The Foster-Greer-Thorbecke Poverty Measures: Twenty Five Years Later

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Abstract Twenty-five years ago the FGT class of decomposable poverty measures was introduced in Foster, Greer, and Thorbecke (1984). The present study provides a retrospective view of the FGT paper and the subsequent literature, as well as a brief discussion of future directions. We begin by describing the context and origins of our paper, and identify three categories of contributions: to measurement, to axiomatics, and to application. A representative subset of the literature generated by the FGT methodology is discussed and grouped according to this taxonomy. The FGT paper has played a central role in several thriving literatures, and has contributed to the design, implementation and evaluation of prominent development programs. The breadth of its impact is evidenced by the many topics beyond poverty to which its methodology has been applied. The paper concludes with a discussion of prospective research topics.

Keywords Poverty, FGT measures, decomposability, axioms, income distribution, subgroup consistency, stochastic dominance

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1 Introduction

Twenty five years ago we introduced a class of poverty measures having the formula $P_\alpha = \frac{1}{n}\sum_{i=1}^{q}(\frac{z-y_i}{z})^\alpha$ where z is the poverty line, y_i is the i^{th} lowest income (or other standard of living indicator), n is the total population, q is the number of persons who are poor, and $\alpha \geq 0$ is a "poverty aversion" parameter. We recognized at the time that the class had certain advantages. Its simple structure – based on powers of normalized shortfalls – facilitated communication with policymakers. Its axiomatic properties were sound and included the helpful property of additive decomposability as well as its conceptual cousin, subgroup consistency. Decomposability allowed poverty to be evaluated across regions or other partitions of the population in a coherent way. Initial responses from colleagues suggested that the new tools might well contribute to the literature on poverty.

However, as time has unfolded, we have been surprised by the impact of the work on theory, application, and policy in the domain of poverty evaluation, and even other unrelated domains. For instance, subsequent research has concluded that the indices are closely linked to stochastic dominance and provide a unifying structure linking poverty, inequality, and wellbeing. The measures have become the standard for international evaluations of poverty, are reported regularly by the World Bank's PovcalNet, by a host of UN agencies, and by individual countries. They are naturally suited for targeting exercises and other policy implementations. They are the basis for a growing statistical literature on stochastic dominance and multidimensional dominance tests. And they have been adapted to measure a host of other phenomena, such as the "graying" of a population, corruption, obesity, the "rich", affordability of low-income housing, food insecurity, and the productivity of economics departments. One crude indicator of the impact of the paper is its citation profile over the intervening 25 years. Data from the *Social Sciences Citation Index* reveals a steadily increasing number of annual citations, rising at an average rate of about 1.5 per year. The cumulative count is now 506, with 126 citations in the last three years alone.[1] More expansive databases, such as Google Scholar, count citations in the thousands. Other authors apply FGT measures without formal attribution, suggesting that they have moved into the realm of the generic.

Now that a quarter century has passed since the publication of our paper, we thought that it could be an opportune time to review its story and the related literature. The resulting essay is largely retrospective – constituting a brief history of the research, its contributions, and subsequent developments – but also includes a shorter prospective exploration. The paper has been a "labor of love" for the coauthors, and we hope that those who are familiar with our original effort will be well entertained, while those unfamiliar will be motivated to explore the literature even further.

We begin in section 2 with a brief discussion of the origin of the class of measures and the context in which they were developed. Section 3 turns to an assessment of the

[1] Citation data as of November 2009. Another indicator is the presence of the measures in standard textbooks in public economics (Myles, 1995) and development (Ray, 1998, Fields, 2001, Todaro and Smith, 2008), in specialized books for development economists (Ravallion, 1994, Deaton, 1997) and collections of papers (Subramanian, 2002, Barrett 2007).

contributions of the paper – to measurement, axiomatics, and empirical work. Section 4 reviews the subsequent work in each of these areas. We conclude with a brief discussion of the road ahead in section 5.

2 Context and origin

By the end of the 1970s, the toolkit for evaluating inequality had been substantially enriched by the work of Atkinson, Kolm, Sen, and many other researchers.[2] The literature was motivated by very practical questions, but was strongly influenced by the theoretical methods and substantive approaches of social choice theory, welfare economics, and risk analysis. A compelling axiomatic framework had been created for inequality measurement and it was influencing the way these concepts were being measured in practice. In turn, practical considerations were feeding back to the construction of new indices. Bourguignon (1979), Shorrocks (1980), and Cowell (1980) converted Theil's (1967) forms of additive decomposition into axioms, which were used to derive and characterize the generalized entropy class of inequality measures, including Theil's two measures.

In contrast to the active research on inequality measurement, the discussion of poverty measurement in the early 1970's largely concerned the selection of a "right" poverty line to identify the poor. Sen (1976) brought into clear focus the importance of a second step in evaluating poverty - the aggregation step. He constructed an axiomatic framework for poverty measures, including two "dominance" axioms: a monotonicity axiom (that requires poverty to rise when a poor income falls), and a transfer axiom (that requires poverty to rise when a poor person transfers income to a richer poor person). The latter requirement – that the measure should reflect the distribution among the poor – was adapted from an analogous axiom in inequality analysis. Sen presented a new poverty measure and described a useful three-step procedure for deriving it. One of the steps assumes poverty to be a normalized weighted sum of shortfalls; a second step selects weights based on the rank order of poor incomes (an "ordinal" approach); a third indirectly sets the normalization factor. He justified the weights using an argument from the literature on relative deprivation. The resulting "Sen measure" can be expressed as

$$S(x;z) = H(I + (1-I)G_p)$$

where x is the income distribution, z is the poverty line, H is the headcount ratio or frequency of the poor, I is the income gap ratio or the average normalized shortfall among the poor, and G_p is the Gini coefficient among the poor.[3] The Sen measure takes into account the depth and distribution of the poor, in addition to the frequency. It reduces to HI (which has come to be known as the poverty gap) in case there is no inequality among the poor; it rises towards H as G_p (or I) tends to 1.

By the end of the decade, the impact of Sen's paper was being seen primarily in theory journals; empirical applications of the measure were not yet common. One exception was a thoughtful piece by Anand (1977) on poverty in Malaysia, which employed the Sen measure and identified practical issues with its use. First, he noted that

[2] See Atkinson (1970), Kolm (1968, 1976a,b) and Sen (1973), the surveys of Foster and Sen (1997) and Cowell (2000), and the retrospective of Kanbur (2008).

[3] See also Shorrocks (1995) and Foster and Sen (1997) for a revised Sen measure along the lines of Thon (1979).

the poverty numbers rendered by the measure might not convey information in the way that, say, H does; instead, levels derive their meaning through comparison with other levels.[4] Second, he observed that the measure is not decomposable across subgroups, thus limiting its usefulness in analyzing regional data. When he constructed what he called a "profile of poverty" (following Orshansky, 1965), he was forced to revert to the headcount ratio. Anand's (1977) paper and subsequent 1983 book exemplified the latent demand for a measure suitably consistent with Sen's axioms, and yet exhibiting greater applicability than the Sen measure.

In the final years of the 1970s, Erik Thorbecke, Joel Greer, and James Foster were affiliated with Cornell University's Department of Economics. Thorbecke had just stepped down as Department Chair; his student Joel Greer was working on a thesis proposal in economic development; and James Foster was writing in economic theory under advisor Mukul Majumdar. The events that led the three to collaborate may be of some interest, so we provide a brief account.

As a development economist with considerable policy experience, Thorbecke came to the project with a very specific policy goal in mind. He had been an advisor to the Kenyan Ministry of Finance and Planning during the formulation of its Development Plan of 1979-1983 and also had been working closely with the International Labor Office (ILO) on poverty issues. The Plan followed in many respects the Basic Needs development strategy, which was first formulated in the mid 1970s as part and parcel of the World Employment Program of the ILO. After the World Employment Conference of 1976, the Basic Needs strategy was adopted by the World Bank and other development agencies and governments, and was the reigning paradigm until the early 1980s.

Since food is perhaps the most fundamental of all basic needs, the Kenyan Development Plan was largely predicated on reducing malnutrition and food poverty. However, a major difficulty in the formulation of an effective anti-poverty development strategy was the almost complete lack of information concerning the magnitude, location and characteristics of the poor households in Kenya. The Central Bureau of Statistics had just completed a 1975 Integrated Rural Survey covering approximately 70 per cent of the Kenyan population, which by contemporary standards was considered excellent coverage. The quantity and quality of information from the survey were also very good; however, by the late 1970s the data had not been systematically analyzed. The ILO invited Thorbecke to proceed with this task, and he brought his student to help in the effort.

Greer would later spend much of 1981 in Nairobi helping to clean and tabulate the data set so that it could be used to analyze food poverty in Kenya consistent with the objective of the Development Plan. This work would also become the basis of his 1985 dissertation (and subsequent publications) on food poverty in Kenya, which identified households and individuals whose income, including own-grown food consumption, appeared inadequate to allow a caloric intake at least equal to the recommended daily allowance for each specific regional group based on provincial food expenditure poverty lines reflecting regional diets.[5] The data allowed caloric shortfalls to be calculated for

[4] A similar observation can be made for members of P_α; this general issue is discussed at greater length in section 5.2 below.

