Recruiting Talent[*]

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Abstract

We propose a model of firm dynamics in which a firm's primary asset is the talent of its workforce. Firms compete in wages to attract applicants, and managers seek to identify the most talented. Over time, a firm's quality evolves as today's recruits become tomorrow's managers. We show that firm-applicant matching is positive assortative, with better firms posting higher wages and attracting better applicants. Initially similar firms then diverge over time and the economy converges to a steady state featuring persistent dispersion in talent, wages and productivity. In the steady state, positive assortative firm-applicant matching offsets regression to the mean and talent constitutes a sustainable competitive advantage. We also show that equilibrium leads to an inefficient selection of talent into the industry, and can be improved by policies that reduce wage dispersion.

1 Introduction

The success of most firms is built upon hundreds of individuals who take thousands of decisions, making it critical to identify and recruit the best talent. Such human capital is a key source of competitive advantage in a wide range of industries, from service to technology. For instance, the standing of a university depends more on the quality of its professors than on its real estate. Indeed, Waldinger (2016) shows that the loss of human capital at German universities had a large, persistent effect on output, whereas the loss of physical capital had a small, temporary effect. Talent is also critical for consultants, salesmen and firms like Netflix, whose human resource manual states "One outstanding employee gets more done and costs less than two adequate employees. We endeavor to have only outstanding employees" (Hastings and McCord, 2009).

Differences in talent across firms perpetuate through hiring because talented managers have better judgment and better information. For example, in our own profession, a star professor is more skilled at assessing a student's job talk than a community college lecturer. Broader evidence for this comes from the literature on referrals. Burks et al. (2015) look at three industries and show that more productive employees make more and better referrals. Beaman and Magruder (2012) support this finding, and additionally show that more productive employees are better able to predict the performance of their referrals.

Based on these tenets – that talent matters and perpetuates through hiring – this paper proposes a model of firm dynamics in which talent takes center stage. We show that firms with many talented employees optimally pay high wages and attract the best applicants, offsetting the natural regression to mediocrity that results from noise in the hiring process. The economy then converges to a steady-state that exhibits persistent heterogeneity in wages, talent and productivity, consistent with the large dispersion found by Abowd, Kramarz, and Margolis (1999) and Foster, Haltiwanger, and Syverson (2008). We also show that welfare is increased by policies that reduce wage inequality, lowering the dispersion of talent and the segregation of workers across firms.

In Section 2, we introduce a static model of labor market competition that serves as a building block for the later, dynamic analysis. A continuum of firms competes for a continuum of workers who have high or low ability. Firms attract applicants by posting wages, and then receive noisy signals about each applicant, helping them screen the wheat from the chaff.[1] Firms with more skilled recruiters receive more accurate signals. We suppose the market is frictionless: The highest-paying firm attracts all applicants and hires the first applicant who produces a good signal (e.g. passes the firm's test). Other firms hire from the remaining, adversely selected pool of workers. The quality of a firm's applicant pool thus depends on its ordinal wage rank, giving rise to a continuous equilibrium wage distribution.

If talent is scarce, firm-worker matching is *positive assortative*. That is, firms with more skilled recruiters post higher wages; they thus attract better applicants and hire more talented recruits. Intuitively, skilled firms have a comparative advantage in hiring from a high-wage applicant pool with a balance of talented workers, rather than hiring from a low-wage pool in which few talented applicants remain. We then show that an increase in screening skills raises the dispersion of wages and the segregation of workers across firms, helping to explain recent trends documented by Card, Heining, and Kline (2013) and Song et al. (2016).

In Section 3, we turn to the full, dynamic model. Workers retire at an exogenous rate and, in order to fill the resulting vacancies, firms compete to recruit from a fresh cohort of job seekers, as in the static model. Motivated by industries such as academia, consulting, or technology, we suppose that a large fraction of a firm's workforce participates in recruiting; specifically, we assume that a firm's recruiting skills corresponds to its fraction of talented workers. This means that both the productivity and recruiting skills of the firm evolve as the firm hires recruits who, in turn, select future recruits.

As in the static model, firms with more talent pay higher wages because of the complementarity between screening skills and applicant quality. Thus, equilibrium is unique with talented firms posting high wages, attracting the best applicants and hiring talented recruits, reinforcing their initial advantage. If all firms start off with similar talent, then the better endowed accumulate

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[1]This "direct screening" differs from the "self-selection" use of the word "screening", e.g. Stiglitz (1975).

talent over time, while the worse endowed hire from poor, deteriorating applicant pools and lose talent. Hence small initial difference are magnified and dispersion arises endogenously. Looking at payoffs, top firms initially pay high wages and lose money; they later recoup this investment when their accumulated talent raises their productivity and provides a competitive edge in identifying future talent. Meanwhile, wages fall over time as firms become differentiated and the lower firms are no longer able to effectively compete in the labor market.

These dynamics converge to a unique steady state, irrespective of the initial distribution of talent. Steady-state dispersion balances two countervailing forces: positive assortative matching which amplifies differences across firms, and imperfect screening which leads to mean regression and equalizes firms. Human capital is thus a source of sustainable competitive advantage. While low-quality firms could in principle catch up by posting higher wages and hiring more talented workers, it is not profitable for them to do so.

In Section 4, we enrich the dynamic model in two directions. First, we introduce heterogeneity in firm fundamentals so that firms differ in both their (endogenous) stock of talent and their (exogenous) marginal product of talent, which we interpret as "technology" that complements talent. Equilibrium wages increase in both technology and talent, so a low-technology firm may initially outbid a high-technology firm if it has significantly more talent. Nevertheless, as talent evolves over time, the economy converges to a unique steady state in which matching is *stratified*, meaning that firms with high technology also accumulate more talent. Hence, the initial two-dimensional matching problem ultimately collapses to a single dimension. This suggests that universities with good faculty but poor fundamentals are inherently unstable in the long-run. We then use this richer model to study a firm's adjustment to an exogenous increase in technology. While such a firm clearly raises its wage, we show that wages and the quality of applicants and recruits stay below the new steady-state level and converge only asymptotically. Intuitively, skilled management complements high-quality applicant pools, so it is a mistake to pay high wages before the management can use the extra resources wisely.

Our second application is to investigate the impact of peer effects on the distribution of talent and wages. We suppose that all workers prefer to work at a firm with more talented colleagues, and show that equilibrium matching is unchanged, as is the evolution of talent. Wages, however, may be quite different. Workers maximize total compensation, i.e. wages plus peer effects, and so high-talent firms can cut their wages, effectively substituting peer effects for monetary compensation. This effect decreases wages in steady state but, if firms are initially similar, intensifies competition to acquire talent in the first place, and drives up wages in early periods.

In Section 5 we study welfare. Since every worker is equally productive at any firm, welfare depends solely on the aggregate sorting of talent into the industry. In contrast to classic matching models (e.g. Shapley and Shubik (1971), Becker (1973)), equilibrium in our model is inefficient. In particular, if a social planner could set the order in which firms screen applicants she would have low-quality firms screen first, that is, matching would be negative assortative. Intuitively, there is a negative *compositional externality* whereby the quality of the applicant pool deteriorates faster if high-quality firms pick first, and we show that this outweighs the private gain from positive assortative matching. We also consider a social planner who can only restrict the set of admissible wages (rather than contracting on firm types directly) and show that she would optimally force all firms to pool at the same wage to induce a random order of screening; the same outcome can be implemented with a suitably chosen wage cap. Hence, while productivity dispersion is not per se

inefficient with our linear model of production, it indicates an inefficiency in the underlying labor market competition and matching process (cf. Hopenhayn (2014)).

1.1 Literature

Our paper is most closely related to the literature on matching with transfers. This was started by Shapley and Shubik (1971) and Becker (1973) who showed that equilibrium is efficient and that, for complementary production functions, matching is positive assortative.[2] A number of recent papers introduce dynamics into matching models. Anderson and Smith (2010) and Anderson (2016) suppose agents match each period and evolve as a function of the match. They prove that equilibrium is efficient, and derive sufficient conditions for matching to be positive assortative. Following Prescott and Boyd (1987), Jovanovic (2014) considers an OLG model where young and old workers work together, enhancing the human capital of the young worker. He shows that noise in the match leads to misallocation, lowering the growth of the economy.

In contrast to this literature, our paper focuses on the hiring process rather than production complementarities.[3] This is important since "the literature has been less successful at explaining how firms can find the right employees" (Oyer and Schaefer, 2011). We propose that hiring is a skill that is heterogeneous across firms, and study how these skills shape wage competition and the resulting allocation of talent and productivity over time. Our model of Bayesian screening provides a micro-foundation for the complementarities that give rise to positive assortative matching. However, unlike classic matching models, equilibrium is inefficient due to the compositional externality.

Our static screening model is related to a variety of models of adverse selection.[4,5] The closest is Kurlat's (2016) model of financial markets with adverse selection in which buyers receive heterogeneous signals about sellers' assets. Our model assumes that firms' signals are independent, and show that firms post different wages, matching is positive assortative and inefficient. In comparison, Kurlat assumes that buyers' signals are nested, meaning that a more informed buyer knows everything that a less informed buyer knows. He shows that buyers post the same price with ties broken in favor of the less-informed buyers, and equilibrium is efficient. In Appendix A.1 we provide a heuristic explanation of how these differences arise.

Our static model also relates to the literature on wage dispersion. Albrecht and Vroman (1992) consider a random-matching model in which agents differ in their reservation wage and show that firms are tempted to raise their wage to appeal to less enthusiastic workers. Burdett and Mortensen (1998) propose a model with homogeneous workers and on-the-job search in which firms pay more to poach workers from their competitors. In these papers, dispersion derives from firms competing for more workers in an economy with matching frictions, whereas our dispersion derives from firms competing for better workers in an economy with adverse selection. Moreover, we focus on how

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[2]In the context of firms, Lucas (1978) and Kremer and Maskin (1996) considered a model where agents are divided into managers and workers, who then interact via a multiplicative production function. Garicano (2000) and Garicano and Rossi-Hansberg (2006) endogenize the production function and number of layers in the hierarchy, while Levin and Tadelis (2005) consider how agents form partnerships.

[3]This view is consistent with Waldinger's (2012, 2016) finding that the loss of star faculty led to a permanent reduction in the quality of hires and thus the quality of the department, but did not affect the productivity of current faculty.

[4]Lockwood (1991) uses such screening in a random search model to argue that the duration of unemployment serves as a signal of worker quality.

[5]There is also a literature on matching with incomplete information and interdependent values. For stability notions, see Chakraborty, Citanna, and Ostrovsky (2010) and Liu et al. (2014).

talent and productivity dispersion evolve over time as a result of selective hiring.

Our paper provides a theory of firm dynamics with a firm's talent as its key strategic asset. Models of firm dynamics were first studied by Lucas and Prescott (1971) and Hopenhayn (1992), where firms' competitive advantage is determined by their technology and demand state. Jovanovic (1982) and Board and Meyer-ter-Vehn (2014) suppose firms' self-esteem and reputation serve as a state variable. Hopenhayn and Rogerson (1993) and Fuchs, Green, and Papanikolaou (2016) suppose that firms differ in their labor and capital stock, respectively, which are slow moving due to firing costs and adverse selection. By focusing on talent and recruiting, our paper provides a new channel through which firms can sustain a competitive advantage that is particularly relevant for service industries (e.g. universities, technology) which make up over 70% of US GDP. It also gives rise to predictions concerning the dispersion and persistence of productivity, wages and employee quality.

Finally, some themes in our paper echo those in dynamic models of political economy. As in our paper, Dewan and Myatt are interested in the evolution of talent within organizations. Dewan and Myatt (2010) focus on firing standards, arguing that as a government ages, its talent pool depletes and standards fall. Dewan and Myatt (2014) focus on recruitment, supposing that a government that can recruit better talent as it becomes more successful, creating a positive feedback loop. Our key assumption – that hiring a worker means hiring his taste – also relates to the literature on dynamic clubs, where today's members must decide who will make decisions tomorrow. Acemoglu and Robinson (2000), Lizzeri and Persico (2004) and Jack and Lagunoff (2006) consider the extension of the voting franchise, while Sobel (2001), Barberà, Maschler, and Shalev (2001) and Roberts (2015) consider general club goods.

2   Static Model

In this section we introduce the static competitive screening model. Section 2.1 describes how heterogeneous firms compete to attract and identify talent. Section 2.2 shows that wages are dispersed in equilibrium. Section 2.3 provides sufficient conditions for positive assortative matching.

2.1   Model

A unit mass of firms, each with one vacancy, competes for a unit mass of workers by posting wages. Workers and firms are both heterogeneous. Workers differ in their talent, with proportion $\bar{q} \in (0, 1)$ talented and the remainder untalented. Firms differ in their recruiting skill. In particular, each firm has a unit mass of recruiters, of which proportion r is skilled, $\theta = H$, and $1 - r$ is unskilled, $\theta = L$. Firms' screening skills r are distributed with pdf f, cdf F, and support $[\underline{r}, \bar{r}]$.[6]

Firms simultaneously post wages; workers then apply to firms in order of wages. As in the Gale-Shapley mechanism, the firm with the highest wage picks a worker; the remainder then apply to the "second" firm, and so on until all firms and workers are matched. Firms select among applicants by picking a random recruiter to administer a pass/fail test to each applicant. Talented workers pass

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[6]We assume that firms are distributed continuously. This is for notational convenience, so we can associate each firm with its screening skill r, and avoid spurious equilibrium multiplicity. But Lemma 1, below, also applies if there were a mass of identical firms; that is, wage dispersion would arise endogenously.

the test, while untalented workers are screened out with probability p_θ, where $0 < p_L < p_H < 1$.[7] The firm randomly hires one applicant who passes the test, which is guaranteed to happen since there is a continuum of applicants.[8] This description assumes firms post different wages, as happens in equilibrium. For concreteness, assume that in case of a tie, all workers break the tie in the same way, as if the firms were infinitesimally differentiated.

Payoffs are as follows. Workers only care about wages, and so accept any job with positive wage. Productivity is μ for talented workers and 0 for untalented workers. Thus, when a firm posts wage w and hires a worker who is talented with probability λ, its expected profits are $\pi = \mu\lambda - w$.[9]

We solve for Nash equilibrium in wages, $w(r)$.

2.2 Preliminary Analysis

We first derive the quality of the recruits hired by firm r. When a recruiter of skill θ screens an applicant pool with expected talent q, Bayes' rule implies that the expected talent of its recruit equals

$$\lambda(q; \theta) := \frac{q}{1 - (1 - q)p_\theta}. \tag{1}$$

The numerator is the proportion of applicants who are talented; the denominator is the proportion of applicants who pass the test. When proportion r of the firm's recruiters have type $\theta = H$, the expected talent of its recruit is thus $\lambda(q; r) := r\lambda(q; H) + (1 - r)\lambda(q; L)$, where we identify $H = 1$ and $L = 0$.

Next, we wish to understand how the quality of the applicant pool depends on the firm's rank in the wage distribution. Suppose all firms post different wages $w(r)$ and write $x = x(w) \in [0, 1]$ for the resulting wage quantiles. In turn, for any quantile x, write $r(x)$ for recruiter skill and $q(x)$ for applicant quality.[10] The highest ranked firm has $r(1)$ skilled recruiters and faces applicant pool $q(1) = \bar{q}$; thus, proportion $\lambda(q(1); r(1))$ of its recruits are talented. Since firms select talented workers disproportionately, lower-ranked firms have an adversely selected applicant pool, meaning $q(x)$ falls as the firm rank x declines. Specifically, at rank x there is a total of $xq(x)$ talented workers, of which firms $[x, x + dx]$ hire $\lambda(q(x); r(x))dx$; hence $d[xq(x)] = \lambda(q(x); r(x))dx$. Rearranging, the talent pool evolves according to

$$q'(x) = \frac{\lambda(q(x); r(x)) - q(x)}{x}. \tag{2}$$

Since screening is imperfect, some talent remains, $q(x) > 0$, for all $x > 0$. However, at the bottom,

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[7]The assumption that talented workers pass the test with certainty is without loss of generality. To see this, assume that talented (untalented) workers pass the test with probability $\bar{p}_\theta$ ($\underline{p}_\theta$) where $\underline{p}_\theta < \bar{p}_\theta < 1$. Then the expected quality of an applicant who passes the test becomes $q\bar{p}_\theta/(q\bar{p}_\theta + (1 - q)\underline{p}_\theta) = q/(1 - (1 - q)\underline{p}_\theta/\bar{p}_\theta)$, which collapses to equation (1) with $p_\theta := \underline{p}_\theta/\bar{p}_\theta$.

