Assessing Some Models of the Impact of Financial Stress Upon Business Cycles**

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1 Introduction

The global financial crisis (GFC) has led to a consideration of how one models the connections between financial stress and the business cycle. Quite a few models and analyses have emerged that aim to elucidate these relationships e.g. Gilchrist et al (2009), Christensen and Dib (2010), Zheng et al (2009), Gertler and Kiyotaki (2010), Greenlaw et al (2008), Liu et al (2009) and Benes at al (2009). These papers deal with a number of issues such as credit availability, collateral and the role of "animal spirits" in initializing and propagating cycles. Questions which naturally arise concern the size of the financial effects upon business cycles, how models can be designed with financial-real linkages and whether such models might be usefully employed to predict recessions. This paper aims to provide a framework to look at such questions.

In section 2 of the paper we review some of the major contributions to the literature that have been concerned with extended financial/real linkages i.e. over and above those coming from interest rates and monetary factors that have always been present in conventional models. We outline the various strategies that have been employed, discussing them according to whether they influence the supply or demand for credit (debt) by agents. In section 3 we subsequently select two of these models - Gilchrist et al (2009)(termed GOZ hereafter) and Iacoviello (2005) that seem to have enjoyed some success, and ask what financial factors in these models contribute to the origin and propagation of recessions. Neither paper looks directly at these questions. GOZ focus upon the decomposition of the level and volatility of the transitory component of GDP into contributions from some identified shocks while Iacoviello reports impulse responses under various scenarios. In contrast our approach is to ask what the models say about the length, duration and other characteristics of the business cycle with and without financial factors. In doing so it is necessary to assemble some "stylized facts" relating to recessions and the role of credit. These are drawn from a number of sources, but principally from the work of the IMF reported in a number of issues of the *World Economic Outlook.* Because many of the measures the IMF use are unfamiliar to the bulk of the applied macroeconomics literature, in that they emphasize turning points in economic activity, we need a method to locate those. For this purpose we adopt the method set out in Harding and Pagan (2002), which has close connections with the NBER dating of business cycles. We find that, while the models replicate some of the features set out in the IMF work, it is clear that neither paper is able to fully capture the effects of financial fragility upon recessions. Against this, in some instances the effects are stronger than might be considered reasonable.

2   Model Designs

It is useful to think about models to handle financial conditions in two stages. First, some "base" model needs to be specified that details how expenditures are determined. Second, this is augmented with a sub-model involving the financial sector and showing how the latter impacts upon financial variables in the base model. This augmentation generally involves the introduction of a financial intermediary (FI) which responds to the demand for credit by supplying it. Sometimes the FI is introduced explicitly and a detailed

description is given of its operation. At other times only a simple summary of what governs the demand for and supply of credit is provided. Throughout we will use the terms "debt" and credit interchangeably, except when it is better to focus upon one side of the balance sheet than the other.

2.1 The Base Macro-economic Model

The selection of a base model will be controversial. Some e.g. Muellbauer (2010) seem to have a preference for what has been termed second generation (2G) models in Fukacs and Pagan (2010). These provide a set of equations describing macroeconomic outcomes which are mostly not designed to be internally consistent in relation to the decisions being made. This may not be a bad thing, as data issues and the need to choose functional forms may mean that a fully consistent model is trying to describe outcomes that we don't observe. However, ultimately it seems to be more of an argument for modifying the structure of some "ideal representation" than completely discarding it.

A popular base model for quite a few macro-economic investigations has been that in Smets and Wouters (2007). This describes the determination of consumption, investment, wages, inflation, monetary policy and the supply side. There are clearly missing items in the model that are likely to be important to macro-economic outcomes e.g. there is a strictly exogenous government expenditure variable. Each of the structural equations for consumption, investment, the price of capital, inflation and wages have effects from expectations about the future as well as past events (z_{t-1}) and other model variables (w_t) i.e. they have the structure

$$z_t = \phi_1 z_{t-1} + \phi_2 E_t z_{t+1} + \phi_3 w_t.$$

In some cases $\phi_1 + \phi_2 = 1$ and, in others, the sum is the discount factor for consumers. Identities are also present and supply is constrained by a production function.

2.2 The Demand For Credit

To augment the base model it is useful to ask what items of expenditure the financial sector would impact upon. Four broad areas are suggested.

1. Fixed Investment by firms.

2. Residential investment by households.

3. Consumer durable expenditures by households.

4. Consumption of goods and services by households.

We review work on how financial conditions have been introduced so as to have an impact upon the expenditures above. Mostly, base models do not specifically distinguish these categories, dealing only with aggregate investment and consumption. Moreover, the four types of expenditures given above do not really exhaust the potential list. For example, there is an extensive use of credit for the financing of inventories, something which showed up strongly in the automobile market during the GFC, where dealers were unable to get credit to hold the vehicles needed to be on display in their saleyards. Even in more normal times inventories need to be financed for the period of time between delivery and sale. Trade credit is also needed in order to pay for raw materials and even labour. But the empirical work on these latter elements has been much less than that on the four areas listed above.

2.2.1 Fixed Investment

This is by far the best developed and involves the financial accelerator. It comes in two versions and implicitly involves a financial intermediary. In the first version, the financial intermediary can be thought of as taking deposits from the household sector and then lending to the business sector that is in need of credit to finance fixed investment. Because there has to be some way of distinguishing between the loaning and credit-using sectors in the augmented model, it is conventional to assign different discount rates to the agents in each of the sectors. The credit-using agents are taken to be less patient than the lending agents. Essentially, this serves to produce two interest rates - one that is connected to the preferences of the lending agents (and which is typically taken to be the policy interest rate) and another that is the rate charged on loans by a financial intermediary.

In the second version there is no precise description of the operations of the intermediary, and its presence is simply summarized by an *external finance premium* charged over the policy rate. Thus credit comes at a cost that is a premium over internal financing resulting from the fact that there is asymmetric information between the borrower and lender. The external finance premium therefore governs the amount of credit that can be obtained,

and so it is necessary to model its determinants. Mostly the premium is simply taken to be increasing in the degree of leverage or decreasing in net worth of the borrower. Therefore increasing amounts of credit are costly, and this impacts on real and nominal quantities. Because the emphasis in this extension is on the demand for credit the external premium equations are often augmented with a shock that is intended to capture variations in supply i.e. the equation is more of a reduced form than a structural equation. This is the strategy used in Gilchrist et al (2010).

Because there are no directly observed series on the external finance premium, either a proxy needs to be constructed or it needs to be left unobservable. Gilchrist et al (2010) take data on the spreads between medium risk long-maturity U.S. corporate bonds and the 10-year Treasury yield to be a measure of this premium. They also utilize data on the leverage ratio of US firms and this series helps to estimate the elasticity of the external finance premium to the leverage ratio.

