Credit Market Imperfections and Business Cycles

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November 2009

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- Imperfect Credit Markets: Some Examples
- Preview of Main Results
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- Business cycle volatilities in reaction to shocks are much higher

Relation with Macro Literature

- Issue: standard real business cycle (RBC) models rely on productivity shocks to generate business cycles.
- But empirical evidence: the true productivity shocks are usually not large enough to generate quantitatively realistic business cycles (Rebelo 2005)
- Therefore, an amplification mechanism is needed. Typically the mechanism is some types of market frictions.
- Credit Market Imperfection: one example of such a mechanism

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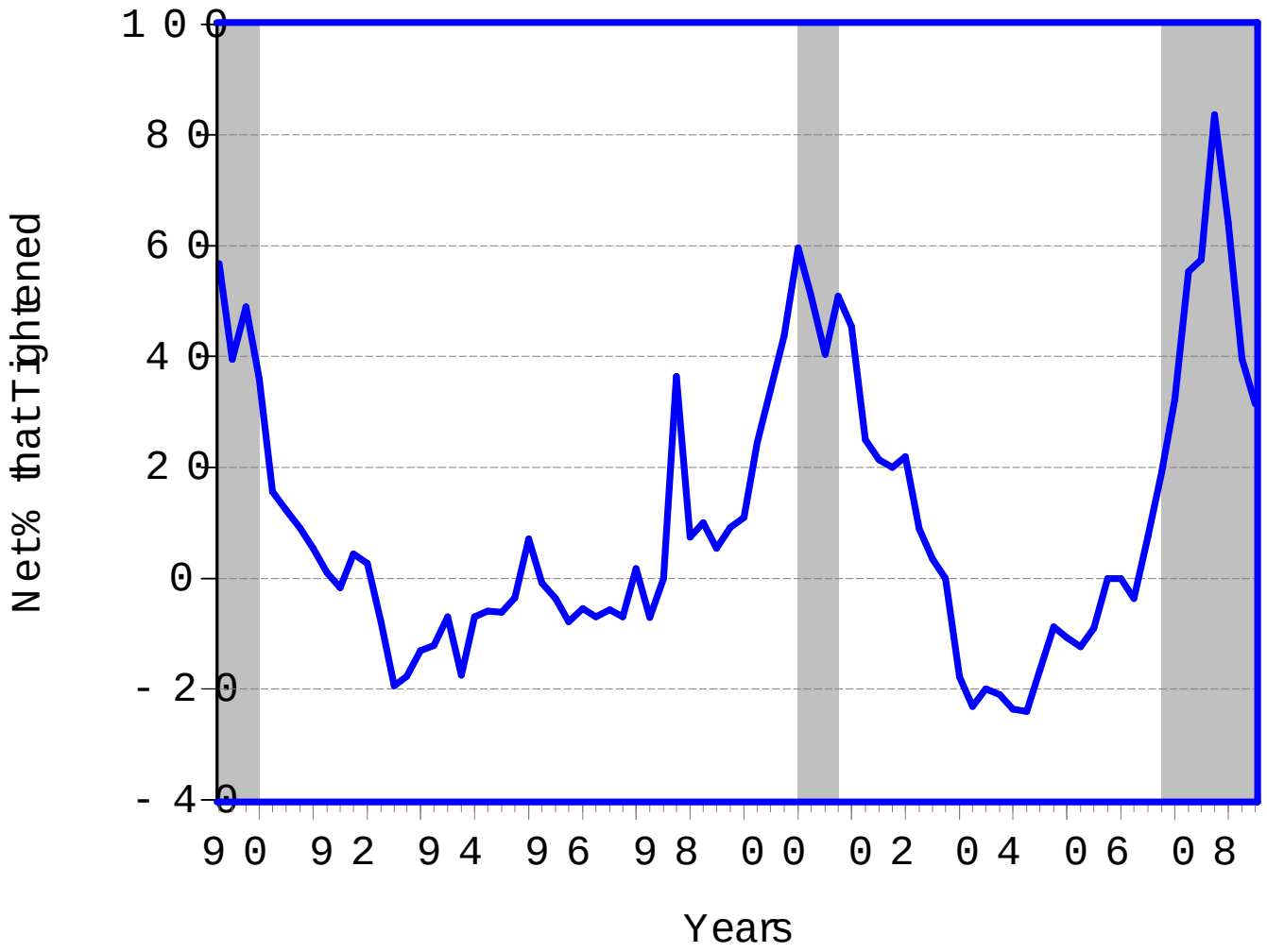
  - shock => net worth => premium => borrowing capacity

