Predictability of Stock Returns and Asset Allocation under Structural Breaks

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Outline I

Predictability of stock returns: Early papers

- Campbell (1987)
- Campbell and Shiller (1988)
- Fama and French (1988, 1989)
- Ferson and Harvey (1991)
- Goetzmann and Jorion (1993)
- Harvey (1989)
- Keim and Stambaugh (1986)

Asset Allocation under Predictability of Returns

- Ait-Sahalia and Brandt (2001)
- Ang and Bekaert (2002)
- Barberis (2000)
- Balduzzi and Lynch (1999)
- Brandt (1999), Brandt, Goyal, Santa-Clara and Stroud (2002)
- Brennan, Schwarz and Lagnado (1997), Brennan and Xia (2001)
- Campbell and Viceira (1999, 2001, 2002), Campbell, Chan and Viceira (2003)
- Detemple, Garcia and Rindisbacher (2003)
- Kandel and Stambaugh (1996)
- Lynch (2001), Lynch and Balduzzi (2000)
- Xia (2001)

Key Questions for Asset Allocation under Predictability

- Parameter Estimation Errors
- Model Uncertainty
- Model Instability

Approaches for solving portfolio choice problems:

1. Plug-in estimation or calibration methods: parameters of the return process are estimated and plugged into the analytical or numerical solution of the investor's portfolio choice
2. Method of moments approach - determine asset allocation off the Euler equation
3. Decision theory approach: integrate model estimation, portfolio selection problem typically by using a Bayesian methodology

Current Return predictability debate

- Pesaran and Timmermann (1995)
- Bossaerts and Hillion (1999)
- Lettau and Ludvigsson (2001)
- Goyal and Welch (2003)
- Cooper, Gutierrez and Marcum (2005)
- Campbell and Thompson (2005)
- Cochrane (2006)

Is the weak out-of-sample predictability due to model instability ("breaks")?

Evidence of structural breaks

- Stock and Watson (1986)
- Bai, Lumsdaine and Stock (1998)
- Ang and Bekaert (2001, 2002)
- Pastor and Stambaugh (2001)

Weakness of out-of-sample return predictability has been linked to model instability

- Lettau, ludvigsson and Wachter (2004)
- Schwert (2003)
- Perez-Quiros and Timmermann (2000)
- Paye and Timmermann (2005)
- Lettau and Van Nieuwerburgh (2005)

Predicting returns under model instability

Pastor and Stambaugh (2001, p. 1207) “Finance practitioners and academics often elect to rely on more recent data ... motivated in part by concerns that the probability distribution of excess returns changes over time, experiencing shifts known as “structural breaks””

- when did the break occur? (Bai, Bai-Perron)
- How much data to use?
- inefficient only to use post-break data (Pesaran and Timmermann (2005, 2006)
- Can breaks happen in the future?

Contributions of this paper

- Develop a methodology to forecast returns and compute asset allocation in the presence of structural breaks
- Introduce a 'meta' distribution for the parameters across regimes and a changepoint model based on a hierarchical Hidden Markov Chain (HMC)
- Consider investment decisions under uncertainty about

1. parameter estimates
2. model specification
3. number, timing (dates) and size of both past and future breaks

Empirical Findings: Predictability

Apply the method to US stock returns, using Dividend Yield or T-bill Rate as predictor variables

- Evidence of multiple breaks in return models
- Break dates coincide with major events such as changes in the Fed's operating procedures (1979, 1982), Great Depression, World War II and the growth slowdown in early 1970s
- Structural breaks influence value and precision of parameter estimates of the return prediction model
- Return Predictability varies greatly over time

Empirical Findings: Asset Allocations

- Structural breaks can have a large effect on a buy-and-hold investor's asset allocation
- Breaks continue to have an important effect under rebalancing
  - Parameter estimation uncertainty becomes more important since the parameters from the current regime are surrounded by larger uncertainty than the full-sample parameters
  - Possibility of future breaks affects investors' optimal allocations even under rebalancing due to incomplete learning since they can only be detected with a lag

Methodology

Changepoint model driven by unobserved discrete state variable
After a break, the new parameters of the return forecasting model are drawn from a meta distribution

$\rightarrow$ we can forecast out-of-sample even in the presence of future breaks

Our approach accounts for structural breaks in return forecasting models, building on

