Forecasting the distribution of economic variables in a data-rich environment

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Abstract

In this paper we investigate the relevance of considering a large number of macroeconomic indicators to forecast the complete distribution of a variable. The baseline time series model is a semi-parametric specification based on the Quantile Auto-Regressive (QAR) model that assumes that the quantiles depend on the lagged values of the variable. We then augment the time series model with macroeconomic information from a large dataset by including principal components or a subset of variables selected by LASSO. We forecast the distribution of the h-month growth rate for four economic variables from 1975 to 2011 and evaluate the forecast accuracy with score functions tailored to evaluate quantile, areas of the distribution, and intervals. The results for the output and employment measures indicate that the multivariate models outperform the time series forecasts, in particular at short horizons and low quantiles, while for the inflation variables the improved performance occurs mostly at the one-year horizon. We also consider an application of the distribution forecasts to predict the probability of a future decline in output and employment and illustrate their practical use at three dates during the last recession.

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1 Introduction

There are several reasons to argue that forecasting the distribution of an economic variable is more interesting and useful compared to forecasting the mean. The most important one is the fact that the distribution or density forecast completely characterizes the uncertain future evolution of the variable, besides providing a gauge of its central tendency similarly to a point forecast. In addition, a distribution forecast is relevant when a decision maker faces asymmetric payoffs over the possible outcomes of the variable. For example, the loss function of a central bank might assess the risks of an increase or a decrease of future inflation differently. This has motivated an increasing number of recent papers that focus on modeling and forecasting the complete distribution of economic variables such as Jore *et al.* (2010), Ravazzolo and Vahey (2010), Clark (2011) Bache *et al.* (2011), and Geweke and Amisano (2012) among others. In particular, Jore *et al.* (2010) and Clark (2011) forecast the distribution of several economic variables and find that allowing for time variation in the conditional variance, through a time series process, is crucial to obtain accurate forecasts. They attribute this result to the decrease in macroeconomic volatility experienced by the U.S. economy after 1984, a period typically referred to as the Great Moderation. Earlier examples of models that assume a stochastic volatility component are Cogley and Sargent (2002) and Stock and Watson (2002b, 2005).

The aim of this paper is to forecast the distribution of a variable using a model that allows for both a time series component and the effect of macroeconomic indicators. The approach that we adopt follows Manzan and Zerom (2009) and assumes that the quantiles of the variable being forecast are a function of its own lags as well as macroeconomic indicators that might be relevant predictors. However, we differ with respect to this earlier paper by including in the forecasting model information about a panel of 141 macroeconomic variables, instead of limiting the analysis to a small set of predictors. The idea of using information about a large number of variables has been extensively considered in economic forecasting since the early work of Stock and Watson (2002a, 2002c) and Forni *et al.* (2000, 2005), and Rossi and Sekhposyan (2012) is a recent application to density forecasting. We consider two approaches to incorporate this vast information set in the quantile regression. The first approach consists of including, as quantile predictors, a small number of factors that are extracted from the panel of macroeconomic variables. The advantage of using this method is that the factors describe, in a parsimonious way, the information contained in the panel about the state of the economy. The flexible nature of the quantile model allows the factors to have heterogeneous effects in different parts of the distribution, such as at the left or right tail or at the center. The second approach that we consider selects a handful of predictors to be included in the quantile regression, while the remaining variables are discarded. The method that we use to select the variables is the LASSO algorithm proposed by Tibshirani (1996), and adopted in a quantile context by Koenker (2004, 2011). In this case, at each forecast date only a subset of variables are selected as predictors which facilitates the interpretation of the relationships and allows to compare the variables selected to those typically used in the forecasting literature. In addition, we select the predictors at each quantile level which could lead to the inclusion of different sets of variables at different parts of the distribution. We also consider a combination of the two methods in which the variables selected by LASSO are then used to construct factors to be included in the quantile regression and refer to this case as targeted factors, in the sense that the factors are obtained from a set of predictors targeted to a specific variable and to a specific quantile level (see Bai and Ng, 2008, for the conditional mean case).