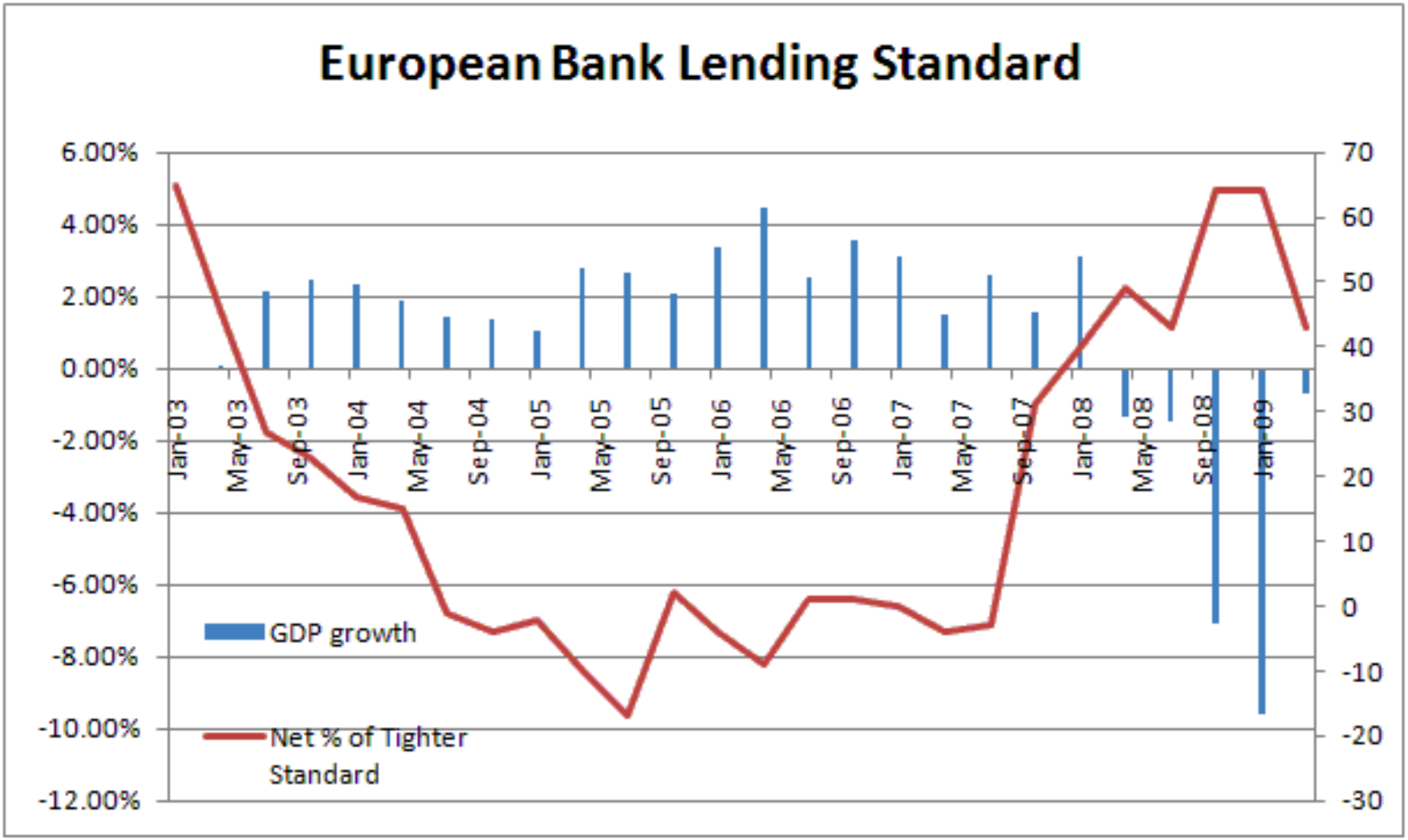
Credit Standards

- "Credit standards" : non-price lending terms specified in the typical bank loans or lines of credit: collateral, loan-to-valuation (LTV) ratio, loan limits, proof of incomes, credit scores, credit ratings, etc.
- Lown and Morgan (2006): Credit standards strongly dominate loan rates in explaining variation in business loans and output.
- Federal Reserve Senior Loan Officer Opinion Survey on Bank Lending Practice: "Over the past three months, how have your bank's credit standards for approving applications for C&I (commercial and industrial) loans or credit lines—other than those to be used to finance mergers and acquisitions—to large and middle-market firms and to small firms changed? Ans: 1) Tightened considerably 2) tightened somewhat 3) remained basically unchanged 4) eased somewhat 5) eased considerably."

FRB Senior Loan Officers Survey



Source: Federal Reserve Board (FRB)



Source: European Central Bank

Quantifying Credit Standards: Maximum LTVs

	1970s	1980s	1990s		1970s	1980s	1990s
Australia	0.70	0.80	0.80	Korea	0.30	0.40	0.40
Belgium	0.65	0.75	0.80	Malaysia	0.65	N/A	0.85
Canada	0.75	0.80	0.80	Netherlands	0.75	0.75	0.75
Chile	N/A	N/A	0.78	New Zealand	0.66	0.80	0.80
Denmark	0.85	0.95	0.80	Norway	0.75	0.80	0.80
Finland	0.80	0.85	0.80	Singapore	N/A	N/A	0.85
France	0.80	0.80	0.80	Spain	0.60	0.80	0.80
Germany	0.65	0.65	0.80	Sweden	0.90	0.95	0.75
Hong Kong	N/A	0.90	0.70	Switzerland	N/A	N/A	0.90
Ireland	0.80	0.90	0.80	Taiwan	0.40	N/A	N/A
Israel	0.50	0.70	N/A	Thailand	0.65	N/A	0.75
Italy	0.50	0.56	0.60	UK	0.81	0.87	0.95
Japan	N/A	0.60	0.55	US	0.80	0.89	0.80

Maximum LTV (Loan-to-Valuation) ratio: the highest mortgage loan that households can obtain from lenders as a % of the value of the property owned.

Source: Almeida, Campello and Liu (2006)

Why LTV less than one? Costly Debt Collection

- Djankov, Hart, McLiesh and Shleifer (2008):
  - 88 developed and less developed countries at January 2006
  - an average of 48% of the collateral value was dissipated in the debt enforcement procedures.
  - Richest 8 countries: average 1.5 years to resolve debt enforcement, and transaction cost = 9% of collateral value
  - Lower middle income countries: 2.8 years and 16%.
- Altman, Brady, Resti and Sironi (2005):
  - 1982-2001: average recovery rate after corporate debt defaults in the US was 37.2%, ranging from 23.4% in 1990 to 62% in 1987.

Preview of Main Results

- Output volatility can be amplified if a borrowing limit is imposed on the agents.
- The increase in output volatility depends on the nature of the borrowing limit.
- Exogenous borrowing limit does not amplify output volatility.
- Endogenous borrowing limit can amplify output volatility, depending on the specification of the LTV.
  - If the LTV ratio is fixed, the amplification is moderate, at most 5%.
  - If the LTV ratio is endogenous (i.e. depending on aggregate output), output volatility can be amplified by up to 39% in the baseline model.
- In the extended model that includes labor-leisure choice, endogenous LTV can still amplify output volatility. The amplification depends on income effect.