2.2.2   Residential Investment

The events preceding the GFC led to an interest in the role of housing investment in the business cycle. Indeed some see it as the key to the latter e.g. Leamer (2005). But inspection of the cycle data has to cast doubt on such a position. Looking at the turning points in the series on quarterly real gross residential investment one finds that the duration of the U.S. residential investment cycle is quite short, on average around 12 quarters, which is almost half what the business cycle length is. This outcome is easily explained by an examination of the data. The growth rate in residential investment is around half that of GDP, while the volatility is about five times as high. Thus, getting negative growth in residential investment is relatively easy, and such growth often results in a turning point in the series. These differences mean that, even if one had the knowledge that residential investment was in a recession, the probability of predicting an NBER-defined recession would just rise to .25 from its unconditional probability of .15. Thus it is hard to subscribe to Leamer's viewpoint that *housing is the business cycle*. This is not to deny that it has a role, but it is not an exclusive one.

It should be observed that Leamer carried out a good deal of massaging of the data before reaching his conclusion. This included smoothing the residential investment data to eliminate some of its peaks and troughs, so that these more closely resembled those of GDP, and eliminating the difference in

the growth rates of the two series. This has a very strong effect as the average growth rate is a key determinant of cycle characteristics - see Harding and Pagan (2002). Furthermore, the smoothing was done by the use of a kernel regression, with the regressor being a time trend. This means that, at time t, one would need to know future data on residential investment in order to compute what the value of the smoothed quantity would be at t i.e. in a predictive context one wouldn't even know at that time whether there was a residential investment slump.

A number of papers have appeared that augment a base model to determine residential investment. Davis and Heathcote (2005) have 3 production sectors while Iacoviello and Neri (2009) augment the standard macro model by adding a second production sector. In the former work the first sector produces consumption and investment goods with capital and labour, while the other creates new houses using capital, labour and land. Some of the calibrated parameters they choose look odd e.g. they would " imply a ratio of non-residential investment to GDP around 27 per cent", which is far larger than in the data, unless one is augmenting private non-residential investment with structures and government investment. The very large rises in housing prices in the late 1970s and in the 2000s are not well explained by Iacoviello and Neri's model, indicating that some extra features would be needed. Apart from the sectorial disaggregation the model features the idea that housing could serve as a collateral asset to finance either investment or consumption, something introduced in Iacoviello(2005), and this is dealt with later in the sub-section on the supply of credit.

Benes et al (2009) is notable for augmenting a base model to capture housing investment in an open economy.[1] Credit is required by the household sector to purchase housing and the financial intermediary raises funds in a foreign market. These are then loaned out to the domestic market. Consequently, the external premium reflects the difference between the domestic and foreign interest rates.

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[1]Their base model is not strictly the Smets and Wouters one but the principles underlying it are the same.

2.2.3 Consumption

In the base macro-economic model the consumption Euler equation (with habit persistence) takes the form (after log-linearization)

$$c_t = \alpha E_t c_{t+1} + (1 - \alpha) c_{t-1} + \theta r_t \tag{1}$$

where r_t is the real rate of interest and small letters denote log departures from a steady state position. Preference shocks may also appear in the structural equation. (1) can be written as

$$\Delta c_t = \frac{\alpha}{1 - \alpha} E_t \Delta c_{t+1} + \frac{\theta}{1 - \alpha} r_t. \tag{2}$$

The term $E_t \Delta c_{t+1}$ in the base model varies with its nature. In general there will be a large number of influences on expected future consumption growth. When the model is extended to incorporate financial influences the number of factors would grow.

Aron et al. (2010) take an empirical approach to assessing how effectively credit might affect consumption. In their work the $E_t \Delta c_{t+1}$ term in (2) does not appear but is replaced by a number of factors involving liquid assets, housing wealth etc. The coefficients on these terms are made functions of a credit conditions index that is constructed differently for the different countries they are examining, but which essentially extracts a common factor from many series chosen to reflect the tightness of credit to some degree. The difference between models that have consumption growth strictly generated by (2), and the relatively unrestricted Aron et al. (2010) specification, might be thought of as revolving around the weights assigned to the variables used in constructing $E_t \Delta c_{t+1}$. Of course the credit conditions index used by Aron et al. is generally constructed from information that is not in the augmented model, but series encapsulating the information could be employed when estimating it by adding them to the observation equations relating to the unknown external finance premium. If a number of series representing credit conditions are added a common factor among them would then be extracted. Another difference is that the base model described above is linear in logs, and so there would be no interaction terms with whatever is used to represent financial stress in the model. Again this might be emulated by performing a second order approximation for the base model, as that will produce interaction terms involving covariances.

2.3 The Supply of Credit

Credit could be rationed. There is no doubt that this was a primary financial mechanism in the models of the 1960s and 1970s, as it reflected the regulated financial markets then in operation. Since that time however the amount of credit supplied by FIs has been more endogenously determined, although some constraints still operate, reflecting asymmetric information. In particular it is often assumed that credit is only supplied if there is an adequate amount of collateral put up by the borrower. This serves to make credit an endogenous variable and not just a rigid constraint. Collateral could be any asset which serves that purpose, such as the capital stock, but mostly a new asset is introduced that is demanded by both entrepreneurs and households. Entrepreneurs use the asset in production, and so it has a role in producing output as well as facilitating the acquisition of credit. Sometimes this asset is referred to as "housing" or "land", since the main component of the value of a house is generally the land value. Households consume housing services and it is often the case that small businesses use mortgages on the proprietor's house as a way of arranging credit. As the price of this collateral asset rises greater quantities of credit can be raised. So it is not an external finance premium that regulates the amount of credit available but rather the price of the collateral asset (given the rate of return to capital). Thus a key informational variable will be something like a loan to value ratio. This need not be fixed (although it is in models such as Iacoviello (2005)) and could be allowed to vary in a stochastic way, although this would probably require a description of how the ratio would be set by a lending institution. Broadly, this is the mechanism at work in Iacovello (2005). It could be desirable to incorporate both the external finance premium and collateral into a single model, and this might be done by having the external finance premium depend on the extent of collateral[2].

