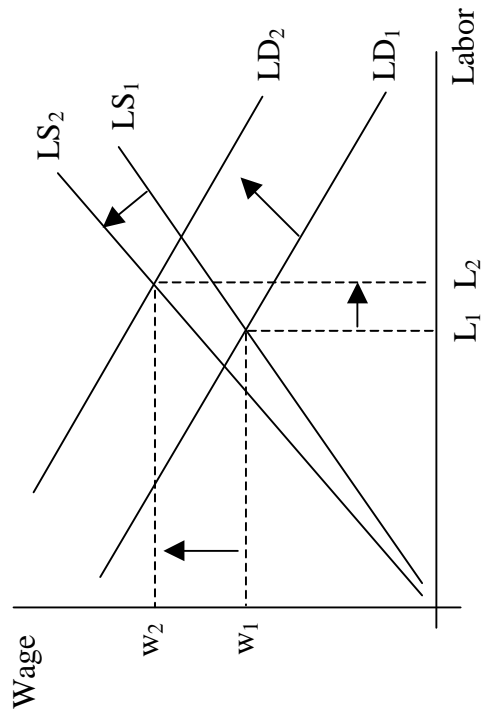


Figure 5 - Labor market of the home country with a positive shock to aggregate productivity.

a. The home labor market with endogenous labor supply



b. The foreign labor market with endogenous labor supply

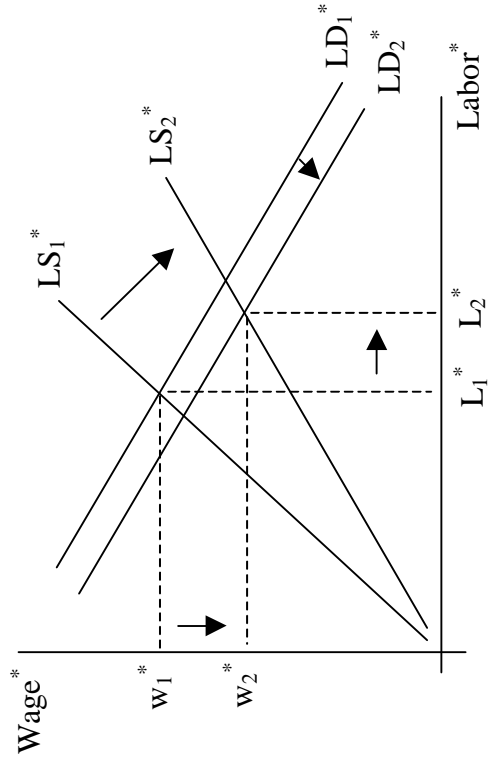


Figure 6 - Labor markets in the home and foreign countries with a positive shock to home productivity.

## A Appendix

### A.1 Balance of Payments

We construct the balance of payments equation by first imposing stock market clearing and bond adjustment costs transfers on the consumer's budget constraint. The budget constraint for the home country becomes:

$$C_t + B_{t+1} + Q_t B_{*t+1} + (N_{ht} \tilde{v}_t) = w_t l_t + (1 + r_t) B_t + Q_t (1 + r_t^*) B_{*t} + N_{dt} \tilde{v}_t + N_{dt} \tilde{d}_t$$

We generate the budget constraint for the foreign agent:

$$C_t^* + B_{t+1}^* + Q_t^{-1} B_{*t+1}^* + (N_{ht}^* \tilde{v}_t^*) = w_t^* l_t^* + (1 + r_t) B_t^* + Q_t^{-1} (1 + r_t^*) B_{*t}^* + N_{dt}^* \tilde{v}_t^* + N_{dt}^* \tilde{d}_t^*$$

We multiply the foreign budget constraint by  $Q_t$  to transform it into home consumption terms and subtract it from the home budget constraint. Imposing the bond market clearing conditions suggest the following equation for net foreign assets:

$$\begin{aligned} 2(1 + r_{t+1}) B_t + 2(1 + r_{t+1}^*) Q_t B_{*t} &= [C_t - Q_t C_t^*] + [N_{et} \tilde{v}_t - Q_t N_{et}^* \tilde{v}_t^*] + 2[B_{t+1} + Q_t B_{*t+1}] \\ &\quad - [w_t l_t - Q_t w_t^* l_t^*] - [N_{dt} \tilde{d}_t - Q_t N_{dt}^* \tilde{d}_t^*] \\ &\quad - [N_{xt} \tilde{d}_{xt} - Q_t N_{xt}^* \tilde{d}_{xt}^*] \end{aligned} \quad (34)$$