[8]Since many workers pass the test, the random prioritization can occur before or after the test. That is, the firm could make all applicants take the test and then choose one, or it could randomly order the applicants and choose the first that passes the test.

[9]As stated, aggregate surplus in our model equals $\mu\bar{q}$ irrespective of firms' strategies since firms have the same, linear production technology and all workers are employed. In Section 5 we endogenize employment, by introducing an entry cost. This introduces a welfare margin, namely the aggregate sorting of talent into the industry.

[10]If an atom of firms posts wage w and workers need to break the tie, our tie-break rule implies that the rank x of a firm with wage w is drawn uniformly from $[\underline{x}(w), \bar{x}(w)]$ where $\underline{x}(w) := \Pr(r : w(r) < w) < \Pr(r : w(r) \le w) =: \bar{x}(w)$. Lemma 1, below, implies that this complication does not arise on the equilibrium path.

firms pick over the workers so many times that in the limit no talent remains, $q(0) = 0$.[11]

The sequential screening equation (2) illustrates the compositional externality of screening discussed in the introduction: Namely, the rate at which $q(x)$ decreases as x falls from 1 to 0 depends on the skill of the screening firms $r(x)$, reflecting the fact that high-skilled firms pick out more talented workers than low-skilled firms, and introduce more adverse selection.

Now, consider equilibrium wages. Since the lowest paying firm $x = 0$ only recruits untalented workers, its productivity is zero, and thus its wage must be zero, too. Moreover, an equilibrium wage distribution can have no atoms or gaps. If an atom of firms $x \in [\underline{x}, \bar{x}]$ offered the same wage w, then – recalling our tie-break rule – a firm could attract discretely better job applicants with a marginal wage raise. On the other hand, if there were a gap so that no firms offered wages $w \in [\underline{w}, \bar{w}]$, then the firm offering $\bar{w}$ could profitably cut its wage to $\underline{w}$ and attract the same applicants. To summarize:

Lemma 1. *An equilibrium wage distribution has no atoms or gaps, with minimum* 0.

2.3 Equilibrium

We have shown that no two firms pay the same wage, but do firms with skilled recruiters pay more or less than those with unskilled recruiters? In Becker (1973), positive assortative matching arises when high types of both market sides are complementary. Similarly, in Lucas (1978) better managers hire more workers since they raise workers' marginal product. Here, the question is whether skilled recruiters have a comparative advantage in screening applicants with higher expected talent.

Formally, any wage profile induces a firm-applicant matching $Q(r) := q(x(w(r)))$, and we say there is *positive assortative matching* (PAM) between firms and applicants if $Q(r)$ is increasing; since $q(x)$ and $x(w)$ are increasing, this is equivalent to the wage function $w(r)$ being increasing. Conversely, we say there is *negative assortative matching* (NAM) if $Q(r)$ is decreasing.

A matching function $Q(r)$ is an equilibrium if no firm r wishes to change its wage w, in particular mimic another firm $\tilde{r}$, by paying wage $\tilde{w} = w(\tilde{r})$ and attracting applicants $Q(\tilde{r})$. The incentive compatibility constraint is then

$$\mu\lambda(Q(r); r) - w \geq \mu\lambda(Q(\tilde{r}); r) - \tilde{w}.$$

Adding to this firm $\tilde{r}$'s IC constraint not to mimic firm r, the wages cancel and we get

$$\lambda(Q(r); r) + \lambda(Q(\tilde{r}); \tilde{r}) \geq \lambda(Q(\tilde{r}); r) + \lambda(Q(r); \tilde{r}).$$

That is, in equilibrium, $\lambda(Q(r); \tilde{r})$ is supermodular in $(r, \tilde{r})$. In other words, defining the *benefit of skilled screeners*

$$\Delta(q) := \frac{\partial}{\partial r}\lambda(q; r) = \lambda(q, H) - \lambda(q, L),$$

$\Delta(Q(r))$ increases in r. This captures the idea that firms target applicants based on the comparative advantage implied by their screening skills.

To understand the form of the equilibrium we show in Appendix A.2 that the benefit function $\Delta(\cdot)$ is single-peaked with peak at $\hat{q} \in (0, 1)$. Intuitively, when all the applicants are talented, $q = 1$,

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[11]To see this, assume by contradiction that $q(0) > 0$. Then $\lambda(q(0); r(0)) - q(0)$ is bounded away from zero and, using equation (2), the derivative $q'(0)$ behaves as $1/x$, contradicting the finiteness of $\int_0^1 q'(x)dx$; thus, $q(0) = 0$.

then any firm can hire a talented recruit, independent of its screening skill, and so $\Delta(1) = 0$. Similarly, when all applicants are untalented, $q = 0$, no firm can hire a talented recruit, and so $\Delta(0) = 0$. In contrast, in the middle, when $q \in (0, 1)$, a firm's recruiting skills help it to separate the wheat from the chaff.

Motivated by industries with relatively few highly productive individuals, such as universities, or technology companies, the body of the paper hereafter focuses on the case of scarce talent.

Assumption. *Talent is scarce,*

$$\bar{q} \leq \hat{q}. \tag{3}$$

Since $\hat{q}$ is a function p_H, p_L, equation (3) is a joint condition on the talent distribution and screening skills. In fact, we show in Appendix A.2 that (3) holds whenever screening is sufficiently noisy, namely a skilled recruiter screening the initial pool is more likely than not to hire an untalented worker, $\lambda(\bar{q}; H) \leq 1/2$. In Appendix A.3 we characterize equilibrium matching in general, without imposing condition (3).

Since applicant talent $q(x)$ ranges from 0 at the lowest-paying firm to $\bar{q}$ at the highest-paying firm, (3) ensures that the benefit function $\Delta(\cdot)$ is increasing at all $q(x)$. That is, skilled firms have a comparative advantage at screening better applicant pools. More formally, in any equilibrium $\Delta(Q(r))$ increases in r; since $\Delta(\cdot)$ is increasing, $Q(r)$ must also increase. We thus have:

Theorem 1. *Equilibrium exists and is unique; matching is positive assortative.*

Proof. The above discussion implies that matching is positive assortative. In particular, firms employ pure strategies $w(r)$ that increase in r. We can then construct the equilibrium. Given PAM, a firm's rank in the skill distribution $F(r)$ equals its equilibrium wage rank x and we can identify a firm by this rank x. The skills $r(x)$ of the rank-x firm is then given by the inverse of $F(r)$, and its applicant quality $q(x)$ is determined by the sequential screening equation (2), and the recruit quality $\lambda(q(x); r(x))$ by Bayes' rule, (1).

Denote the equilibrium wage required to attract applicants of quality q by $W(q)$. At the bottom, $W(q(0)) = 0$. Firm x's first-order condition

$$W'(q(x)) = \mu \lambda'(q(x); r(x)) \tag{4}$$

determines marginal wages, where $\lambda'(q; r)$ denotes the partial derivative $\partial \lambda(q; r)/\partial q$. Conversely, since marginal profits $\mu \lambda'(q; r) - W'(q)$ single-cross in r and matching is positive assortative, the FOC (4) implies global optimality. Thus equilibrium exists and is unique. $\square$

When talent is scarce, firms with skilled recruiters pay higher wages than those with unskilled recruiters. Equilibrium wages are determined by integrating the first-order condition (4), $w(r) = \int_{\underline{r}}^{r} \mu \lambda'(Q(\tilde{r})), \tilde{r}) Q'(\tilde{r}) d\tilde{r}$. Equilibrium profits in turn are determined via the envelope theorem as the integral over incremental screening skills, $\pi(r) = \int_{\underline{r}}^{r} \mu \Delta(Q(\tilde{r})) d\tilde{r}$. Intuitively, total value $\mu \lambda(q; r)$ depends on both the applicant pool quality and the screening ability; workers capture the marginal benefit of the former and firms capture the marginal benefit of the latter.

Our analysis has interesting predictions for wage and productivity dispersion. Figure 1 illustrates our benchmark simulation and exhibits a decreasing density of productivity, $1/\lambda'(x)$. Intuitively, when the test is relatively accurate and talent is scarce, a few top firms hire the bulk of the talent, and the remaining firms are left with untalented workers. Similarly, the density of wages is decreasing since firms compete fiercely for the top-ranks, but see little differences among the bottom ranks.
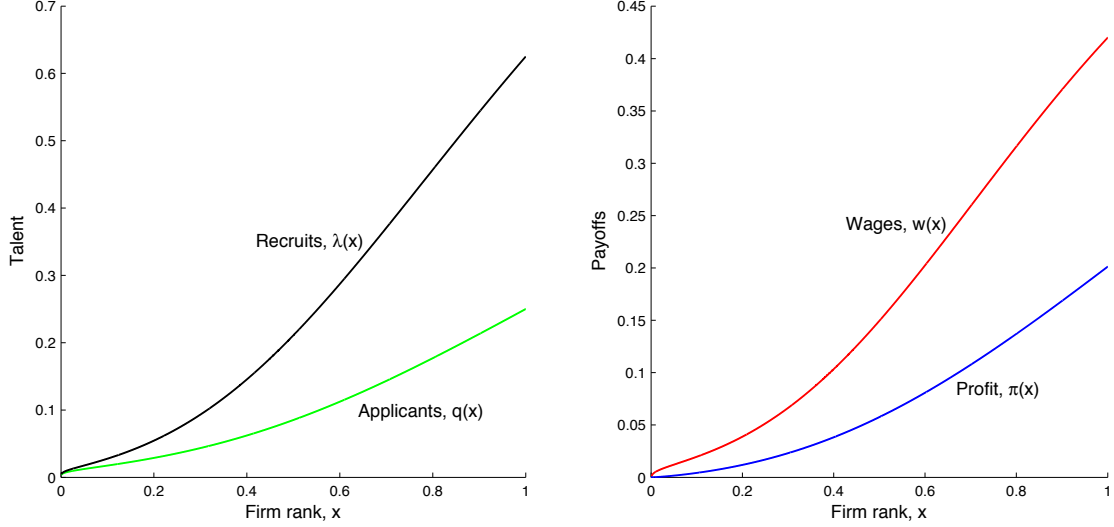
Figure 1: **Equilibrium with Scare Talent.** The **left** panel shows the quality of applicants $q(x)$ and recruits $\lambda(x) := \lambda(q(x); r(x))$ as a function of the wage rank $x = F(r)$. The **right** panel shows the resulting wages $w(x)$ and profits $\pi(x)$. These figures assume $p_H = 0.8$, $p_L = 0.2$, $r \sim U[0,1]$, $\mu = 1$ and $\bar{q} = 0.25$.

Figure 2(a) shows the effect of change in screening technology (e.g. the information content of networks, or development of new assessment techniques) that alters the distribution of recruiter skills $F(r)$. First, we compare the benchmark simulation with $r \sim U[0,1]$ to a setting where all firms have $r = 0.5$. Since the firms become undifferentiated, profits vanish, and wages rise to equal productivity. In addition, as screening ability shifts from top firms to middling firms, productivity drops for top-ranked firms; this reduces adverse selection and productivity rises for middle and lower-ranked firms. Second, suppose the screening ability of firms rises from $r = 0.5$ to $r = 0.8$. The productivity/wages of the top firms rise as they select better workers; this lowers applicant quality for lower firms and reduces talent among their recruits, even though their screening skills have improved just the same. Thus inequality rises, as measured by the productivity/wage ratio between the top recruit and the average recruit $\lambda(\bar{q}; r)/\bar{q}$.[12] Rising screening skills may thus explain the increasing segregation of workers across firms seen in the US and Europe, whereas a rise in complementarities does not seem to be consistent with the data (Card, Heining, and Kline (2013), Song et al. (2016)).

Figure 2(b) shows that skilled-biased technological change also leads to an increase in inequality. Specifically, suppose an innovation doubles productivity μ, but halves the number of productive agents $\bar{q}$, such that aggregate production $\mu\bar{q}$ remains unchanged. For the top firm, as the number of talented workers halves, the number of applicants passing the test falls, meaning that $\lambda(\bar{q})$ drops by less than half, and productivity $\mu\lambda(\bar{q})$ increases. The top firm thus hires proportionally more of the available talent, and proportionally less talent remains for lower firms, raising inequality.[13] The figure also shows that profits of high-ranked firms fall as it becomes more important to be at the top of the distribution, making them fight harder for these slots.

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[12]Appendix A.4 offers a variant of this comparative static, that pertains to all firm ranks $x \in [0,1]$, rather than just comparing the top $x = 1$ to the average: Restricting attention to homogeneous recruiters, i.e. $p_L = p_H =: p$, a rise in p increases the productivity ratio $\lambda(x')/\lambda(x)$ for all $x' > x$.

[13]For the case of identical recruiters, Appendix A.4 formalizes this argument.

(a) Changes in Screening Skills
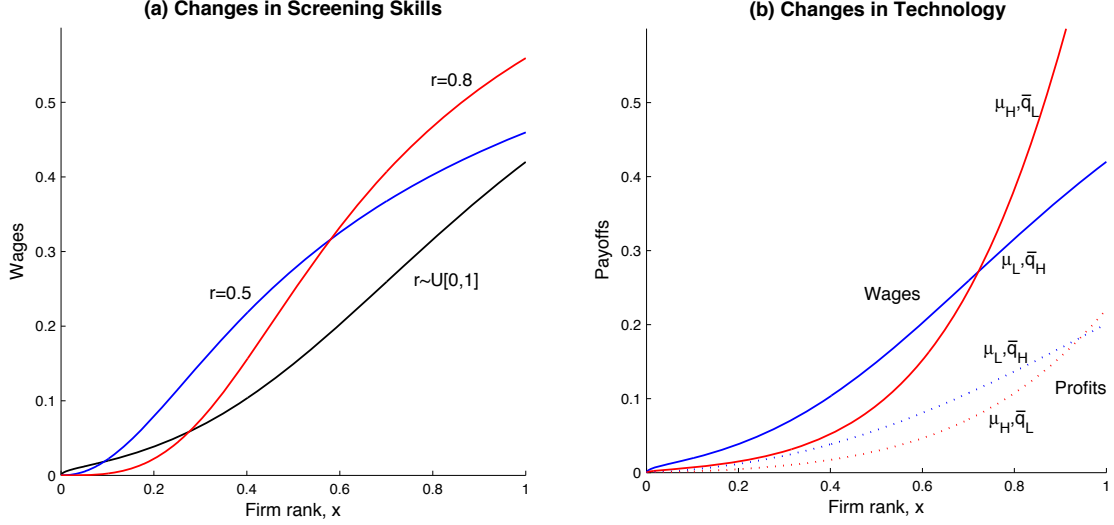
(b) Changes in Technology

Figure 2: **Comparative Statics.** The **left** panel illustrates changes in the distribution of skill, r. The **right** panel illustrates changes in technology, with $(\mu_L, \bar{q}_H) = (1, 0.25)$ and $(\mu_H, \bar{q}_L) = (5, 0.05)$. The other parameters are the same as in Figure 1.

3 Dynamic Model

We now embed our static labor market into a model of firm dynamics, allowing us to endogenize the distribution of talent within and across firms. The key premise is that the employees make the recruiting decisions, with talented employees being more skilled at recruiting. Firms thus desire talented workers both for the immediate increase in productivity, and for the benefit of having skilled recruiters in the future.