- Chib (1998)
- Pastor and Stambaugh (2001)
- Pesaran, Pettenuzzo and Timmermann (2005)

Return Prediction Model

- Restricted VAR:

$$z_t = B' \widetilde{x}_{t-1} + u_t$$

- $z_t = (r_t, x_t)'$, $\widetilde{x}_{t-1} = (1, x_{t-1})'$

r_t : excess return at time t
x_{t-1} : return predictor(s)
$u_t \sim N(0, \Sigma)$

- μ_r, μ_x : intercepts in the return and predictor equation
- β_r, β_x coefficients on lagged predictor:

$$r_t = \mu_r + \beta_r x_{t-1} + \varepsilon_{rt}$$
$$x_t = \mu_x + \beta_x x_{t-1} + \varepsilon_{xt}$$

Methodology (continued)

- Break process tracked by integer-valued state variable, S_t. Conditional on K breaks

$$
\begin{array}{llll}
z_t = B_1' \widetilde{x}_{t-1} + u_t, & E[u_t u_t'] = \Sigma_1 & 1 \leq t \leq \tau_1 & s_t = 1 \\
\vdots & \vdots & \vdots & \\
z_t = B_{K+1}' \widetilde{x}_{t-1} + u_t, & E[u_t u_t'] = \Sigma_{K+1} & \tau_K + 1 \leq t \leq T & s_t = K+1
\end{array}
$$

- $\Upsilon_K = \{\tau_0, ..., \tau_K\}$: collection of break points
- Covariance matrix, Σ_j, decomposed as follows:

$$\Sigma_j = diag(\psi_j) \times \Lambda_j \times diag(\psi_j)$$

Both volatilities and correlations can vary across regimes

Transition Probabilities

- Break dynamics is modeled through the transition probability matrix P :

$$\tilde{P} = \left(\begin{array}{ccccc|c} p_{11} & p_{12} & 0 & \cdots & 0 & \\ 0 & p_{22} & p_{23} & \cdots & 0 & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \\ 0 & \cdots & 0 & p_{KK} & p_{K,K+1} & \\ 0 & 0 & \ldots & 0 & p_{K+1,K+1} & p_{K+1,K+2} \\ \hline 0 & 0 & \ldots & & 0 & p_{K+2,K+2} \\ & & & & & \ddots \end{array} \right)$$

- $p_{j,j}$ is assumed to be independent of $p_{i,i}$, for $j \neq i$, and is drawn from a beta distribution: $p_{j,j} \sim Beta\,(a, b)$

Meta Distributions

- Apply hierarchical prior setup to forecast returns out-of-sample
- Location and scale parameters within each regime, (B_j, Σ_j), are drawn from common "meta" distributions
- Data from previous regimes carry information relevant for current data and for the new parameters after a future break
- By using meta distributions that pool information from different regimes, historic information is used efficiently in estimating the parameters of the current regime
- Special cases:
  - pooled scenario (parameters are identical across regimes)
  - regime-specific scenario (parameters unrelated across regimes)

Meta Distributions (continued)

To characterize the parameters of the meta distribution, we assume

$$
\begin{aligned}
vec(B)_j &\sim N\left(b_0, V_0\right), j = 1, ..., K+1 \\
\psi_{j,i}^{-2} &\sim Gamma\left(v_{0,i}, d_{0,i}\right) \\
\lambda_{j,ic} &\sim N(\mu_{\rho,ic}, \sigma^2_{\rho,ic}),
\end{aligned}
$$

$$
\begin{aligned}
b_0 &\sim N\left(\underline{\mu_\beta}, \underline{\Sigma_\beta}\right) \\
V_0^{-1} &\sim W\left(\underline{v_\beta}, \underline{V_\beta^{-1}}\right),
\end{aligned}
$$

$W(.)$: Wishart distribution
$\underline{\mu_\beta}$, $\underline{\Sigma_\beta}$, $\underline{v_\beta}$, $\underline{V_\beta^{-1}}$: prior hyperparameters

Meta Distributions (continued)

$$v_{0,i} \sim Exp\left(\underline{\rho_{0,i}}\right)$$
$$d_{0,i} \sim Gamma\left(\underline{c_{0,i}}, \underline{d_{0,i}}\right)$$