### A.2 Equilibrium Conditions

The model is characterized by the following nonlinear system of equations:

1. *Price and Profit definitions:*

$$\tilde{\rho}_{dt} = \left( \frac{a}{a-1} \frac{w_t}{Z_t \tilde{z}_d} \right) \quad (35)$$

$$\tilde{\rho}_{dt}^* = \left( \frac{a}{a-1} \frac{w_t^*}{Z_t^* \tilde{z}_d} \right) \quad (36)$$

$$\tilde{\rho}_{xt} = \left( \frac{a}{a-1} \frac{w_t}{Z_t \tilde{z}_{xt}} \frac{\tau}{Q_t} \right) \quad (37)$$

$$\tilde{\rho}_{xt}^* = \left( \frac{a}{a-1} \frac{w_t^*}{Z_t^* \tilde{z}_{xt}} \tau Q_t \right) \quad (38)$$

$$\tilde{d}_{dt} = \frac{1}{a} C_t \tilde{\rho}_{dt}^{1-a} \quad (39)$$

$$\tilde{d}_{dt}^* = \frac{1}{a} C_t^* \tilde{\rho}_{dt}^{*1-a} \quad (40)$$

$$\tilde{d}_{xt} = \frac{1}{a} C_t^* Q_t \tilde{\rho}_{xt}^{1-a} - \frac{w_t F_{Xt}}{Z_t} \quad (41)$$

$$\tilde{d}_{xt}^* = \frac{1}{a} C_t Q_t \tilde{\rho}_{xt}^{*1-a} - \frac{w_t^* F_{Xt}}{Z_t^*} \quad (42)$$

2. *Price Index*<sup>30</sup>:

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<sup>30</sup>Using the equation for the price index of consumption, we substitute in average prices.

$$1 = (N_{dt}\tilde{\rho}_{dt}^{1-a} + N_{xt}^*\tilde{\rho}_{xt}^{*1-a}) \quad (43)$$

$$1 = (N_{dt}^*\tilde{\rho}_{dt}^{*1-a} + N_{xt}\tilde{\rho}_{xt}^{1-a}) \quad (44)$$

3. *Expected Profit:*

$$\tilde{d}_t = \tilde{d}_{dt} + \left(\frac{N_{xt}}{N_{dt}}\right)\tilde{d}_{xt} \quad (45)$$

$$\tilde{d}_t^* = \tilde{d}_{dt}^* + \left(\frac{N_{xt}^*}{N_{dt}^*}\right)\tilde{d}_{xt}^* \quad (46)$$

4. *Free Entry:*

$$\tilde{v}_t = w_t F_{Et}/Z_t \quad (47)$$

$$\tilde{v}_t^* = w_t^* F_{Et}^*/Z_t^* \quad (48)$$

5. *Zero-Profit Intermediate Exporter*<sup>31</sup>:

$$\tilde{d}_{xt} = \left[\frac{a-1}{k+1-a}\right] \frac{w_t F_{Xt}}{Z_t} \quad (49)$$

$$\tilde{d}_{xt}^* = \left[\frac{a-1}{k+1-a}\right] \frac{w_t^* F_{Xt}^*}{Z_t^*} \quad (50)$$

6. *Share Exporting Firms:*

$$(1 - G(z_{xt})) = \frac{N_{xt}}{N_{dt}} = z_{\min}^k \left(\frac{k}{k+1-a}\right)^{k/(a-1)} \tilde{z}_{xt}^{-k} \quad (51)$$

$$(1 - G(z_{xt}^*)) = \frac{N_{xt}^*}{N_{dt}^*} = z_{\min}^k \left(\frac{k}{k+1-a}\right)^{k/(a-1)} \tilde{z}_{xt}^{*-k} \quad (52)$$

7. *Number of Firms:*

$$N_{dt} = (1 - \delta)(N_{dt-1} + N_{et-1}) \quad (53)$$

$$N_{dt}^* = (1 - \delta)(N_{dt-1}^* + N_{et-1}^*) \quad (54)$$

8. *Euler Equation for Domestic Bonds:*

$$C_t^{-1}(1 + nB_{t+1}) = (1 + r_{t+1})\beta E_t C_{t+1}^{-1} \quad (55)$$

$$C_t^{*-1}(1 - nB_{*t+1}) = (1 + r_{t+1}^*)\beta E_t^* C_{t+1}^{*-1} \quad (56)$$