We forecast the h-month percentage change ($h = 3$, 6 and 12) of Industrial Production Index (INDPRO), Nonfarm Payroll Employment (PAYEMS), the Consumer Price Index for all urban consumers (CPIAUCSL), and the Personal Consumption Price Index that excludes Food and Energy (PCEPILFE) starting in January 1975 until June 2011 (438 forecasts). The models that we consider are the Quantile Auto-Regressive (QAR) model proposed by Koenker and Xiao (2006), in which the conditional quantiles are only a function of past values of the variable being forecast, as well as the QAR model augmented by either the factors, the LASSO selected variables, or the targeted factors. The model forecasts are evaluated based on their accuracy relative to a benchmark, which we chose to be the QAR estimated on a rolling window. The benchmark choice is inspired by the evidence of the Great Moderation which suggests that a rolling window might be more suitable to account for the distributional changes occurred in many macroeconomic variables. We measure the accuracy of the distribution forecasts using several score functions that have been proposed in the literature to evaluate individual quantiles, parts of the distribution (e.g., left or right tail, center), and interval forecasts. The testing results indicate that augmenting the QAR model with factors and LASSO selected variables delivers distribution forecasts that are more accurate compared to the rolling QAR benchmark. In particular, we find that for output and employment the better performance occurs in the lower part of the distribution and at the shorter horizons considered, although for PAYEMS the augmented models estimated on a rolling window outperform the benchmark also at the center and right tail at the three-month horizon. Instead, for the inflation measures we find that the macroeconomic variables matter the most at the one-year horizon, in particular when considering headline CPI inflation.

In addition to evaluating the accuracy of the distribution forecasts over the complete out-of-sample period, we also consider the possibility that the forecast accuracy of the models might have changed over time. An example of the relevance of testing this hypothesis is given by the comparison of the recursive and rolling QAR forecasts that, for most variables considered, are found to be equally accurate in the full out-of-sample period. However, when we perform a fluctuation test that compares the performance of the 10, 50, and 90% quantile forecasts on a 10-year rolling window we find that for PAYEMS (at all horizons) and for PCEPILFE (at the 12-month horizon) the rolling QAR quantile forecasts outperform the recursive starting from the second-half of the 1980s. In addition, the results suggest that this evidence is limited to the top quantile, rather than to the median and the lowest quantile considered. By comparing the top quantile forecasts of the two estimation schemes we find that those based on the rolling window are systematically lower compared to the recursive case in the Great Moderation period. This evidence is thus consistent with the literature discussed earlier, although it points to the fact that the decline in volatility was mostly driven by a downward shift of the right tail of the distribution, rather than by a symmetric shift in both tails. In addition, the fluctuation test also shows the models augmented with macroeconomic information seem to perform better in the tails of the distribution up to the beginning of the 1990s and in particular at short horizons, except for CPI inflation where the outperformance occurs mostly at the one-year horizon.

Furthermore, the LASSO selection delivers a subset of variables that can be considered as the most relevant predictors of a specific series at a given quantile level. For INDPRO and PAYEMS we find that among the top predictors there are several Producer Price Indices (e.g., capital goods and intermediate materials), the 3-month T-bill rate spread over the federal fund rate, housing variables (e.g., building permits and housing starts), employment variables (e.g., non-durable manufacturing employment), banking variables (e.g., saving deposits and consumer credit outstanding), as well as some of the NAPM Indices, namely New Orders, Production, and Prices. Instead, for the inflation measures LASSO selects some Producer Prices Indices mostly related to consumer goods and commodities, the spread of the bank prime rate over the federal fund rate, several money and banking variables (e.g., saving deposits, commercial and real estate loans, and some monetary aggregates) and some housing and employment variables. In addition, also for the inflation measures several NAPM indicators are selected, in particular the Price Index, the Supplier Deliveries Index and the Employment Index.

A potential application of the distribution forecasts is to produce probability forecasts of events of interest, e.g., that the variable is higher/lower than a certain value in the future. We consider the case of negative growth in INDPRO and PAYEMS as a proxy for a recession and calculate the probability forecasts for this event from the quantile models as well as from a probit model that uses the yield spread as a predictor (see Rudebusch and Williams, 2009). The evaluation of the probability forecasts shows that adding the factors and the LASSO selected variables to the QAR specification significantly improves the accuracy of recession probability forecasts over the time series QAR forecasts for both variables, but only occasionally they outperform the yield spread forecasts. The time series plot of the probability forecasts of negative growth for INDPRO reveals that the LASSO quantile model had difficulty anticipating the decline in output during the last two recessions, as opposed to the yield spread that indicated an increasing probability of a recession. This can be partly explained by the nature of the latest two recessions which were characterized by weak warnings from the real side of the economy of the approaching contraction (see Stock and Watson, 2003, for a detailed account of the 2001 recession). Finally, we illustrate and discuss the practical use of the distribution forecasts at the inception and during the latest recession by producing forecasts for the growth rate of the four variables based on the information available in January 2007, 2008, and 2009 and for 1 up to 16 month ahead.