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- To preclude Ponzi-schemes, the growth of borrowing is limited:
- $\lim_{j \to \infty} E_t \frac{B_{t+j}}{(1+r)^j} = 0$

Problem of the Agent

$$\max_{C_t, K_{t+1}, L_{t+1}, B_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma} \quad \text{subject to}$$

$$C_t + K_{t+1} - (1-\delta) K_t + Q_t L_{t+1} + B_t R = K_t^{\alpha_1} L_t^{\alpha_2} Z_t + B_{t+1} + Q_t L_t$$

where
C_t: consumption
Q_t: price of land in terms of consumption goods
$\delta \in (0,1)$: depreciation rate
$\sigma > 0$ is the constant coefficient of relative risk aversion.
$\beta \in (0,1)$ is the discount factor

where K_0 , Z_0 and B_0 are given.

and **Borrowing Constraints ...**

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- ,where $\theta \in [0, 1]$ is the LTV ratio

FOC: Euler Equations

Trade-off between consumption and capital acquisition

- $C_t^{-\sigma} = E_t \beta C_{t+1}^{-\sigma} \left(\alpha_1 K_{t+1}^{\alpha_1 - 1} Z_{t+1} + 1 - \delta \right)$ (All Cases)

Trade-off between consumption and land acquisition

- Exogenous Borrowing Constraint:
  - $C_t^{-\sigma} Q_t = E_t \beta C_{t+1}^{-\sigma} \left(\alpha_2 K_{t+1}^{\alpha_1} Z_{t+1} + Q_{t+1} \right)$
- Endogenous Borrowing Constraint (collateral evaluated at Q_t)
  - $C_t^{-\sigma} (1 - \theta) Q_t = E_t \beta C_{t+1}^{-\sigma} \left(\alpha_2 K_{t+1}^{\alpha_1} Z_{t+1} + Q_{t+1} - \theta Q_{t+1} R \right)$
- Endogenous Borrowing Constraint (collateral evaluated at $E_t Q_{t+1}$)
  - $C_t^{-\sigma} (Q_t - E_t \theta Q_{t+1}) = E_t \beta C_{t+1}^{-\sigma} \left(\alpha_2 K_{t+1}^{\alpha_1} Z_{t+1} + Q_{t+1} - \theta Q_{t+1} R \right)$
- If $\theta = 0$, then these cases are the same $=>$ same dynamics

Baseline Model: Competitive Equilibrium

- Definition: a sequence of prices $\{Q_t\}_{t=0}^{\infty}$ and per capita allocations $\{C_t, K_{t+1}, L_{t+1}, B_{t+1}\}_{t=0}^{\infty}$, such that, given the initial allocations of capital goods, borrowing/ lending positions, the loan-to-valuation ratio θ and the world interest rate r, the following conditions hold:

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- Goods market: the goods produced $K_t^{\alpha_1} L_t^{\alpha_2} Z_t$ is equal to the sum of consumption C_t and investment $K_{t+1} - (1-\delta) K_t$.

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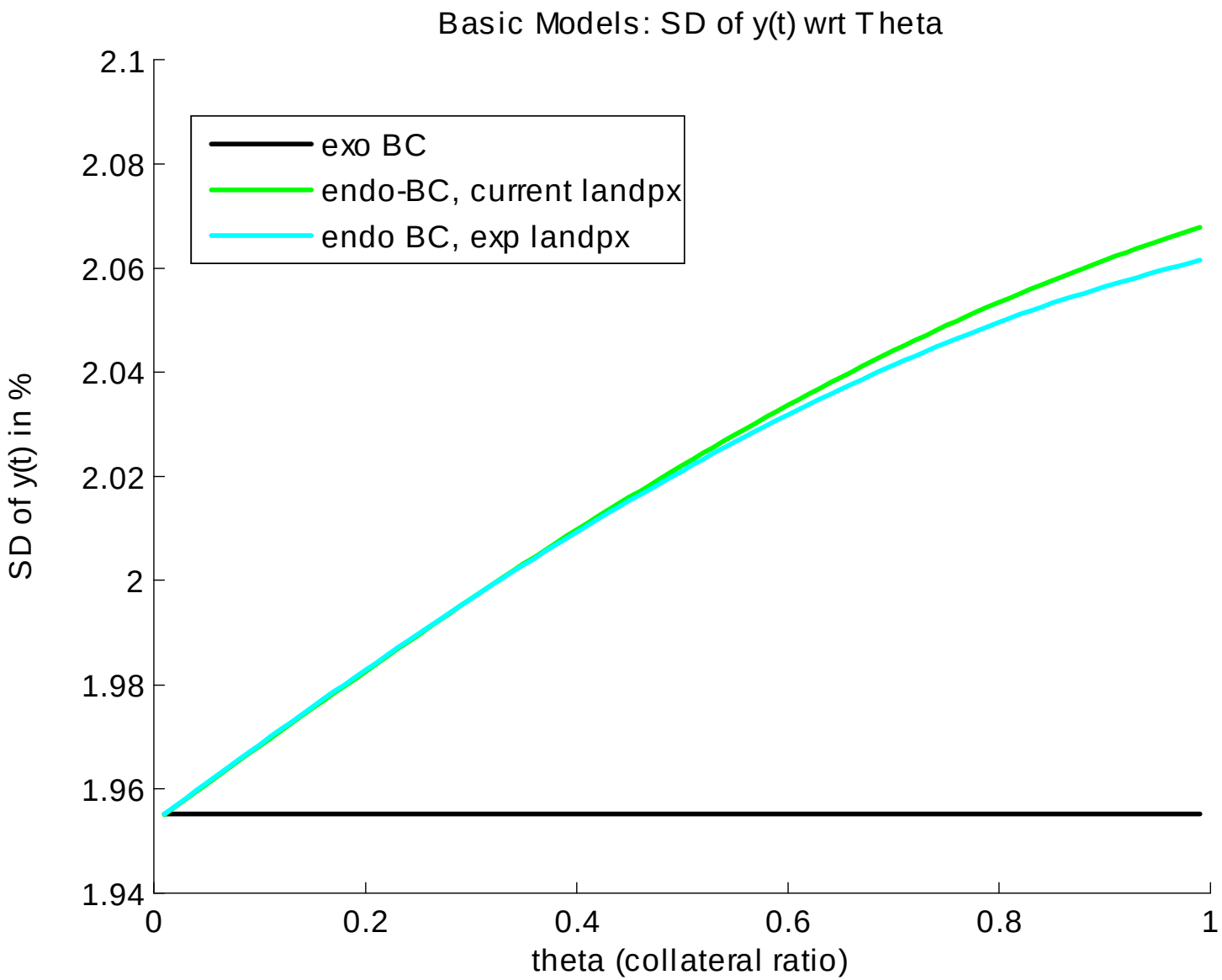
- Goods market: the goods produced $K_t^{\alpha_1} L_t^{\alpha_2} Z_t$ is equal to the sum of consumption C_t and investment $K_{t+1} - (1-\delta) K_t$.
- Land market: the demand for land L_{t+1} is equal to the land supply, which is fixed at 1.