$\underline{\rho_{0,i}}$, $\underline{c_{0,i}}$ and $\underline{d_{0,i}}$: prior hyperparameters
Hyperparameters of correlation matrix (truncated to lie on $(-1, 1)$):

$$\mu_{\rho,ic} \sim N\left(\underline{\mu_{\mu,ic}}, \underline{\tau^2_{ic}}\right)$$
$$\sigma^{-2}_{\rho,ic} \sim Gamma\left(\underline{a_{\rho,ic}}, \underline{b_{\rho,ic}}\right)$$

$\underline{\mu_{\mu,ic}}$, $\underline{\tau^2_{ic}}$, $\underline{a_{\rho,ic}}$, $\underline{b_{\rho,ic}}$: prior hyperparameters

$$a \sim Gamma\left(\underline{a_0}, \underline{b_0}\right)$$
$$b \sim Gamma\left(\underline{a_0}, \underline{b_0}\right)$$

Prior elicitation

Impose two constraints on the parameters:

- $\beta_x < 1$
- $0 \leq \mu_x/(1 - \beta_x) \leq \bar{\mu}_x$

- $\underline{\mu_\beta} = 0_{m^2}$
- $\underline{\Sigma_\beta} = sc \times I_{m^2}$
- $\underline{V_\beta} = diag(0.1, 10, 0.01, 0.1)$
- $\underline{c_{0,i}} = 1$, $\underline{d_{0,i}} = 1/\infty$ and $\underline{\rho_{0,i}} = \infty$
- $\underline{\mu_{\mu,12}} = 0$, $\underline{\tau^2_{12}} = \infty$, $a_{\rho,12} = 1$ and $b_{\rho,12} = 0.01$
- $\underline{a_0} = 1$ and $\underline{b_0} = 0.02$

Empirical Results

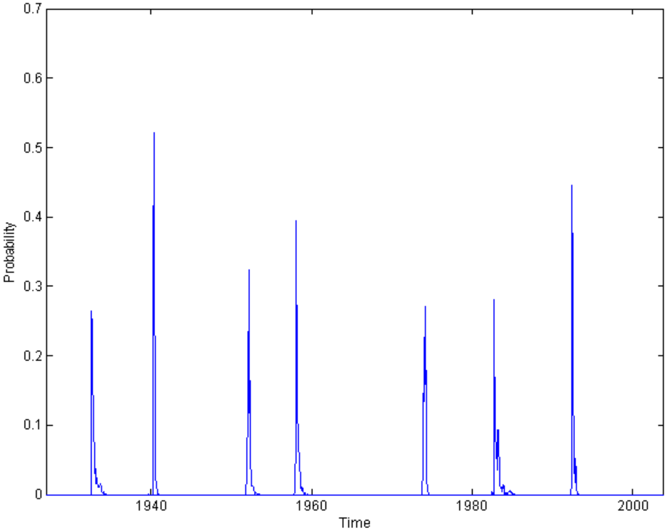
Data

- Monthly data on a portfolio of US stocks comprising firms listed on the NYSE, AMEX and NASDAQ
- Sample: 1926:12-2003:12
- Data source: CRSP

Evidence of Breaks

I. Excess returns - Dividend Yield							
# breaks	log lik.	marg. log lik.	Post. Prob.	Break locations			
0	1368.18	1319.34	0.00				
1	1433.09	1369.33	0.00	Apr-52			
2	1445.83	1383.28	0.00	Feb-52	Sep-54		
3	1454.38	1387.49	0.00	May-40	Apr-52	Jul-92	
4	1459.03	1390.04	0.00	May-40	Feb-58	Jan-74	Jul-92
5	1462.11	1391.51	0.00	May-40	Apr-52	Feb-58	Mar-74
				Jul-92			
6	1476.60	1404.75	0.01	Aug-32	May-40	Feb-58	Mar-74
				Oct-82	Jul-92		
7	1482.55	1408.74	0.69	Aug-32	May-40	Apr-52	Feb-58
				Mar-74	Oct-82	Jul-92	
8	1482.64	1407.89	0.30	Jun-32	May-40	Apr-52	Feb-58
				Mar-74	Oct-82	Apr-92	Jul-92