The paper is organized as follows. Section (2) describes the quantile forecasting models while in Section (3) we discuss the score functions and the statistical test of equal predictive accuracy used in evaluating the forecasts. Section (4) provides the details of the empirical application and discusses the results of the tests, their fluctuation version, the variables selected by LASSO, and the comparison of the probability forecast of negative output and employment growth relative to the yield spread probit model. The practical use of the distribution forecasts during the latest recession is discussed in Section (5), and Section (6) draws the conclusion of the paper.

2 Models

Denote by Y_t (for $t = 1, \cdots, T$) the variable we are interested to forecast in period T and h-step ahead which is assumed to be stationary. The baseline (time series) h-step forecasting model that we consider is the Quantile Auto-Regressive (QAR) specification considered in Koenker and Xiao (2006):

$$Q^{\mathrm{QAR}}_{t+h|t}(\tau) = \alpha(\tau) + \sum_{i=1}^{p_\tau} \beta_i(\tau) Y_{t-i+1} \tag{1}$$

where $Q^{\mathrm{QAR}}_{t+h|t}(\tau)$ indicates the τ-level conditional quantile of Y_{t+h}, $\alpha(\tau)$ and $\beta_i(\tau)$ are parameters, and p_τ is the lag order used to model the τ quantile. The QAR model extends to a quantile regression setting the Auto-Regressive (AR) model used for the conditional mean. The model allows the dynamic relationship of the variable to possibly vary at different parts of its distribution. Since the parameters $\beta_i(\tau)$ (for $i = 1, \cdots, p_\tau$) could vary across quantiles τ, the model allows for heterogeneous degrees of persistence of the variable. This is for example the case in the application to interest rates in Koenker and Xiao (2006). The selection of the lag order at each quantile is performed by a Schwarz-like criterion by choosing the p_τ that minimizes the following quantity:

$$SIC_\tau\left(p_\tau\right) = T \log \hat{\sigma}_\tau\left(p_\tau\right) + \frac{p_\tau}{2}\log(T)$$

where $\hat{\sigma}_\tau\left(p_\tau\right)$ is the average τ quantile loss function of the estimated model with p_τ lags.

The aim of this paper is to evaluate the relevance, from a forecasting point of view, of augmenting the time series quantile model in Equation (1) with information about a large number of macroeconomic variables rather than relying on a few, although relevant, variables. We consider three of the several approaches that have been proposed to reduce the dimensionality of the problem. The first approach has been widely used in the conditional mean forecasting literature and consists of extracting principal components from the panel of macroeconomic variables and use them as predictors. Instead, the second approach selects the most relevant predictors in the panel by means of shrinkage methods. Finally, we also consider a combination of the previous methods in which the principal components are obtained from a subset of variables in the panel selected by the shrinkage method. An alternative approach that we do not examine in this paper is the combination of forecasts from bivariate models as discussed in Huang and Lee (2010) for the conditional mean case and in Rossi and Sekhposyan (2012) for density forecasts. Furthermore, an additional refinement to the current setup is bagging the quantile predictors as proposed in Lee and Yang (2006, 2008) which represents an effective approach to account for parameter estimation uncertainty and model uncertainty and result in higher prediction accuracy. For all models, we re-arranged the quantiles to avoid their crossing as proposed in Chernozhukov *et al.* (2010).

Factor-Augmented Quantile Auto-Regression (FA-QAR)

A popular approach in forecasting is to augment a time series model with principal components obtained from a large panel of macroeconomic variables. This approach is proposed, among others, by Stock and Watson (2002a, 2002c) and Forni *et al.* (2000). Stock and Watson (2006) provide a comprehensive survey of the application of the method in macroeconomic forecasting. Denote by $f_{k,t}$ the k-th principal component obtained from the variables $X_{j,t}$ $(j = 1, \cdots, J)$ that are assumed to be stationary; the Factor-Augmented QAR (FA-QAR) is given by

$$Q_{t+h|t}^{\text{FA-QAR}}(\tau) = \alpha(\tau) + \sum_{i=1}^{p_\tau} \beta_i(\tau) Y_{t-i+1} + \sum_{k=1}^{K} \gamma_k(\tau) f_{k,t} \tag{2}$$