Benchmark Parameter Values

Baseline Model	Description	Benchmark Values
β	discount factor	0.95
σ	inverse of relative risk aversion	1
α_1	capital share of output	0.3
α_2	land share of output	0.1
δ	depreciation rate	0.1
θ	loan-to-valuation ratio (LTV)	1
$\bar{B}$	exogenous borrowing limit	1.2218
r	exogenous interest rate	0.0526
ρ	persistence of tech shock	0.9
σ_ε^2	variance of innovation to tech shock	0.005

Output Volatility: Baseline Model



Volatilities: Baseline Model

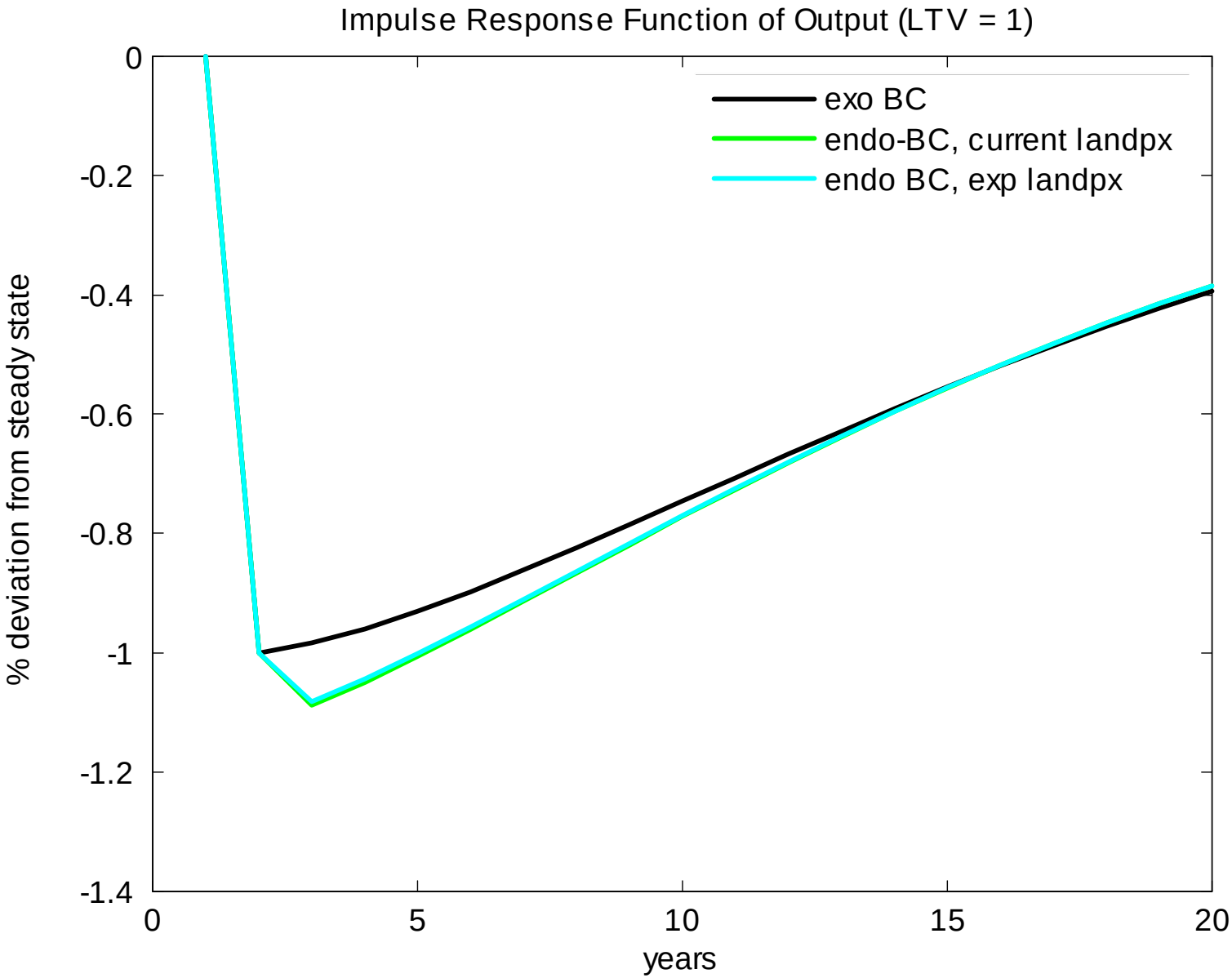
LTV θ	**LTV=0**			**LTV=1**		
Case	1	2a	2b	1	2a	2b
σ_{y_t}	1.96	1.97	1.97	1.97	2.07	2.06
σ_{c_t}	1.39	1.39	1.39	1.57	1.54	1.54
σ_{i_t}	3.86	3.87	3.87	3.96	5.34	5.24
σ_{b_t}	n.a.	1.42	1.41	n.a.	0.89	0.83
σ_{q_t}	1.43	1.42	1.42	1.52	0.89	0.89
σ_c/σ_y	0.7	0.7	0.7	0.8	0.8	0.8
σ_i/σ_y	2.0	2.0	2.0	2.0	2.6	2.6
σ_b/σ_y	n.a.	0.7	0.7	n.a.	0.4	0.4
$\sigma_{q_t}/\sigma_{y_t}$	0.7	0.7	0.7	0.8	0.4	0.4