We show that, as in the static model, equilibrium is unique with high-talent firms offering high wages that complement their superior screening skills; they thus attract superior applicants, and sustain their talent advantage. We characterize firm values, wages, and profits over time and simulate this equilibrium for initially identical firms. Eventually, the talent distribution converges to a steady state, where the positive assortative firm-applicant matching offsets the regression to mediocrity that results from imperfect screening.

3.1 Model

Time $t \geq 0$ is continuous. There is a unit mass of firms, each with a unit mass of jobs. At time t, a firm is described by its proportion of talented workers r_t; initially, the distribution of r_0 is exogenous.

At every instant $[t, t+dt]$, proportion αdt workers retire, leaving firms with vacancies. In the job market, there are then αdt open jobs and αdt applicants, of whom fraction $\bar{q}$ are talented. Analogous to the static model, firms compete for these applicants by posting life-time wages w_t;[14] applicants who do not find a job immediately leave the industry.

We assume that talented workers become skilled recruiters, and untalented workers become unskilled recruiters, so the firm's skill coincides with its stock of talent, r_t. If a firm with talent r_t

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[14]Equivalently, one can think of worker compensation as constant flow wage $(\alpha + \rho)w_t$ until retirement, but it simplifies the accounting to have firms incur these costs up-front.

posts a wage w_t with rank x_t, then it attracts applicants $q_t = q_t(x_t)$ and hires recruits of quality $\lambda(q_t; r_t)$. Writing $r_t(x)$ for the talent of the firm with wage rank x at time t, the quality of the applicant pool $q_t(x)$ is determined by

$$q_t'(x) = \frac{\lambda(q_t(x); r_t(x)) - q_t(x)}{x} \text{ and } q_t(1) = \bar{q}, \tag{5}$$

as in Section 2. The evolution of a firm's talent r_t is then given by

$$\dot{r}_t = \alpha(\lambda(q_t; r_t) - r_t). \tag{6}$$

As for payoffs, workers maximize lifetime wages w_t, while firms' flow profits equal revenue μr_t minus wages. A firm's problem is to choose wages to maximize total discounted profits. Denoting the discount rate by $\rho \geq 0$, its value function is

$$V_s(r_s) = \max_{\{w_t\}_{t \geq s}} \int_s^\infty e^{-\rho(t-s)}(\mu r_t - \alpha w_t)dt, \tag{7}$$

where r_t evolves according to (6).

An equilibrium is given by a wage path $\{w_t\}_{t \geq 0}$ for every firm,[15] so that given the induced wage ranks $x_t(w)$ and applicant qualities $q_t(x)$, every firm's wage path is optimal. We say an equilibrium is *essentially unique*, if the induced distribution over equilibrium trajectories $\{r_t\}_{t \geq 0}$ is unique.[16]

3.2 Firm's Problem

First, we study a firm's optimal wage path $\{w_t\}_{t \geq 0}$ for any given applicant function $q_t = q_t(x_t(w))$, that is, without imposing equilibrium restrictions on other firms. As in Section 2, it is convenient to write $W_t(q)$ for the wage required to attract applicants q at time t, and let the firm optimize directly over the applicant pool q_t. After this change of variable, the firm's Bellman equation becomes

$$\rho V_t(r) = \max_q \{\mu r - \alpha W_t(q) + \alpha(\lambda(q; r) - r)V_t'(r) + \dot{V}_t(r)\}. \tag{8}$$

Firm value is determined by its flow profits plus appreciation due to talent acquisition or a secular trend. Assuming wages are differentiable, the first-order condition is

$$W_t'(q) = \lambda'(q; r)V_t'(r). \tag{9}$$

Intuitively, the cost of attracting better applicants (the LHS) must balance the gains of a higher quality applicant pool which increases the recruit quality and thereby firm value (the RHS).

Compared to the first-order condition in the static model (4), the additional factor $V_t'(r)$ on the RHS of (9) captures the fact that the firm in our dynamic model is only replacing an infinitesimal part of its workforce at every instant. Importantly, both terms on the RHS rise in r: The marginal

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[15]As in the static model, the restriction to deterministic wages is without loss. In principle, a firm might mix between two wages by switching between them arbitrarily fast. To avoid measurability issues associated with such strategies, we allow for "distributional wage strategies" but show in Section 3.3 that equilibrium strategies are almost always pure.

[16]This definition avoids two spurious notions of multiplicity. First, in continuous time, any firm's optimal strategy $\{w_t\}_{t \geq 0}$ can be unique only almost always. Second, if two or more firms are initially identical but then drift apart, only the distribution of trajectories can be determined uniquely.

benefit of better applicants $\lambda'(q; r)$ because of the complementarity as in Section 2, and the marginal value of higher talent $V_t'(r)$ because the value function $V_t(r)$ is convex in r. Convexity follows since discounted payoffs (7) for given wages $\{w_t\}_{t \geq s}$ are linear in current talent r_s, and the upper envelope of linear functions is convex. Intuitively, intermediate levels of recruiting skills r have the draw-back that the firm's wage must strike a compromise between the firm's low-skill and high-skill recruiters, while a firm with homogeneous recruiters, $r = 0$ or 1, can choose the optimal wage for all.

Firms with more talent thus have a higher marginal benefit from attracting better applicants, yielding positive assortative matching. Hence, the firm's wages are dynamic complements: an increase in today's wage raises tomorrow's talent, and thereby tomorrow's optimal wage.

To compute equilibrium wages (below) from the first-order condition (9), we next analyze the marginal value of talent via the envelope theorem

$$V_t'(r_t) = \frac{\partial}{\partial r_t} \int_t^\infty e^{-\rho(s-t)} [\mu r_s - \alpha W(\hat{q}_s)] ds = \mu \int_t^\infty e^{-\rho(s-t)} \frac{\partial r_s}{\partial r_t} ds, \tag{10}$$

where the partial derivative $\partial r_s / \partial r_t$ takes the optimal strategy $\{\hat{q}_s\}_{s \geq t}$ as fixed. Since r_t evolves according to (6) we can calculate this derivative to obtain[17]

$$V_t'(r_t) = \mu \int_t^\infty e^{-\int_t^s \rho + \alpha(1 - \Delta(\hat{q}_u)) du} ds. \tag{11}$$

Intuitively, the future benefit of better employees is discounted both at the interest rate ρ and the retirement rate α; but selective recruiting raises the persistence of firm talent, or equivalently, reduces the talent decay rate by a factor $(1 - \Delta(q))$.

3.3 Equilibrium

Given the single-firm analysis, it is straightforward to characterize equilibrium. The benefit of attracting talented applicants on the RHS of (9) increases in the firm's talent r, so firms with more talent post higher wages and attract better applicants. More strongly, even if firms share the same talent r_0 initially they post different wages, as in the static model, recruit different types of workers, and diverge immediately (see Appendix B.1). Thus, in equilibrium, each firm is characterized by a rank x, which describes the firm's position in the talent, applicant, and wage distribution at all times $t > 0$.

Equilibrium is then characterized in two simple steps

(1) Allocations. At any point in time, applicant quality $q_t(x)$ is determined by sequential screening (5). The evolution of a firm's talent $r_t(x)$ is then given by equation (6).

(2) Payoffs. Firms marginal value of talent is determined by (11), with $\hat{q}_u = q_u(x)$. Using this, wages $W_t(q)$ are given by the first-order condition (9), with $r = r_t$, $\hat{q}_u = q_u(x)$, and $W_t(0) = 0$.

Given these wages, the FOC (9) implies global optimality of the HJB (8) because the net benefit of attracting marginally better applicants $\lambda'(q; r(x)) V_t'(r(x)) - W_t(q)$ single-crosses in x. Standard verification theorems then imply that the policy functions are indeed optimal. To summarize:

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[17]To see this, write the solution of the ODE $\dot{r}_t = \phi(r_t) = \alpha(\lambda(q_t; L) - r_t(1 - \Delta(q_t)))$ as a function of the initial value $r_0 = z$. Then, $\frac{\partial^2}{\partial t \partial z} r_t(z) = \frac{\partial}{\partial z} \phi(r_t(z)) = -(1 - \Delta(q_t)) \frac{\partial}{\partial z} r_t(z)$, or $\frac{\partial}{\partial t} \log \frac{\partial}{\partial z} r_t(z) = -(1 - \Delta(q_t))$. Integrating up and de-logging we get $\frac{\partial}{\partial z} r_t(z) = \exp(-\int_0^t (1 - \Delta(q_s)) ds)$ since $\frac{\partial}{\partial z} r_0(z) = 1$. See, for example, Hartman (2002) Section V, Theorem 3.1.

Theorem 2. *Equilibrium exists and is essentially unique. Firm-applicant matching is positive assortative and the distribution of talent has no atoms at $t > 0$.*

This result shows that even if firms start off with identical talent, some post higher wages than others, attract better applicants, and hire better recruits. These firms accumulate talent, continue to pay high wages, and the distribution of talent disperses over time.

The evolution of talent, wages and profits are illustrated in Figure 3. This simulation assumes that, at $t = 0$, all firms employ average workers, with quality $\bar{q} = 0.25$, and flow wages equal average productivity, $\mu\bar{q}$.

Panel (a) illustrates the evolution of $r_t(x)$ and $q_t(x)$. The "vertical" lines represent the cross-sectional distribution of (r, q) at different times, while the "horizontal" lines represent the sample-paths of selected firms. The top-ranked firm recruits from the constant applicant pool $q_t(1) = \bar{q}$, and so (6) implies that its talent grows monotonically and converges to a steady state. For lower-ranked firms, the dynamics are more subtle. For example, firm $x = 0.5$ initially improves as its recruits are more talented than retirees. However, as the firms above become better at identifying talent, its applicant pool deteriorates and its quality eventually falls back. Panels (b) and (c) show the resulting time paths for talent and applicant pool quality for three sample firms.

Panel (d) shows the evolution of firm value net of legacy wages.[18] Initially all firms are identical and have zero value. However, as some post high wages and acquire more talented workers, they become differentiated and the value of highly ranked firms rises. On the flip-side, these firms incur short-term losses while they invest in talent. Panel (e) shows that profits are initially zero at all firms since both revenue and wages reflect their legacy employees. As highly-ranked firms pay high wages to acquire talent, profits fall. However, this talent raises productivity and gives the firms a comparative advantage in recruiting talented workers in the future, so eventually profits turn positive. Panel (f) then shows that wages (and wage dispersion) fall over time. Initially, firms are identical and compete aggressively for talent to gain a competitive advantage in the future. Later, the differentiation between firms softens the competition. Moreover, wages fall most for middling firms, mirroring the deterioration of applicant quality at these firms seen in panel (c).[19]

Figure 3 indicates that the economy converges to a steady state. Steady-state talent among recruits and applicants $\{r_*(x), q_*(x)\}$ is easily characterized. First, the talent of each firm's recruits and retirees balance:

$$\lambda(q; r) = r. \tag{12}$$

Since $\partial\lambda(q; r)/\partial r = \Delta(q) < 1$, (12) has a unique fixed point, which we denote by $r = \xi(q) := \lambda(q; L)/(1 - \Delta(q))$. Naturally, firms with better applicants have higher talent; formally, $\xi(q)$ increases since $\lambda(q; r)$ rises in q. Second, substituting $r(x) = \xi(q(x))$ into the sequential screening equation (5) we obtain:

$$q'(x) = \frac{\lambda(q(x); \xi(q(x))) - q(x)}{x}. \tag{13}$$

Together, replacement (12) and sequential screening (13) determine equilibrium talent and applicants $\{r_*(x), q_*(x)\}$.

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[18]At time t, the legacy wage bill ω_t evolves according to $\dot{\omega}_t = \alpha(w_t - \omega_t)$, with initial condition $\omega_0 = \mu r_0 = \mu\bar{q}$. The firm's net value is then given by $V(r_t) - \omega_t/(\alpha + \rho)$. Similarly, flow profits are $\mu r_t - \omega_t$.

[19]This echoes the wage effect of a mean-preserving spread in recruiting skills, from $r \equiv 0.5$ to $r \sim U[0, 1]$, in the static model illustrated in Figure 2(a)
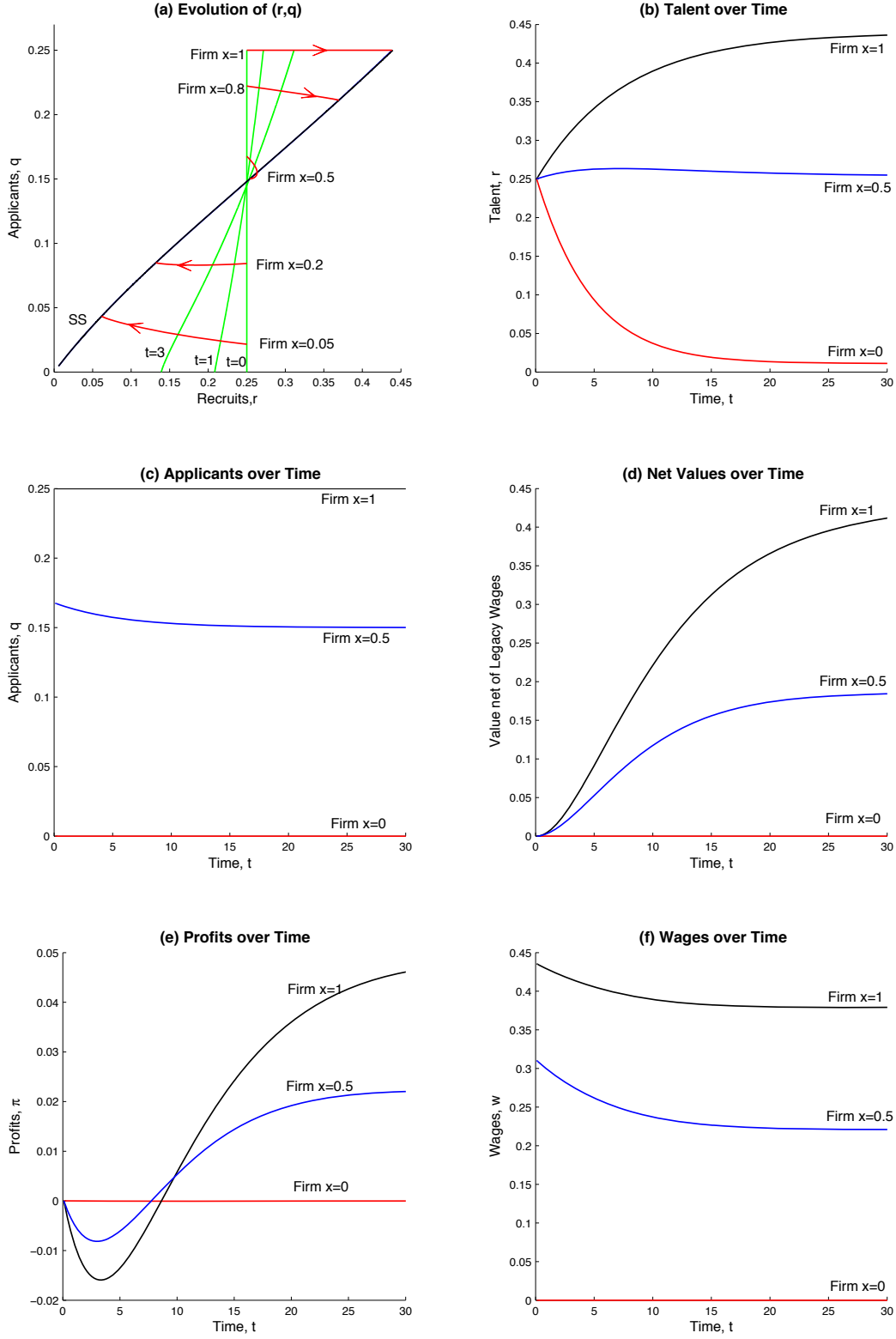
Figure 3: **Equilibrium Dynamics.** At $t = 0$, all firms have talent $\bar{q}$ and pay wages $\mu\bar{q}$. Later, they choose wages optimally as characterized in the text. As in Figure 1, we assume $p_H = 0.8$, $p_L = 0.2$, $\mu = 1$, and $\bar{q} = 0.25$. Thinking of t in years, we assume the interest rate is $\rho = 0.1$, and turnover is $\alpha = 0.2$.