where K indicates the number of factors included in the regression. The advantage of this approach is that it reduces the dimensionality of the problem by concentrating the informational content of a large number of J macroeconomic variables in a small number K of factors. The FA-QAR represents a straightforward extension to the quantile framework of the FA-AR model that is often used in forecasting the conditional mean of economic indicators. In the empirical application that follows, we use the same p_τ selected for the QAR model as discussed above, while we fix the number of factors included in the quantile regression K to be equal to 3 and 5. If the macroeconomic factors are relevant predictors of the dynamics of Y_{t+h}, then we expect the predictive quantiles of the FA-QAR model to be (relatively) more accurate compared to the QAR model in a sense that will be discussed later in the forecast evaluation Section.

LASSO Quantile Regression (LASSO-QAR)

An alternative approach to use the information contained in the large panel of macroeconomic indicators is to select a subset of them that are relevant to forecast the variable. There are several shrinkage methods proposed in the literature for the linear regression model and, prominent among them, is LASSO (Least Absolute Shrinkage and Selection Operator) proposed by Tibshirani (1996). The idea is to estimate the parameters of the model by adding a penalization term which is proportional, in the case of LASSO, to the sum of the absolute value of the parameters. In this way, the parameters of the irrelevant variables are shrunk to zero thus allowing to identify a subset of variables with non-zero coefficients. This approach has been adopted by Koenker (2004) in the context of a quantile panel data model where LASSO is used to select the fixed effects, and by Koenker (2011) for the case of additive nonparametric quantile regression. De Mol *et al.* (2008) consider LASSO selection in a comparison with other shrinkage methods and principal components in the case of conditional mean forecasting.

In this paper, we use LASSO to select a subset of the J macroeconomic variables that are useful in forecasting the τ quantile of Y_{t+h}. The LASSO-QAR model is given by

$$Q_{t+h|t}^{\text{LASSO-QAR}}(\tau) = \alpha(\tau) + \sum_{i=1}^{p_\tau} \beta_i(\tau) Y_{t-i+1} + \sum_{j=1}^{J} \delta_j(\tau) X_{j,t} \tag{3}$$

where the parameters $\alpha(\tau)$, $\beta^{'}(\tau)$, and $\delta^{'}(\tau))$ are estimated by minimizing the following quantity:

$$\sum_{t=1}^{T} \rho_\tau \left(Y_{t+h} - \hat{\alpha}(\tau) - \sum_{i=1}^{p_\tau} \hat{\beta}_i(\tau) Y_{t-i+1} - \sum_{j=1}^{J} \hat{\delta}_j(\tau) X_{j,t} \right) + \lambda \frac{\sqrt{\tau(1-\tau)}}{2} \sum_{i=1}^{J} |\hat{\delta}_j(\tau)|$$

with $\rho_\tau(u) = u(\tau - I(u < 0))$ denoting the piecewise linear quantile loss function. The non-negative parameter λ represents a shrinkage parameter that controls the amount of penalization that is applied in estimation. The two leading cases are $\lambda = 0$, which represents the case of no penalization such that the estimates are equal to the standard quantile estimates of Koenker and Bassett (1978), and $\lambda \to \infty$ in which the estimates of $\delta(\tau)$ are shrunk to zero for all macroeconomic variables. Hence, in the first case the parameter estimates are likely to be poorly estimated due to the large number (J) of variables included, while in the second case the LASSO-QAR model reduces to the pure time series QAR model.

The choice of the LASSO penalty is extremely important for the performance of the forecasting model. A small value of λ leads to the inclusion of a large number of irrelevant variables which causes more volatile forecasts, while a large value might lead to the exclusion of some relevant variables. We select the value of λ following the proposal of Belloni and Chernozhukov (2011). It consists of simulating the J-dimensional vector $S_{\tau,b} = \sum_{t=1}^{T} (\tau - I(U_{t,b} - \tau)) X_t$, where $U_{t,b}$ is a random uniform draw in the $[0,1]$ interval and $b = 1, \cdots, B$, with B the total number of replications. The penalty λ is then selected as c times the $(1-\alpha)$ empirical quantile of the B simulated values of $\max_\tau ||S_{\tau,b}||_\infty$. In the application that follows, we set α equal to 0.05 and provide results for the constant c equal to 1, 2, and 3 in order to evaluate the sensitivity of the performance of the LASSO-QAR forecasts to the penalization parameter.