Case 1 Exogenous Borrowing Constraint

Case 2a Endogenous Borrowing Constraint, collateral eval. at current px

Case 2b Endogenous Borrowing Constraint, collateral eval. at future px

Impulse Response Function



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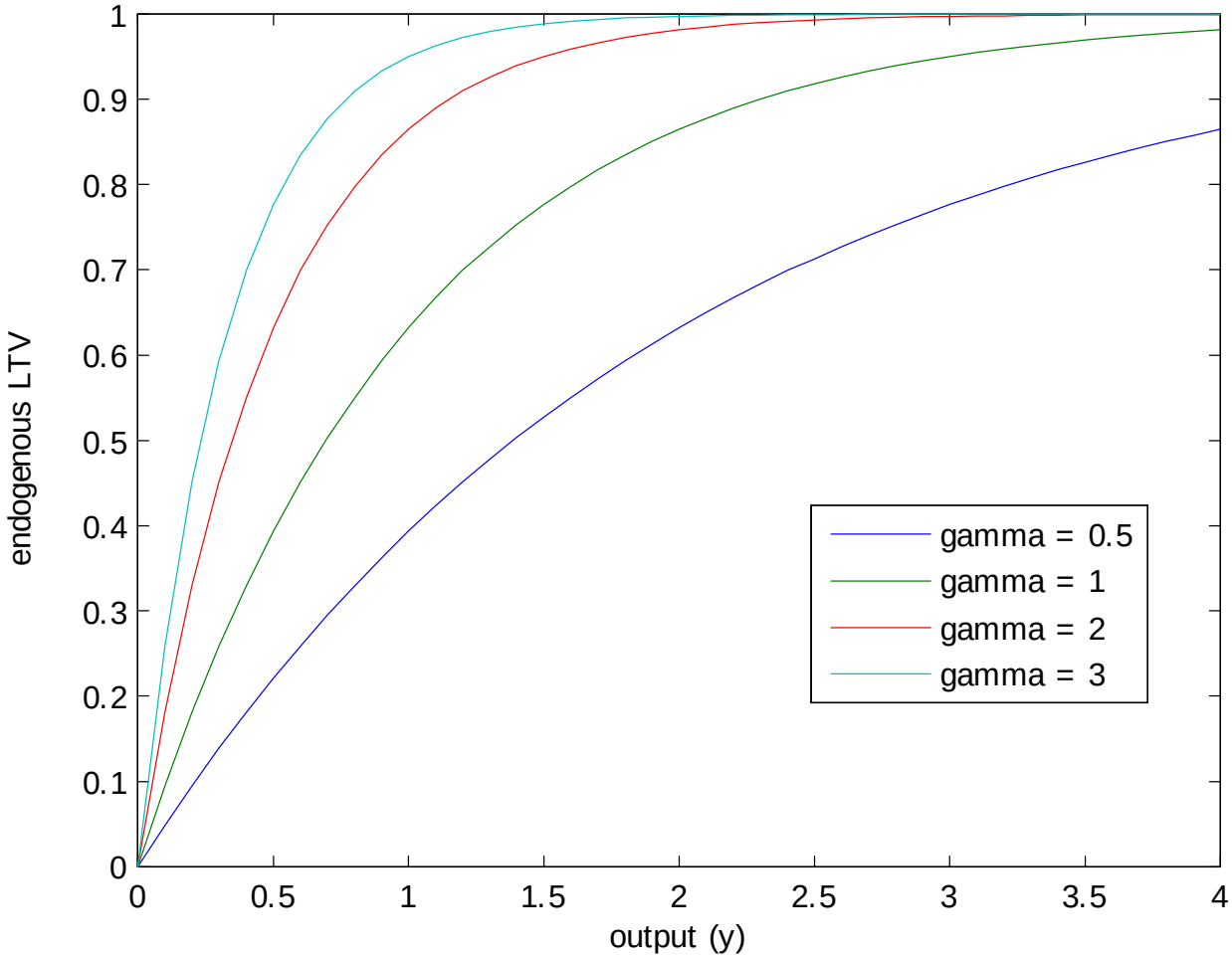
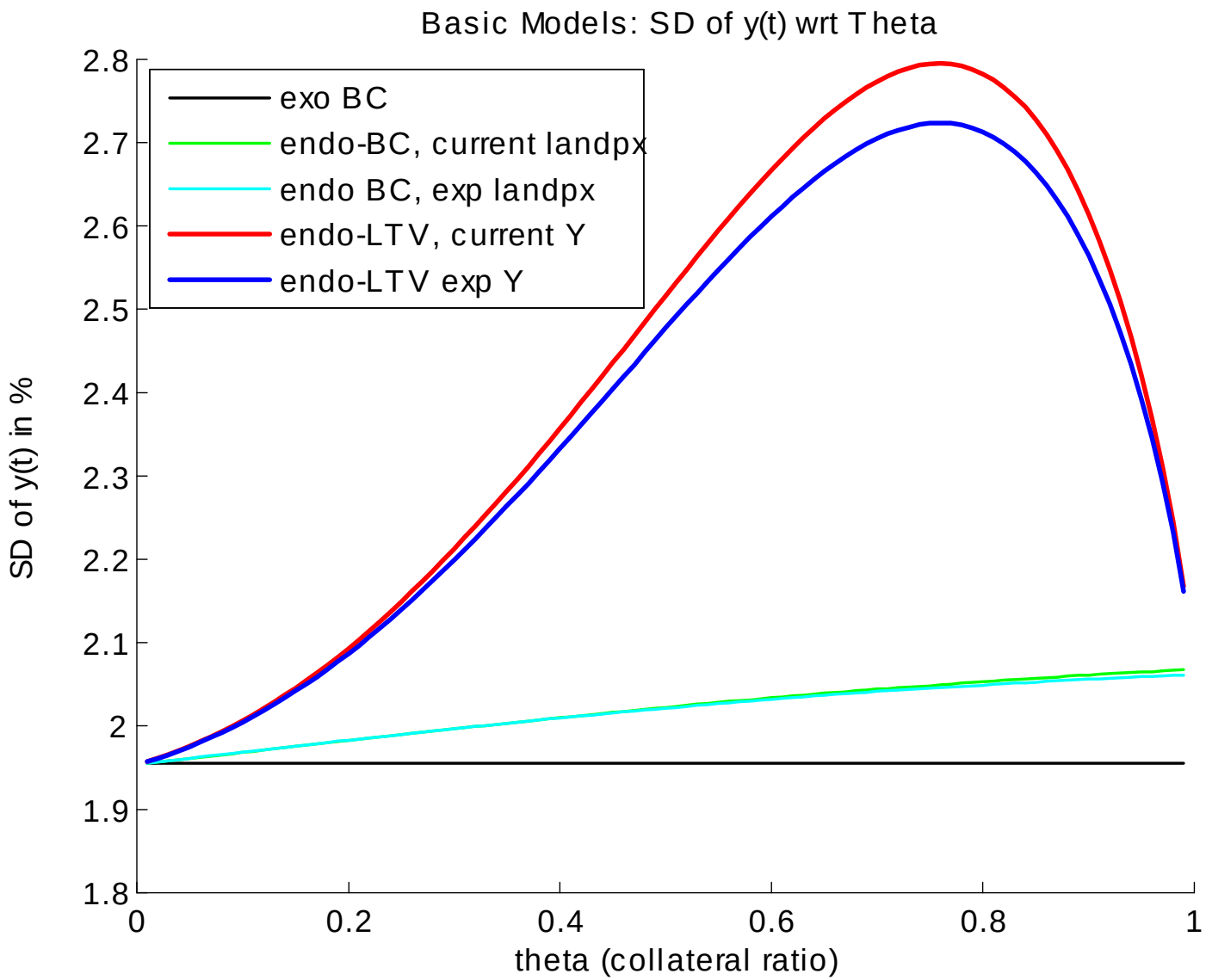