[5] A different dataset – the Nairobi Household Survey of 1970 – was used in the paper, which was drafted before Greer's 1981 trip.

each household in the sample and so it would be possible to go beyond the headcount ratio and calculate aggregate poverty measures along the lines of Sen. The question which faced Greer and his advisor as they planned out this research was how best to measure aggregate poverty.

Foster, by contrast, came to the project as a theoretician with a particular interest in the axiomatic method. By the fall of 1979, he had published a paper in social choice theory and a second was forthcoming; and although his advisor Mukul Majumdar was not in this specific area of theory, he was supportive of these research efforts. Moreover, as Chair of the Department, Majumdar was also instrumental in bringing Amartya Sen from Oxford to Cornell for a week-long visit as Andrew D. White Professor-at-Large, which proved to be pivotal for Foster's future research direction. During several conversations over the course of the visit in October of 1979, Sen encouraged Foster to work on inequality and poverty measurement.[6] Foster subsequently immersed himself in the literature and, with Majumdar's insightful assistance, produced the first of a number of pieces in this area - a thesis chapter characterizing Theil's inequality measure using a decomposability axiom.

Sen's visit to Cornell turned out to be an important catalyst for the FGT project. Thorbecke had discussions with Sen about the Kenyan project and his search for a poverty measure. He expressed his doubts about Sen's rank weighting procedure and mentioned an alternative approach – that would later become the basis for the FGT measures. Later, during his presentation on poverty measurement at Cornell,[7] Sen made reference to these discussions by identifying a "Thorbecke position" which used nonlinearity and separability in contrast to Sen's own non-separable rank order weighting. Shortly thereafter, Thorbecke asked Greer to explore this approach as part of his thesis, and he drafted a thesis proposal around a measure that weighted shortfalls with shortfalls – what would later be known as the FGT measure P_2 (or the squared gap measure). Greer knew Foster was working on decomposable inequality measures and discussed the project with him. After several productive interactions, Foster came on board and a draft was produced in July of 1980. By early 1981, Cornell Department of Economics Working Paper No. 243 was circulated, which included several elaborations (such as the Pen's Parade diagram given below) that were later excised in the published version. The key concept of "subgroup monotonicity" (now called "subgroup consistency") was absent from earlier versions, but appeared in the final draft in response to a referee's call for a conceptual justification for additive decomposability. The paper was accepted in 1983 and appeared in 1984.[8]

The FGT poverty measures were thus born and formalized at Cornell University. They were motivated by applied research and technical assistance undertaken in Kenya and were substantively influenced by methods drawn from social choice theory and

[6] In a 1976 exchange of letters, Sen had provided encouragement and advice to Foster on his first published paper in social choice theory.

[7] The presentation of October 3, 1979 was based on the poverty measurement survey published as Sen (1979).

[8] Its acceptance was perhaps surprising given the initial responses of the referees and editor. One referee expressed doubt "whether this contribution is sufficient to warrant publication" and the second was not particularly enthusiastic: "I tend to agree with the authors that their poverty measure…has obvious advantages over some of the other proposed measures." The editor remarked "I am not anxious to publish 'another poverty index'" and "I would put the probability at around one half of acceptance."

welfare economics. The project was a joint and synergetic effort involving two empirically inclined development economists and a theorist, and was shaped by the ideas and advice of Amartya Sen.

3 Contributions

The FGT paper contributed to the literature on poverty in three ways: (i) it introduced a new class of poverty measures that is understandable, theoretically sound, and applicable, (ii) it helped justify the measures using new and practical axioms, and (iii) it provided a concrete illustration of the new technology. We now describe these contributions in greater detail.

3.1 Measurement

The FGT class is based on the *normalized gap* $g_i = (z-y_i)/z$ of a poor person i, which is the income shortfall expressed as a share of the poverty line.[9] Viewing g_i^α as the measure of *individual poverty* for a poor person, and 0 as the respective measure for non-poor persons, P_α is the average poverty in the given population. The case $\alpha = 0$ yields a distribution of individual poverty levels in which each poor person has poverty level 1; the average across the entire population is simply the headcount ratio P_0 or H. The case $\alpha = 1$ uses the normalized gap g_i as a poor person's poverty level, thereby differentiating among the poor; the average becomes the poverty gap measure P_1 or HI. The case $\alpha = 2$ squares the normalized gap and thus weights the gaps by the gaps; this yields the squared gap measure P_2. As α tends to infinity, the condition of the poorest poor is all that matters.

The parameter α has an interpretation as an indicator of "poverty aversion" in that a person whose normalized gap is twice as large has 2^α times the level of individual poverty. Alternatively, α is the elasticity of individual poverty with respect to the normalized gap, so that a 1% increase in the gap of a poor person leads to an α% increase in the individual's poverty level.[10] The parametric class of measures gave analysts and policymakers an instrument to evaluate poverty under different magnifying glasses with varying sensitivity to distributional issues.

The FGT paper emphasized the squared gap measure P_2, noting its simplicity and the fact that many arguments used in support of Sen's measure also apply to P_2. Sen had used a general additive form for poverty measures in which poverty is a normalization factor times the weighted sum of the normalized gaps of the poor. He used rank orders as weights – so that the poorest person in a population of q poor persons is assigned a weight of q, the next has a weight of q-1, and so forth until the least poor person is assigned a weight of 1. Although this approach has been used to great effect in social

[9] Donaldson and Weymark (1986) distinguish between a "weak" definition of the poor, which takes the poverty line as the minimum *non-poor* income, and a "strong" definition in which z is the maximum *poor* income. The original FGT paper followed Sen (1976) and used the strong definition; here we use the weak definition.

[10] The interpretation of the parameter clearly drew its inspiration from the Atkinson (1970) class of inequality measures and its "inequality aversion" parameter.

choice theory (especially in the Borda count method), in the present context it could be viewed as being rather unresponsive to one's own condition and perhaps too responsive to the precise conditions of the other poor persons. By contrast, P_2 simply weights an individual's normalized gap by the normalized gap.

Sen argued that poverty should have an absolute deprivation component (as represented by the normalized gap) and a relative deprivation component (given by the weight) and then, invoking an example from the relative deprivation literature, posited that the latter was naturally represented by the rank order of a person in the group of the poor. One could argue, however, that this was not the only conception of relative deprivation. For example, an alternative notion of the *magnitude* of relative deprivation, which is "the extent of the difference between the desired situation and that of the person desiring it" (Runciman, 1966), suggests the use of the normalized gap itself. Hence, it could be argued that P_2 is also composed of absolute and relative components.

The Sen measure could be expressed in terms of H, I, and G_p – where each component provides relevant information on the frequency, depth, and distribution of poverty.[11] The squared gap measure can likewise be expressed as $P_2 = H[I^2 + (1\text{-}I)^2 C_p^2]$, where C_p^2 is the squared coefficient of variation (a decomposable inequality measure from the generalized entropy class) among the poor. Figure 1 from the working paper version of FGT provides a graphical representation of the link between C_p^2 and P_2 via this formula. A distribution is depicted in the style of Pen's Parade, but with ordered incomes given in poverty units and the total population normalized to 1. The quantities H, I, and 1-I are as noted, while C_p^2 is just the variance of the poverty-unit incomes of the poor about the mean. Suppose, instead, that all poor households had this mean level. Then each normalized shortfall g_i would be identical to I, and P_2 would be $\Sigma_{i=1}^{q} I^2 / n = HI^2$, as given by the formula when $C_p^2 = 0$. However, since the poor incomes are distributed unequally, P_2 is larger by $H(1\text{-}I)^2 C_p^2$. When H and I are held constant, P_2 varies with C_p^2 according to this expression. This intuitive formula and the other justifications paralleling Sen helped with the interpretation of the new measure and gave it a certain credence. However, the *properties* satisfied by P_2 and the rest of the FGT class set it apart from its predecessor and gave it broader applicability.[12]

3.2 Axioms

Axioms for poverty measures can be usefully grouped under three general headings: invariance, dominance, and subgroup axioms (Foster, 2006). The invariance category includes symmetry, replication invariance, scale invariance, focus, and continuity.[13] All

[11] Foster and Sen (1997) use the term "partial indices" to describe the components, since none is a fully-fledged aggregate measure of poverty satisfying the basic axioms for poverty measurement.

[12] Ideas do not appear in isolation, and it is not surprising that the FGT paper overlapped with others that appeared (or resurfaced) in the early 80s. Kundu (1981) did not mention decomposability or the P_α class, but defined P_2 and derived several of its properties. Clark, Hemming and Ulph (1981) proposed two classes: one that can be converted to an additive decomposable class which includes Chakravarty's (1983) family; a second equivalent to $H(P_\alpha)^{1/\alpha}$ for $\alpha > 1$. While browsing in the Lionel Robbins library, Foster (1984) discovered the paper by Watts (1968) that proposed a distribution sensitive poverty measure many years before the rest of the literature.

[13] Each invariance axiom states that poverty should be unaffected by some change in incomes and/or the poverty line: for symmetry, it is a permutation or switching of incomes; for replication invariance, it is a

the P_α measures satisfy the invariance axioms, with the exception of P_0 which has a discontinuity at the poverty line.