9. *Euler Equation for Foreign Bonds:*

$$Q_t C_t^{-1}(1 + nB_{*t+1}) = (1 + r_{t+1}^*)\beta E_t Q_{t+1} C_{t+1}^{-1} \quad (57)$$

$$Q_t^{-1} C_t^{*-1}(1 - nB_{t+1}) = (1 + r_{t+1})\beta E_t^* Q_{t+1}^{-1} C_{t+1}^{*-1} \quad (58)$$

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<sup>31</sup>To generate this condition, we utilize the cutoff condition,  $\frac{1}{a}C_t^*Q_t\rho_{xjt}^{1-a} = \frac{w_t F_{Xt}}{Z_t}$ , the useful transform,  $z_{xt} = \left(\frac{k}{k+1-a}\right)^{1/(1-a)}\tilde{z}_{xt}$ , and the definition for exporter profit,  $\tilde{d}_{xt}$ .

10. *Euler Equation for Mutual Fund Shares:*

$$\tilde{v}_t = \beta(1 - \delta)E_t\left(\frac{C_{t+1}}{C_t}\right)^{-1}(\tilde{d}_{t+1} + \tilde{v}_{t+1}) \quad (59)$$

$$\tilde{v}_t^* = \beta(1 - \delta)E_t^*\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-1}(\tilde{d}_{t+1}^* + \tilde{v}_{t+1}^*) \quad (60)$$

11. *Labor Market Clearing*<sup>32</sup>:

$$C_t^{-1/\lambda}w_t^{1/\lambda} = \frac{a-1}{w_t}(N_{Dt}\tilde{d}_{dt} + N_{xt}\tilde{d}_{xt}) + \frac{(aN_{xt}F_{Xt} + N_{Et}F_{Et})}{Z_t} \quad (61)$$

$$C_t^{*-1/\lambda}w_t^{*1/\lambda} = \frac{a-1}{w_t^*}(N_{Dt}^*\tilde{d}_{dt}^* + N_{xt}^*\tilde{d}_{xt}^*) + \frac{(aN_{xt}^*F_{Xt} + N_{Et}^*F_{Et})}{Z_t^*} \quad (62)$$

12. *Net Foreign Assets:*

$$\begin{aligned} 2(1 + r_{t+1})B_t + 2(1 + r_{t+1}^*)Q_tB_{*t} &= [C_t - Q_tC_t^*] + [N_{et}\tilde{v}_t - Q_tN_{et}^*\tilde{v}_t^*] + 2[B_{t+1} + Q_tB_{*t+1}] \\ &\quad - [w_t l_t - Q_t w_t^* l_t^*] - [N_{dt}\tilde{d}_{dt} - Q_t N_{dt}^* \tilde{d}_{dt}^*] \\ &\quad - [N_{xt}\tilde{d}_{xt} - Q_t N_{xt}^* \tilde{d}_{xt}^*] \end{aligned} \quad (63)$$

Note that the mutual fund market clearing conditions in addition to the zero net supply of bonds conditions have already been included in the above equilibrium equations. The 29 equations are log-linearized around the symmetric steady state to form 29 linear equations in terms of percentages (denoted by  $\hat{\cdot}$ )<sup>33</sup>.

In addition to these equations, the stochastic processes for the technology innovations must be specified:

$$\begin{aligned} \begin{bmatrix} \hat{Z}_t \\ \hat{Z}_t^* \end{bmatrix} &= \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} \hat{Z}_{t-1} \\ \hat{Z}_{t-1}^* \end{bmatrix} + \begin{bmatrix} \xi_{Zt} \\ \xi_{Zt}^* \end{bmatrix} \\ E_t(Z_t) &= E_t^*(Z_t^*) = \bar{Z} \\ \sigma_{\xi_{Zt}} &= \sigma_{\xi_{Zt}^*} = \sigma_\xi \\ \rho_{\xi_{Zt}\xi_{Zt}^*} &= \rho_{\xi_Z\xi_Z^*} \end{aligned} \quad (64)$$

<sup>32</sup>When labor is supplied inelastically, we set the left-hand side of the equations to 1.

<sup>33</sup>Since the variable,  $B_t$ , has an expected value of zero in equilibrium, it is linearized with respect to steady state consumption:  $\hat{B}_t = \frac{B_t - 0}{C} = \frac{B_t}{C}$ .

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