   Iacoviello (2005) has some quantitative work on this. Liu et al (2009) also deal with it, although in their case the empirical work might be regarded as more problematic, because their model has unconventional elements e.g the degree of impatience of the entrepreneurial sector is not fixed and varies stochastically as a unit root process. As well, there are two technology related shocks, one describing the general level of technology and the other being connected to an investment-specific technology. These are also taken to be

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[2]In a sense it does that already since net worth affects the external premium, but this is essentially treating capital as the collateral asset.

unit root processes, and so there are three permanent components in Liu et al.'s model. This contrasts with the assumption in Smets and Wouters that technology is a stationary stochastic process around some deterministic trend. There is much to be said for the unit root in general technology. Moreover, the relative prices for certain types of investment seems to behave in a non-stationary way, and this may lead to the need for a unit root in technology in that sector, so as to generate a permanent component in relative prices. Such a strategy has been employed by Fisher (2006), among others. However, it is not clear whether this is a good solution for aggregate investment.

Two other ways of modelling the supply of credit by FIs should be mentioned. Gertler and Kiyotaki (2010) have many financial intermediaries which are aggregated. This serves to provide both a retail market for funds and a wholesale (inter-bank) market. Because one can observe data on the inter-bank market this extension looks promising for empirical work. Another important feature that might need to be captured in models was pointed out by Greenlaw et al. (2008). They effectively observed that the credit supplied by financial institutions would likely vary with the Value at Risk (VaR) of their portfolio, as that has become the standard method of determining the limit on the amount of loans that can be supplied. Because the VaR is based on the probability of returns being less than a given value this will rise in a recession, and so the "credit multiplier" would be smaller.

3   Evidence of Financial Effects on Aggregate Activity

What is the evidence concerning the impact of financial factors upon the aggregate level of activity? Here we exclude questions relating to the impact of the short-term interest rate as these generally appear in the base model. Instead we ask about the evidence on the impact of credit conditions upon aggregate activity. A good summary of this evidence has been provided in sources such as International Monetary Fund (2009), and here we select five conclusions from that document. There are more 'stylized facts" but these seem a useful starting point.

1. In the first two years of an expansion after a financial crisis credit grows quite weakly, much more weakly than output does.

2. Restrictions on the supply of credit have a significant impact on the strength of the recovery. Here strength is measured as the cumulative output growth one year after the expansion begins.

3. The probability that an economy will stay in a recession beyond a certain number of quarters is higher when the emergence of a recession was accompanied by a financial crisis. A crude interpretation of this would be that recessions with a financial crisis are of longer duration.

4. The probability of a recession should increase markedly once the external finance premium exceeds some "crisis level".

5. Annual output growth can be predicted by utilizing a measure of financial stress. Moreover, real investment growth shows even greater predictability. The latter seems to imply that the effects of financial factors will be greater on investment than on aggregate economic activity. In particular the cycle in investment expenditure should be more closely related to credit conditions.

To look at these outcomes in the context of a model one needs a measure of financial stress. In the IMF work the dates of financial crises were taken to be those identified by Reinhart and Rogoff (2008), who used a "narrative" approach to find them. Here we need a measure that can be generated by any augmented model. Because the financial stress measure aims to measure the extra costs that firms have to encounter if they are required to borrow, one measure of this would be the size of the external finance premium. Thus, ideally, one would define a crisis as occurring when the finance premium gets above a certain level but, as this is unlikely to be easy to determine, we simply investigate relationships as the level of the premium rises.

The features noted above require that one locate turning points in the level of economic activity before they can be computed i.e. to locate the dates when an expansion or a recession started. For this purpose we use the BBQ program, which is a quarterly version of the method for locating turning points set out in Bry and Boschan (1971). The program is described in Harding and Pagan (2002).[3]

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[3]A modified version written by James Engel is used and is available at http://www.ncer.edu.au/data/

4 Two Models with Financial-Real Linkages

4.1 The Gilchrist et al (2010) (GOZ) Augmentation of the SW Model

As outlined in the previous section there are many items that might be influenced by credit and many models that might be constructed to elucidate the interaction between credit and business cycles. A large model capturing all the possibilities might be desirable, but in this paper we focus upon some suggested ways of capturing a number of these linkages. The first model we examine, due to Gilchrist et al. (2010), incorporates the effects of credit upon fixed investment, but does not have any specific role for collateral. It uses the Smets-Wouters model as the base model and then augments it with four equations

$$E_t \tilde{r}_{t+1}^K = \frac{1-\delta}{\bar{R}_K + (1-\delta)} E_t \tilde{q}_{t+1} + \frac{\bar{R}_K}{\bar{R}_K + (1-\delta)} E_t \widetilde{mpk}_{t+1} - q_t \quad (3)$$

$$\tilde{s}_t = E_t \tilde{r}_{t+1}^K - (\tilde{r}_t - E_t \tilde{\pi}_{t+1}) \quad (4)$$

$$\tilde{s}_t = \chi(\tilde{q}_t + \tilde{k}_t - \tilde{n}_t) + \varepsilon_t^{fd} \quad (5)$$

$$\tilde{n}_t = \frac{\bar{K}}{\bar{N}}(\tilde{r}_t^K - E_{t-1}\tilde{r}_t^K) + E_{t-1}\tilde{r}_t^K + \theta \tilde{n}_{t-1} + \varepsilon_t^{NW}, \quad (6)$$

where the over-bar indicates a steady state value, r_t^K is the rate of return on capital, q_t is Tobin's Q, mpk_t is the marginal product of capital, s_t is the external finance premium, K_t is the capital stock, N_t is entrepreneurs' net worth and tildes indicate log (level) deviations from steady state. Of the coefficients δ is the depreciation rate of capital and θ is the survival rate of entrepreneurs. The first term in (6) is the leveraged return to entrepreneurs and the second is the cost of debt to them. The shocks ε_t^{fd} and ε_t^{NW} are meant to capture credit supply disruptions and matching of the model variable with net worth data. (5) shows how the external finance premium varies with the degree of leverage.

4.2 The Iacoviello (2005) (IAC) Model

There are three types of agents in this model - a patient consumer who lends and two borrowers; one of the latter is an impatient consumer and the other is

an entrepreneur. There is a collateral asset (housing) that is in fixed supply. Its services are consumed and also used in production. Because housing is in fixed supply there is no residential investment but there is fixed capital investment. One can use the housing asset as collateral and so a key feature of the model is how much credit can be borrowed based on the value of the asset used as collateral i.e. the loan to value ratio. This ratio differs between households and firms. When Iacoviello estimated these parameters he found them to be .89 for entrepreneurs and .55 for households. The decisions made by households and firms are much the same as those in the GOZ model except that credit constraints can limit expenditures, and so changes in the value of collateral can potentially have effects on business cycles. There is no external finance premium however.

5 Business Cycle Characteristics of the Models

5.1 The GOZ Model

Using the GOZ parameter values we simulate their variant of the Smets-Wouters (SW) model and study the resulting business cycle. Table 1 contains the cycle output along with what we would get when BBQ is applied to quarterly U.S. GDP data over the period 1973:1-2009:4[4].