We now show that from any initial condition, the economy converges to the steady state.

Theorem 3. *The steady-state talent distribution $r_*(x)$ is unique and has no gaps or atoms. For any distribution of initial talent r_0, firm x's equilibrium talent $r_t(x)$ converges to $r_*(x)$.*

Proof. See Appendix B.2. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ □

In steady state, the distribution of talent and productivity $r_*(x)$ is dispersed. This may be surprising since firms' talent is subject to regression to mediocrity. With imperfect matching, when a skilled worker retires there is chance he is replaced by an unskilled worker; similarly an unskilled worker may be replaced by a skilled worker. Thus, if each firm hired from the same pool, then firms' talent levels would converge to the population average $\bar{q}$ at the talent decay rate, $\alpha(1 - \Delta(q))$, as reflected by the exponent in (11). Countering this decay is the effect of positive assortative matching: firms with skilled employees post higher wages and recruit from a better pool. As a result, the steady state supports permanent heterogeneity in firm quality, productivity and profits.

Talent thus generates a sustainable competitive advantage. One might wonder why a firm with untalented workers doesn't compete more aggressively in wages to build its talent over time. While this is a feasible strategy, it is simply too expensive. High-talent firms have a higher marginal benefit from raising wages because recruiting skills and applicant quality are complements in the job market.

To show convergence, Figure 3(a) suggests a "proof by induction". The top firm recruits from a pool of constant quality, so equation (6) implies that its talent converges exponentially to steady state. Then consider firm x close to 1. As the talent of higher firms converges, firm x's applicant pool converges, and then firm x's talent converges, too, by equation (6). The formal proof is more complicated because x is continuous; it proceeds by showing that the steady state satisfies a contraction property, and then applies the contraction mapping theorem over a small interval, akin to the proof of the Picard-Lindelöf theorem.

The model generates predictions about how value is shared between workers and firms. In steady state the marginal value of talent (11) simplifies to

$$V'_*(r_*(x)) = \frac{\mu}{\rho + \alpha(1 - \Delta(q_*(x)))}. \tag{14}$$

As in (11), the marginal product of talent μ is annuitized at the interest rate ρ and the talent decay rate, which is now constant, equal to $\alpha(1 - \Delta)$. Substituting this marginal value into the first-order condition (9), marginal flow wages are then given by

$$(\rho + \alpha)W'_*(q) = \frac{\rho + \alpha}{\rho + \alpha(1 - \Delta(q))}\mu\lambda'(q; r). \tag{15}$$

Steady-state profits equal output minus flow wages, $\Pi_*(q) = \mu\xi(q) - (\alpha + \rho)W_*(q)$. Differentiating (12) and substituting into (15), the ratio of returns to capital and labor are given by

$$\frac{\Pi'_*(q)}{(\alpha + \rho)W'_*(q)} = \frac{\rho\Delta(q)}{(\rho + \alpha)(1 - \Delta(q))} \tag{16}$$

As in the static model, profits derive from the differences in screening skills between recruiters $\Delta(q)$. In the dynamic model this effect is reinforced by the fact that screening skills raise the persistence of the talent stock, as captured by the $1 - \Delta(q)$ term in the denominator of (16).

As discussed in Section 2, improved screening may help explain rising between-firm inequality. These forces are magnified in the dynamic model. To see this, suppose we start with screening skills $\mathbf{p} = (p_L, p_H)$ and steady-state talent $r_*(q)$. In the short-run, if we fix firms' talent and increase signal accuracy $\mathbf{p}$, productivity dispersion rises as high-paying firms fish out more of the talented workers from the applicant pool. Formally, $\frac{\partial}{\partial \mathbf{p}} \lambda(\bar{q}; r) > 0$ for any fixed r, so there is an increase in ratio of the highest productivity to the average productivity, $\lambda(\bar{q}; r)/\bar{q}$ for $r = r_*(1)$. Then, over time, this increase in dispersion is amplified as talent accumulates at the top firms, further raising their screening ability and the talent of their recruits. Formally, $r_*(1)$ increases in $\mathbf{p}$, meaning that the long-run increase in productivity dispersion, $\lambda(\bar{q}; r_*(1))/\bar{q}$ exceeds the short-run increase by a factor $1/(1 - \Delta(\bar{q}))$.[20]

It is also interesting to study the effect of the turnover rate (e.g. comparing the low-turnover 1970s and the high-turnover 2000s, or comparing history and economics departments). In our model, an increase in turnover, α, has no impact on the steady-state distribution of talent, but does raise the rate at which the economy converges to the steady state. Equation (16) then implies that profits fall while wages and wage dispersion rise (see Appendix B.3 for details). Intuitively, the comparative advantage of a firm is its stock of talent; when turnover is high, this stock depletes quickly, and firms get a small share of the (constant) output. Indeed, as turnover increases, a low-talent firm can mimic a high-talent firm and get almost the same profits as the past quickly becomes irrelevant; this intensifies competition and drives up wages.

4   Applications

In this section we consider two applications of the dynamic model. First, we suppose firms differ in both exogenous technology and endogenous talent. Second, we allow for peer effects, so that workers care about both wages and the average talent of the firm.

4.1   Heterogeneous Technology

We extend the basic dynamic model and suppose that firms differ in both their initial talent r_0 and also in their technology, μ. While both jointly determine productivity, the former is endogenous, and evolves over time as the firm hires new recruits, while the latter is exogenous. In the case of universities, we interpret μ as the value a university places on research, perhaps because of its charter. In the case of tech firms, heterogeneity in μ reflects the idea that, in the words of Netflix's HR manual, "In procedural work, the best are two times better than average. In creative/inventive work, the best are ten times better than average" (Hastings and McCord, 2009).

Initially, talent and technology may differ in arbitrary ways across firms, and equilibrium must map this two-dimensional heterogeneity into one-dimensional wage rankings. However, we show that over time the two dimensions of heterogeneity collapse into one, and the economy converges to a unique, stratified steady state with positive assortative technology-talent matching. We then use this steady state to study how an individual firm optimally adjusts its talent pool after an unexpected technology shock.

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[20]Indeed, recalling the steady state $r_*(1) = \lambda(\bar{q}; r_*(1))$, we have $\frac{d}{d\mathbf{p}} \lambda(\bar{q}; r_*(1)) = \frac{\partial}{\partial \mathbf{p}} \lambda(\bar{q}; r_*(1)) + \Delta(\bar{q}) \frac{d}{d\mathbf{p}} r_*(1) = \frac{\partial}{\partial \mathbf{p}} \lambda(\bar{q}; r_*(1))/(1 - \Delta(\bar{q}))$.

Assume that technology is binary, $\mu \in \{\mu_L, \mu_H\}$, and that fraction ν of firms have the low technology, $\mu_L < \mu_H$. We first argue that the target applicant pool $\hat{q}_t$ is increasing in both technology and talent. Fixing an aggregate wage schedule, $W_t(q)$, the firm's first-order condition is given by (9). As before, an increase in talent r increases both the marginal value of talent $V_t'(r; \mu)$ and the marginal quality of recruits $\lambda'(q; r)$, and thus increases the RHS of (9). Similarly, an increase in technology μ increases the marginal value of talent $V_t'(r; \mu)$ and the RHS of (9). We prove this in Appendix C.1; intuitively, it follows because technology and talent are complements in the profit function.

Consider two firms, "A" and "B". If A has both more talent and better technology than B, then A pays higher wages and attracts a better applicant pool. The more interesting case is when A has more talent, while B has better technology; then, the ranking is ambiguous. If A posts higher wages than B, it attracts better applicants, reinforcing its lead in talent. Conversely, if A posts lower wages than B, it attracts worse applicants and may fall behind B in both talent and technology. Thus, in contrast to the homogeneous technology case in Section 3.3, firms' wage rank may change over time.

We now characterize the steady state. In steady state, wage ranks x are constant, and so talent $r_*(x)$ and applicant quality $q_*(x)$ are determined by the replacement equation (12) and sequential screening (13) from the homogenous model, and are thus independent of the distribution of technology. We next need to specify the technology level $\mu(x)$ of firm x. We say that the steady state is *stratified* if $\mu(x)$ equals μ_H for high wage firms $x > \nu$, and μ_L for low wage firms $x < \nu$. Anticipating Theorem 4, we denote this step function by $\mu_*(x)$. The steady-state marginal value of talent (14) becomes

$$V_*'(r; \mu) = \frac{\mu_*(x)}{\rho + \alpha(1 - \Delta(q_*(x)))}. \tag{17}$$

Finally, marginal wages in the first-order condition (15) become

$$W_*'(q) = \lambda'(q_*(x); r_*(x))V_*'(r_*(x); \mu_*(x)), \tag{18}$$

with initial condition $W_*(0) = 0$.

Theorem 4. *The stratified steady state $\{r_*(x), q_*(x), \mu_*(x), W_*(q)\}$ is the unique steady-state equilibrium. Any equilibrium converges to this steady state.*

Proof. To see that the stratified steady state indeed constitutes an equilibrium, note that the FOC (18) implies global optimality of the HJB because the net benefit of attracting marginally better applicants $\lambda'(q; r_*(x))V_t'(r_*(x); \mu_*(x)) - W_t(q)$ single-crosses in x.

To show uniqueness of the steady state, suppose technology-talent matching is not stratified; then $\mu(x)$ has a downward jump at some $\tilde{x}$. Observe that workforce and applicant talent, $r_*(x), q_*(x)$, are continuous in x. Thus, the marginal benefit of attracting better applicants, $\lambda(q; r_*(x))V'(r_*(x); \mu(x))$ also jumps downwards at $\tilde{x}$. Hence, high-technology firms just below $\tilde{x}$ want to offer higher wages than low-technology firms just above, contradicting steady-state equilibrium.

The convergence proof in Appendix C.2 proceeds in three steps. First, we argue that the equilibrium talent ranking across firms eventually settles down since high-technology firms' ranks always rise, while low-technology firms' ranks fall. Next, we argue that the wage ranking across firms eventually settles down, too. Then, by the uniqueness of steady-state equilibrium and continuity

arguments, the equilibrium converges to the stratified steady state. □

Theorem 4 shows that differences in exogenous technology eventually prevail over differences in endogenous talent. A high-tech firm has a higher marginal benefit from talent and posts higher wages than nearby low-tech firms, overcoming its talent deficit. Hence the high-tech firm recruits better workers and, in the long run, ends up with more talent.

One often sees universities and firms that have particularly good employees relative to their "fundamentals", often due to historic accidents. Theorem 4 shows that talent provides a temporary competitive advantage, but ultimately fundamentals prevail. For example, Arai (2003) shows that capital-intensive firms tend to pay higher wages; our model predicts that such firms will accumulate talent and provide them with a further comparative advantage in the job market.

We next consider how a single firm adjusts in response to an increase in technology. For example, a university obtains a new donor who particularly values quality, or Netflix changes business models from shipping DVDs to streaming. We then wish to study how the firm's wage, recruits and talent change over time.

Formally, we suppose the economy is in steady state at time $t = 0$ and a low-technology firm, "firm A", with talent $r_0 < r_*(\nu)$ receives an unexpected shock that makes it high-technology. Write r_t for firm A's post-shock talent, and $\tilde{W}(r)$ for steady-state wages of a firm with talent r, i.e. $\tilde{W}(r_*(x)) := W_*(q_*(x))$; also recall that $r_*(\nu)$ is the lowest steady-state talent among high-technology firms.

Theorem 5. *After its technology shock, firm A posts wages $w_t \in (\tilde{W}(r_t), \tilde{W}(r_*(\nu))]$. Talent r_t increases over time, and converges to (but never reaches) its steady-state level $r_*(\nu)$.*

Proof. Since the marginal value of a wage raise $\lambda(q; r)V'(r; \mu)$ increases in a firm's talent r and technology μ, firm A sets its wage above the steady-state wage of a low-tech firm with the same talent, $w_t > \tilde{W}(r_t)$, but is below the wage of a high-technology firm with talent $r_*(\nu)$, $w_t \leq \tilde{W}(r_*(\nu))$. The first inequality is strict since equilibrium wages are smooth at $\tilde{W}(r_t)$, but the second may be weak because the wage function has a kink at $r_*(\nu)$. Thus, firm A's talent r_t is bounded above $r_*(\nu)$, but rises over time, and hence converges. By the proof of convergence in Theorem 4, its limit exceeds the steady-state talent of all low-technology firms. However, since its recruits are weakly worse than firm $r_*(\nu)$'s, its talent does not reach the steady-state level $r_*(\nu)$ in finite time. □

When a low-technology firm experiences an increase in productivity, its wages jump up. However, if its talent is sufficiently below steady state, its wages do not jump all the way up to its steady-state level. Intuitively, the firm does not initially have the skills to spend a wage boost wisely. The firm thus hires people below its aspired steady-state level of talent both because it pays (weakly) lower wages, and because of its inferior recruiting skills. In the long-run, the wage and applicant pool rises to the steady state, and the talent level follows along. These dynamics differ notably from the "burst hiring" whereby a firm spends extravagantly to quickly build a complementary group of employees, and re-adjusts wages downwards after the burst.

4.2 Peer Effects

In the baseline model, a worker chooses the firm that pays the highest wages. In this section, we suppose his preferences also depend on the fraction of talented workers at the firm, r. Formally, we

assume that a hired worker's *effective compensation* is given by $w + \gamma r$, where $\gamma > 0$ measures the relative importance of peer effects. This could represent the prestige effect of passing the toughest recruiting test, or the increase in human capital when a new worker first joins the firm. We show that the introduction of peer effects has no effect on equilibrium matching or the evolution of talent, but does lower steady-state wages.[21]

First, consider the static model of Section 2. As before, incentive compatibility implies that $\Delta(Q(r))$ increases in r, so matching is positive assortative; i.e., firms with higher skills offer higher effective compensation and attract better applicants. Intuitively, peer effects make high-skill firms more attractive but, unlike in Becker (1973), do not make them relatively more attractive for talented agents. Since workers care about effective compensation, equilibrium wages simply decrease by γr, with slope $w'(r) = \mu \lambda'(Q(r), r)Q'(r) - \gamma$. This is the *static peer effect*: Firms substitute peer effects for wages, and so wages and wage dispersion fall. In fact, the wage schedule $w(r)$ may flip if firms are sparsely distributed (e.g. for top universities) or if γ is large; for simplicity, our discussion focuses on the opposite case, where equilibrium wages increase in talent.

Consider now the dynamic model of Section 3. In equilibrium, a firm must offer lifetime compensation $U_t(q)$ to attract applicants of quality q; the required wage for firm r equals $W_t(q; r) = U_t(q) - \gamma r$. The Bellman equation is again given by (8), with first-order condition

$$U_t'(q) = \lambda'(q; r)V_t'(r), \tag{19}$$

meaning that more talented firms attract better applicants, hire better recruits and stay ahead of less talented firms. Hence, talent and applicant quality $(r_t(x), q_t(x))$ converge to the same steady state as before, $r_*(x) = \xi(q_*(x))$.

Peer effects increase the marginal value of talent. Applying the envelope theorem, the steady-state value is given by,

$$V_*'(r_*(x)) = \frac{\mu + \alpha\gamma}{\rho + \alpha(1 - \Delta(q_*(x)))}. \tag{20}$$

Intuitively, a new worker who increases talent by dr allows the firm to cut the wage of future recruits by γdr while this worker is employed. In addition, the new worker helps identify future talent who can be used to lower wages even further.