The dominance category includes various versions of monotonicity, the transfer axiom, and transfer sensitivity.[14] Higher α reflects greater aversion to poverty, and this is reflected by the dominance properties satisfied by P_α for various ranges of α: monotonicity for $\alpha > 0$, with P_0 just violating it (as increased individual poverty does not affect P_0); the transfer axiom for $\alpha > 1$, with P_1 just violating it; and transfer sensitivity for $\alpha > 2$, with P_2 just violating it. This also signals the versatility of the FGT class in that one can select a specific measure with a desired degree of sensitivity to each of the underlying forms of transformation, namely, increased individual poverty, increased inequality among the poor, or increased inequality among the poor matched by decreased inequality at higher poor incomes.

The final category of axioms draws on the intuition that subgroups of populations are also populations whose poverty levels may be evaluated – and these levels should bear some relationship to the poverty level of the overall population. The additive decomposability axiom outlined in the 1984 paper requires overall poverty to be a population share weighted average of subgroup poverty levels. The origins of this axiom are found in the work of Anand (1977) in poverty and Theil (1967), Bourguignon (1979), Shorrocks (1980), and Cowell (1980) in inequality – although inequality decompositions have an additional between-group term. Measures satisfying additive decomposability have greater applicability – allowing consistent profiles of poverty to be constructed, identifying the characteristics or factors that contribute to poverty, and estimating the contribution of a subgroup to overall poverty.

A second axiom introduced in the paper, called subgroup monotonicity there, but now more commonly known as subgroup consistency, grapples with the fundamental link between subgroup and overall poverty without specifying the structure of that link. It requires overall poverty in a population to rise whenever (i) poverty in a subgroup increases, (ii) poverty in the rest of the population remains the same, and (iii) there is no migration across subgroups. This consistency property gets to the heart of a central policy relevant requirement: that successful regional or targeted poverty policies should, in fact, improve *overall* poverty. The Sen index and other poverty measures that are more sensitive to context and relative position will, in certain cases, violate subgroup consistency. All of the FGT measures satisfy both additive decomposability and subgroup consistency.

3.3 Application

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replication or multiple cloning of the entire population; for scale invariance, it is a scaling up or down of all incomes and the poverty line; for the focus axiom, it is a change in a non-poor income which leaves it non-poor; and for continuity it is invariance to whether a limiting process takes place before or after the measure has been applied.

[14] In its simplest form, monotonicity requires poverty to rise if a poor person's income falls; the transfer axiom says this is also true even if the decrement is matched by a same sized increment to a richer poor person, hence is a "regressive" transfer; transfer sensitivity says this is true even if the regressive transfer among the poor is matched by a "same sized" progressive transfer among the poor that is higher up the distribution, resulting in a "composite" transfer. The three axioms are associated with the three orders of stochastic dominance.

The final section of the paper provided an example using data from the 1970 Nairobi Household Survey to illustrate the use of P_2 in evaluating overall poverty and constructing a profile of poverty. The published example was brief with a table listing the poverty levels for subgroups identified by the length of time lived in Nairobi, and showed the subgroup poverty levels and percentage contributions to total poverty. The working paper version offered a much more extensive analysis with subgroups defined by several other characteristics and six values of α. Additional figures depicted how rankings among groups are altered as poverty aversion is raised. The longer version also echoed Anand (1977) in noting that the values of P_α obtained for $\alpha \geq 2$ "…will not become particularly meaningful until more studies are done which allow comparisons." As noted in Section 4.3.1, examples were soon available, beginning with a series of follow-up studies by Greer (1985) and Greer and Thorbecke (1986a-c) on food poverty in Kenya. These studies provided key empirical demonstrations of the FGT methodology and, in fact, grew out of the same research agenda that helped motivate the original paper.

4 Subsequent developments

In the 25 years since the publication of the FGT paper, an impressive body of research has arisen making use of its results. Much of this research has applied the measurement techniques to assess trends in poverty or identify characteristics associated with being poor. Several lines of research have evaluated policy options either theoretically or empirically using the measures. A number of studies have added to the pool of measurement techniques by adapting or extending the FGT measures. Others have focused on the axiomatic foundations of poverty measurement or proposed a characterization of the FGT class. This section surveys some of the key results from the subsequent literature. We do not aim to be comprehensive nor particularly unbiased in our presentation; instead, we present a few representative studies from the many that we find interesting.

4.1 Measurement

The FGT paper contributed to poverty measurement by developing a parametric class of measures having certain desirable characteristics, and one that policymakers could understand. Many authors have continued along this line of research by proposing other evaluation tools including statistical tests for interpreting estimates. A dominance approach to poverty has been developed to understand when poverty comparisons agree over a range of poverty lines, or when many different poverty measures agree.[15] With the advent of Sen's capability approach, the literature has constructed measures of multidimensional poverty based on the FGT class; the related measures of chronic poverty and vulnerability are easily obtained by considering time and risk. Finally, there

[15] Many of these techniques are described in the book *Poverty Comparisons* by Ravallion (1994) whose use of the terms "poverty incidence, depth and severity" for the three main FGT measures P_0, P_1 and P_2, also became common parlance at the World Bank and among policymakers. Other accessible presentations are found in Deaton (1997), Foster and Sen (1997), Zheng (2000), and Duclos and Araar (2006).

are some interesting applications of the FGT measures to other non-poverty measurement domains. These varied contributions to measurement are now discussed.

4.1.1 Direct extensions

Sen (1979) has noted that the choice of a measure depends on the purpose for which it is to be used and, accordingly, several modifications of the basic FGT formula have be proposed to address certain questions. Foster and Shorrocks (1991) consider the absolute FGT poverty measure $z^{\alpha}P_{\alpha}$ (used in 4.1.2 below). Rodgers and Rodgers (1991) obtain a measure of poverty intensity in a subgroup by dividing subgroup P_{α} by the overall P_{α}. Fields and Bourguignon (1997) propose the discontinuous measure $\delta P_0 + P_{\alpha}$ for $\delta > 0$ and $\alpha > 1$, to obtain more flexible allocative properties than the usual P_{α} class for use in public finance exercises. Ray (2006) uses the FGT measures to derive a metric to evaluate the efficiency of transfer systems in reaching their intended targets

The FGT measures, like all poverty measures satisfying replication invariance, are normalized by the total size of the population and therefore do not themselves provide information on the absolute number of poor.[16] In certain contexts, however, the total number of poor and their aggregate (not average) conditions may be exactly what is important. This observation prompted Kanbur (2005, p. 228) to pose the following question: "If the total number of poor goes up but, because of population growth, the percentage of the poor in the total population goes down, has poverty gone up or down?" To take a case in point, in Sub-Saharan Africa the headcount ratio corresponding to the $1.25 a day poverty line fell from 53.4% in 1981 to 50.9% in 2005 while the absolute number of poor almost doubled from 212 million to 388 million over the same period. The economist's instinct, influenced by the replication invariance axiom that undergirds the FGT and other standard measures of poverty (and inequality and wellbeing), is to state that poverty has gone down. In contrast, those who work directly with the poor and are burdened by rising numbers may argue that poverty has instead gone up. A simple solution, of course, is to "denormalize" the FGT (or multiply it by the population size) to obtain a measure that is "absolute" in this respect (Foster, 1998). Chakravarty, Kanbur, and Mukherjee (2002) provide an extension of the FGT index that goes further: it has a parameter that reflects the relative weights to be given to "absolute numbers" versus the "fraction in poverty".

The case of Sub-Saharan Africa suggests a second, more substantial, reason for extending the FGT class. As emphasized in section 4.1.4, the FGT measures are static (using data from a single time period) and unidimensional (using data on a single indicator of wellbeing), which does not make them very sensitive to major demographic trends, such as the HIV-AIDS epidemic, or more generally to the positive correlation between premature mortality and poverty that may be influencing the data used by the measures. Kanbur (2005, p. 228) raises the premature mortality issue in the following terms: "Suppose the incidence of poverty (and/or the number of poor) goes down because the poor die at a faster rate than the non-poor. Is this a legitimate 'decrease' in poverty?" Kanbur and Mukerjee (2007) argue that it is not conceptually correct for differentially higher mortality rates among the poor to reduce measured poverty. Indeed, regarding the

[16] That is, unless total population size is known.

death of a poor individual due to malnutrition occasioned by low income and lack of assets as an improvement in poverty is at the very least counterintuitive. Similarly, should the increase in the measured level of static, unidimensional poverty that would arise as a result of a successful public health campaign, be viewed as an increase in "actual" poverty? This is an important critique of the entire enterprise of measuring poverty as a function of current income shortfalls – ignoring other dimensions such as health, that are highly complementary to income, and time. However, the problem is even more intractable for two reasons: it involves the transition of death, which is not explicitly addressed by usual assessment methods; it requires an empirical determination of causality and a counterfactual state, which can be both challenging and controversial.