Location of Breaks: Yield Model



Predictability from the Dividend Yield

- Evidence of seven Breaks
- Break dates reasonably precisely determined
- Break locations are associated with major events
  - Great Depression (1932)
  - Beginning of World War II (1940)
  - Major oil price shocks and growth slowdown (1974)
  - End of the change in the Fed's operating procedures (1982)
  - Beginning of the bull market of the nineties (1992)
- Remaining break dates (1952 and 1958) harder to interpret

Estimates (Dividend Yield Model)

	Regimes								
	Full sample	27-32	32-40	40-52	52-58	58-74	74-82	82-92	92-03
				μ_r					
mean	-0.003	-0.028	-0.026	-0.016	-0.028	-0.062	-0.086	-0.056	-0.020
s.d.	0.005	0.029	0.033	0.015	0.024	0.021	0.033	0.028	0.011
				β_r					
mean	0.197	0.392	0.737	0.479	0.825	2.090	1.999	1.747	1.408
s.d.	0.120	0.489	0.701	0.262	0.508	0.680	0.724	0.782	0.576
				σ_r					
mean	0.055	0.105	0.086	0.038	0.034	0.038	0.050	0.046	0.045
s.d.	0.001	0.009	0.007	0.002	0.003	0.002	0.004	0.003	0.003
				μ_x					
mean	0.001	0.005	0.004	0.002	0.002	0.003	0.004	0.002	3.0E-04
s.d.	0.001	0.002	0.002	0.001	0.001	0.001	0.002	0.001	2.0E-04
				β_x					
mean	0.983	0.916	0.901	0.967	0.951	0.919	0.908	0.940	0.979
s.d.	0.006	0.033	0.035	0.016	0.022	0.021	0.034	0.026	0.009

Estimates (continued)

					$\sigma_x \times 100$				
mean	0.286	0.714	0.443	0.235	0.148	0.111	0.226	0.149	0.065
s.d.	0.007	0.063	0.036	0.014	0.013	0.006	0.016	0.010	0.004
					ρ_{rx}				
mean	-0.872	-0.930	-0.936	-0.858	-0.928	-0.955	-0.963	-0.941	-0.946
s.d.	0.008	0.021	0.022	0.023	0.023	0.020	0.017	0.021	0.020
					p				
mean		0.982	0.986	0.990	0.983	0.992	0.987	0.988	N.A.
s.d.		0.013	0.010	0.008	0.013	0.006	0.010	0.010	N.A.

Hyper Parameters

Hyperparameters of Meta distributions				
I Return equation				
Mean Parameters				
	mean	s.d.	95% conf interval	
$b_0(\mu_r)$	-0.042	0.038	-0.123	0.033
$b_0(\beta_r)$	1.218	0.508	0.225	2.209
II Dividend Yield equation				
Mean Parameters				
	mean	s.d.	95% conf interval	
$b_0(\mu_x)$	0.003	0.002	1.0E-04	0.008
$b_0(\beta_x)$	0.918	0.033	0.839	0.972
Correlation parameters				
	mean	s.d.	95% conf interval	
μ_ρ	-0.920	0.042	-0.984	-0.831
Transition Probability parameters				
	mean	s.d.	95% conf interval	
a_0	33.258	15.015	10.498	73.091
b_0	0.806	0.308	0.357	1.462

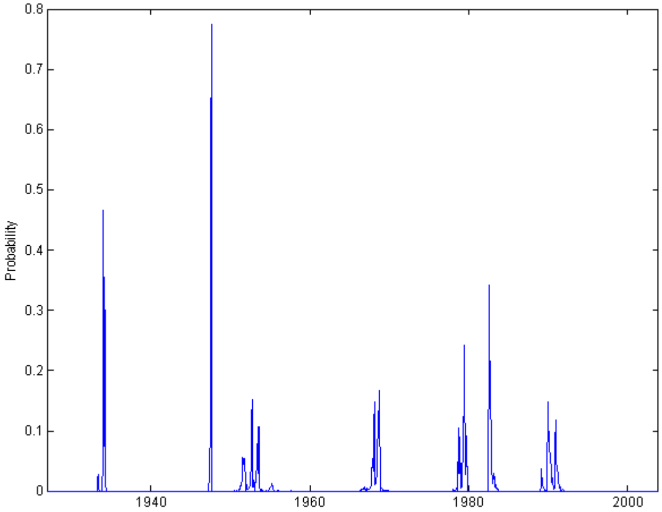
Variation in Parameter Estimates Across Regimes