Output Volatility: Baseline Model



Volatilities: Baseline Model, LTV=0.75

Case	1	2a	2b	3a	3b	3b vs. 1
σ_{y_t}	1.97	2.05	2.05	2.79	2.72	**1.39**
σ_{c_t}	1.52	1.50	1.50	1.49	1.47	0.97
σ_{i_t}	3.93	4.96	4.93	21.90	21.35	5.43
σ_{b_t}	8.07	1.03	0.99	8.67	7.98	0.99
σ_{q_t}	1.50	1.03	1.03	8.16	7.50	5.01
σ_{θ_t}	n.a.	n.a.	n.a.	1.27	1.22	n.a.
σ_c/σ_y	0.8	0.7	0.7	0.5	0.5	
σ_i/σ_y	2.0	2.4	2.4	8.0	8.0	

Case 1 Exogenous Borrowing Limit $\bar{B}$

Case 2a Endogenous Borrowing Limit, collateral evaluated at Q_t

Case 2b Endogenous Borrowing Limit, collateral evaluated at $E_t Q_{t+1}$

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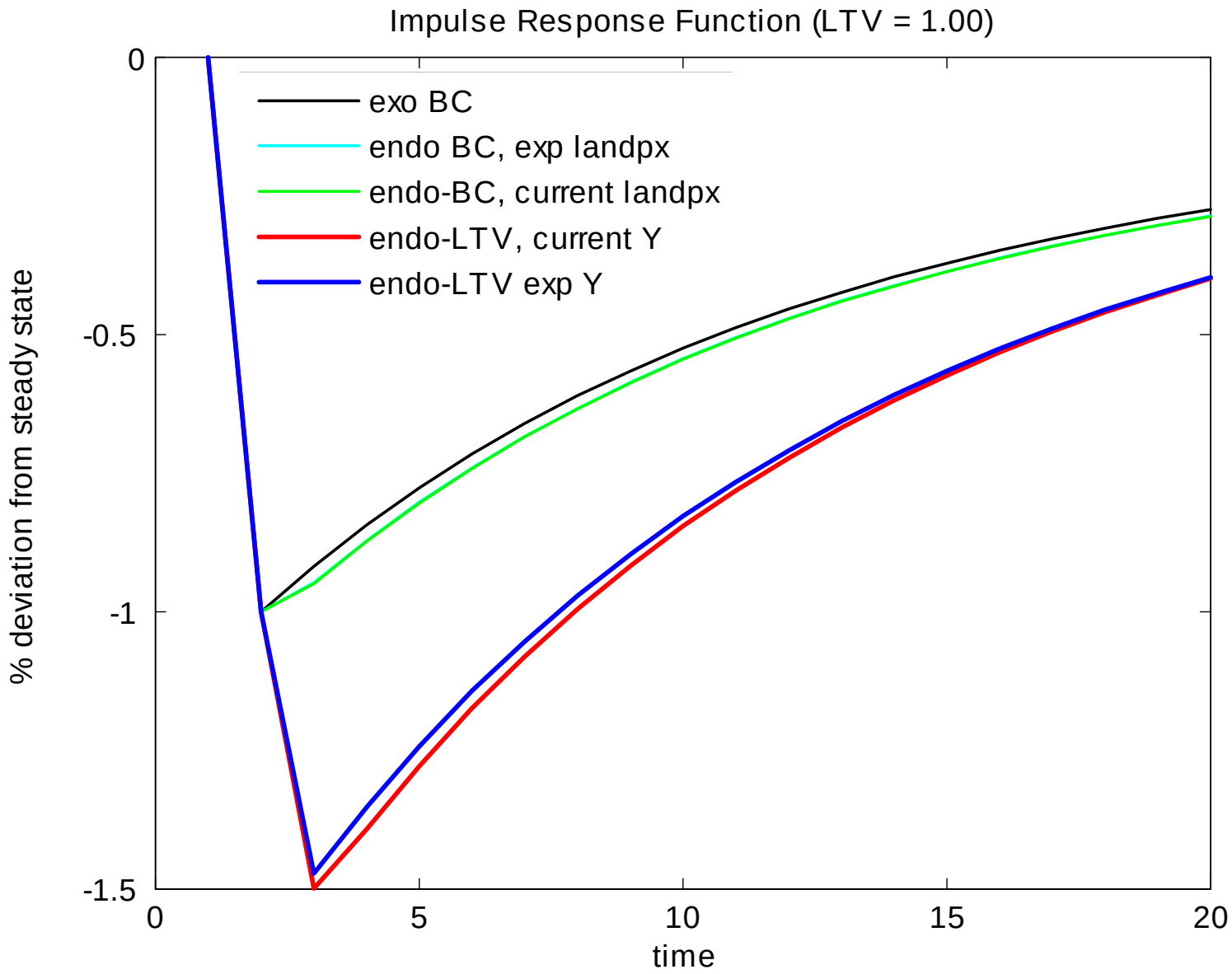
Impulse Response Function



Extension: Labor-Leisure Choice

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- The marginal rate of substitution $N^{\psi-1}$ only depends on labor and not on consumption. In other words, the income effect brought by a change in wage rate is precluded.