Kanbur and Mukherjee (2007) modify the FGT measure to make it sensitive to premature mortality. They specifying a normative lifetime close to the range observed in rich countries today. If a poor person dies before this normative age of death, she continues to "live" in the data, with the expected low level of income, until reaching the normative lifetime age. Consequently, the measured level of poverty does not automatically register a decrease in poverty as a result of the premature death. The extension is conceptually attractive and addresses a troubling issue, but has some major practical difficulties if it is to be implemented. Estimating how many years of an individual's life were "lost" as a consequence of poverty is quite problematic. Estimating the counterfactual income profile of the prematurely dead individual between the actual time of death and the postulated normative lifetime is clearly difficult. And yet both steps are needed for the solution they propose.[17]

The FGT measures have also been used to define and quantify other concepts related to poverty. One methodological issue of great interest to policymakers relates to the quantification of the relative contribution of growth versus redistribution to observed changes in poverty. Datt and Ravallion (1992) provide formulae for each of the three main FGT poverty measures that allow an overall change in poverty to be broken down into these two effects (plus a residual term). Their estimates for India and Brazil in the 1980s suggest that the structure of growth was very different in the two countries. A related concept is "pro-poor growth" which gauges the extent to which growth in the mean income is reaching the poor. Son and Kakwani (2008), for instance, propose an indicator called the "poverty equivalent growth rate" defined as the rate of growth that would have resulted in the same level of poverty reduction as the present growth rate if the growth process had not been accompanied by a change in inequality. They show that it is a product of the usual growth rate and a pro poor growth index, and compute the rate for the three main FGT measures. Other approaches to pro-poor growth include Foster and Szekely (2008) and Duclos (2009).

4.1.2 Dominance

[17] The discussion is reminiscent of the classic difference between inequality, per se, and inequality due to some other factor (such as a socioeconomic indicator), a distinction that is particularly salient in the context of health. Usually the other factor acts to increase inequality; but in the present context, the disproportionate mortality among the poor serves to lower poverty. Kanbur and Mukerjee (2007) would like to evaluate poverty net of the mortality impact.

The main purpose of the FGT paper was to address the aggregation step in poverty measurement – not the identification step. And yet the measures presented there turned out to be a key component in a strategy to avoid the inherent arbitrariness in the identification step by using stochastic dominance. The idea behind the approach of Foster and Shorrocks (1988a,b) is simple: In some cases it may be possible to make unambiguous determinations of poverty with respect to the choice of a poverty line; in others, the judgment may depend on the specific line chosen. One can systematically study the former situation to derive the underlying (variable line) poverty ordering for a given poverty measure.[18]

Foster and Shorrocks focus on the three main FGT measures and find that the associated orderings are, in fact, the three orders of stochastic dominance developed in risk analysis, with P_0 yielding first order dominance, P_1 yielding second order dominance (used by Atkinson, 1970), and P_2 yielding third order dominance. They note that the three poverty orderings are linked to welfare orderings associated respectively with three classes of additive welfare functions: those with positive marginal welfare of income (or efficiency preferring); those also with diminishing marginal welfare from income (or equity preferring); and those with convex marginal welfare (or transfer sensitive). Consequently, for example, unambiguously lower poverty according to P_1 is equivalent to higher welfare for the Atkinson equity preferring class; and, in the fixed mean case, this implies that the Lorenz curve will be lower (and vice versa).[19] They also observe that the three poverty orderings are nested (in that the poverty ordering for P_0 implies the poverty ordering for P_1, and it in turn implies the poverty ordering for P_2) and obtain analogous results for restricted ranges of poverty lines below some upper threshold.

Atkinson (1987) began with these results and made the following interesting observation for continuous additive poverty measures: if the first order dominance holds over a restricted range of lines, then not only can an unambiguous comparison be made for P_0, P_1, and P_2, it must hold for all the measures satisfying monotonicity. This means that the variable line poverty orderings are also variable measure poverty orderings. Second order dominance likewise ensures that all the measures satisfying monotonicity and the transfer axiom will agree. This surprising result broadened the reach of the FGT measures, since their variable line poverty orderings were enough to ensure dominance for entire classes of poverty measures. Ravallion (1994) saw the potential power of these results for real-world analyses of poverty, and provided a guidebook for practitioners to apply and interpret dominance techniques. He coined the terms "poverty incidence curve", "poverty deficit curve" and "poverty severity curve" for the curves that depict the level of P_α poverty as a function of the poverty line, for $\alpha = 1$, 2, and 3. Jenkins and Lambert (1997) recast the exercise in terms of poverty gaps and construct analogous diagrams that neatly illustrate the dimensions of poverty and indicate dominance. For a review of the subsequent literature on poverty orderings, see Zheng (2000).

[18] The origin of the Foster-Shorrocks approach to poverty orderings is closely related to the theme of this paper. After a presentation of the FGT measure at Cornell by Joel Greer, Nick Kiefer noted that P_2 was similar to the lower partial variance measure of downside risk and suggested a paper by Bawa (1975). Foster followed up on the suggestion and sketched out some tentative dominance results. While on leave at the LSE in 1982 he discussed the results with Shorrocks and they began their collaboration.

[19] Foster and Shorrocks (1988c) emphasize the value of the partial ordering approach in unifying the literatures on poverty, inequality and welfare measurement. See also Zheng (2000) and Duclos and Makdissi (2004).

4.1.3 Statistical tools

When measuring poverty in a world of imperfect data, it is useful to formulate statistical tests in order to gauge the confidence of a given comparison. The original FGT paper did not present the associated tools, but since then the literature has provided a steady stream of inference based research for poverty estimation. It exploits the fact that the FGT measures take the form of a simple mean of a function of incomes given the poverty line. One line of research assumes that the poverty line is fixed and given. Kakwani (1993) evaluated the distribution of the estimated poverty values and showed that it asymptotically follows a normal distribution. This allows poverty levels to be compared using a simple difference of means test. Xu (2007) suggests an alternative route that shows that the sample counterparts to the FGT and other decomposable measures can be represented as a "U-statistic" (or some function of a U-statistic). Dia (2008) first estimates the distribution of income using kernel estimation techniques and then applies a poverty measure to obtain an estimated value.

A second line of research allows the poverty line to be measured with error. Ravallion (1984) provides an estimation technique that applies when an absolute poverty line is itself measured with some error, as might occur if the line is itself estimated using, say, a basic needs approach. He provides the distribution of the test statistics appropriate for comparing poverty levels in this situation. Zheng (2001) discusses the asymptotic properties of measures of poverty when the poverty line is endogenous. The proposed estimate depends on the underlying population income distribution, and he uses a kernel estimation technique to arrive at the distribution of income based on the incomes of individuals in the sample. Complementing these approaches are tests that explicitly account for the sampling design of survey data. Kakwani (1994), Cowell and Victoria-Feser (1996), and Zheng (2004) among others, have modified the estimation techniques to account for the difficulties that stem from using sample data rather than census data.

One of the most active areas for statistical research associated with the FGT measures concerns tests for poverty orderings or, equivalently, stochastic dominance. Many papers in this literature utilize an FGT measure or some transformation to characterize the dominance criteria. Key papers include Anderson (1996), Davidson and Duclos (2000) and Barrett and Donald (2003), each of whom was clearly motivated by the connection of dominance to the FGT measures. The tests involve the estimation and comparison of population parameters; they essentially use an FGT measure as the sample counterpart for estimation purposes. There are several modules available for standard statistical packages that provide test statistics for the FGT and the associated dominance conditions under various conditions.[20]

4.1.4 Multidimensional poverty

The FGT measures' axiomatic properties, and additive decomposability in particular, make it a useful instrument for extending poverty measurement beyond its traditional bounds. One key direction is to incorporate other dimensions, such as health, education,

[20] See, for example, the comprehensive software for distributive analysis (DAD) of Duclos and Araar (2006), which highlights the FGT measures and their dominance techniques.

and nutrition, into the definition of poverty. A second is to include observations from many time periods to capture dynamic aspects of poverty and to discern between the chronically poor and the transient. A third is to incorporate risk into the discussion, noting that many persons who are not poor are nevertheless vulnerable. Each of these directions takes a broader, multidimensional view of poverty and, as we shall see, the FGT methodology is repeatedly used to construct appropriate measures.

Multiple domains The original FGT measures were unidimensional, relying on shortfalls in a single variable to reflect poverty. Yet the extent to which a person is well off or poor may well depend on achievements in several distinct and independently important dimensions and, if so, this calls for the development of new multidimensional measures that reflect the complex nature of wellbeing and poverty. Sen's capability framework provides the most comprehensive starting point.[21] In this framework, wellbeing depends on "capabilities" or one's freedom to achieve certain valuable "doings and beings" called functionings, and poverty is viewed as capability (or functioning) deprivation, which for measurement purposes might be indicated by levels of achievement that fall short of minimum levels. Under the impetus of this approach, the analysis and measurement of multidimensional poverty has progressed significantly, and in 1998 two significant papers emerged. The first, a mimeo by Brandolini and D'Alessio (1998) outlines a range of possible measurement methodologies that could be drawn upon to construct a multidimensional measure of functioning (and mentions a "work in process" by Bourguignon and Chakravarty). The second, by Chakravarty, Mukherjee, and Ranade (1998), proposes a multidimensional extension of the FGT class and specifies a set of appropriate axioms. A number of papers succeeded these including: Tsui (2002), which draws quite heavily on the subgroup consistency approach of Foster and Shorrocks (1991); Atkinson (2003), which contrasts welfare based and "counting" approaches; Chakravarty and D'Ambrosio (2006), which measures the related concept of "social exclusion"; a survey by Chakravarty and Silber (2008); and the twin volumes of Kakwani and Silber (2008a,b). Each contains multidimensional extensions of the FGT methodology. Thorbecke (2008) provides an intuitive overview of the related literature; here we discuss three papers in greater depth.