- Dividend yield parameter highly persistent - mean autoregressive parameter varies from 0.90 to 0.98
- Correlation estimates for the innovations to stocks and the lagged dividend yield range from -0.96 to -0.85
- Stayer probabilities are high, mean durations ranging from 70 to 140 months
- Greater uncertainty about slope of dividend yield coefficient: parameter centered on 1.2 with a standard deviation of 0.50
- Greatest variability in parameters across regimes is associated with the effect of the dividend yield on stock returns

Predictability from the Short Interest Rate

- Evidence of seven breaks again
- Break dates more dispersed than for the dividend yield model
- Overlap in break dates
  - Great Depression (1934)
  - End of World War II (1947)
  - Vietnam War (1968)
  - Beginning and end of the change to the Fed's operating procedures (1979 and 1982)
  - Beginning of the nineties' bull market (1990)
- Greatest uncertainty about slope of T-bill rate in return equation

Location of Breaks: T-bill Model



Estimates (T-bill model)

	Regimes								
	Full sample	27-34	34-47	47-52	52-68	68-79	79-82	82-90	90-03
				μ_r					
mean	0.009	-0.001	0.007	0.016	0.027	0.037	0.092	0.021	0.009
s.d.	0.003	0.018	0.005	0.011	0.007	0.019	0.034	0.024	0.010
				β_r					
mean	-1.402	-0.439	-3.343	-4.353	-8.215	-8.203	-9.976	-2.452	-0.942
s.d.	0.731	6.886	11.343	10.542	3.015	4.074	3.656	4.086	2.724
				σ_r					
mean	0.055	0.109	0.059	0.036	0.033	0.047	0.047	0.049	0.044
s.d.	0.001	0.008	0.003	0.004	0.002	0.003	0.006	0.004	0.002
				$\mu_x \times 100$					
mean	0.004	0.006	0.001	0.017	0.011	0.032	0.132	0.049	0.006
s.d.	0.002	0.004	0.001	0.006	0.005	0.014	0.069	0.021	0.004
				β_x					
mean	0.986	0.958	0.924	0.844	0.959	0.937	0.832	0.914	0.974
s.d.	0.005	0.021	0.027	0.065	0.020	0.030	0.075	0.037	0.012

Estimates (continued)

				$\sigma_x \times 100$					
mean	0.040	0.033	0.004	0.011	0.027	0.045	0.129	0.047	0.025
s.d.	0.001	0.003	2.7E-04	0.002	0.001	0.003	0.017	0.004	0.001
				ρ_{rx}					
mean	0.005	0.067	0.003	0.170	-0.011	-0.189	-0.311	0.302	0.067
s.d.	0.032	0.026	0.026	0.046	0.033	0.034	0.033	0.037	0.030
				p					
mean		0.985	0.991	0.981	0.992	0.989	0.976	0.987	N.A.
s.d.		0.012	0.007	0.015	0.006	0.009	0.019	0.010	N.A.

Economic Significance of Breaks

Assume power utility over terminal wealth:

$$u(W_{T+h}) = \frac{W_{T+h}^{1-\gamma}}{1-\gamma}, \quad \gamma > 0.$$

$T \rightarrow T+h$: Holding period
γ : coefficient of relative risk aversion
W_{T+h} : Terminal wealth ($W_T = 1$)

$$W_{T+h} = (1-\omega)\exp(r_f h) + \omega \exp(r_f h + r_{T+1} + ... + r_{T+h})$$

Buy-and-hold Investor

Buy-and-hold investor solves the following program

$$\max_{\omega} E_T \left(\frac{\left((1-\omega)\exp(r_f h) + \omega \exp(r_f h + R_{T+h})\right)^{1-\gamma}}{1-\gamma} \right)$$

subject to the no short-sales constraints $0 \leq \omega < 1$

No Breaks, no Estimation Uncertainty

$p(R_{T+h}|\widehat{\Theta}, S_{T+h} = 1, \mathcal{Z}_T)$: predictive return distribution ignoring parameter estimation uncertainty and breaks

$\Theta = (\mu, \beta_0, \Sigma)$: VAR parameters
$\widehat{\Theta}$: Parameter estimate
$\mathcal{Z}_T$: Information set