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[4]Alberto Ortiz kindly provided us with a Dynare program that simulated the model they use. The parameter values set in that code are different to those reported in their paper, but this is most likely due to the fact that a longer period of data, 1973:1-2009:4 was now available. Because we did not have their data on leverage we were not able to re-estimate it.

Table 1: Cycle Characteristics: Data and SW Model				
		Data		SW
Expan Dur		13.6		15.4
Contract Dur		4.8		4.5
Expan Amp		9.2		9.5
Contract Amp		-2.8		-1.7
Expan Cum Amp		132.4		125.5
Contact Cum Amp		-8.1		-5.98

The basic macro model shows quite a good match to the business cycle characteristics, although expansions are longer and recessions less severe than they should be. This suggests that there could be a role for credit as a determinant of cycle characteristics. Table 2 produces the same statistics as for Table 1, but now for the GOZ model. In general the effects of credit upon the average cycle is relatively small, with only about a two quarter reduction in its duration. Of course credit may be important in *particular cycles* rather than on average.

Table 2: Cycle Characteristics: GOZ and SW Models				
		GOZ		SW
Expan Dur		14.2		15.4
Contract Dur		4.3		4.5
Expan Amp		8.9		9.5
Contract Amp		-1.6		-1.7
Expan Cum Amp		107.9		125.5
Contact Cum Amp		-5.6		-5.9

Some experiments can be conducted here. Doubling the standard deviation of the credit supply shocks has a very small effect upon the cycle. It is necessary to make much bigger changes in order to have an impact, well outside the range of values of the external finance premium that has been ob-

served. Thus, quadrupling the standard deviation reduces expansion length to 12.8 quarters and raises the amplitude of recessions, although only to -1.9%. But it does this by producing premia that can go to 1000 basis points. At those levels the probability of a recession is .72, but one might think that this is rather low for such an extreme case. Doubling the coefficient χ in (5) also has relatively small effects, but does move the durations and amplitudes closer towards what is in the data.

5.2   The Iacoviello Model

Using the parameter values provided in the paper we simulate the model. As business cycles involve the *level* of output we add on to the output coming from the model a trend that equals the 3.0% p.a. growth in GDP observed over his sample period and then study the resulting business cycle.[5] Table 3 contains the cycle output for the IAC model along with what we would get when using the NBER business cycle states over the period 1973:1-2003:4. He seems to work with 1974:1-2003:4 but, as the starting point here was in a recession, we began the dating a little earlier. Clearly the fit is quite good, although expansions are not as long or as strong as seen in the data.

Table 3: Cycle Characteristics: Data and SW Model				
		Data		IAC
Expan Dur		17.7		14.2
Contract Dur		3.4		2.8
Expan Amp		16.5		15.2
Contract Amp		-1.8		-2.3
Expan Cum Amp		-5.0		-4.6
Contact Cum Amp		265		165

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[5]Iacoviello "detrended" the GDP data with a band-pass filter before estimating the parameters of his model. Ideally we would like to add the "trend" associated with such a filter back on to the simulated output from the model. To do that an algorithm to invert the filtered series to obtain the original levels would be needed. At present we do not have a formula to perfrom this inversion. A similar problem exists in relation to the Hodrick-Prescott filter and a solution in that context was proposed by Landon-Lane (2002).

6 Correspondence of GOZ Model Business Cycle Outcomes with Stylized Facts

We now seek to examine some of the characteristics listed in the preceding sub-section. It will be necessary to find the credit growth rates implied by the model. Appendix A shows that the growth in dividends $\Delta \ln D_t$ equals

$$
\begin{aligned}
\Delta \ln D_t^* &= \Delta q_t^* + \Delta k_t^* + \gamma^* + 100\Delta \ln(1 - l_t^{-1}) \qquad (7) \\
l_t &= \exp((q_t^* + k_t^* - n_t^*/100) + \ln(1.3634))
\end{aligned}
$$

where q_t is the log of the price of capital, k_t is the log of the capital stock, γ is the long-term rate of growth of per capita output, l_t is leverage and an asterisk indicates these are measured in percentage form. The adjustment term $\ln(1.3634)$ is based on the GOZ data.

6.1 Relative Credit and Output Growth During an Expansion after a Financial Crisis

To assess the first characteristic we computed the credit growth and GDP growth over the first eight quarters of expansions. Let these be Δc_j and Δz_j respectively, where j indexes an expansion. Forming $\phi_j = \Delta z_j - \Delta c_j$, the stylized facts would be that the average of ϕ_j from expansions that came after a financial crisis would be positive. Given that we use the size of the external finance premium (s_t) as an index of the extent of a financial crisis we would expect some relation between ϕ_j and s_j (where s_j is the external premium before the $j'th$ expansion began). We used a number of measures of the level of external finance premium ψ_j, namely the value at the origin of the expansion $(s_{t_j^*}$ at time $t_j^*)$, and two averages of that based on current and and past values, $\frac{1}{2}\sum_{k=0}^{1} s_{t_j^*-k}$ and $\frac{1}{3}\sum_{k=0}^{1} s_{t_j^*-k}$. As the conclusions were the same in all cases we present the relation with the first of the three measures. Then the regression of ϕ_j upon ψ_j and a constant gives

$$
\phi_j = -4.3 + 6.3\psi_j,
$$
$$
(1.2) \quad (3.8)
$$

suggesting that credit growth is weaker than output growth over the first two years of expansion when the external premium at the beginning of the expansion is higher.

It is worth noting that the average credit growth over the first eight quarters of all expansions is -2.6 versus the 4.8 in output. But this hides an enormous variation. There are many simulations in which the growth in credit over the first two years of an expansion exceeds that of output. This remains true for a significant number of expansions which are preceded by a high external finance premium, which we would interpret as a financial crisis. The growth rates in credit are extremely volatile, with a standard deviation of quarterly growth of 5.86 versus only .69 for output. The extreme volatility comes from the factor $100\Delta\ln(1 - l_t^{-1})$ in the growth of credit as set out in (7). If one computes the standard deviation of $100\Delta\ln(1 - l_t^{-1})$ from the data and the model we find these to be 5.52 and 8.83 respectively, so that the volatility in the implied credit growth is also in the data used by GOZ.

6.2 Credit Restrictions and Output Growth

The same exercise was performed with respect to the growth in output over the year following the end of a recession. There is little evidence that the annual growth is much affected by the size of the external finance premium. So there does not seem to be much of an impact upon the first year's output growth in an expansion beginning with a high external finance premium.