Now consider wages. Starting with identical firms, we show in Appendix C.3 that the introduction of peer effects increases initial wages and wage dispersion. This *dynamic peer effect* captures the idea that talent allows a firm to lower future wages, as reflected in (20), increasing incentives to accumulate it in the first place. Steady-state wages reflect both the static peer effect, which allows talented firms to pay less, and the dynamic peer effect that intensifies competition for future talent. Since the dynamic effect is discounted, the static effect dominates and so steady-state wages and wage dispersion fall in γ, as shown in Appendix C.3. Hence, with initially identical firms, the introduction of peer effects shifts wages from later recruits to those who enter the industry earlier, exacerbating the trend of falling wages seen in Figure 3(d).

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[21]We view this extension as a reduced form that shows the robustness of our results. In a full reputation model, the agent might care about the market's inference of his quality, λ. In a full peer effects model, the worker might care about the talent over time, not just when hired.

5 Welfare

In order to keep the model as simple as possible, we have so far abstracted from the question of firm entry and the level of employment. We now address these questions, and in particular whether equilibrium matching sorts talent into the industry efficiently. Aggregate sorting is an important issue in our applications. In the academic context, some students may be a better fit for a professional capacity than for academic research. Similarly, Netflix's HR policy states "we endeavor to have only outstanding employees", (Hastings and McCord, 2009).

To study aggregate sorting, we assume that employment has a fixed cost $k \geq 0$. This implies a net value of $\mu - k$ for talented workers, and $-k$ for untalented workers, who should thus not enter the industry. Equivalently, one can interpret k as an opportunity cost for the employee (adjusting wages appropriately). Under either interpretation, untalented workers should not enter the industry and the effectiveness of this sorting depends on the organization of the market.

5.1 Static (In)efficiency

We first consider the static model of Section 2 with a fixed cost $k \geq 0$. In equilibrium, profits are increasing in r, so firms above some cutoff $\tilde{r} > \underline{r}$ enter the market; matching is then positive assortative. Intuitively, if all firms entered the market, the lowest applicant pool would consist solely of untalented workers, $q(0) = 0$, and the lowest-wage firm would not recover its fixed cost. The cutoff is then determined by the zero-profit condition, $\mu\lambda(Q(\tilde{r}); \tilde{r}) = k$, and mass $F(\tilde{r})$ of workers are unemployed, of whom fraction $Q(\tilde{r})$ are talented.

We now consider the surplus maximization problem of a social planner, who can decide which firms enter the market, and the order in which firms select applicants. Optimal entry is characterized by some threshold $\check{r}$ with quantile $\check{x} = F(\check{r})$. Given $\check{x}$, the planner chooses a measure-preserving map r from $[\check{x}, 1]$ to $[\check{r}, \bar{r}]$ to maximize social surplus $\int_{\check{x}}^{1}[\mu\lambda(q(x); r(x)) - k]dx$, where $q(x)$ is given by sequential screening (2). For example, PAM corresponds to $r(x) = F^{-1}(x)$, and NAM to $r(x) = F^{-1}(1 + \check{x} - x)$.

Equilibria in continuous matching markets with transferable utility are typically efficient (e.g. Gretsky, Ostroy, and Zame (1992)). Surprisingly, this welfare theorem fails in our model. One can see the downside of PAM in a two firm variant of our model. Suppose there is a good firm with some skilled recruiters, and a bad firm with only unskilled recruiters who hire randomly, i.e. $p_L = 0$. Under PAM, the good firm has applicants $\bar{q}$ while the bad firm faces an adversely selected pool $q < \bar{q}$. But under NAM, both firms face the original pool quality $\bar{q}$ since the bad firm hires randomly, generating no adverse selection. Thus, firms jointly hire better recruits under NAM than under PAM.

The key difference to standard matching models with exogenous types is that the quality of low applicant pools $q(x)$ is endogenous, subject to adverse selection induced by the screening of better paying firms. This compositional externality is maximized by PAM and minimized by NAM, so the supermodularity of $\lambda(q; r)$ and adverse selection work at cross purposes. Surprisingly, we can show that adverse selection unambiguously dominates, and NAM is efficient.

Theorem 6. *In the static model with positive costs $k > 0$, the planner's solution is characterized by some entry threshold $\check{r} > \underline{r}$ and NAM for firms $r \in [\check{r}, \bar{r}]$.*

Proof. For a fixed entry threshold $\check{r}$ and unemployment $\check{x} = F(\check{r})$, maximizing surplus is equivalent

to maximizing employed talent $m(\check{x})$, where $m(x) = \bar{q} - q(x)x$. Now, relax the planner's problem by letting her allocate additional recruiting skills to the firms. Specifically, for any threshold $\check{x}$ and screening skills $r(x)$ for $x \geq \check{x}$, we define $\zeta(x)$ as the derivative of employed talent $m(\check{x})$ with respect to $r(x)$. Marginally better screening skills at firm x increase employed talent at rank x by $dm(x) := \Delta(q(x))$. On the flip-side, removing talent from the applicant pool at rank x acts as a negative compositional externality on lower-ranked firms $\tilde{x} \in [\check{x}, x]$.

To compute the effect on employed talent $m(\check{x})$, note first that, by definition, total employed talent evolves according to $m'(x) = -\lambda(q(x); r(x)) = -\lambda((\bar{q} - m(x))/x; r(x))$. Thus, incremental employed talent $dm(\tilde{x})$ depreciates at rate $\lambda'(q(\tilde{x}); r(\tilde{x}))/\tilde{x}$ as $\tilde{x}$ decreases from x to $\check{x}$,[22] and the effect of rank x screening skills on aggregate employed talent equal

$$\zeta(x) := \frac{dm(\check{x})}{dr(x)} = \frac{dm(\check{x})}{dm(x)} \frac{dm(x)}{dr(x)} = \exp\left(-\int_{\check{x}}^{x} \frac{\lambda'(q(\tilde{x}); r(\tilde{x}))}{\tilde{x}} d\tilde{x}\right) \Delta(q(x)). \tag{21}$$

The key argument of the proof is that $\zeta(x)$ strictly decreases: Differentiating (21) and multiplying through with $\exp(\int_{\check{x}}^{x} [\lambda'(q(\tilde{x}); r(\tilde{x}))/\tilde{x}] d\tilde{x})x$, we get

$$\Delta'(q(x))q'(x)x - \lambda'(q(x); r(x))\Delta(q(x)). \tag{22}$$

In this equation, the first, positive term captures the idea that allocating screening skills to higher-ranked firms is efficient because of the complementarity, as in Becker (1973). That is, shifting screening skills up by ϵ ranks, where applicant talent is $q'(x)\epsilon$ higher, increases talent intake by $\Delta'(q(x))q'(x)\epsilon$. The second, negative term captures the idea that better screening at higher ranked firms exacerbates adverse selection for lower ranked firms. That is, better skills at rank $x + \epsilon$ reduces quality of the applicant pool of the x-ranked firm by $\Delta(q(x))\epsilon/x$, and talent intake by $\lambda'(q(x); r(x))\Delta(q(x))\epsilon/x$. These two terms are illustrated in Figure 4.

To show that $\zeta(x)$ is decreasing, we wish to show (22) is negative. Using (2) to substitute for $q'(x)x = \lambda(q(x); r(x)) - q(x)$, and observing that $-\Delta'(q(x))q(x) < 0$, it is sufficient to show that

$$\Delta'(q(x))\lambda(q(x); r(x)) - \lambda'(q(x); r(x))\Delta(q(x)) < 0.$$

Writing partial derivatives of $\lambda(q; r)$ as $\lambda_q := \lambda'$, $\lambda_r = \Delta$, and $\lambda_{qr} = \Delta'$, the LHS becomes $\lambda_{qr}\lambda - \lambda_q\lambda_r$, which is negative because $\lambda(q; r)$ is log-submodular in q and r, as we show in Appendix D.1.

The fact that $\zeta(x)$ decreases implies that NAM is efficient given any entry threshold $\check{r} > \underline{r}$: If the matching function is anything other than NAM, we can find $r' > r$ with wage ranks $x' > x$. Swapping the ranks x and x' effectively shifts screening skills from rank x' to rank x. Since $\zeta(x)$ decreases in x, this raises aggregate surplus.

Finally, we argue that the planner's cutoff $\check{r}$ exceeds $\underline{r}$. If all firms did enter, $\check{r} = \underline{r}$, then all workers would be employed and the lowest-ranked firms contributes negative social surplus, $\lambda(q(x); r(x)) - k$. Welfare is thus increased by having some firms exit (in fact, the lowest skilled firms, since $\zeta(x)$ is positive). $\qquad \square$

Theorem 6 is interesting for two reasons. First, it shows that equilibrium exhibits excessive dispersion in productivity and talent. Second, it traces this inefficiency to the compositional externality

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[22]To see this, echoing footnote 17, write the solution of the ODE $m'(\tilde{x}) = \phi(\tilde{x}, m) = -\lambda((\bar{q} - m)/\tilde{x}; r(\tilde{x}))$ as a function of the initial value $\bar{m} = m(x)$. Then, $\frac{\partial^2}{\partial \tilde{x} \partial \bar{m}} m(\tilde{x}, \bar{m}) = \frac{\partial}{\partial \bar{m}} \phi(\tilde{x}, m) = \frac{\partial}{\partial \bar{m}} m(\tilde{x}, \bar{m})\lambda'((\bar{q} - m)/\tilde{x}; r(\tilde{x}))/\tilde{x}$, or $\frac{\partial}{\partial \tilde{x}} \log \frac{\partial}{\partial \bar{m}} m(\tilde{x}, \bar{m}) = \lambda'((\bar{q} - m)/\tilde{x}; r(\tilde{x}))/\tilde{x}$.
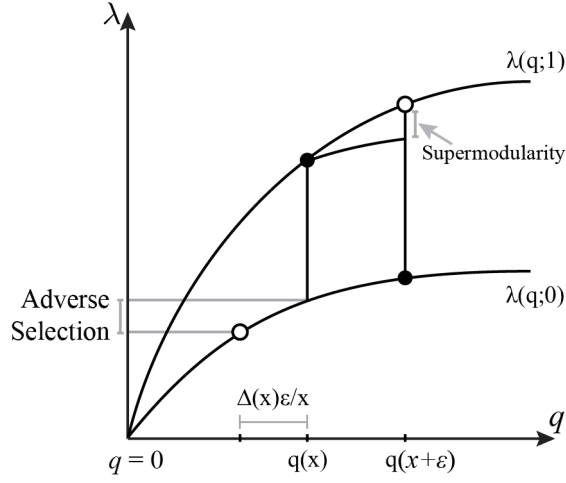
Figure 4: This figure illustrates the supermodularity and adverse selection forces arising in Theorem 6. The solid dots represent talent intake under NAM, where the rank-x firm has high skill and the rank-$x + \epsilon$ has low skill. When shifting skills to the rank-$x + \epsilon$ firm, we get PAM with talent intake represented by the empty dots.

that high-wage firms exert on low-wage firms by adversely selecting the talent pool. This compositional externality is absent in classical matching models since the choice set of a firm depends only on its rank in the wage distribution, and not the identity of the firms above it.[23]

Theorem 6 has practical implications. Negative assortative matching resembles the draft in professional sports leagues, e.g. Major League Baseball or the NFL, where the lowest-ranked football teams have first pick. Theorem 6 suggests that the reverse draft maximizes sorting of talent into the league, in addition to lowering productivity dispersion and thus leveling the playing field.

In most other labor markets, the recruiting order cannot be designed centrally. Consider for instance a planner who cannot observe firms' skills and who can only affect the recruiting order by restricting the set of admissible wages. In equilibrium, high-skill firms offer weakly higher wages than low-skill firms by Theorem 1. Given this constraint, and the fact that $\zeta(x)$ is decreasing, the planner's optimal policy is a fixed wage so that firms choose in a random order. This optimum can also be implemented by a wage cap.[24] This argument suggests that the NCAA's ban on paying college athletes raises talent in competitive college athletics. It does not do so by lowering wages - the marginal athletes would be paid zero wages anyhow - but rather by preventing colleges with the best scouts bidding away the best athletes and lowering the quality of the marginal programs,

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[23]Compositional externalities (and their associated inefficiency) can also arise in directed search models, e.g. when firms' wage offers affect the distribution of workers entering the labor market (Albrecht, Navarro, and Vroman (2010)), or when worker's job search creates "phantom vacancies" because employers fail to remove their filled vacancies from the market (Albrecht, Decreuse, and Vroman (2015)).

[24]When using a fixed wage $\check{w}$, the planner must choose $\check{w}$ so the marginal firm breaks even. That is, $\check{w} = \mu E_x[\lambda(q(x); \check{r}) | x \geq F^{-1}(\check{r})] - k$, where the expectation is taken over ranks in the atom, $x \in [F^{-1}(\check{r}), 1]$. If $\check{w}$ is not a fixed wage, but rather a wage cap that firms are free to underbid, we need to check additionally that no firm wants to do so. It suffices to check that the marginal firm $\check{r}$, which is most tempted to cut wages, does not want to cut its wage to 0, which is the most profitable deviation. To see this, note that at $w = 0$ the firm exerts no externality on other firms or workers, so captures its full contribution to social surplus. By definition of $\check{r}$, this social surplus is zero when the firm is placed inside the atom, as instructed by the planner. When posting $w = 0$, the contribution to social surplus is thus negative, meaning the firm's profits from setting $w = 0$ are also negative.

causing them to exit.[25]

5.2 Dynamic (In)efficiency

In the dynamic model of Section 3, the inefficiency of positive assortative matching is magnified. At any give time, PAM harms the quality of recruits in the industry. Moreover, under PAM, high-talent firms accumulate more talent, while low-talent firms fall behind, which raises talent dispersion and that amplifies the static inefficiency.

The following result captures these forces. Suppose that each potential firm starts off with exogenous talent r_0 and chooses whether or not to enter at cost k/ρ. Labeling firms by their talent rank x, flow social surplus $\mu R_t - (1 - \check{x})k$ then depends on the entry threshold $\check{x}$ and aggregate employed talent $R_t := \int_{\check{x}}^1 r_t(x)dx$. Under PAM, firm x has wage rank x, and there is steady-state dispersion of talent, as in Section 3. Under NAM, firm x has wage rank $1 - x + \check{x}$, so talent at any firm converges to the same constant in finite time; once a set of firms has reached this constant talent level, we interpret NAM as all those firms posting the same wage distribution.

Theorem 7. *Consider the dynamic model with positive costs $k > 0$. For any entry threshold $\check{x} > 0$, social surplus is strictly higher under NAM than under PAM at all times $t > 0$.*

Proof. We wish to show that aggregate employed talent is higher under NAM than under PAM, $R_t^N > R_t^P$, at all times $t > 0$. Let $q_t^P(x)$ be applicant pool quality and $m_t^P := \bar{q} - q_t^P(\check{x})\check{x}$ be aggregate talent intake under PAM, and similarly for NAM using "N" superscripts. At $t = 0$, we have $R_0^N = R_0 = R_0^P$. For all $t \geq 0$, we claim that whenever $R_t^N \geq R_t^P$ then $m_t^N > m_t^P$. To see this observe that, by assumption, there is more recruiting talent under NAM, and it places more weight on lower ranks of the wage distribution. Since $\zeta(x)$ in equation (21) is positive and decreasing in x, this means that more talented workers are employed under NAM than PAM, $m_t^N \geq m_t^P$. Moreover, since $\zeta(x)$ is strictly decreasing, this inequality is strict unless the talent distribution under PAM and NAM is both identical and degenerate, equal to one atom of identical firms, which can only arise at $t = 0$. Since aggregate employed talent evolves according to $\dot{R}_t^P = \alpha(m_t^P - R_t^P)$, and similarly for NAM, the above claim and the single-crossing lemma implies $R_t^N > R_t^P$ for all $t > 0$, as required. □

As in the static model, this result has practical implications. Suppose the planner cannot observe the firms' initial talent and impose a screening order directly, but can choose a set of admissible wages at each point in time. In equilibrium, high-talent firms pay higher wages than low-talent firms, so the planner maximizes aggregate employed talent at all times $t > 0$ by forcing all firms to offer the same wage w_t at time t.[26]

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[25]Another possible policy to increase quality in the market, namely entry barriers, is not effective in our model. In models where privately informed firms decide whether or not to enter (e.g. Atkeson, Hellwig, and Ordoñez (2015)) such barriers can increase welfare by screening out low-quality firms. In contrast, in our model equilibrium entry is optimal given positive assortative matching. Intuitively, the marginal firm does not impose an externality on others and enters if and only if entry is efficient. With this said, if the planner imposes NAM or pools the firms together, then an entering firm does have a negative externality on other firms. This is why the optimal single wage must be strictly positive to prevent excessive entry. Indeed, this suggests that a low, fixed wage is optimal for the NCAA, such as paying for tuition.