Investor maximizes

$$\max_{\omega} \int u(W_{T+h}) p(R_{T+h}|\widehat{\Theta}, S_{T+h} = 1, \mathcal{Z}_T) dR_{T+h}.$$

No Breaks, Parameter Estimation Uncertainty

Integrating over the posterior distribution, $\pi(\Theta|S_{T+h} = 1, \mathcal{Z}_T)$, leads to the predictive distribution of returns conditioned only on the observed sample and the assumption of no breaks:

$$p(R_{T+h}|S_{T+h} = 1, \mathcal{Z}_T) = \int p(R_{T+h}|\Theta, S_{T+h} = 1, \mathcal{Z}_T) \times \pi(\Theta|S_{T+h} = 1, \mathcal{Z}_T) d\Theta.$$

Investor solves the asset allocation problem

$$\max_{\omega} \int u(W_{T+h}) p(R_{T+h}|S_{T+h} = 1, \mathcal{Z}_T) dR_{T+h}.$$

Past Breaks Only

Condition on K historic breaks but no new break in $[T+1, T+h]$

$$p(R_{T+h}|S_{T+h} = S_T = K+1, \mathcal{Z}_T)$$
$$= \int p\left(R_{T+h}|\,\Theta_{K+1}, S_{T+h} = K+1, S_T = K+1, \mathcal{Z}_T\right)$$
$$\times\, \pi\left(\Theta_{K+1}|\, H, p, \mathcal{S}_T, \mathcal{Z}_T\right) d\Theta_{K+1}$$

$\Theta_{K+1} = (vec(B)_{K+1}, \psi_{K+1}, \Lambda_{K+1})$: parameters from state $K+1$
$H =$ hyperparameters of the meta distribution =
$\left(b_0, V_0, v_{0,1}, d_{0,1}, ..., v_{0,m}, d_{0,m}, \mu_{\rho,12}, \sigma^2_{\rho,12}, ..., \mu_{\rho,m-1m}, \sigma^2_{\rho,m-1m}, a, b\right)$
Investor solves the portfolio problem

$$\max_{\omega} \int u(W_{T+h}) p(R_{T+h}|S_{T+h} = S_T = K+1, \mathcal{Z}_T) dR_{T+h}$$

Past and Future Breaks

Condition on up to n_b breaks over $[T, T+h]$. Let j track the date where a break occurs. Probabilities of zero, one, up to n_b breaks:

$$p(S_{T+h} = K+1 | S_T = K+1, \mathcal{Z}_T) = p^h_{K+1,K+1}$$

$$p(S_{T+h} = K+2 | S_T = K+1, \mathcal{Z}_T) = \sum_{j_1=1}^{h} (1 - p_{K+1,K+1})\, p^{j_1-1}_{K+1,K+1}$$

$$p(S_{T+h} = K+3 | S_T = K+1, \mathcal{Z}_T) = \sum_{j_1=1}^{h-1} \sum_{j_2=j_1+1}^{h} p^{j_1-1}_{K+1,K+1} (1 - p_{K+1,K+1}) \times p^{j_2-j_1-1}_{K+2,K+2} (1 - p_{K+2,K+2})$$

$$\vdots$$

$$p(S_{T+h} = K+n_b+1 | S_T = K+1, \mathcal{Z}_T) = \sum\nolimits_{j_1=1}^{h-n_b+1} \ldots \sum\nolimits_{j_{n_b}=j_{n_b-1}+1}^{h} \left(\prod_{j=1}^{n_b} p^{d_j}_{K+j,K+j} (1 - p_{K+j,K+j}) \right).$$

Model uncertainty

Use T-bill rate or Dividend Yield as predictor variable?

- Avramov (2002)
- Cremers (2002)

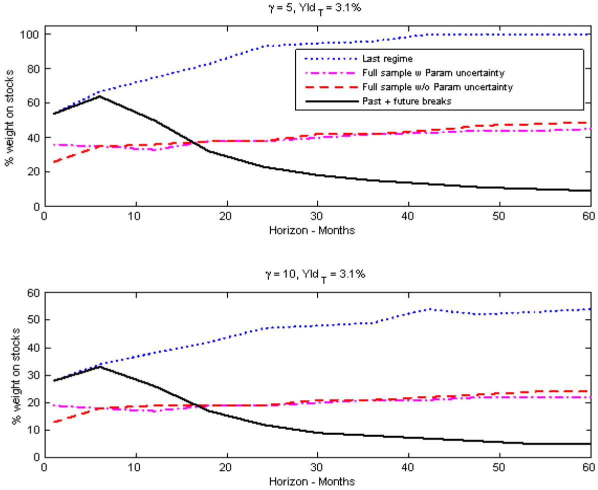
BMA over predictor variables, X, and number of breaks, $\bar{K}_x$:

$$p(R_{T+h}|\mathcal{Z}_T) = \sum_{x=1}^{\overline{X}} \sum_{k=0}^{\bar{K}_x} p_x(R_{T+h}|S_T = k_x + 1, \mathcal{Z}_T) p(M_{k_x}|\mathcal{Z}_T).$$

Empirical Asset Allocation Results

- Mean-reverting component in returns means that the risk of stock returns grow slower than in the IID model (Barberis, 2000)
- Parameter estimation uncertainty generally reduces a risk averse investor's demand for stocks: New information leading investors to revise downward their belief about mean stock returns is similar to a permanent negative dividend shock
- Interaction between parameter estimation uncertainty and structural breaks:
  - Breaks mean that bad draws of the parameters of the return model will eventually cease to affect returns following future breaks
  - Breaks lower the precision of current parameter estimates and increase the importance of parameter estimation uncertainty
  - Which effect dominates depends on the variability in the parameters across regimes and on the average duration of the regimes

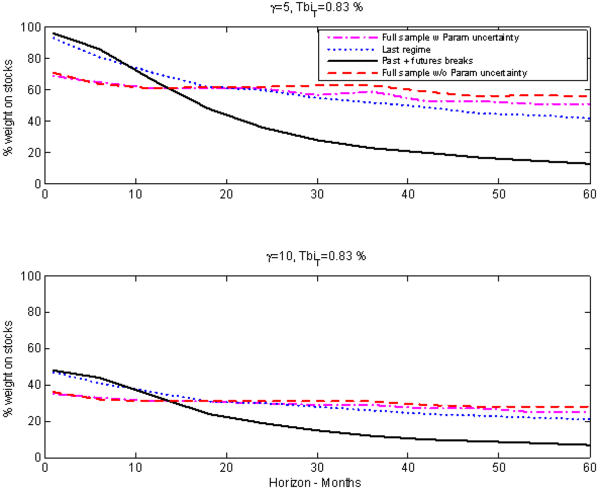
Asset Allocation under Predictability from Yield



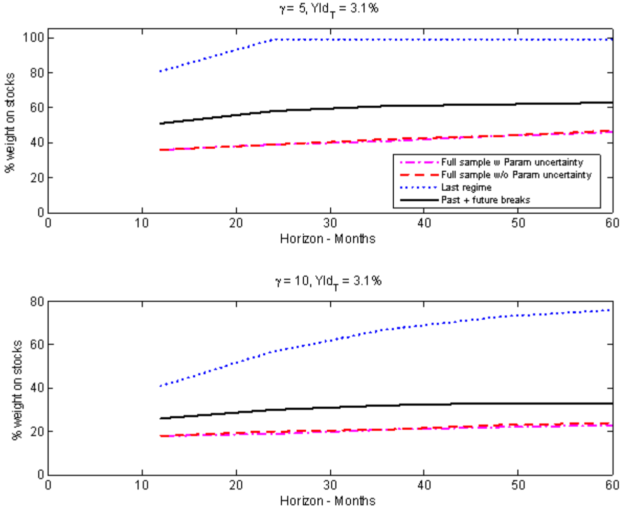
Results Based on the Dividend Yield

- Initial value of (persistent) predictor variable matters a lot
- Allocation to stocks increases in the horizon if the initial value of the dividend yield is very low and breaks are ignored
- If past breaks are accounted for but future breaks are ignored, the asset allocation can be flat or increasing in the horizon
- If both past and future breaks are modeled, we see a non-monotonic or sometimes strongly declining allocation to stocks, the longer the investment horizon
- Parameter instability has a larger effect on a buy-and-hold investor's optimal asset allocation than parameter estimation uncertainty