Bourguignon and Chakravarty (2003) take as a fundamental starting point that "a multidimensional approach to poverty defines poverty as a shortfall from a threshold on each dimension of an individual's well being" (p. 27). They focus on the two-dimensional case, and include persons who are deprived in at least one dimension in their analysis (later called the "union" approach to identifying the poor). They build a class of multidimensional poverty measures that extends the FGT class to many dimensions. The new measures satisfy a number of desirable axioms and are consistent with the possibility of interacting attributes, a feature they discuss at some length. In the case of substitutes, a person with a larger quantity of attribute k experiences a smaller drop in poverty with an increase in attribute j: for example, the reduction in poverty caused by a unit increase in income would be greater for people with very low education levels than for individuals nearer to the cutoff for education. For complements, the opposite is true: the drop in poverty would be larger for individuals endowed with more education. They argue that,

[21] See, Foster and Sen (1997) for a concise presentation of the capability approach.

in theory, their poverty indices could be generalized to any number of attributes, but note that this would require assuming the same elasticity of substitution between attributes, which may lessen the appeal of their approach. Their measures are sensitive to the correlation between dimensions, but they require cardinal data, which may restrict their use in practice.

Alkire and Foster (2007) present a comprehensive methodology that combines a new method of identifying the poor and a multidimensional extension of the FGT class. The poor are identified using two forms of thresholds: "first a cutoff within each dimension to determine whether a person is deprived in that dimension; second a cutoff across dimensions that identifies the poor using a (weighted) count of the dimensions in which a person is deprived." This identification method extends the "union" approach (where poor persons are deprived in one or more dimensions) and the intersection approach (where the poor are deprived in all dimensions) and it can be used with ordinal data.[22] The aggregation step across individuals employs the FGT measures, appropriately adjusted to account for multidimensionality.

We note that the search for a multidimensional poverty measure is not purely an academic pursuit. In 2007, the National Council of Evaluation of the Social Development Policy of Mexico (CONEVAL) was mandated and given the responsibility by the Government of Mexico to develop an operational multidimensional poverty measure that could be used to monitor national poverty and allocate central government funds. The associated law specified the dimensions to be incorporated into the measure. CONEVAL is currently deciding between several options adapted from the above literature.

Duclos, Sahn and, Younger (2006ab) develop a dominance approach to multi-dimensional poverty. They extend the concept of a poverty line in one dimension to a poverty frontier in multiple dimensions, which in turn defines the set of cutoff vectors over which dominance is examined. The measure employed in their dominance exercise is essentially a generalization of the FGT index with separate poverty aversion parameters for each dimension. Their methodology is sensitive to the covariance (or association) across dimensions, but cannot be applied when variables are complements.

Time and risk The original FGT class is static in that it captures poverty at one point (or period) in time and ignores possible fluctuations in consumption across periods.[23] Yet many of the remaining unresolved issues in poverty analysis relate directly or indirectly to the dynamics of poverty. Policymakers are particularly interested in the conditions under which some households remain chronically poor and others are temporarily in poverty, since the types of interventions needed to alleviate the two types of poverty are very different. Appropriate insurance schemes (such as crop insurance) and other consumption-smoothing measures can be effective in reducing temporary poverty, but are likely to be less effective against chronic poverty. Reducing the latter might require significant investments in human and health capital and some redistribution of assets – particularly land.

McCulloch and Calandrino (2003) distinguish three types of chronic poverty: (1) mean consumption across time being below the poverty line; (2) a high frequency of

[22] In principle, dimensional cutoffs could be determined using a participatory method, while the policymaker could select the number of dimensions required to be considered poor.
[23] This subsection draws on Thorbecke (2003).

being in poverty over some time (or a high probability of being poor) and; (3) a high degree of persistence in poverty. The first two papers considered here use (1) to identify the chronically poor and apply FGT measures to evaluate its aggregate level. Rodgers and Rodgers (1993) regard persistent poverty as a state in which income is less than needs in many consecutive years. Their measurement of chronic poverty is based on a comparison of "permanent income" and "permanent needs". Given a multi-year observation period, Rodgers and Rodgers (1993) measure an agent's permanent income as "the maximum sustainable annual consumption level that the agent could achieve with his or her actual income stream over the same period if the agent could save and borrow at prevailing interest rates". In turn, transitory poverty is defined as the difference between annual poverty in a given year and chronic poverty. An agent may be chronically poor but temporarily out of poverty in a given year. Alternatively, an agent who is not chronically poor may experience transitory poverty in some year. The decomposition into the two types of poverty is based on the P_2 (squared poverty gap) measure. In an application to China, Jalan and Ravallion (2000) define transient poverty as the contribution of consumption variability over time to expected consumption poverty. The non-transient component is the poverty that remains when inter-temporal variability in consumption has been smoothed out (to the mean consumption level), and this is what they call chronic poverty. Again the measure uses P_2. Foster (2006) employs the second definition, and regards as chronically poor a person who is in poverty no less than τ share of the time, where τ is a fixed number between 0 and 1. He measures chronic poverty using the FGT measures appropriately adjusted for duration.

The second definition also leads directly to the concept of vulnerability and the extent to which households can protect themselves against a variety of shocks. A number of contributions have attempted to define and operationalize the concept of vulnerability. Christiaensen and Boisvert (2000) contrast poverty and vulnerability in the following way. Poverty is concerned with not having enough now, whereas vulnerability is about having a high probability now of suffering a future shortfall. Their notion of vulnerability is the risk of a future shortfall and is expressed as a probability statement regarding the failure to attain a certain threshold of wellbeing in the future. They measure vulnerability as the probability of falling below the poverty line z, multiplied by a conditional probability-weighted function of a shortfall below this poverty line. Consistent with the FGT poverty measure they use a vulnerability-aversion parameter α such that by setting $\alpha > 1$, households with a higher probability of large shortfalls become more vulnerable. A key question at this stage is whether vulnerability and consequent risk-aversion is part and parcel of multi-dimensional poverty in the sense that certain sets of shortfalls of attributes (deprivations) generate vulnerability or whether vulnerability is a separate dimension of poverty. In a conceptual breakthrough, Ligon and Schechter (2003) - also relying on FGT - break down vulnerability into two components reflecting poverty and risk, respectively. The first component is supposed to represent that part of vulnerability due to (chronic) poverty, while the second reflects risk and uncertainty and, presumably, transitory poverty. While this distinction is ingenious and useful in estimating the utility gain that could accrue to the poor, if there were a means to remove their risk-aversion through some social insurance program, it could mask the fact that certain types of current poverty (portfolios of deprivations) render those households more vulnerable. In

turn, higher risk by altering the behavior of the poor pushes some of them further into a poverty trap. In this sense, vulnerability (risk) and poverty are inherently inter-related.

4.1.5 Other domains

The FGT measures can be applied to other domains in which there is a well-defined cutoff and shortfalls are considered undesirable. Denny (2002), for example, applies the FGT methodology to the domain of education, or more specifically, literacy. He uses test scores from the International Adult Literacy Survey to obtain measures of illiteracy applicable for comparisons across participating countries. The headcount ratio is the usual illiteracy rate; P_1 and P_2 account for the depth and severity of illiteracy. He provides examples where the new measures present a different picture than the literacy rate, and shows how decompositions help clarify the structure of illiteracy within a country. Sahn and Stifel (2002) use P_α to measure child malnutrition where the variable is standardized height-for-age z-scores in various countries. Among other results, they find that India has the highest level of malnutrition, whether measured by the traditional incidence, depth or severity measures. Their analysis includes dominance tests and statistical evaluations. Gundersen (2008) applies P_α to the US Department of Agriculture's "food security scale" to obtain the incidence, depth and severity measures of food insecurity for the USA.

The FGT measures can also be modified to address the case where observations *above* a cutoff are of concern and the "excess" above the target is used in place of the shortfall below. For example, Basu and Basu (1987) apply a modified FGT measure to the distribution of ages in a country to measure the "graying" of the country. Chaplin and Freeman (1999) consider measures of the affordability of public housing in the UK based on the distribution of the "rent-to-income" ratios of tenants. The cutoff is set to 25%, and all tenants with rents exceeding 25% of income fail the affordability test. The usual aggregate measure is a headcount ratio of the households failing the test; they select a modified P_3 because of its superior properties and show how the rankings of regions in the UK are altered. Lubrano et al (2003) measure the aggregate productivity of economics departments using a modified P_α measure, where the cutoff is a minimum level of productivity for an individual researcher to be considered active. Jolliffe (2004) uses a modified P_α to evaluate the extent to which a population is overweight. The variable here is the distribution of the BMI and the cutoff is the WHO standard of 25. He argues that the traditional headcount ignores policy relevant information provided by the "overweight gap" index and the "squared overweight gap" index. Peichl, Schaefer, and Schleicher (2008) evaluate the conditions of the rich in Germany using modified FGT measures. Foster, Horowitz and Mendez (2009) adapt the FGT measures to assess the aggregate corruption in a region, where a transaction is not considered corrupt until it exceeds a certain cutoff level. These examples illustrate the applicability of the FGT methodology, and show that its impact extends well beyond the domain of poverty measurement.