[26]Beyond the current model, another way of endogenizing recruiting skills is to let firms simultaneously invest into their screening ability r at cost $c(r)$ before posting wages. For example, this might represent hiring more HR professionals, conducting more interviews or more background checks. In equilibrium, firms post different wages, and

6   Conclusion

This paper proposes a new model of firm dynamics in which talent takes center-stage. The model is based on the idea that talented workers are both more productive and better at identifying talented applicants. In the static labor market, matching is positive assortative when talent is scarce, with high-skill firms posting higher wages and attracting better applicants. We then embed this in a dynamic model in which today's recruits become tomorrow's managers and hire the next generation of workers. The economy converges to a unique steady state in which talent, wages and productivity are dispersed, with talent as the source of sustainable competitive advantage. Finally, we show that equilibrium is inefficient and can be improved by policies that reduce wage dispersion.

The paper offers a launch pad to address questions related to organization within firms, and the competition between firms. As written, our model assumes that the task of recruiting is shared equally among all of a firm's employees, making recruit quality $\lambda(q; r)$ a linear function of the firm's productivity r. This is reasonable if firms never learn the talent of their workers, or if the identity of the recruiter is defined by their job title (e.g. theorists hire theorists, while macroeconomists hire macroeconomists). But usually firms do learn about the talent of their workers and can assign authority to the talented ones. For example, a firm on an upward trajectory as in Section 4.1, would want to place more authority in the hand of the recent, high-quality recruits or seek help with recruiting from professional head-hunters. We can incorporate such ideas in our model by supposing that recruit quality $\lambda(q; z)$ depends on recruiting skills z, that depend on non-linearly on talent r as well as on other, endogenous parameters. As long as skills z increase in talent r, our insights on positive matching, convergence, and steady state dipsersion carry over to such richer models. Moving forward, it would be natural to study how firms set objectives for workers and assign authority to make decisions, and how employees can take actions to influence that authority.

Our model also assumes that jobs offer lifetime employment. It is straightforward to allow agents to leave at some exogenous rate and re-enter the job market. Assuming firms cannot discriminate based on age, the applicant pool then consists of both new workers and those who recently quit. One can go a step further, and allow firms to learn the values of their workers (e.g. at a Poisson rate) and fire the bad ones. These workers would then re-enter the pool, lowering the overall quality $\bar{q}$. A more ambitious extension of our model would allow for on-the-job search or poaching. This is particularly interesting because job-to-job moves help firms aggregate information and can improve welfare. Any such analysis would depend on whether recruiters can discriminate between employed and unemployed applicants (e.g. Board and Meyer-ter-Vehn (2015)) and whether employers know the quality of their workers (e.g. Greenwald (1986)). Moreover, if workers are informed of their own quality, firms may be able to improve selection via performance-pay, such as piece rates or backloaded pay (e.g. Lazear (2000)).

Finally, our model is silent on firm size. Following Lucas (1978), it would be natural to endogenize firm size and study how competition on the intensive margin for better employees interacts with competition on the extensive margin for more employees. It would also be interesting to study stochastic productivity μ, meaning that firms would need to constantly adjust their talent pool to their current standing. A more ambitious extension of our model would endogenize the growth of

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pick different skill levels to complement these choices, with high-wage firms investing more in their skill. However, such an equilibrium is inefficient: adverse selection means that firms have excessive incentive to invest, since this worsens the quality of the application pool for firms posting lower wages. This result contrasts with the finding of efficient investment found in classic matching models, e.g. Cole, Mailath, and Postlewaite (2001).

μ by making it a function of workers' ideas. We leave this for future work.

Appendix

A Analysis from Section 2

A.1 Nested Signals

In this section we clarify the relationship between our model and that of Kurlat (2016) with "false positives". The most significant difference is that our model assumes that firms' tests are conditionally independent, whereas Kurlat assumes "nested" signals. In the context of our model, nested signals mean that recruiters with the same skill administer the same test, and that an untalented worker who passes the test of a skilled recruiter also passes the test of an unskilled recruiter.[27] Workers thus fall into one of four categories: A) talented workers, who automatically pass both tests, B) untalented workers who pass both tests, C) untalented workers who pass the test of the unskilled recruiter but fail the test of the skilled recruiter, and D) untalented workers who fail both tests.[28] Assume also for simplicity that firms have homogeneous recruiters, either all skilled or all unskilled, that there are more talented workers than skilled firms, and that there is a small entry cost as in Section 5. This guarantees that in equilibrium all skilled firms and some unskilled firms enter. Also, since equilibrium wages are not distributed smoothly in this model (see below) we assume that workers adopt an endogenous tie-breaking rule that guarantees equilibrium existence.

Since unskilled firms hire proportionally from categories A, B, and C, they do not impose a compositional externality on other firms. But skilled firms, who hire proportionally from categories A and B, but screen out C workers, impose a negative externality on unskilled firms. Thus, unskilled firms have an incentive to outbid skilled firms but not vice versa (and neither type of firm has an incentive to outbid its own type). The equilibrium wage distribution is thus degenerate, with all entering firms offering some wage w^* and workers endogenously breaking ties in favor of unskilled firms. The wage w^* is determined by the entry condition of unskilled firms, and the proportion of unskilled firms offering wage w^* is determined by the condition that all workers in categories A and B are employed. This equilibrium has negative assortative matching and efficient aggregate sorting, in sharp contrast to the positive assortative matching and inefficient aggregate sorting of our baseline model with independent signals.

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[27]Other differences between our model and Kurlat's are superficial. His focus on asset markets and ours on labor markets is purely semantic. The impatience of his distressed sellers would correspond to a reservation wage of talented workers in our model. Finally, his model allows for richer interaction across many markets, but the unique equilibrium described in his Proposition 1 features a single market and is equivalent to the equilibrium in an auction where buyers bid for assets, and then sellers apply from top to bottom bid with buyers screening applying sellers and accepting the first seller who passes their test, exactly as in our model.

[28]Category A corresponds to the "green assets" in Kurlat's table I, category C to "red assets", and category D to the "black assets".

A.2 Conditions for Positive Assortative Matching

Here we show that $\Delta(q)$ is single-peaked, with maximum at $\hat{q} \in (0,1)$. Differentiating, we have

$$\lambda'(q;\theta) = \frac{1 - p_\theta}{[1 - (1-q)p_\theta]^2}$$

$$\lambda''(q;\theta) = \frac{-2(1-p_\theta)p_\theta}{[1 - (1-q)p_\theta]^3} = \lambda'(q;\theta)\frac{-2p_\theta}{[1 - (1-q)p_\theta]}$$

and so $\lambda''(q;\theta)/\lambda'(q;\theta)$ decreases in p_θ. Thus, if $\Delta'(\hat{q}) = \lambda'(\hat{q};H) - \lambda'(\hat{q};L) = 0$, then

$$\Delta''(\hat{q}) = \lambda''(\hat{q};H) - \lambda''(\hat{q};L) = \lambda'(\hat{q};H)\left[\frac{\lambda''(\hat{q};H)}{\lambda'(\hat{q};H)} - \frac{\lambda''(\hat{q};L)}{\lambda'(\hat{q};L)}\right] < 0,$$

and so Δ is single-peaked.

We can compute $\hat{q}$. Since $\lambda'(\hat{q};H) = \lambda'(\hat{q};L)$, we have $[1 - (1-\hat{q})p_L]\sqrt{1 - p_H} = [1 - (1 - \hat{q})p_H]\sqrt{1 - p_L}$, which implies

$$\hat{q} = \frac{(1-p_L)\sqrt{1-p_H} - (1-p_H)\sqrt{1-p_L}}{p_H\sqrt{1-p_L} - p_L\sqrt{1-p_H}}. \tag{23}$$

We next argue that

$$\lambda(\bar{q};H) \leq 1/2 \tag{24}$$

is a sufficient condition for $\Delta'(\bar{q}) \geq 0$, and hence for (3), $\bar{q} < \hat{q}$. Thus, (3) requires not only that talent is scarce but also that screening skills are limited. Intuitively, a perfect recruiter with $p_H = 1$ (outside the model) has a comparative advantage at screening applicants with quality close to zero since he can identify the very last talented worker remaining in the pool; hence, matching cannot be positive assortative. Formally, (23) vanishes as p_H approaches one, and so (3) fails for any $\bar{q}$.

To see that (24) implies $\Delta'(\bar{q}) \geq 0$, it is useful to write recruit quality as a function of applicant quality and screening skill, $\lambda = \hat{\lambda}(q,p) = q/(1 - (1-p)q)$ for general $p \in [p_L, p_H]$.

Lemma 2. $\hat{\lambda}_{qp}(q,p) \gtreqless 0$ *iff* $\hat{\lambda}(q,p) \lesseqgtr 1/2$.

Proof. Since Bayes' rule is multiplicative in likelihood ratios, it is useful to transform the probabilities p and q into $z(q) := q/(1-q)$, the likelihood ratio of talented vs untalented applicants, and $y(p) := 1/(1-p)$, the likelihood ratio with which talented and untalented applicants pass the test. The likelihood ratio of talented vs untalented recruits then equals yz, which we then transform back to the expected talent of the recruit $\tilde{q}(yz) := yz/(1 + yz)$; we note the derivatives $\tilde{q}'(yz) = 1/(1 + yz)^2, \tilde{q}''(yz) = -2/(1 + yz)^3$. Then

$$\begin{aligned}\hat{\lambda}_{qp}(q,p) &= \partial_{pq}\tilde{q}(z(q)y(p))\\ &= z'(q)y'(p)[\tilde{q}'(z(q)y(p)) + z(q)y(p)\tilde{q}''(z(q)y(p))]\\ &= z'(q)y'(p)\frac{1 - z(q)y(p)}{(1 + z(q)y(p))^3}\end{aligned}$$

is positive iff $z(q)y(p) > 1$, that is iff $\hat{\lambda}(q,p) = \tilde{q}(z(q)y(p)) < 1/2$. □

The likelihood ratio transformation traces the supermodularity of $\hat{\lambda}(q,p)$ at low levels of quality where $\hat{\lambda}(q,p) < 1/2$ to the multiplicative nature of Bayes' rule. At high levels of quality where
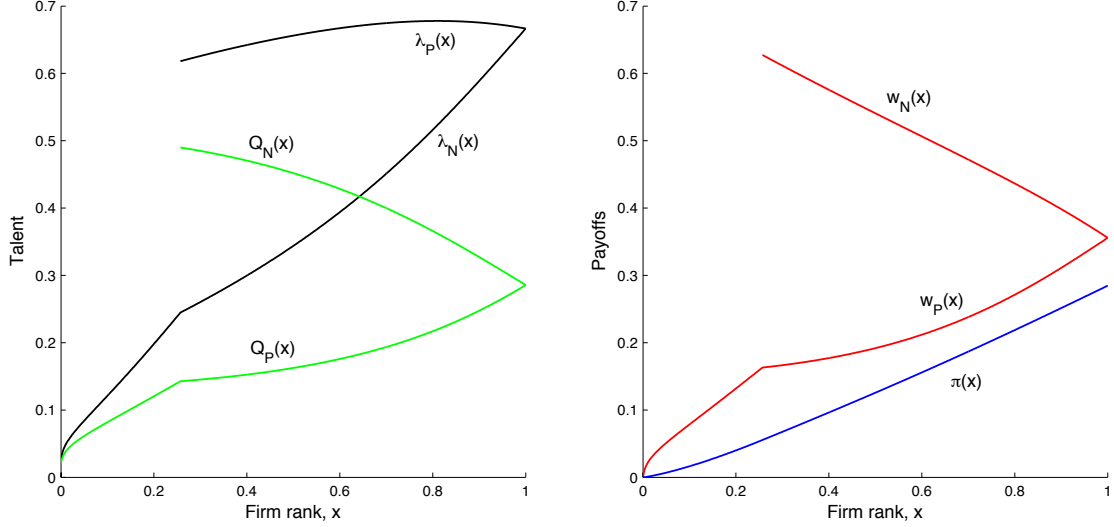
Figure 5: **Equilibrium with Abundant Talent.** The **left** panel shows the quality of applicants and recruits. The **right** panel shows the resulting wages and profits. The parameters are the same as Figure 1, except that $\bar{q} = 0.5$.

$\hat{\lambda}(q, p) < 1/2$, this supermodularity is overwhelmed by the concavity of the reverse transformation $\tilde{q}$, which captures the diminishing marginal benefit of increasing the likelihood ratio when recruit quality is already high.

Now, (24) implies $\hat{\lambda}(\bar{q}, p) \leq \hat{\lambda}(\bar{q}, p_H) = \lambda(\bar{q}; H)\lambda(\bar{q}; H) \leq 1/2$ for every $p \leq p_H$, and hence $\hat{\lambda}_{qp}(\bar{q}, p) \geq 0$ by Lemma 2. Thus, $\Delta'(\bar{q}) = \hat{\lambda}_q(\bar{q}, p_H) - \hat{\lambda}_q(\bar{q}, p_L) = \int_{p_L}^{p_H} \hat{\lambda}_{qp}(\bar{q}, p) dp \geq 0$.

For a partial converse, assume that p_L is close enough to p_H. Then, if (24) fails, we have $\hat{\lambda}(\bar{q}, p) > 1/2$ and hence $\hat{\lambda}_{qp}(\bar{q}, p) < 0$ for all $p \in [p_L, p_H]$, and so $\Delta'(\bar{q}) < 0$. In this sense, (24) is necessary as well as sufficient for positive assortative matching.

A.3 General Matching

In the body of the paper we assume that talent is scarce, $\bar{q} \leq \hat{q}$, implying PAM by Theorem 1. In fact, $\bar{q} \leq \hat{q}$ is necessary as well as sufficient for PAM; indeed, with $\bar{q} > \hat{q}$ and PAM, $\Delta(Q(r))$ falls in r once $Q(r) \geq \hat{q}$, violating incentive compatibility. Here, we characterize the equilibrium if talent is abundant, $\bar{q} \in (\hat{q}, 1)$.

In equilibrium the advantage of skilled screeners $\Delta(Q(r))$ increases in r. Since $\Delta(q)$ is single-peaked around $\hat{q}$, equilibrium has the following form. First, the most skilled firm $\bar{r}$ is matched with an applicant pool of quality $\hat{q}$ that maximizes the benefit of screening skills. Second, less-skilled firms over the range $[r^*, \bar{r})$ post two wages and are matched with both worse applicant pools $Q_P(r) < \hat{q}$, and better applicant pools $Q_N(r) > \hat{q}$ such that $\Delta(Q_N(r)) = \Delta(Q_P(r))$. As we describe below, this indifference condition, determines the weight that firms put on the two wages. Finally, the least-skilled firms $[\underline{r}, r^*)$ are matched with bad applicant-pools $Q_P(r) < Q_P(r^*) < \hat{q}$, where the cutoff r^* satisfies $\Delta(Q_P(r^*)) = \Delta(\bar{q})$. Since $\Delta(\cdot)$ increases for low talent $q < \hat{q}$, the lower branch $Q_P(r)$ increases, representing PAM. Conversely, $\Delta(\cdot)$ decreases for high talent $q > \hat{q}$ and so the upper branch $Q_N(r)$ increases, representing NAM. These functions are illustrated in Figure 5. More formally:

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Theorem 8. *Suppose talent is abundant, $\bar{q} \in (\hat{q}, 1)$. There exists an equilibrium. Equilibrium is characterized by a threshold $r^* \in (\underline{r}, \bar{r})$, an increasing PAM function $Q_P(r)$ for $r \in [\underline{r}, \bar{r}]$, and a decreasing NAM function $Q_P(r)$ for $r \in [r^*, \bar{r}]$.*

Proof. Unlike in the case with scarce talent, $\bar{q} < \hat{q}$, where equilibria must be in pure strategies, the preamble to this theorem shows that we must now consider mixed strategies. Any mixed strategy profile induces a joint distribution over screening skills r and wages w, which in turn induces a joint distribution over r and wage ranks x. Writing $r(x)$ for the expected skills at rank x, applicant pools $q(x)$ are determined by sequential screening (2) as usual. The resulting firm-applicant matching $Q(r)$ may now be set-valued. The optimality requirement $\Delta(Q(r)) \leq \Delta(Q(r'))$ then means $\Delta(q) \leq \Delta(q')$ for all $q \in Q(r), q' \in Q(r')$ with $r \leq r'$.