6.3 Recession Duration and Financial Crises

We can investigate the dependence of recessions upon the external finance premium by taking advantage of the binary nature of the recession indicator R_t ($R_t = 1$ if the economy is in recession but zero otherwise). One possibility is to compute $P(R_t = 1|s_t)$. Assuming this has a Probit form it will be $\Phi(\alpha + \beta s_t)$, where $\Phi_j(\cdot)$ is the cumulative standard normal distribution function. Figure 1 plots this (PRREX) as a function of s_t from simulations of the GOZ model. An alternative comes from recognizing that the recession states R_t generated by BBQ (and also true for NBER recession indicators) follow a recursive process of the form (see Harding (2010))

$$
\begin{aligned}
R_t &= 1 - (1 - R_{t-1})R_{t-2} - (1 - R_{t-1})(1 - R_{t-2})(1 - \wedge_{t-1}) \\
&\quad - R_{t-1}R_{t-2}\vee_{t-1},
\end{aligned}
$$

where $\wedge_t$ is a binary variable taking the value unity if a peak occurs at t and zero otherwise, while $\vee_t$ indicates a trough. By definition $\wedge_t = (1 - R_t)R_{t+1}$

and $\vee_t = (1 - R_{t+1})R_t$ and, in BBQ,

$$\wedge_t = 1(\{\Delta y_t > 0, \Delta_2 y_t > 0, \Delta y_{t+1} < 0, \Delta_2 y_{t+2} < 0\})$$
$$\vee_t = 1(\{\Delta y_t < 0, \Delta_2 y_t < 0, \Delta y_{t+1} > 0, \Delta_2 y_{t+2} > 0\}),$$

where $\Delta_2 y_t = y_t - y_{t-2}$ will be six monthly growth. One might then ask whether the probability $\Pr(R_t|s_t)$ is the most informative measure. An alternative is to conditional upon the previous states i.e. to (say) ask what is the probability of going into a recession at time t given that we were in expansion at $t-1$ and $t-2$? This probability will be

$$
\begin{aligned}
\Pr(R_t|R_{t-1} &= 0, R_{t-2} = 0, s_t) = 1 - (1 - E(\wedge_{t-1}|s_t)) \\
&= E\{1(\Delta y_t < 0, \Delta_2 y_{t+1} < 0)|s_t\} \\
&= E\{1(\Delta y_t < 0)1(\Delta_2 y_{t+1} < 0)|s_t\} \\
&= E(\Psi_t|s_t)
\end{aligned}
$$

Again assuming a Probit specification $E(\Psi_t|s_t) = \Phi_2(s_t)$. Figure 2 contains a plot of this (PRREX1) against s_t. There is clearly a big difference in the two conditional probabilities. If it is known that one is in an expansion in the preceding two periods the rise in the probability of a recession, even for large values of the external finance premium, is quite small. Indeed, the result suggests that the external finance premium will not be very useful for predicting recessions, a point we come back to in a later sub-section. Note that the unconditional probability of a recession over 1953:2-2009:3 was .12 so that, although the rise in the external premium does increase the probability quite substantially, it never gets to a standard critical value often used in predicting recessions of .5.

6.4 The Probability of a Recession and the Size of the External Finance Premium

The next question we seek to examine is whether the duration of the recession, once one is in a recession, depends upon the magnitude of the external finance premium. There are two ways one might do this. One is to just relate the durations of recessions to the external premium. A linear regression shows a positive relation, but the connection is not strong, with even large changes in the premium only causing the duration to increase by a quarter. Another
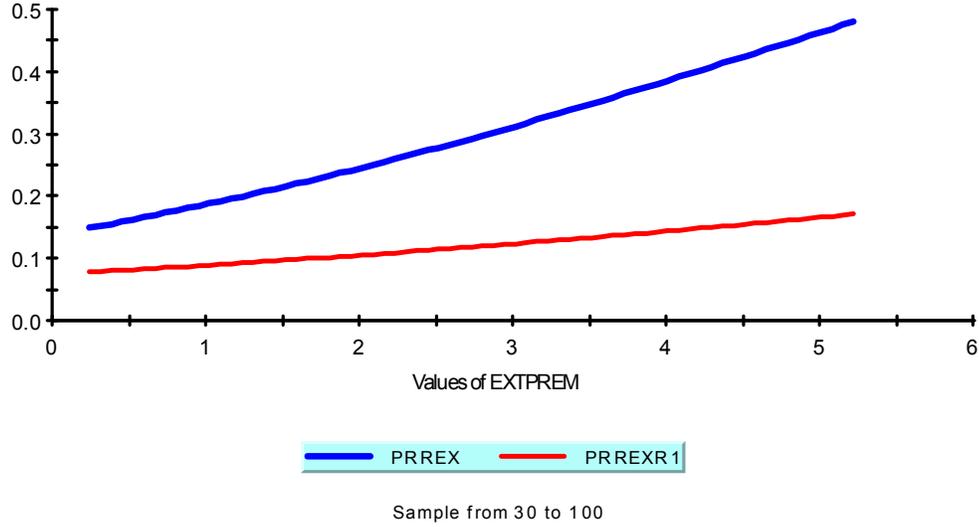
0.5
0.4
0.3
0.2
0.1
0.0

0   1   2   3   4   5   6

Values of EXTPREM

PRREX   PRREXR1

Sample from 30 to 100

Figure 1: Plots of Pr(R=1|s) and Pr(R=1|s,R(-1)=0,R(-2)=0)

method which is instructive is to compute $\Pr(R_{t+m} = 1|R_t = 1, s_t)$ i.e. the probability that, in m periods time, the economy will still be in a recession which began at time t. Table 4 shows what this probability is for three levels of s_t and for $K = 1, 2, 3$. It should be noted that, since BBQ has a restriction that recessions and expansions must last at least two quarters, the only reason that $\Pr(R_{t+1} = 1|R_t = 1, s_t) \neq 1$ is that there will be an $R_{t+1} = 1$ when the recession ends at t[6]. Table 3 shows this probability for various levels of the external risk premium and m.

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[6]This fact also means that for $m \leq 3$ the value of m must be the duration of a recession since, if $R_t = 1$, then $R_{t+1} = 1$, owing to the fact that recessions last two quarters. If $R_{t+3} = 1$ then $R_{t+2} = 1$, otherwise we would have a one period expansion.