4.2 Axioms

The axiomatic method played a central role in understanding and communicating the advantages of the FGT class of measures. Many subsequent authors have further

investigated the axiomatic structure for poverty measurement, and used the framework for discerning among alternative measures.[24] Additive decomposability and subgroup consistency play central roles in this discussion. Decomposability was motivated primarily by applications and has had a great impact in the empirical and policy uses of poverty measures. The related property of subgroup consistency has revealed itself to be a potent theoretical tool, with links beyond the measurement of poverty to inequality, welfare, and living standards, and even cooperative game theory. In addition, a number of axiomatic characterizations of the FGT class of measures have been presented. In what follows, we discuss a few of these contributions.

4.2.1 Decomposability and Subgroup Consistency

The decomposability property used to motivate the FGT measures is now seen as the standard formula for linking subgroup and overall poverty levels. A lengthy list of empirical applications of decomposable measures now exists – some examples are given below in section 4.3.1 – and a parallel body of work has explored the conceptual implications of the property. In an early discussion of decomposability, Foster (1984) stressed that while the property is not needed for every case where subgroup poverty levels are evaluated, it becomes essential when the goal is an assessment of the contribution of subgroup poverty to overall poverty. Several papers restrict attention to general classes of decomposable measures. Atkinson (1987), for example, defines an "additively separable" class of poverty measures over continuous income distributions that are also decomposable.[25] Foster and Shorrocks (1991) axiomatically derive several general classes of decomposable poverty measures using the allied property of subgroup consistency.

The subgroup consistency axiom, which appeared in the FGT paper in a slightly less demanding form, was originally motivated by examples from inequality analysis found in earlier versions of Cowell (1984, 1988) and in Mookherjee and Shorrocks (1982)). A similar property, with restrictions on subgroup means, leads to the most natural characterization of the generalized entropy measures of inequality (Shorrocks, 1988). Foster and Shorrocks (1991) explored the implications of subgroup consistency for poverty measurement and showed that subgroup consistent indices are in essence monotonic transformations of additively decomposable measures. These results justify decomposable indices, not just for their empirical usefulness, but for their unique role in ensuring consistency between subgroup and overall levels. More recently, and in a different context, the property has been shown to select Atkinson's (1970) parametric class of equally distributed equivalent income (ede) functions and the other "general means" from among all possible income standards.[26] Sen (2006), however, has argued

[24] See for example Foster (1984), Hagenaars (1987), Seidl (1988) and Zheng (1997).

[25] Atkinson (1987) uses an additive form to mirror the structure of the welfare functions in Atkinson (1970) and does not mention decomposability.

[26] An income standard summarizes an entire distribution as a single representative income level and satisfies various invariance axioms, a normalization axiom, and linear homogeneity; functions of this sort underlie nearly all inequality and poverty measures (Foster, 2005, 2006).

against subgroup consistency in part because of its unlimited applicability to all partitions of the population, whether salient or not.[27]

4.2.2 Characterizations

A first approach to characterizing the FGT can be found in Basu and Basu (1991), who modify Sen's three-step procedure to derive their variation of the FGT indices. Foster and Shorrocks (1991) assume subgroup consistency and derive the class of relative measures (satisfying scale invariance) and the class of absolute measures (satisfying translation invariance).[28] They show that the intersection only includes the headcount ratio or certain transformations (see also Zheng, 1994, who drops subgroup consistency). They then ask if there might be pairs of measures, one relative and one absolute, that are compatible in that they render the same judgments when the poverty line is fixed. They prove that $(P_\alpha, z^\alpha P_\alpha)$ is essentially the only such pair, where the latter is the absolute FGT measure. Ebert and Moyes (2002) extend these results to the orderings represented by poverty measures at a given poverty line (each ordering does not make comparisons across poverty lines). They convert the two invariance axioms into ordinal forms, and note that the FGT orderings generated by P_α (and $z^\alpha P_\alpha$) are consistent with both axioms (indeed they are the same ordering). Their main theorem shows that the FGT orderings are essentially the only ones consistent with the two axioms. Ebert and Moyes also introduce a new concept analogous to an income standard or ede in inequality analysis. The equivalent societal income (ESI), or the amount of income which, if received by all individuals in the population, will yield the same poverty level as the actual income distribution, is a cardinal representation of the underlying "lower poverty" ordering that could be useful for policy purposes. Chakraborty, Pattanaik, and Xu (2008) characterize measure P_2 with the help of a property they call "equivalent transfer". As noted in Foster, Greer, and Thorbecke (1984), P_2 just violates transfer sensitivity, in that a transfer of the same size between two poor persons the same distance apart has the same effect on poverty, no matter the location of the pair. This equivalent transfer axiom states this in the form of a requirement, and their main result shows that given a range of standard properties, this property uniquely identifies P_2.

4.3 Applications

The FGT class has proven to be very useful for evaluating the extent of poverty across space and time. Empirical applications abound, and virtually every country has been analyzed using P_2, P_1, and P_0, at one time or the other. Most of the applications make use of the decomposability property to analyze the important correlates of the incidence, depth and severity of poverty and set the stage for informed discussion of policies to confront poverty. Accordingly, the FGT class has been instrumental in the design of these

[27] See also Foster and Sen (1997). Sen's argument is based on a rejection of symmetric unidimensional poverty measurement, and hence is better seen as a call for multidimensional measurement than as a rejection of subgroup consistency, per se. Despite this disagreement on subgroup consistency, one of the first papers to cite FGT was Sen (1983).

[28] See Donaldson and Weymark (1980) for a discussion of absolute measures. The absolute FGT measures, defined by $z^\alpha P_\alpha$, are routinely employed in tests of stochastic dominance.

policies, in part due to its decomposable structure, but also because of its specific functional form that generates straightforward formula for policy design. The following provides a sample of the broad and varied literature on the applications of the FGT class of measures.

4.3.1 Empirical applications

The first empirical applications of the FGT methodology were undertaken by Greer and Thorbecke using the Integrated Rural Household Survey data for Kenya (Greer, 1985, Greer and Thorbecke, 1986a-c). A detailed food poverty profile of Kenyan smallholders (constituting over 70 per cent of the total population) was derived. National food poverty, as measured by P_2, was decomposed according to six sets of characteristics: region of residence, household size and composition, household landholding size, cropping pattern and degree of market involvement, type of employment, and by gender and marital status of the head of the household. Regional food poverty lines were established based on the diet composition actually consumed by the poor reflecting the different regional tastes and preferences and local food prices. These studies illustrated and demonstrated the decomposable property of the P_2 measure based on population-share weights and set the stage for a myriad of subsequent poverty profiles based on the FGT methodology world-wide.

The literature contains hundreds of empirical applications using the FGT methodology, and we will not attempt a survey here. Instead, we highlight briefly some selective examples of sectoral and regional poverty decompositions.

Sectoral Decompositions The sectoral pattern of growth is a key determinant of the impact of growth on poverty. Thus, in an early application of all three FGT measures, Huppi and Ravallion (1991) examine the structure of poverty in Indonesia by sector of employment, and how it changed during the adjustment period 1984 to 1987. They found that gains to the rural sector in key regions were quantitatively important to Indonesia's success in alleviating poverty. Most poverty exists – and most gains in alleviating poverty were made – in the rural farming sector. These gains were associated with crop diversification and continued growth in off-farm employment. The government's adjustment program favored rural areas and were crucial to Indonesia's evident success at maintaining momentum in alleviating poverty.

The impact of a sector's output on poverty alleviation can be direct through the increase in incomes accruing to the poor households who contributed through their labor or land or other resources to the sector's growth of output. But another part of poverty reduction results from the interdependence of economic activities (the socio-economic groups' spending and re-spending effects), which can be estimated within a general equilibrium framework such as a Social Accounting Matrix (or SAM). Thorbecke and Jung (1996) applied the FGT measure to a relatively highly disaggregated Indonesian SAM (including 75 sectors, 8 household groups and 24 different production activities). They found that total poverty reduction effects originating from agricultural production activities are highest, followed by services and informal activities - a finding that can be generalized to most settings. Among manufactures, food processing and textiles, which

have closer inter-production linkages with agriculture or are more labor intensive (especially of unskilled labor), made relatively large contributions to poverty alleviation.

Spatial and Regional Extensions A critical issue in regional poverty decompositions relates to the choice of poverty lines. For instance, in the light of distinct dietary regimes and differences in relative prices, would it be more appropriate to use regional poverty lines rather than the same national poverty line?

There are currently two main monetary methods of setting the poverty line, i.e. the Cost of Basic Needs (CBN) and the Food-Energy-Intake (FEI) methods. The CBN approach has the advantage of ensuring consistency (treating individuals with the same living standards equally) while the FEI approach has the advantage of specificity reflecting better the actual food consumption behavior of individuals around the caloric threshold given their tastes, preferences, and relative prices.