We first argue that only one firm is screening at a given wage rank x, and thus the skills of this firm equals $r(x)$. Assume otherwise, that two firms $r < r'$ both screen at rank $x \in Q(r), Q(r')$. Then $\Delta(Q(r)) = \Delta(Q(r'))$, and so incentive compatibility implies that any intermediate firm $\tilde{r} \in (r, r')$ must share the same value of Δ. However, a mass of firms cannot share the same value of Δ since since $q(x)$ strictly increases and $\Delta(\cdot)$ has no flat spots.

Next, we characterize the form of matching. The monotonicity of $\Delta(Q(r))$ implies that $r(x)$ is single-peaked, increasing for $q(x) < \hat{q}$ and decreasing for $q(x) > \hat{q}$. Along the increasing branch, let firm r be matched with rank $X_P(r)$, and let $Q_P(r) = q(X_P(r))$ be the corresponding applicant pool. Along the decreasing path, let firm r be matched with rank $X_N(r)$ and let $Q_N(r) = q(X_N(r))$ be the corresponding applicant pool.

We construct equilibrium as follows. The top firm is matched with $\hat{q}$, meaning that $Q_P(\bar{r}) = Q_N(\bar{r}) = \hat{q}$. A priori we don't know the rank of firm $\bar{r}$, so we denote it by $\hat{x} := X_P(\bar{r}) = X_N(\bar{r})$. For lower firms, we have the indifference condition

$$\Delta(q(X_P(r))) = \Delta(q(X_N(r))) \tag{25}$$

and the availability of firms with type r

$$\left| X_N'(r) \right| + X_P'(r) = f(r). \tag{26}$$

We interpret $X_P'(r)$ as the weight firm r places on the PAM branch. Since $\Delta(q)$ and $q(x)$ are continuous and the distribution of r has no gaps, these supports are intervals with upper bound $\bar{r}$. Moreover, since $\Delta(q(0)) = \Delta(0) = 0 < \Delta(\bar{q}) = \Delta(q(1))$, the support of X_P is all of $[\underline{r}, \bar{r}]$, while the support of X_N is truncated below, at some $r^* \in (\underline{r}, \bar{r})$. We have thus established the equilibrium characterization of Theorem 8.

To establish equilibrium existence (and also show how to compute an equilibrium), we now show that there exist functions $\{Q_N(r), Q_P(r), X_N(r), X_P(r)\}$ satisfying (25) and (26), and wages $W(q)$ so that firm r finds it optimal to attract applicants $Q_N(r)$ and $Q_P(r)$. For the PAM branch, we have to adjust the sequential screening equation (2) by the weight that the firms place on the PAM branch,

$$Q_P'(r) = \frac{\lambda(Q_P(r), r) - Q_P(r)}{X_P(r)} X_P'(r). \tag{27}$$

and similarly for the NAM branch. Differentiating (25), and substituting for $Q_P'(r)$ and $Q_N'(r)$, and

then using (26) yields

$$\Delta'(Q_P(r)) \left[\frac{\lambda(Q_P(r), r) - Q_P(r)}{X_P(r)} \right] X_P'(r) = \Delta'(Q_N(r)) \left[\frac{\lambda(Q_N(r), r) - Q_N(r)}{X_N(r)} \right] \left(X_P'(r) - f(r) \right).$$
(28)

This equation then yields the weight $X_P'(r)$ and, integrating from $\hat{x}$, yields the mass functions X_N, X_P. One can then derive the applicant functions Q_N, Q_P from (27).

To ensure this ODE solves the original problem, it needs to satisfy the boundary condition $Q_N(r(1)) = \bar{q}$, which says that the top ranked firm recruits from the unadulterated pool. We therefore study $Q_N(r(1))$ as a function of the free variable $\hat{x}$, and apply the intermediate value theorem. At the top, when $\hat{x} = 1$, $\bar{r}$ is the highest ranked firm by assumption implying $Q_N(r(1)) = \hat{q} < \bar{q}$. At the bottom, as $\hat{x} \to 0$ we claim that $Q_N(r(1)) \to 1 < \bar{q}$. To see why, observe that as $X_P(\bar{r}) = \hat{x} \to 0$ firms place all their weight on the NAM branch of the equilibrium. For this to be true, the indifference equation (25) means that $Q_N(r) > \hat{q}$ for all $r < \bar{r}$ which requires $\bar{q} \to 1$ so that $Q_N(r) \to 1$ pointwise. Thus by continuity, for some $\hat{x} \in (0, 1)$, there exists a solution to the ODE with the original boundary conditions and $X_P(\bar{r}) = X_N(\bar{r}) = \hat{x}$.

Finally, we turn to wages. Given the functions $\{Q_N, Q_P, X_N, X_P\}$, wages $W(q)$ are given by the first-order condition. For the PAM branch, this means

$$W'(Q_P(r)) = \lambda'(Q_P(r); r)$$

with boundary condition $W(0) = 0$.

Now, to see that Q_N, Q_P, along with wages $W(q)$ constitutes an equilibrium, consider firm r, say in the joint support of X_P and X_N. The marginal value of attracting better applicants $\lambda'(q; r) - W'(q) = \lambda'(q; r) - \lambda'(q; r(q)) = \Delta'(q)(r - r(q))$ is (1) positive for $q < Q_P(r)$ since $\Delta' > 0$ and $r > r(q)$; (2) negative for $q \in (Q_P(r), \hat{q})$ since $\Delta' > 0$ and $r < r(q)$; (3) positive for $q \in (\hat{q}, Q_N(r))$ since $\Delta' < 0$ and $r < r(q)$; and (4) negative for $q > Q_N(r)$ since $\Delta' < 0$ and $r > r(q)$. Thus, the two local maxima are $Q_P(r)$ and $Q_N(r)$. To see that these are equally profitable, we write their difference as

$$\int_{Q_P(r)}^{Q_N(r)} \Delta'(\tilde{q})(r - r(\tilde{q}))d\tilde{q} = \int_{Q_P(r)}^{Q_P(\bar{r})} \Delta'(\tilde{q})(r - r(\tilde{q}))d\tilde{q} + \int_{Q_N(\bar{r})}^{Q_N(r)} \Delta'(\tilde{q})(r - r(\tilde{q}))d\tilde{q}$$

$$= \int_r^{\bar{r}} \Delta'(Q_P(\tilde{r}))(r - \tilde{r})Q_P'(\tilde{r})d\tilde{r} + \int_{\bar{r}}^r \Delta'(Q_N(\tilde{r}))(r - \tilde{r})Q_N'(\tilde{r})d\tilde{r}$$

which is zero by (28). $\qquad\qquad\square$

It is worth noting that this model is highly tractable. In general, models with PAM and NAM regions are hard to characterize (e.g. Chiappori, McCann, and Nesheim (2010)). For us, the key to tractability is the fact that the screening advantage $\Delta(q)$ is independent of r, which follows from our assumption that a firm's screening skills are a linear combination of its recruiters' skills.

A.4 Comparative Statics

Here we show that wage and productivity dispersion rise when either screening skills rise (due to improved technology), or aggregate talent falls (due to skill-biased technological change). In order to simplify the analysis, we assume that all recruiters are identical, screening out untalented

workers with probability p. This means that firms are also identical, so profits are zero and wages, productivity and talent all coincide.

We say that *dispersion rises* when the talent ratio between the x'-ranked firm and the x-ranked firm rises for all $x' > x$. Writing the recruit quality $\lambda(x, p, \bar{q})$ as a function of rank x, screening skills p, and aggregate talent $\bar{q}$, we wish to show that $\lambda(x', p, \bar{q})/\lambda(x, p, \bar{q})$ falls in $\bar{q}$ and rises in p. That is, a fall in $\bar{q}$ or an increase in p allows the top firms to acquire a proportionately larger share of the talent. This is formalized by the following result:

Lemma 3. (a) :$\lambda(x, p, \bar{q})$ *is log-submodular in* $(x, \bar{q})$ *and* (b), *if* $p(1 + \bar{q}) \leq 1$, *log-supermodular in* (x, p).

Proof. Part (a): Let us write the applicant quality by $q(x, p, \bar{q})$ and it's logarithm by $\vartheta(x, p, \bar{q}) = \log q(x, p, \bar{q})$. We claim that $q(x, p, \bar{q})$ is log-submodular in $(p, \bar{q})$. By (1) and (2), log-applicant quality obeys $\vartheta_x(x, p, \bar{q}) = \frac{q_x(x, p, \bar{q})}{q(x, p, \bar{q})} = \frac{1}{x}(\frac{1}{1 - p(1 - e^{\vartheta})} - 1) =: \psi(x, \vartheta, p)$. Differentiating gives us the result

$$\vartheta_{x\bar{q}} = \frac{d}{d\bar{q}} \psi(x, \vartheta(x, p, \bar{q}), p) = \psi_\vartheta \vartheta_{\bar{q}} < 0$$

since $\psi_\vartheta < 0$ and $\vartheta_{\bar{q}} > 0$. Intuitively, when applicant talent $\bar{q}$ drops by half, the talent of top firms drops by less than half since these firms screen out some of the newly untalented applicants, meaning they hire proportionally more of the available talent.

Next, we claim that $\lambda(x, p, \bar{q})$ is log-submodular in $(x, \bar{q})$. To see this, write log recruit quality, $\Lambda := \log \lambda$, as a function of screening skills p and log applicant quality ϑ, $\Lambda(\vartheta, p) = \log \frac{e^\vartheta}{1 - p(1 - e^\vartheta)} = \vartheta - \log(1 - p(1 - e^\vartheta))$. First note that $\Lambda(\vartheta, p)$ increases in ϑ with $\Lambda_\vartheta = 1 - \frac{qp}{1 - p(1 - q)} = \frac{1 - p}{1 - p(1 - q)} > 0$, and increases in p with $\Lambda_p = \frac{1 - q}{1 - p(1 - q)} = \frac{1}{1/(1 - q) - p} > 0$. The submodularity in $(x, \bar{q})$ then follows easily because both factors in

$$\frac{d}{d\bar{q}} \Lambda(\vartheta(x, p, \bar{q}), p) = \Lambda_\vartheta(\vartheta(x, p, \bar{q}), p) \vartheta_{\bar{q}}(x, p, \bar{q})$$

are positive and fall in x. Intuitively, the larger proportional decline of applicant quality at lower-ranked firms is aggravated by the concavity of recruit quality Λ in applicant quality ϑ.

Part (b): We first show that $q(x, p, \bar{q})$ is decreasing in p and log-supermodular in (x, p). To verify monotonicity, observe that for $x = 1$, $q(x, p, \bar{q}) = \bar{q}$, independent of p. Fixing $p' > p$, (1) and (2) imply $q_x(x, p, \bar{q}) = \frac{1}{x}\left(\frac{(1 - q)p}{1 - (1 - q)p}\right)$, so $q_x(x, p', \bar{q}) > q_x(x, p', \bar{q})$ when $q(x, p', \bar{q}) = q(x, p', \bar{q})$. The single-crossing lemma then implies $q(x, p', \bar{q}) < q(x, p, \bar{q})$ for all $x < 1$. To verify log-supermodularity, differentiating yields

$$\vartheta_{xp} = \frac{d}{dp} \psi(x, \vartheta(x, p, \bar{q}), p) = \psi_\vartheta \vartheta_p + \psi_p > 0 \tag{29}$$

since $\psi_\vartheta, \vartheta_p < 0$ and $\psi_p > 0$, and thus falls by a greater amount at lower-ranked firms when screening skills p increase.

Next, we claim that $\lambda(x, p, \bar{q})$ is log-supermodular in (x, p) if $p(1 + \bar{q}) \leq 1$. This is harder to establish since the greater proportional drop of applicant quality at low-ranked firms is counter-acted by a greater benefit of improved screening skills at those firms, as witnessed by $\Lambda_{\vartheta p} < 0$. For the aggregate effect to favor high-ranked firms, we need

$$\frac{d}{dp} \Lambda(\vartheta(x, p, \bar{q}), p) = \Lambda_\vartheta \vartheta_p + \Lambda_p = \frac{(1 - p)\vartheta_p + (1 - q)}{1 - p(1 - q)} \tag{30}$$

to rise in x. Indeed, differentiating (30) with respect to x, multiplying through with $\frac{x}{1-q}(1 - p(1 - q))^3 > 0$, recalling from (29) that $\vartheta_{xp} \geq \psi_p$ and $\vartheta_p \leq 0$, and then substituting $\psi_p = \frac{1}{x}\frac{1-q}{(1-p(1-q))^2}$ and $q_x = \frac{1}{x}(\frac{q}{1-p(1-q)} - q) = \frac{1}{x}\frac{pq(1-q)}{1-p(1-q)}$, we get

$$\frac{d^2}{dxdp}\Lambda(\vartheta(x,p,\bar{q}),p) \stackrel{\text{sgn}}{=} \frac{x(1-p(1-q))^2}{1-q}\frac{\partial[(1-p)\vartheta_p + (1-q)]}{\partial x} - \frac{x(1-p(1-q))}{1-q}pq_x[(1-p)\vartheta_p + (1-q)]$$

$$\geq \frac{x(1-p(1-q))^2}{1-q}[(1-p)\psi_p - q_x] - x(1-p(1-q))pq_x$$

$$= 1 - p - pq(1 - p(1-q)) - p^2q(1-q)$$

$$= 1 - p(1+q) \geq 0$$

where the last inequality follows by the assumption that $p(1+\bar{q}) \leq 1$. $\qquad\square$

B   Proofs from Section 3

B.1   Proof that Talent Distribution is Smooth

Here we argue that if there is an atom of initially identical firms, these firms diverge immediately. Assume to the contrary, that at time $t > 0$ an atom of firms has the same worker quality r_t. Writing x for the rank of r_t, let this atom be $[\underline{x}, \bar{x}]$. Since optimal wages rise in talent and hence talent differences never vanish, firms in the atom must have identical talent r_s for all $s \in [0, t]$. At any time $s \in [0, t]$ the wage distribution must be smooth by the arguments in Section 2. If firms in the atom post different wages, they drift apart. Hence the firms must employ non-degenerate distributional strategies,[29] posting both high and low wages to attract good and bad applicants; they must thus be indifferent across a range of applicants $[\underline{q}_s, \bar{q}_s]$ for all $s \in [0, t]$. Thus, the first order condition (9) must hold with equality on $[\underline{q}_s, \bar{q}_s]$ for all $s \in [0, t]$ and the atom quality r_s.

To see that such distributional strategies cannot be optimal, consider a firm that deviates by always attracting the best applicants in the atom $\bar{q}_s$, rather than mixing over good and bad applicants. At time $s = 0$, the choice $\bar{q}_0$ is optimal. Moreover, over time the firm's quality rises above r_s since it attracts better applicants. Since the marginal benefit of attracting better applicants, the RHS of (9), strictly increases in r, this deviation strictly improves on the posited distributional strategy. This proves that initially identical firms diverge immediately.