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Table 4 Probability of Recession for m Periods as Ext Premium Varies				
Ext premium (basis points)		$m = 1$	$m = 2$	$m = 3$
25		.70	.38	.16
300		.72	.42	.20
485		.74	.46	.23

It is clear that there is an increase in the probability of the duration of a recession as the external finance premium rises, but what is striking is how small the rise in this probability is. Essentially both computations address the often-quoted result that a recession associated with a financial crisis is around twice as long as one that does not have one. As we would think that a crisis would involve a high external interest rate premium, given that there would be little credit available, the GOZ model would fail to deliver such a prediction. One would certainly associate a crisis with a high probability of recession, as seen in Table 1, but its duration does not seem to depend much on that. Two reasons might be advanced for this. One is the use of per capita output rather than the level of output to date cycles, as recessions are longer with the per capita measure. Another arises from the degree of persistence in the growth in credit. It is interesting to note that the persistence in $\{\ln(1 - (1/r_t)) - \ln(1 - (1/r_{t-1}))\}$ used to form credit growth in (7) is quite different in the data than the model.

6.5   Credit Growth and Recession Prediction

One might ask if there is any evidence that a recession can be predicted with GOZ model. The probability is different to what was computed above as we are now looking at $\Pr(R_{t+1}|s_t)$ and not $\Pr(R_t|s_t)$. We might also be interested in $\Pr(R_{t+1}|R_t = 0, R_{t-1} = 0, s_t)$. The latter equals

$$E\{1(\Delta y_{t+1} < 0, \Delta_2 y_{t+2} < 0)|s_t\}$$

and points to the fact that predicting a recession involves successfully predicting *negative* quarterly and six monthly growth over the two quarters following on from the prediction point.[7] This is much stronger than the ability

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[7]Because there are two interest rates in the model and one of these is the policy rate it might be better to use the spread over a three month T-Bill rate. But doing this does not change the results very much.

to predict growth rates of output *per se*. It may be that we predict positive ones well, and this can make the prediction record for output growth look rather good, even though recession prediction is a dismal failure.

There are some current difficulties in determining the predictions of the GOZ model since it was estimated using data that was not available to us. In particular, the credit spread was one they constructed. To gain some idea of how useful it will be we assume that the latter is well represented by the Baa spread. The graphs in GOZ suggest the Baa spread is related to their indicator, although they argue that the latter is a better predictor. With the Baa spread as s_t we evaluate $\Pr(R_{t+1}|s_t)$ from 1973 onwards. Here the Baa spread used is that available at the beginning of the quarter a prediction is to be made about. Although the spread is highly significant in a Probit model fitted to the recession indicator (t ratio of 4), it is clear from the graph in Figure 2 that it adds little to the predictive power. Even in the 2008 recession it was not indicating one until the recession was well under way (the predicted probability in the first quarter of 2008 was just .27). This finding is something which concurs with that found by Harding and Pagan (2010b) for many series recommended as useful for predicting recessions.[8]

Why might we expect the GOZ model to be ineffective at predicting recessions? To understand the limits of using models such as this for predicting recessions we observe that at time $t-1$ we would be predicting $1(\Delta y_t < 0, \Delta_2 y_{t+1} < 0)$ using the information available at $t-1$ i.e. we aim to predict future growth outcomes. A check on whether a model would be able to predict such a quantity is to ask how important future shocks are to these growth outcomes. By this we will mean the unpredictable part of future shocks i.e. if shocks like technology have an autoregressive structure it is the innovation whose impact upon the business cycle we wish to determine. We therefore simulate the GOZ model turning off the contemporaneous innovations i.e. the model is run with current shocks set to zero, although they are re-set to their actual values in later periods. To illustrate what is done take an AR(1) $z_t = \rho z_{t-1} + e_t$, where e_t is white noise. Defining $z_t^- = \rho z_{t-1}$ we note that z_t and z_t^- differ only by the current shock and that z_t^- will be

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[8]We also experimented with using the GOZ model itself to predict recessions. Because we did not have data on leverage etc we constructed an estimate of $E(\Delta y_{t+1}|F_t)$ from the GOZ model, deleting the influence of the external finance premium. Then $E(\Delta y_{t+1}|F_t)$ and the Baa spread were both used in the Probit model. Each variable was significant but there was no improvement in predictive power for recessions over that shown for the Baa spread. Indeed, with one exception, there was less predictive power.
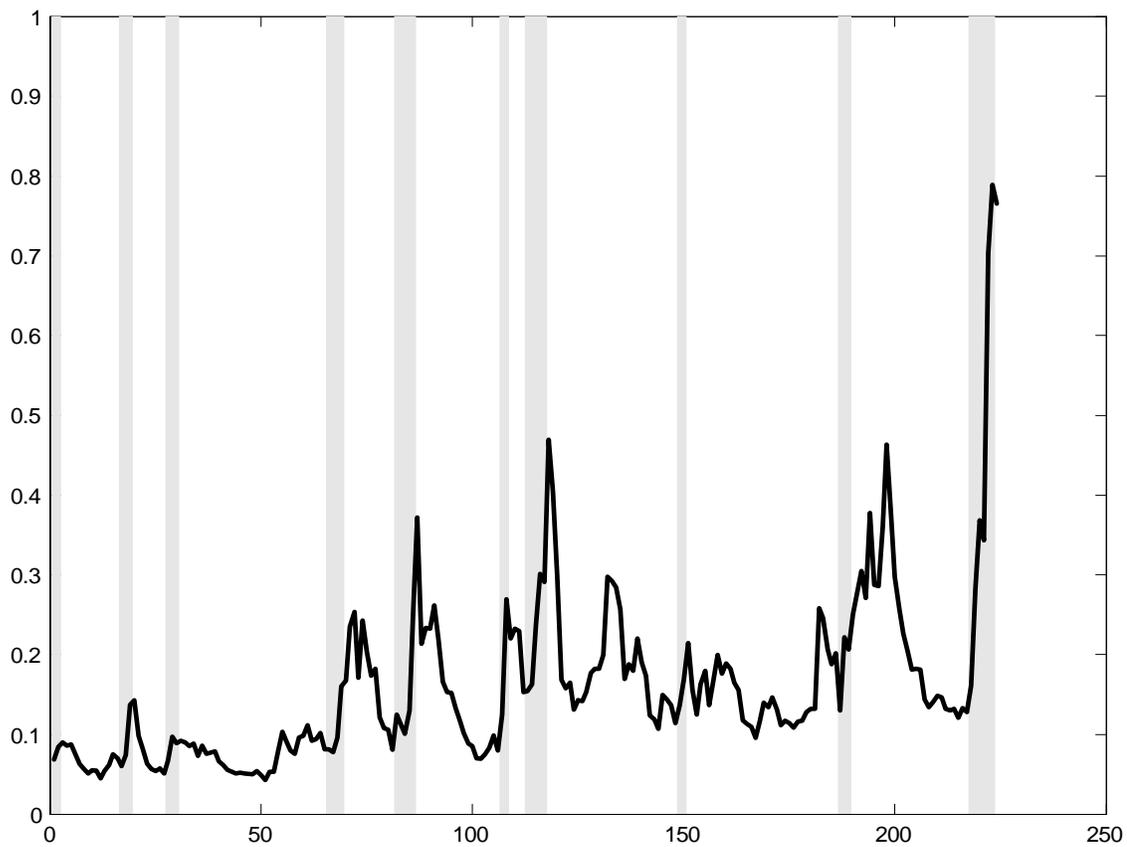
Figure 2: Prob US GDP Recession Given Baa Spread