The LASSO coefficient estimates $\hat{\delta}(\tau)$ might be biased for the variables with non-zero parameters. Hence, it is common in the literature to re-estimate the quantile regression model only including the subset of variables with non-zero coefficients. Given the values of c and κ, the selected macroeconomic variables at quantile τ are denoted by $\tilde{X}^\tau_{l,t}$ (for $l = 1, \cdots, L_\tau$, with $L_\tau \leq J$), and the QAR augmented by these variables (denoted as POST-LASSO-QAR) is given by

$$Q^{\text{POST-LASSO-QAR}}_{t+h|t}(\tau) = \alpha(\tau) + \sum_{i=1}^{p_\tau} \beta_i(\tau) Y_{t-i+1} + \sum_{l=1}^{L_\tau} \delta_l(\tau) \tilde{X}^\tau_{l,t} \tag{4}$$

Targeted Factor-Augmented QAR (TFA-QAR)

The last approach that we consider combines the two previous methods. Based on the subset of variables selected by LASSO, we extract K principal components from the 10 variables with the largest absolute coefficient[1]. The approach is similar to the proposal in Bai and Ng (2008) of constructing targeted factors, that is, factors based on a subset of the variables included in the large panel which have been selected with the specific target of forecasting the variable of interest. Furthermore, in this application the factors are also targeted to the specific quantile τ under consideration since the variable selection is quantile-specific. Denote by $tf_{k,t}(\tau)$ the targeted factor ($k = 1, \cdots, K$) at quantile τ obtained from the the first step of LASSO selection. Then, the conditional quantile model is given by:

$$Q^{\text{TFA-QAR}}_{t+h|t}(\tau) = \alpha(\tau) + \sum_{i=1}^{p_\tau} \beta_i(\tau) Y_{t-i+1} + \sum_{k=1}^{K} \gamma_k(\tau)\ tf_{k,t}(\tau) \tag{5}$$

Also in this case we fix the number of factors K to equal 3.

[1] In case the number of variables with non-zero coefficient is less than 10 we include all variables.

3 Forecast accuracy

The aim of this paper is to evaluate and compare the models discussed in the previous Section in terms of their ability to forecast (out-of-sample) the conditional distribution of the variable of interest. The first forecast is produced for month T based on information available up to $T-h$ and the last is for month $T+F$ (total of F forecasts). We consider both a recursive and a rolling scheme to generate the forecasts. In the first case the estimation window expands as new observations are added to the sample, while for the rolling scheme the estimation window is kept constant and new observations replace the oldest ones in the sample. The literature on testing density and distribution forecasts has rapidly expanded from the early proposal of Diebold *et al.* (1998) and a recent survey is provided in Corradi and Swanson (2006). We measure the accuracy of the forecasts using several of the score functions considered in Gneiting and Raftery (2007) and Gneiting and Ranjan (2011) that aim at evaluating different characteristics of the forecasts and are discussed below.

Since the models in Section (2) produce forecasts for the quantiles of Y_{t+h}, it seems natural to consider the quantiles also in the evaluation and comparison of the out-of-sample performance. The *Quantile Score* (QS) is proposed to evaluate quantile forecasts based on the same loss function $\rho_\tau(u)$ that is used in quantile estimation. The QS for the quantile forecast of model i (i=QAR, FA-QAR, LASSO-QAR, TFA-QAR), denoted by $Q^i_{t|t-h}(\tau)$ (for $t = T, \cdots, T+F$), is given by the piecewise linear asymmetric loss function

$$QS^i_{t|t-h}(\tau) = \left[Y_t - Q^i_{t|t-h}(\tau)\right]\left[\mathcal{I}(Y_t \leq Q^i_{t|t-h}(\tau)) - \tau\right] \tag{6}$$