It has been argued by Ravallion and Bidani (1994) that in order to make valid welfare comparisons, the reference basket (bundle) yielding the caloric threshold should remain constant. The monetary poverty line at any point in time is then obtained by multiplying the constant quantitative reference basket by the variable price vector to obtain z at current (nominal) prices and then deflating it by an appropriate price index (often the consumer price index, CPI) to express z in real terms. Given the crucial importance of context-specific conditions in shaping the perception of poverty, it can be argued that the setting of z at a more location - specific level would lead to a more accurate appraisal of poverty. The use of a (normative) national or even provincial poverty line in the light of major intra-regional and inter-village differences in socio economic conditions can distort the poverty diagnosis at the local level. Again, this illustrates the inherent conflict between the specificity and consistency criteria. It is not possible to satisfy both simultaneously. The trade-off between welfare consistency (using the same national food basket in all regions) and being realistic and faithful to different regional preferences and diets (which calls for specific regional poverty lines) is unambiguous (Thorbecke, 2004). The earliest applications of FGT (in particular P_2) to measuring food poverty in Kenya by Geer and Thorbecke adopted the FEI methodology to derive the different food baskets actually consumed by the poor around the caloric RDA and corresponding regional food poverty lines. The welfare consistency criterion was violated in favor of capturing the actual consumption behavior of households in different settings. Subsequently, most attempts at deriving regional poverty estimates relied on the CBN approach on the ground that this method besides being welfare consistent was also more robust. One of the most elaborate critical comparisons of these two methods in a regional application to Mozambique was undertaken by Tarp et al (2002) and it is probably fair to state that the choice of approach is still being debated.

4.3.2 Policy applications

In contrast to the empirical applications described above, which seek to understand the patterns of poverty observed in the real world, the policy applications considered in this section are explicitly normative, with a poverty measure replacing a welfare function as the objective function. The framework has led to a rich literature on the optimal design of policies to minimize P_α under varying assumptions about feasible policy instruments.

This, in turn, has helped guide the design and evaluation of a prominent contingent cash transfer program and other development programs.

The idea that P_α could be used in policy design was not explicitly discussed in the FGT paper, although it was apparent that decomposability and subgroup consistency were partially motivated by policy considerations. Much of the literature on optimal design of poverty programs makes use of the specialized form of the P_α class of measures and, in particular, the individual poverty function g_i^α underlying the aggregate measure. Differentiation shows that for $\alpha > 0$, the marginal impact of a transfer of income on i's poverty is proportional to $-g_i^{\alpha-1}$; hence, the marginal impact on P_α of equal sized transfers to all members of a population subgroup is proportional to $-P_{\alpha-1}$. This special property of the FGT class – that the impact on P_α (or g_i^α) depends on $P_{\alpha-1}$ (or $g_i^{\alpha-1}$) – has been exploited repeatedly in the literature that followed the important work of Kanbur (1987ab). We now briefly discuss this literature.

Consider a world in which a government (or some other entity) is to allocate funds across a population in such a way as to maximally reduce poverty, but is limited by information and policy tools. This is similar to a traditional optimal taxation exercise (Kanbur, 2008), but with a more focused poverty objective instead of a social welfare function. The simplest case of "perfect targeting" is where the government has full information about consumption (or income) levels and can transfer funds freely to individuals. How would it allocate a limited budget of welfare benefits? Bourguignon and Fields (1990) take up this "perfect targeting" scenario and derive the optimal (poverty minimizing) form of direct income transfers from a fixed budget when poverty is measured by the FGT class. They find that for P_α having $0 \leq \alpha < 1$, the optimal policy raises the richest poor out of poverty; for P_1 a best policy transfers funds indiscriminately among the poor, so long as none is lifted above the poverty line; and for P_α having $\alpha > 1$, the optimal policy begins with the poorest person and raises this income level until the next poorest income level is reached, at which point both are raised up, and so forth. This exercise provides an additional reason why the headcount ratio is not a very good objective: it yields a remarkably regressive policy. In contrast, the distribution sensitive measure P_2 always targets the poorest of the poor.

The above scenario assumes perfect information – and yet information is rarely, if ever, perfect, and is always costly. One can avoid information costs by providing equal benefits to all households regardless of their incomes and characteristics (the no targeting policy) but only at the expense of large leakages that would allow a significant part of the anti-poverty budget to be used in a way that does not directly reduce poverty. An alternative between the extremes of perfect information and none at all is the case where the population can be partitioned into identifiable subgroups. It is assumed that the policymaker can adjust welfare benefit levels across subgroups in response to group attributes, but cannot vary benefits within a subgroup. Assuming that there is a fixed budget devoted to the reduction of poverty as measured by P_α for $\alpha > 1$, how should the budget be allocated among the mutually exclusive groups?

One might naturally expect the policymaker to target the subgroup with the highest level of poverty according to P_α in order to minimize P_α. However, Kanbur (1987ab) showed that the optimal budgetary rule would allocate funds first to the subgroup whose $P_{\alpha-1}$ level is highest. A similar optimal policy is obtained in a model of poverty alleviation and region-specific food subsidies (Besley and Kanbur, 1988), a

model that would apply equally well to any commodity consumed by the poor. Ravallion and Chao (1989) provided a useful numerical algorithm for this framework, and also discuss the optimal targeting mix across urban and rural areas to minimize P_2. They note that the headcount ratio P_0 in many countries is many times higher in rural than urban areas, but that this does not imply that urban poverty should *not* be targeted. This decision should depend on P_1 levels, which in principle could be higher in urban areas (but as noted by Levy (1991), typically are not).

Thorbecke and Berrian (1992) considered the optimal budgetary rule in a SAM based computable general equilibrium model that allows for interactions between subgroups, and shows how the rule becomes more complex. In particular, if P_2 as the objective function to be minimized, it may not be optimal to target the group with the highest P_1, due to the differential patterns of spending across subgroups and the subsequent impact of the interactions.

In an influential paper, Levy (1991) considered the design of a poverty program for the extremely poor in Mexico. A key part of his discussion concerned the choice of poverty measure:

> "Indicators of poverty should incorporate concerns about its severity and distribution; the head-count ratio fails to do this, as do other 'indices of marginalization' computed by government agencies and currently used to identify the poor. The Foster-Greer-Thorbecke poverty index [P_α] satisfies axioms with respect to severity and distribution of poverty, can be separably decomposed, allows measurement of the contribution of each region to total poverty, can serve to rank regions for delivery of benefits, and can be used to monitor progress in poverty alleviation." (Levy, 1991, p. 83)

He argued for poverty information to be gathered at the state and county level; the resulting geographical poverty profile should then used to target poverty programs that "exploit the complementarities among nutrition, health and education," where targeting is done in accordance with Besley and Kanbur (1988), and also "could be made contingent on (parents) bringing their children periodically for inoculations and other … medical attention." (Levy, 1991, p. 63). This was the template for the Progresa program, which was implemented by Levy under President Zedillo in 1997. This pioneering contingent cash transfer (CCT) program was designed with the goal of minimizing P_2.[29]

More recently, FGT measures have been used by Morduch (1999, 2000) to understand the poverty impact of different microfinance strategies. Suppose that a self-sustaining institution is only able to target the "richer" poor (say with incomes at 90% of the poverty line), while a program receiving an external subsidy can target poorer households (say with 50% of the poverty line). Assume further that both are able to achieve the same (marginal) increase in household income. Then poverty measured by P_0 would be unchanged for both; P_1 poverty would fall equally for both; but P_2 would register a five-fold decrease for the subsidized program as compared to the self-sustaining

[29] The Mexican law implementing *Progresa* required the program to be targeted to P_2 applied to an aggregated variable and, indeed, the formula of P_2 appears in the statement of the law. As noted above, CONEVAL in Mexico is currently considering multidimensional poverty measures based on the FGT measures.

one (with an even greater differentiation when P_3 is used).[30] Viewing the subsidies as the cost of better targeting, and the associated larger decrease in poverty as the benefit, the net benefit may well be larger for a subsidized microfinance institution than one that is fully sustainable.

Two final examples link the FGT indices and two policy instruments for poverty alleviation. One concerns the Millennium Development Goals (MDGs) of the United Nations, adopted in 2001 by the UN member nations. The first MDG is "to eradicate extreme poverty and hunger"; its first target is "to halve…the proportion of people whose income is less than one dollar a day"; and measures P_0 and P_1 are the first two indicators for monitoring progress of the target. One question concerns the cost of achieving a country-by-country percentage reduction as compared to achieving the same overall reduction in a "poverty efficient" way. Anderson and Waddington (2007) consider this question for P_0, P_1 and P_2 under certain simplifying assumptions, and show that for the headcount the country-by-country targets entail much greater poverty reductions for the poorest countries than in the poverty efficient allocation; whereas, for the poverty gap P_1 and squared gap P_2 this difference appears to be much smaller. In contrast, Collier and Dollar (2002) show that the existing allocation of aid across countries is quite far from an optimal allocation, and that the optimal allocation appears to be robust to the choice between the three FGT measures P_0, P_1 and P_2.

A second example concerns the Poverty Reduction Strategy Papers (PRSPs) introduced in 1999 by the World Bank and International Monetary Fund, to help low income countries assess and combat poverty. Constructing a PRSP is particularly challenging for the poorest countries due to the specialized research capabilities that are needed.[31] It therefore was fortunate that over the period 1995-2005 approximately 150 African economists were trained in poverty analysis under the auspices of a large scale collaborative project initiated by the African Economic Research Consortium, and co-directed by Erik Thorbecke. National research teams from 15 countries in Sub-Saharan Africa completed poverty profiles and assessments that relied extensively on the FGT methodology. These country case studies typically became the basis for the country PRSPs.