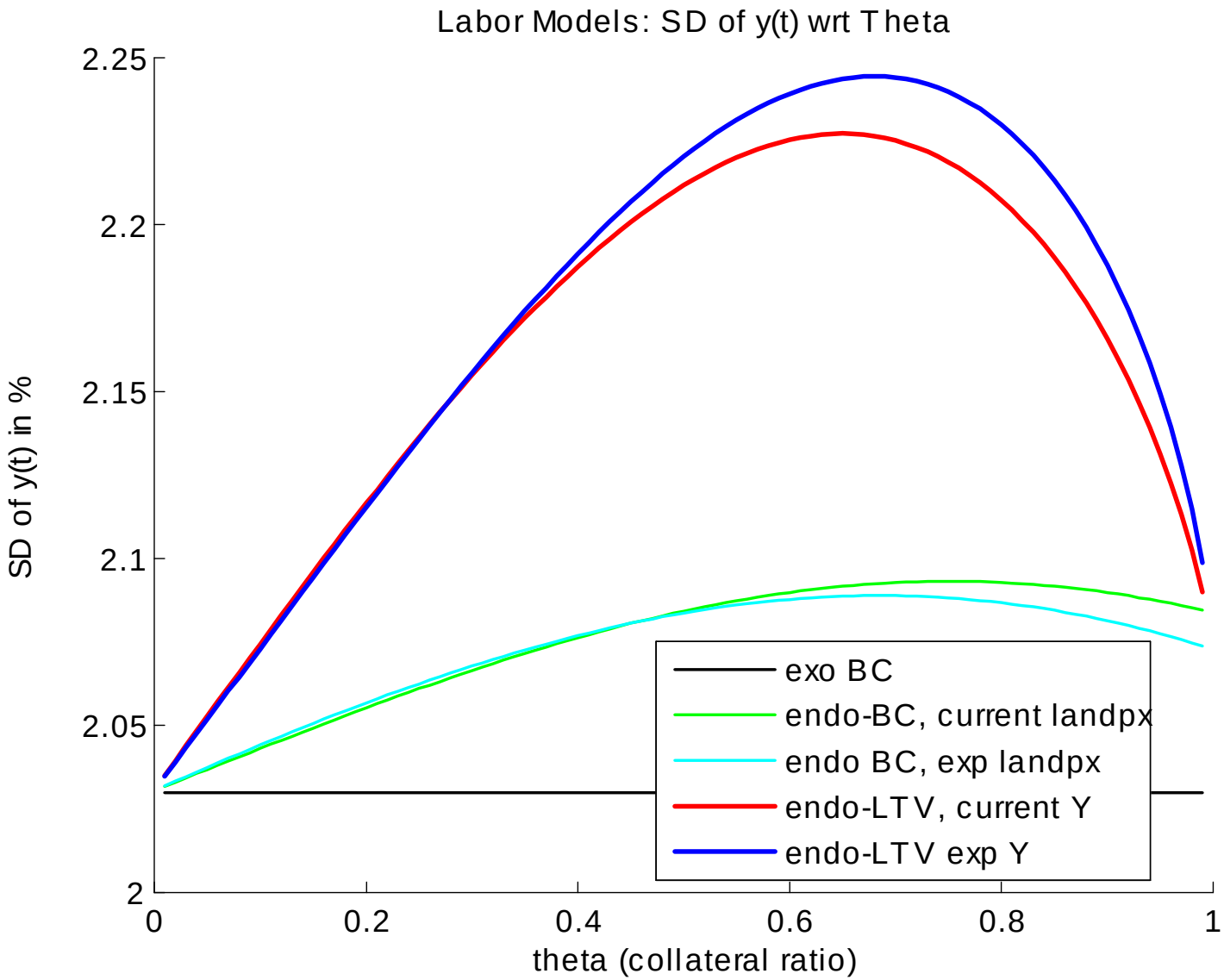
Calibration of the Labor-Leisure Choice Model

Standard Labor Model		Baseline Values
α_3	labor share of output	0.6
χ	inverse of labor supply elasticity	1
A	disutility of labor	7
GHH Labor Model[1]		
ψ	parameter of utility function	1.4555
v	parameter of utility function	2

[1] from Schmitt-Grohe and Uribe (2003)

Output Volatility

Standard Preference



Volatilities: Labor Model

Standard Preference, LTV=0.7

Case	1	2a	2b	3a	3b	3b vs. 1
σ_{y_t}	2.03	2.14	2.14	2.28	2.30	**1.13**
σ_{c_t}	1.86	1.90	1.90	1.96	1.96	1.06
σ_{i_t}	4.02	5.56	5.52	7.55	8.32	2.07
σ_{n_t}	0.26	0.21	0.21	0.19	0.21	0.79
σ_{b_t}	n.a.	1.11	1.05	2.28	2.35	n.a.
σ_{q_t}	1.75	1.11	1.10	1.19	1.19	0.68
σ_{θ_t}	0.00	0.00	0.00	1.18	1.24	n.a.