the predicted value of z_t when the shock is set to zero. Table 5 shows the business cycles from the GOZ model with current shocks present (equivalent to basing the computation on z_t) and with them suppressed (equivalent to z_t^- and hence designated GOZ^-). It is clear that the current shocks have an enormous effect upon the cycle characteristics. Expansions are now very long, and so that there will be fewer recessions, leading to our conclusion that the GOZ model will predict fewer recessions if future shocks are not known. In summary, the issue of good prediction of recessions is always whether a model or an indicator can capture the unknown future shocks coming into the system.

Table 5: Impact of Current Shocks on Business Cycles in GOZ Model				
		GOZ	GOZ^-	
Expan Dur		14.2	30.8	
Contract Dur		4.3	3.7	
Expan Amp		8.9	14.6	
Contract Amp		-1.6	-.74	
Expan Cum Amp		107.9	506	
Contact Cum Amp		-5.6	-2.1	

6.6   Relative Performance of Investment Prediction

One of the observed cycle characteristics listed above was that there would be a stronger response by investment than output. We therefore studied the investment cycle present in the data on U.S. non-residential investment. Here expansions were 12 quarters long on average and recessions were 6.5. So, while the investment cycle length is not far from that of GDP, the recessions are longer and the expansions shorter. Figure 3 presents the probability of an investment recession as a function of the Baa spread, and it is apparent that it rises very quickly with the spread. This is also true of the model, where the probability of an investment recession is .45 when there is an (annualized) spread of 200 basis points, .58 when the spread is 300 points, and .83 when it is 520 basis points. Indeed, the model predicts an even stronger effect than seen in the data (of course the Baa spread is not the external finance
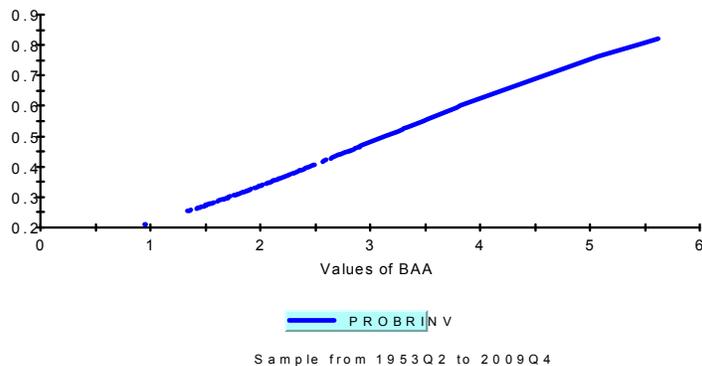
Figure 3: Pr(R=1|Baa Spread) for U.S. Investment Data

premium used by GOZ and so it might be different if that was used).

Given that figure 3 understates the probability of an investment recession as a function of the spread, a comparison of Figures 3 and 4 leads to the query of why the probability of an output recession for a given spread is so much lower than that for investment? One reason is that investment in the GOZ model is only 10% of GDP, so a very large negative growth rate in investment is needed to cause a negative growth in output. This suggests that one needs to work with a broader set of investment expenditures i.e. housing and consumer durables could be crucial to getting the quantitative financial effects right. In turn this implies that collateral effects will be important. Integrating housing and consumer durables into the GOZ model would seem a useful extension of it.

7 Correspondence of Iacoviello Model Business Cycle Outcomes with Stylized Facts

Because there is no external finance premium in Iacoviello's model it is not clear how one would define a crisis. One might think of it as whether asset prices are a long way below their steady state levels. Accordingly we focus upon examining the stylized facts with that as a proxy. A regression of the same type as was done for the GOZ model, but with ψ_j now being the log
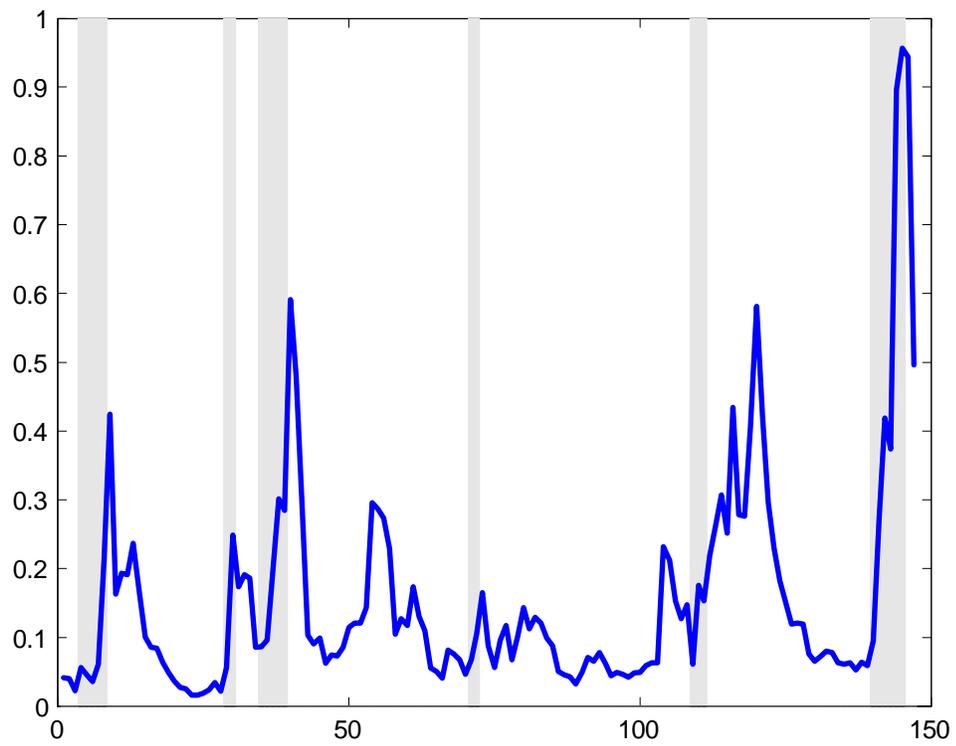
24

Figure 4: Prob Investment Recession and Recession Periods for US Using Baa Spread

deviation of the asset price from its steady state value, gives

$$\phi_j \;\;=\;\; -13.7 + 2.0\psi_j,$$
$$(\text{-7.2}) \;\; (3.4)$$

so the model does seem to generate one of the stylized facts relating to the relative growth of credit and output after a "crisis". We can also look at how the probability of a recession varies with the asset price deviation.[9] Figure 5 shows that it is very high when there is a negative deviation. In many ways this seems to be too strong a result since asset prices are only 10% below their steady state value at the left-hand end of the figure.