5 Prospective areas

The technology of poverty measurement continues to improve as it is adapted to address new conceptual, empirical, and policy challenges. Many of the topics discussed above are quite active areas for research. In particular, multidimensional poverty measurement has risen in importance due in part to Sen's capability approach and increased demand by individual countries and international organizations. The statistical literature on dominance is also expanding as are applications to poverty evaluation and other fields relying on this technology. Policy and empirical applications are ongoing in many countries and sectors. What will be the next direction for expanding this useful technology? Where will the literature take us over the next 25 years? We conclude our

[30] The derivative of the individual poverty function for P_0 is 0; for P_1 it is proportional to -1; and for P_2 it is proportional to $-g_i$.

[31] For example, the guide to preparing a PRSP includes a heavy dose of FGT measurement technology, and even summarizes the Bourguignon and Fields (1990) results.

paper with a brief outline of some possible directions for future work. We do not attempt to provide a definitive response, but rather suggest a number of topics that are especially of interest to us.

5.1 Qualitative data

As poverty measurement is taken to other domains or extended into a multidimensional environment, a crucial emerging issue is how to measure poverty when data do not have the characteristics of income, which is typically taken to be cardinal and comparable across persons. How is poverty or deprivation to be measured in the presence of ordinal, categorical, or qualitative data?[32] Must we retreat to a headcount ratio, or can we continue to evaluate the depth or distribution of deprivations – key benefits provided by the higher order FGT measures when the variable is cardinal? This issue can also arise in unidimensional studies but is almost inevitable in discussions of multidimensional poverty where data on capabilities and functionings can have the most rudimentary of measurement characteristics.

5.2 Axioms versus intuition

The original Sen critique of the headcount ratio and the income gap ratio motivated the development of an array of distribution sensitive poverty measures, including the squared poverty gap measure P_2. These measures satisfy additional axioms and thus are deemed to be superior; but in moving from "partial" indices (cf, Foster and Sen, 1997, p. 168) to aggregate measures, has something been lost? Partial index values are often inherently meaningful and convey information on an important aspect of poverty (eg, H = 0.40 indicates that 40% of the population is poor). In contrast, the numerical values of distribution sensitive measures are typically become meaningful when compared to other values (as emphasized by Anand, 1977, and Foster, Greer, and Thorbecke, 1981). The numerical values of P_2, which are often close to zero due to the squaring of the normalized gaps, can be difficult to interpret even though the formula is clear and even transparent. And this problem is even greater for P_α with $\alpha > 2$, which satisfy an additional axiom – transfer sensitivity – and yet are rarely if ever used.[33] Is there a trade-off between axiomatic desirability and the information conveyed by a poverty measure? How can this tradeoff be evaluated and when should the balance tilt in favor of axioms or, alternatively, in favor of simplicity?

It should be remembered that the headcount ratio is *still* the most commonly used measure of poverty; it would be interesting to understand the factors behind its prevalence despite its failure of fundamental criteria for measuring poverty. Alternatively, might there be ways of making the numerical values of P_α more transparent

[32] See for example the discussion in Alkire and Foster (2007) or Kanbur (2003).

[33] For example, consider the following quote from the IDRC website on poverty measurement: "Notwithstanding the above, interpreting the numerical value of FGT indices for α different from 0 and 1 can be problematic. We can easily understand what is meant by a proportion of the population in poverty or by an average poverty gap, but what, for instance, can a squared-poverty-gap index actually signify? And how to explain it to a government Minister?..." See http://www.idrc.ca/en/ev-103707-201-1-DO_TOPIC.html.

– especially for larger α? One approach that is inherent in Ravallion (1994) is to use the three measures P_0, P_1, and P_2 in tandem to help clarify the additional information each brings to the table. The poverty incidence measure gives information on frequency, the poverty depth measure adds information on average gaps, while the poverty severity index includes information on the distribution by concentrating on the poorest poor. Another possibility suggested by Subramanian (2004) and Foster (2005) is to consider the subgroup consistent transformation $(P_\alpha)^{1/\alpha}$ for $\alpha > 0$ that is not additively decomposable, but has values that are easier to interpret.[34] This is analogous to the tradeoffs between the decomposable generalized entropy inequality measure (whose values are difficult to interpret) and the Atkinson class of inequality measures (whose values have an intuitive interpretation via welfare), which is not decomposable. It would be very useful to explore the tension between axiomatic acceptability and meaningfulness in the context of poverty measurement.

5.3 Relativities and absolutes

As observed by Foster (1998), there are many competing notions of "absolute" and "relative" in the measurement of poverty. For example, P_2 is relative in terms of its invariance property and its treatment of population sizes; it typically uses an absolute poverty line that does not adjust when incomes rise. It would be useful to subject the now traditional assumptions to a more systematic analysis, and to explore alternatives that may be more appropriate in certain circumstances. The axiom of scale invariance, for example, might be the subject of additional scrutiny. For example, this axiom is only applicable when analyzing distributions at different poverty lines, and yet one might argue that only common line comparisons are meaningful. At the same time, the sharp divide between relative poverty lines, which are typically used in the more developed countries, and absolute poverty lines, which are often used in developing countries, needs to be bridged. How this is to be done, and how the resulting cutoff will interact with the poverty measure, are important issues for investigation. Some progress in this direction includes work by Ali and Thorbecke (2000), Atkinson and Bourguignon (2001), Foster and Szekely (2008), and Ravallion (2009); for a treatment of a related line of research in the measurement of welfare, see Atkinson and Brandolini (2008).

As noted above, the AIDS epidemic has brought to the fore a related difficulty: If a disease disproportionally affects poorer persons, and through death removes them from consideration by the measure, then, indeed, average poverty may well be seen to diminish. Who is to be taken into account when measuring poverty? Over what time frame? This problem is certainly related to the multidimensional aspect of poverty (including the problem of time), but it can also be viewed as an example of a broader category of fundamental issues concerning the "denominator" of poverty measurement: who is to be included in the calculation of the poverty value? The complexity of the relative/absolute divide expands in the face of a multidimensional approach to poverty, and is related to the issue of how to weigh deprivations in one dimension against the deprivations in another. Could one dimension be measured against a relative cutoff while a second used an absolute one? What might this imply for the trade off between

[34] As a generalized mean of normalized gaps it measures poverty in the space of normalized gaps, and has higher values at higher α. It is related to the second measure in Clark, Hemming and Ulph (1981) and to the ESI of Ebert and Moyes (2002).

dimensions as the relative cutoff changes? There are many questions that need to be addressed.

5.4 Identification versus aggregation

Sen's (1976) observation that the aggregation step is an important component of poverty analysis has led to the development of many alternative aggregation methods. The identification step, by contrast, has changed very little in structure from the time that Rowntree (1901) constructed a poverty line of just over a pound a week to identify poor families in York. Given the achievements in aggregation, it may be an appropriate time to re-evaluate the notion of a poverty line and its role in identifying and targeting the poor. Some have suggested that the entire construct of a poverty line should be thrown out.[35] This is an intriguing possibility, which if accepted would lead to a radical overhaul of the way poverty is envisioned and would, in turn, provoke many questions. If the identification step is to be dropped, what should replace it? If it is to be retained, how can it be accomplished without a poverty line? How is targeting to be defined and implemented? Some authors criticize the abrupt 0-1 nature of the poverty line, and replace it with a fuzzy approach. Is this a good solution or does it simply multiply the arbitrariness? If the identification step is altered, what then is to become of the focus axiom as a property of aggregate poverty measures? Foster and Szekely (2008) use an inequality averse general mean or ede as a "poor income standard" having no poverty line but undeniably emphasizing the poorest incomes. Is this an effective way of combining concerns for poverty and inequality in one measure, or is there a superior approach?[36] Further work is needed to evaluate practical alternatives to poverty line identification.

Second, although there is a perception that the identification step is straightforward in the unidimensional case, some interesting and subtle points remain concerning the link between the identification step, the poverty line, and the aggregation step. Imagine a methodology that used one poverty line to identify the poor and a second, higher poverty line as the standard against which to aggregate. This might be quite natural if the goal were to determine the conditions of the poorest poor as a separate group within the larger set of the poor.[37] Of course, this would not matter if the measure were the headcount ratio, since it relies only on the frequency of the poor, not how far they fall below the poverty line. However, the values given by P_1 and P_2 can be greatly altered by having a different line for identification and aggregation. What rules out such a disconnect between the two steps of poverty measurement? Can axioms be reformulated to apply to the overall methodology of poverty measurement, including the identification and aggregation step? This is a subtle problem, but one that deserves additional thought.

Third, the importance of the identification grows much larger when poverty is taken to be a function of many multiple dimensions. Now the set of the poor can assume a variety of shapes including the two extremes of the union and the intersection and all

[35] Deaton (1997, p. 144), for example, expresses deep skepticism about poverty lines: "…I see few advantages in trying to set a sharp line, below which people count and above which they do not." However, he also notes the practical advantages of poverty lines and continues to use them and the FGT measures in his empirical work.

[36] Fields (2005) considers other ways of combining inequality and poverty measures.

[37] See Levy (1991). The integrated measurement of the "ultra-poor" is an important area for future research.

possibilities in between. Which form of identification is the most natural? Can axioms be devised for identification methods alone? How should axioms for measures be reformulated to apply to the entire methodology, including the identification and aggregation steps? Some initial steps have been taken by Alkire and Foster (2007), but much remains to be done.

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Figure 1 P_2 and the distribution