B.2   Proof of Theorem 3

Here we show that worker quality converges. First rewrite firm x's talent evolution (6) as $\dot{r}_t(x) = \alpha(\lambda(q; L) - (1 - \Delta(q_t(x)))r_t(x))$. For constant applicants q, quality thus drifts towards $\xi(q) = \lambda(q; L)/(1 - \Delta(q))$ at rate $\alpha(1 - \Delta(q))$. Of course, for any firm x, the applicant quality $q_t(x)$ also changes over time, and $q_t'(x)$ is given by (5). We now show that these two nested ODEs, converge to the steady state $q_*(x), r_*(x)$.

First, we derive a contraction property. Define the limits $\underline{q}(x) := \liminf_t q_t(x)$, $\bar{q}(x) := \limsup_t q_t(x)$, $\underline{r}(x) := \liminf_t r_t(x)$, and $\bar{r}(x) := \limsup_t r_t(x)$. Next, interpret (5) as an operator Q, mapping

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[29]When using a distributional strategy, a firm posts an entire distribution of wages $\nu_t = \nu_t(w)$ of wages at any time t; we then interpret $r_t(x)$ as the weighted-average talent of firms posting the x-ranked wage, and solve for the firm's evolution of talent by taking expectations over the RHS of (6).

firm quality functions $r_t(x)$ into applicant quality functions $q_t(x) = Q[r_t(\cdot)](x)$. We claim that:

$$Q[\xi(\bar{q}(\cdot))](x) \leq \underline{q}(x) \leq \bar{q}(x) \leq Q[\xi(\underline{q}(\cdot))](x). \tag{31}$$

To understand (31), first observe that that Q is antitone: if $r(x) \geq \hat{r}(x)$ for all x then $q(x) = Q(r(\cdot))(x) \leq Q(\hat{r}(\cdot))(x) = \hat{q}(x)$, since $q(x) = \hat{q}(x) = \bar{q}$ and the RHS of (5) increases in r. Intuitively, better recruiters introduce more adverse selection. Inequalities (31) then state that if applicant quality was equal to one of its limits, $\underline{q}$ and $\bar{q}$, and quality r was in steady state $r = \xi(q)$, then the induced difference in applicant pools is larger than the original difference.

We prove (31) in two steps. First, since $r_t(x)$ drifts towards $\xi(q_t(x))$, which is asymptotically bounded by $\xi(\underline{q}(x))$ and $\xi(\bar{q}(x))$, we have

$$\xi(\underline{q}(x)) \leq \underline{r}(x) \leq \bar{r}(x) \leq \xi(\bar{q}(x)) \tag{32}$$

for all x. Second,

$$\underline{q}(x) = \lim_{t\to\infty} \inf_{t'>t} q_{t'}(x) = \lim_{t\to\infty} \inf_{t'>t} \{Q[r_{t'}(\cdot)](x)\} \geq \lim_{t\to\infty} Q[\sup_{t'>t}\{r_{t'}(\cdot)\}](x) = Q[\bar{r}(\cdot)](x)$$

where the first inequality is the definition of the lim inf, the second the definition of the operator Q, the inequality uses the antitonicity of Q — since $r_{t'}(\hat{x}) \leq \sup_{t'>t}\{r_{t'}(\hat{x})\}$ for all t' and $\hat{x}$, we have $Q[r_{t'}(\cdot)](x)$ exceeds $Q[\sup_{t'>t}\{r_{t'}(\cdot)\}](x)$ for all t' and x, and hence so does $\inf_{t'>t} Q[r_{t'}(\cdot)](x)$— and the last inequality uses the dominated convergence theorem to exchange the limit $t \to \infty$ and the operator Q, as well as the definition of the limsup, $\bar{r}(x) = \lim_{t\to\infty} \sup_{t'>t} r_{t'}(x)$. Together with the analogue argument for $\bar{q}(x)$, we get

$$Q[\bar{r}(\cdot)](x) \leq \underline{q}(x) \leq \bar{q}(x) \leq Q[\underline{r}(\cdot)](x). \tag{33}$$

Since Q is antitone, (32) and (33) imply (31).

To complete the proof of convergence, suppose "inductively" that applicant and firm quality converge above some $\hat{x} \in (0, 1]$, i.e. $\underline{q}(x) = \bar{q}(x)$, and hence $\underline{r}(x) = \bar{r}(x)$, for all $x \in (\hat{x}, 1]$. Fix ϵ, and let $\delta(\epsilon) := \max_{x\in[\hat{x}-\epsilon,\hat{x}]} |\bar{q}(x) - \underline{q}(x)|$ be the maximum distance the liminf and limsup drift apart over $[\hat{x} - \epsilon, \hat{x}]$. Since $\xi(q)$ is Lipschitz in q, and the RHS of (5) is locally Lipschitz in q and r,

$$\max_{x\in[\hat{x}-\epsilon,\hat{x}]} |Q[\xi(\underline{q}(\cdot))](x) - Q[\xi(\bar{q}(\cdot))](x)| \leq K\epsilon\delta(\epsilon)$$

for some constant K. Letting $\epsilon < 1/K$, this equation states the exact opposite of (31). Hence we must have $\underline{q}(x) = \bar{q}(x)$ and hence $\underline{r}(x) = \bar{r}(x)$, for all $x \in [\hat{x} - \epsilon, 1]$, and thus for all $x \in [0, 1]$.

B.3   Proof that Wage Dispersion rises in Turnover

Formally, (15) implies that $W_*(q)$ rise in α, while (15) and (16) imply that $\Pi_*(q)$ increases. Moreover, $W_*(q)$ is log-supermodular and supermodular in (q, α), and so wage dispersion rises in both absolute and proportional senses. To see this observe that $\frac{\partial^2}{\partial\alpha\partial q} \log[W'_*(q)] \overset{\text{sgn}}{=} \Delta'(q) > 0$ which means that $W'_*(q_H)/W'_*(q_L)$ increases in α, for $q_H > q_L$, and hence implies $\frac{\partial^2}{\partial\alpha\partial q} \log[W_*(q)] = \frac{\partial}{\partial\alpha}[W'_*(q)/\int_0^q W'_*(\hat{q})d\hat{q}] > 0$. Finally note that a log-supermodular function that is increasing in both arguments is also supermodular.

C   Proofs from Section 4

C.1   Proof that Wages Rise in Productivity

To verify that $V_t'(r; \mu)$ increases in μ, fix an initial workforce r_t and consider the optimal wage policies $\{\hat{q}_s^{H,L}\}$ and the associated worker trajectories $\{r_s^{H,L}\}$ of a high and low productivity firm. If $\hat{q}_t^H = \hat{q}_t^L$, the statement is true. Thus we can assume $q_t^H \neq q_t^L$ and define T as the first time (possibly ∞) after t with $r_T^H = r_T^L$. Thus, the trajectories $\{r_s^H\}_{s \in [t,T]}$ and $\{r_s^L\}_{s \in [t,T]}$ are strictly ordered, in that we either have $r_s^H < r_s^L$ for all $s \in (t,T)$, or $r_s^H > r_s^L$ for all $s \in (t,T)$. Write $R_{H,L} = \int_t^T e^{-\rho(s-t)} r_s^{H,L} ds$ and $W_{H,L} = \alpha \int_t^T e^{-\rho(s-t)} W(\hat{q}_s^{H,L}) ds$ for the aggregate workforce and wage bill of firm μ. Since the trajectories r_s^H and r^L reunite at $s = T$, optimality by firms H and L imply $\mu_H R_H - W_H \geq \mu_H R_L - W_L$ and $\mu_L R_H - W_H \leq \mu_L R_L - W_L$. Thus, $(\mu_H - \mu_L)(R_H - R_L) \geq 0$, and hence $r_s^H > r_s^L$ for all $s \in (t,T)$.

C.2   Proof of Theorem 4

Fix an equilibrium. For any time t, let y_t be a firm's rank in the distribution of talent r, and x_t its rank in the distribution of wages w. Since wages rise in talent and technology, high-technology firms post higher wages than their talent rank, $x_t \geq y_t$, and so y_t (weakly) increases; conversely, low productivity firms have $x_t \leq y_t$, and y_t (weakly) decreases. In either case, y_t converges to some y. Since y_t converges uniformly for any countable set of firms - which can be chosen to be dense in the set of all firms' initial ranks - the convergence is indeed uniform for all firms. Clearly, the distribution of limit ranks y is uniform on $[0,1]$, and we can identify firms with their productivity and their limit rank y.

Let F_H, F_L be the distribution of limit ranks for high and low productivity firms. To establish stratified technology-talent matching, we will show that the support of F_H lies above the support of F_L. Let $\bar{y} := \max \operatorname{supp} F_L$ for the highest limit rank of a low productivity firm, we need to show that there is no "mismatched" high productivity firm, i.e. with $y < \bar{y}$. Assume otherwise, and let $\tilde{y} = \sup\{y \in \operatorname{supp} F_H : y < \bar{y}\}$ be the highest mismatched firm; note that $\tilde{y} \leq \bar{y} \leq 1$, but neither inequality need be strict. We will derive a contradiction by showing that high-technology just below $\tilde{y}$ outbid low-technology firms just above. However, in order to do this, we first have to show that for wage ranks converge to talent ranks on $[\tilde{y}, 1]$.

First, consider firms $[\bar{y}, 1]$. Almost all of these firms are high-technology, so we are in the setting of Section 3.3, and Theorem 3 implies that applicant and worker quality (q,r) converge to their steady-state levels $(q_*(y), r_*(y))$ for all $y \in [y, 1]$.

Next, consider firms $[\tilde{y}, \bar{y}]$. Since $\tilde{y}$ is in the support of both F_H and F_L, there are both high- and low-technology firms with this limit rank. Write $\tilde{x}_t^H, \tilde{y}_t^H$ and $\tilde{x}_t^L, \tilde{y}_t^L$ for the wage rank and talent rank of the high and low technology firms that converge to $\tilde{y}$. Since both the high wage ranks of the high-technology firm $\tilde{x}_t^H \geq \tilde{y}_t^H$ and the low wage ranks of the low-technology firm $\tilde{x}_t^L \leq \tilde{y}_t^L$ must sustain the same limit quality $\lim \tilde{y}_t^H = \lim \tilde{y}_t^L = \tilde{y}$, the wage ranks, too, must converge to the same limit $\lim \tilde{x}_t^H = \lim \tilde{x}_t^L = \tilde{y}$. Then, since almost all firms with $y \in [\tilde{y}, \bar{y}]$ are low-technology and the order of their wage ranks is time-invariant with the lowest converging to $\tilde{y}$ and the highest being bounded above by $\bar{y}$, the wage rank of any such firm y must converge to y. Thus, applicant and worker quality converge to the steady-state quantities, $q_*(y)$ and $r_*(y)$, also for these firms.

To obtain the contradiction, consider a low-technology firm with talent r_t^L limit talent rank $\tilde{y}$,

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and a high-technology firm with talent r_t^H and limit talent rank $\tilde{y} - \epsilon$, and compare their marginal benefit of better applicants $V_t'(r; \mu)\lambda'(q; r)$. Since talent ranks converge to close-by limits and talent is continuous in rank, the talent gap $r_t^L - r_t^H$ vanishes for small ϵ and large t. Moreover, wage quantiles and hence applicants qualities $\hat{q}_t^L$ and $\hat{q}_t^H$ are ϵ-close for large t. Recalling

$$V_t'(r; \mu) = \mu \int_t^\infty e^{-\int_t^s \rho + \alpha(1 - \Delta(\hat{q}_u))du} ds$$

from (11), the ratio $V_t'(r_t^H; \mu_H)\lambda'(q; r_t^H)/V_t'(r_t^L; \mu_L)\lambda'(q; r_t^L)$ converges to $\mu_H/\mu_L > 1$ for small ϵ and large t. This contradicts the assumption that the low-technology firm pays higher wages.

C.3 Wage Dispersion and Peer Effects

To study wage dispersion, observe that (19) give us,[30]

$$\frac{d}{dx} W_t(q_t(x), r_t(x)) = \lambda'(q_t(x); r_t(x))V_t'(r_t(x))q_t'(x) - \gamma r_t'(x)$$

Now, suppose that all firms start off with identical talent. We claim that initial wage dispersion is larger with peer effects. At $t = 0$, $r_0(x) \equiv \bar{q}$, and

$$\frac{d}{dx} W_0(q_0(x), \bar{q}) = \lambda'(q_0(x); \bar{q})V_0'(\bar{q})q_0'(x)$$

The Envelope Theorem implies that the marginal value of talent is

$$V_t'(r_t) = (\mu + \alpha\gamma) \int_t^\infty e^{-\int_t^s \rho + \alpha(1 - \Delta(\hat{q}_u))du} ds$$

is increasing in γ. Hence wages become more dispersed as γ grows.

In steady state, the marginal value is given by (20), while the applicant and talent pool are related by $\lambda(q_*(x), r_*(x)) = r_*(x)$. Substituting these in yields,

$$\frac{d}{dx} W_*(q_*(x), r_*(x)) = \left[\frac{\mu(1 - \Delta(q_*(x))) - \gamma\rho}{\rho + \alpha(1 - \Delta(q_*(x)))} \right] r_*'(x)$$

Hence if the wage schedule is increasing, as occurs when peer effects are sufficiently small, an increase in γ lowers dispersion.

D Proofs from Section 5

D.1 Log-submodularity of $\lambda(q; r)$

Here we show that recruit quality $\lambda(q; r)$ is log-submodular in applicant quality q and the proportion of skilled recruiters r. Log-submodularity may come as a surprise since $\Delta(q) = \lambda_r(q; r)$ increases, that is, λ is *supermodular*. Supermodularity captures the idea that an increase in recruiting skills raises the *absolute* effect of applicant quality on recruit quality. Log-submodularity in contrast captures the effect of recruiting skills on the *relative*, percentage increase in recruit quality.

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[30]To determine the level of wages, observe that the lowest firm will choose wages so that utility is zero, i.e. $W_t(q_t(0), r_t(0)) = -\gamma r_t(0)$. In steady state, $r_*(0) = 0$.

We first argue log-submodularity of λ as a function of q and p

$$\frac{\partial^2}{\partial q \partial p} \log \frac{q}{1 - (1 - q)p} = -\frac{\partial^2}{\partial q \partial p} \log(1 - (1 - q)p) = -\frac{\partial}{\partial q} \frac{p}{1 - (1 - q)p} = -\frac{\partial}{\partial q} \frac{1}{1/p - (1 - q)} < 0.$$

Intuitively, recruit quality equals the ratio of talented test-passers q to all test passers $1 - (1 - q)p$. The number of talented test-passers q does not depend on screening skills. But the elasticity of $1 - (1 - q)p$ with respect to q rises in screening skills p: For with no screening skills, $p = 0$, all applicants pass the test and so the number of test-passers does not depend on q, but this number rises in the number of talented applicants q when the test screens out some untalented applicants, $p > 0$.

Next, we establish that λ is log-submodular also as a function of q and p

$$
\begin{aligned}
\frac{\partial^2 \lambda(q; r)}{\partial q \partial r} \lambda(q; r) - \frac{\partial \lambda(q; r)}{\partial r} \frac{\partial \lambda(q; r)}{\partial q} &= \Delta'(q)\lambda(q; r) - \Delta(q)\lambda'(q; r) \\
&= \Delta'(q)(\lambda(q; 0) + r\Delta(q)) - \Delta(q)(\lambda'(q; 0) + r\Delta'(q)) \\
&= \Delta'(q)\lambda(q; 0) - \Delta(q)\lambda'(q; 0) \\
&= (\lambda'(q; 1) - \lambda'(q; 0))\lambda(q; 0) - (\lambda(q; 1) - \lambda(q; 0))\lambda'(q; 0) \\
&= \lambda'(q; 1)\lambda(q; 0) - \lambda(q; 1)\lambda'(q; 0)
\end{aligned}
$$

which is negative since log-submodularity in q and p implies $\frac{\lambda'(q;1)}{\lambda(q;1)} < \frac{\lambda'(q;0)}{\lambda(q;0)}$.

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