The feature noted in connection with the GOZ model about the importance of current shocks also holds here. If these are removed expansions have very long durations (31.6 quarters) and the over-riding importance of current shocks in initiating a downturn suggests that it will be very hard to predict investment recessions with Iacoviello's model. Over his sample period the duration of investment cycle expansions and contractions were 8 and 4.3 quarters respectively, while the model implied they would be 8 and 3.7 - so duration was well matched. This was not so true of amplitude, as expansions had a 23% rise in output on average in the data and the model predicted only 17% i.e. the model implied cycle was not as strong as in the data. It was the case that the impact on investment of asset price deviations was much the same as for GDP, marking a difference to the GOZ model. In the Iacoviello model total fixed private investment seems to be the series modelled and this is about 17% of GDP. Of course in Iacoviello's model consumption is influenced by collateral effects as well and this is a large percentage of GDP.

The Iacoviello model is however of most interest for what it tells us about the impact of variations in the loan-to-value ratios. One might think of this as an index of how easy it is to get credit. In the Iacoviello model the two loan-to-value ratios are set at .89 (entrepreneurs) and .55 (households), so we multiply these with a constant k in order to emulate a range of credit

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[9]The calculation is based on non-parametrically estimating $E(R_t|q_t)$, where R_t is the binary variable that is unity in recessions and q_t is the deviation of the asset price from its steady state value. As one might expect from the shape of Figure 5 there is a big difference between this and what one gets from fitting a Probit model. To ensure that the number of observations is sufficient we dropped data for large negative and positive values of q_t. As apparent the estimated probability for large negative values fell from what is presented. At the other end the probability went up for large positive values.
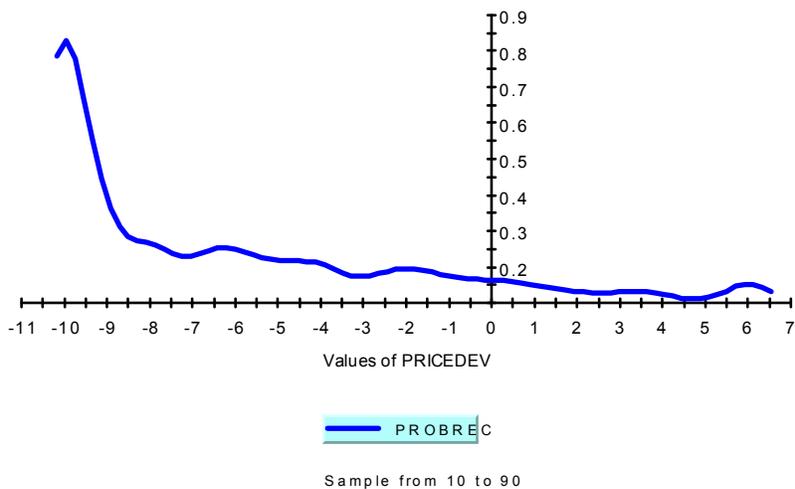
Figure 5: Probability of Recession for Iacoviello Model as Asset Price Departs from Steady State

conditions. The values of k chosen are .5, .9, 1.0 and 1.1, so that the third of these values uses the ratios in Iacoviello's work. Table 6 shows how the (unconditional) probability of a recession changes for different values of k. It is possibly surprising that easier credit leads to a greater probability of a recession. Moreover, even very tough credit conditions ($k = .1$) lead to a probability of recession that is much the same as when $k = .5$. It might have been expected that the recession probability would have increased but this does not seem to be the case. Easier credit leads to a higher probability of recession since it produces a greater volatility in GDP growth. Consequently, because a recession is associated with negative GDP growth, it is easier to obtain such an outcome when volatility is higher (expected mean growth is of course constant). In a sense this is a story about imbalances.

Table 6: Probability of Recession in IAC Model	
For Degrees of Credit Availability	
k	Prob
.5	.05
.9	.09
1.0	.15
1.1	.24

8 Conclusion

9 Appendix A: Derivation of Debt Growth Equation

Let leverage be $l_t = \frac{Q_t K_t}{N_t}$. Then

$$l_t = \frac{Q_t K_t}{Q_t K_t - D_t} = \frac{1}{1 - d_t}$$

where $d_t = D_t / Q_t K_t$, and

$$d_t = 1 - l_t^{-1}$$
$$\implies D_t = Q_t K_t (1 - l^{-1}).$$

It immediately follows that

$$\Delta \ln D_t = \Delta \ln(Q_t K_t) + \Delta \ln(1 - l_t^{-1})$$
$$= \Delta q_t + \Delta k_t + \gamma + \Delta \ln(1 - l_t^{-1})$$

where $q_t = \ln(Q_t / \bar{Q}_K), k = \ln(K / \bar{K}\gamma^t)$, since the steady state growth rate of capital will be the same as output (γ). Designating the ratio $\frac{K_t}{N_t}$ as $R_{KN,t}$ an expression for l_t is available from

$$
\begin{aligned}
\ln l_t &= \ln Q_t + \ln R_{KN,t} \\
&= \ln Q_t - \ln \bar{Q}_K + \ln R_{KN,t} - \ln \bar{R}_{KN} \\
&= q_t + k_t - n_t + \ln \bar{Q} + \ln \bar{R}_{KN} \\
\therefore \quad l_t &= \exp(q_t + k_t - n_t + \ln \bar{Q} + \ln \bar{R}_{KN})
\end{aligned}
$$

Now Gilchrist et al use measure variables in percentage changes so, designating these by a " $*$ ", we get

$$
l_t = \exp((q_t^* + k_t^* - n_t^*)/100 + \ln \bar{Q} + \ln \bar{R}_{KN}).
$$

$\ln \bar{Q} + \ln \bar{R}_{KN}$ is available from the GOZ code as $\ln(1.3634)$. Hence we have

$$
\begin{aligned}
\Delta \ln D_t^* &= \Delta q_t^* + \Delta k_t^* + \gamma^* + 100 \Delta \ln(1 - l_t^{-1}) \\
l_t &= \exp((q_t^* + k_t^* - n_t^*/100) + \ln(1.3634)).
\end{aligned}
$$

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