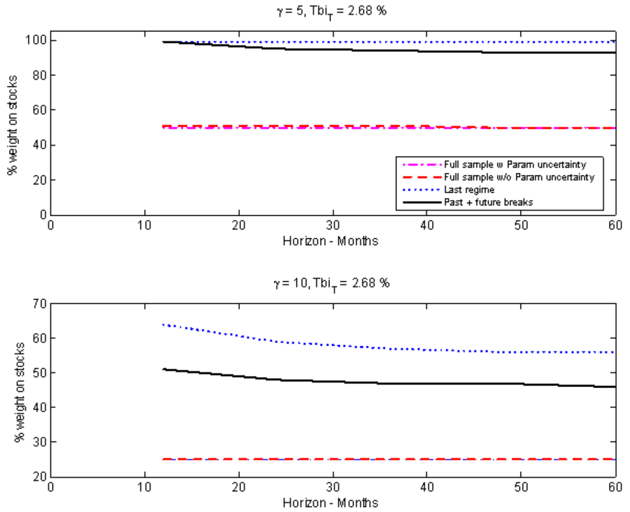
Asset Allocation under Predictability from T-bill Rate



Rebalancing under Predictability from the Yield



Rebalancing under Predictability from the T-bill rate



Rebalancing Results

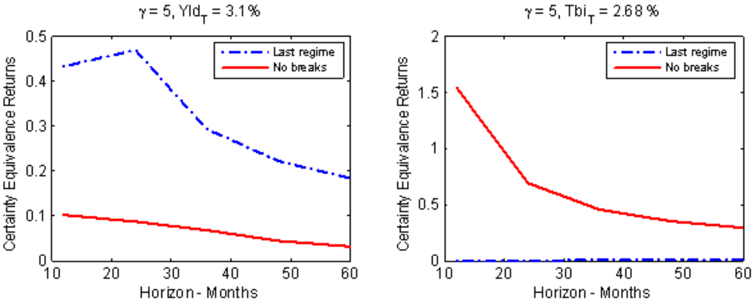
- Allocation to stocks goes from being decreasing under no rebalancing to being marginally increasing under rebalancing.
- Rebalancing provides an efficient way for investors to adjust their asset allocation in case a future adverse shock hits the parameters of the stock return equation
- Optimal asset allocations with and without breaks continue to be very different due to differences in the parameter estimates
- Breaks have a very significant effect on asset allocations even under rebalancing

Welfare Costs of ignoring Breaks

Compute certainty equivalent return (CER) for investor who ignores breaks versus an investor who accounts for breaks

- For the dividend yield model, the CER lies between 0.1 and zero percent
- The CER grows to 1.5 percent at short horizons under predictability from the T-bill rate

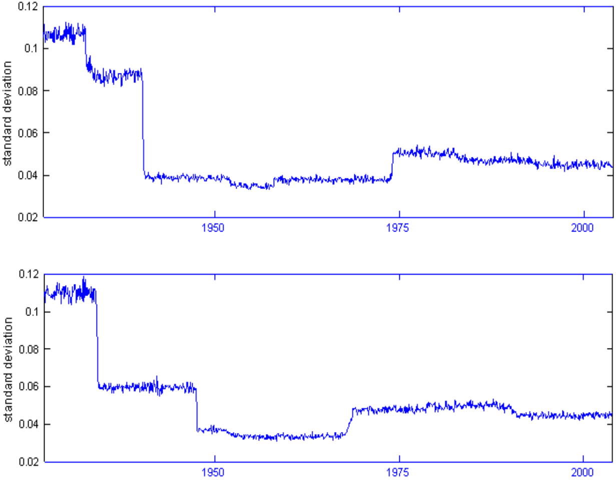
CER under rebalancing and Predictability



Time-varying Volatility of Returns

- Our model allows for breaks to the covariance matrix of returns and is capable of accounting for heteroskedasticity in returns insofar as this coincides with the identified regimes
- The mean value of the standard deviation of returns varies significantly from a level around 10% around the Great Depression to a level near 3-4% in the middle of the sample

Time-variations in Volatility



Conclusion

Our analysis accounts for

1. model uncertainty
2. parameter uncertainty
3. uncertainty about the number and size of historical breaks
4. uncertainty about future (out-of-sample) breaks

Empirical results

- Parameters of standard forecasting models appear to be highly unstable and subject to multiple shifts
- Many of the breaks coincide with important historical events
- Once such breaks are accounted for, the possibility of future breaks has a large impact on the optimal asset allocation