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Correlation with Output: Labor Model

Standard Preference, LTV=0.7

Case	1	2a	2b	3a	3b
$corr\,(c_t, y_t)$	0.96	0.99	0.98	0.99	0.99
$corr\,(i_t, y_t)$	0.90	0.74	0.74	0.59	0.53
$corr\,(n_t, y_t)$	0.45	0.65	0.64	0.85	0.84
$corr\,(b_t, y_t)$	n.a.	1.00	1.00	1.00	0.99
$corr\,(q_t, y_t)$	0.97	1.00	1.00	1.00	0.99
$corr\,(\theta_t, y_t)$	n.a.	n.a.	n.a.	1.00	0.99

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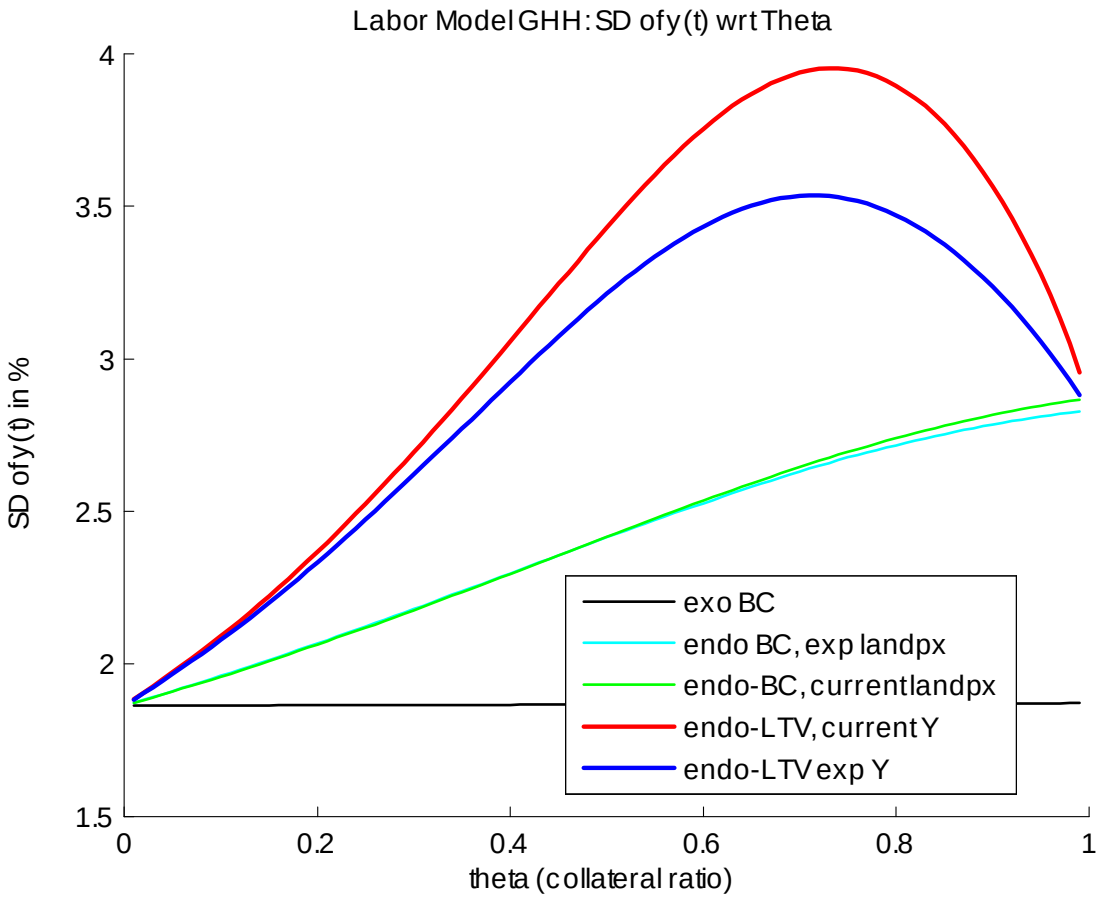
Output Volatility: Labor Model

GHH Preference



Volatilities: Labor Model

GHH Preference, LTV=0.7

Case	1	2a	2b	3a	3b	3b vs. 1
σ_{y_t}	1.73	2.49	2.51	3.70	3.28	**1.89**
σ_{c_t}	1.48	1.95	1.96	2.69	2.44	1.64
σ_{i_t}	1.61	3.41	3.44	6.11	5.91	3.68
σ_{n_t}	1.03	1.49	1.50	2.21	1.97	1.90
σ_{b_t}	n.a.	1.49	1.53	3.75	2.96	n.a.
σ_{q_t}	1.66	1.52	1.53	2.13	1.91	1.15
σ_{θ_t}	n.a.	n.a.	n.a.	1.71	1.10	n.a.

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- The relationship between the endogenous LTV ratio and output volatility is typically non-monotonic. When the steady-state value of the LTV is calibrated to approximately 0.75, the output volatility reaches its maximum.
- In the model with labor-leisure choice, the endogenous LTV still amplifies output volatility. The amplification depends on the magnitudes of income effect.

Appendix 1: Steady State of Baseline Model

- $K = (\beta\alpha_1)^{\frac{1}{1-\alpha_1}} \left[1 - \beta\left(1-\delta\right)\right]^{-\frac{1}{1-\alpha_1}}$
- $Y = K^{\alpha_1}$
- $Q = \frac{\alpha_2 K^{\alpha_1}}{r}$
- $B = \theta Q$
- $C = K^{\alpha_1} - \left(1-\delta\right)K - Br$
- $\theta = 1 - \exp\left(-\gamma Y\right)$

Note: For models with labor-leisure choice, closed-form solutions for the steady state are not available.

Log-Linearized System: Baseline Model

Case: Endo-BC, expected landpx, endo-LTV on current output

$$\sigma c_t = E_t\left[\sigma c_{t+1} + (1-\alpha_1)\left[1-\beta(1-\delta)\right]k_{t+1} - \left[1-\beta(1-\delta)\right]z_{t+1}\right]$$
$$\sigma(1-\theta)c_t - q_t = E_t\left[\sigma(1-\theta)c_{t+1} - \beta\alpha_1 rk_{t+1} - \beta r z_{t+1} - \beta q_{t+1}\right]$$
$$\left[\alpha_1 + s_k(1-\delta)\right]k_t + z_t - s_c c_t - s_b R b_t = s_k k_{t+1} - s_b b_{t+1}$$
$$b_{t+1} - E_t q_{t+1} = 0$$
$$\tilde{\theta}_t = \left(\tfrac{1-\theta}{\theta}\right)\gamma Y\left(\alpha_1 k_t + z_t\right)$$
$$\rho z_t = z_{t+1} - \tilde{\varepsilon}_{t+1}$$

, where $s_k = K/Y$, $s_c = C/Y$ and $s_b = B/Y$

, where x_t or $\tilde{x}_t$ is % deviation from steady state, $x = c, k, b, q, z, \theta$