Another related score function proposed by Gneiting and Raftery (2007) is the *Weighted Quantile Score* (WQS) which targets the evaluation to an area of the distribution as opposed to a single quantile as in the case of the QS. The WQS consists of integrating the QS in Equation (6) over the possible values of τ with the score multiplied by a weight function that focuses the evaluation on a specific area of interest of the distribution. The WQS is given by

$$WQS^i_{t|t-h}(w) = \int_0^1 QS^i_{t|t-h}(\tau)\omega(\tau)d\tau \tag{7}$$

where $\omega(\tau)$ denotes the weight function. We consider several weight functions to evaluate specific areas such as the left tail ($\omega(\tau) = (1-\tau)^2$), right tail ($\omega(\tau) = \tau^2$), tails ($\omega(\tau) = (2\tau-1)^2$), and the center of the distribution ($\omega(\tau) = \tau(1-\tau)$). An additional weight function that we consider is a uniform weight function ($\omega(\tau) = 1$) which provides an overall measure of accuracy of the distribution forecasts. This represents an alternative way to assess the overall goodness of a distribution forecast compared to evaluating density forecasts using the *Logarithmic Score* (LS) function which has been extensively considered in economics and finance by, among others, Amisano and Giacomini (2007), Rossi and Sekhposyan (2010), Geweke and Amisano (2010, 2011). If we denote by $f^i_{t|t-h}(\cdot)$ the density forecast for target date t of model i, the LS function is defined as

$$LS^i_{t,h} = -\log f^i_{t|t-h}(Y_t) \tag{8}$$

and represents the (negative) value of the density forecast evaluated at the forecast realization Y^h_t. The

LS function is often used because it has the useful property of rewarding density forecasts that have higher mass at the realization of the variable. On the other hand, the LS function narrowly focuses on just one aspect of the density forecasts, and, for example, does not reward a forecast that assigns high probability to outcomes close to the realization. The negative orientation of the function is introduced to have an interpretation in terms of loss which is consistent with the other score functions discussed. The last score function that we consider is the *Interval Score* (IS) function proposed by Gneiting and Raftery (2007) that evaluates two important characteristics of an interval given by the coverage, i.e., the frequency of forecasts that fall in the interval, and the length of the interval. Define the lower and upper bound of the interval by the predictive quantiles at levels $\theta_l = (1-\theta)/2$ and $\theta_u = (1+\theta)/2$. Then, the score for a $(100 * \theta)\%$ level interval for model i, denoted by $IS^i_{t|t-h}(\theta)$, is defined as

$$IS^i_{t|t-h}(\theta) = \left[Q^i_{t|t-h}(\theta_u) - Q^i_{t|t-h}(\theta_l)\right] + \frac{2}{1-\theta} * \left[Q^i_{t|t-h}(\theta_l) - Y_t\right] \mathcal{I}\left(Y_t < Q^i_{t|t-h}(\theta_l)\right) + \frac{2}{1-\theta}\left[Y_t - Q^i_{t|t-h}(\theta_u)\right] \mathcal{I}\left(Y_t > Q^i_{t|t-h}(\theta_u)\right) \tag{9}$$

We statistically evaluate the performance of the competing models in relative terms, that is, by comparing the score of a model to the score of another model that takes the role of benchmark. Denote by $S^i_{t|t-h}$ the score of model i and $S^j_{t|t-h}$ the score of model j, where the score can be the QS, WQS, LS, or IS discussed above. Then, we follow Giacomini and White (2006) and Amisano and Giacomini (2007) and test the null hypothesis of equal forecast accuracy of the two models, $S^i_{t|t-h} = S^j_{t|t-h}$ (for $t = T, \cdots, T+F$), using the test statistic

$$t = \left(\overline{S^j_h} - \overline{S^i_h}\right)/\sigma \tag{10}$$

where $\overline{S^i_h}$ and $\overline{S^j_h}$ denote the sample average of the scores in the forecasting period, and σ denotes the HAC standard error of the difference in scores. The test statistic t is asymptotically standard normal and rejections for negative values of the statistic indicate that model j significantly outperforms model i (and vice-versa for positive values). In the next Section we present results also for models estimated on a recursive and rolling window, although the recursive estimation is not consistent with the theoretical assumption of non-vanishing estimation error required by the test of Giacomini and White (2006). Hence, the results when a recursively estimated model is involved should be considered as approximate.

4 Application

We forecast four economic variables at the monthly frequency that represent closely watched business cycle and inflation indicators: Industrial Production Index (INDPRO), Total Non-farm Payroll Employment (PAYEMS), Consumer Price Index for all urban consumers (CPIAUCSL), and Personal Consumption Expenditure chain-type Price Index less food and energy (PCEPILFE). For all these variables we assume that they are non-stationary and forecast the annualized h-period growth rate which is defined as $Y_{t+h} = (1200/h)[\ln I_{t+h} - \ln I_t]$, where I_t indicates the level of the variable or index in month t. The sample starts in January 1960 and ends in June 2011 (618 observations) and we begin the out-of-sample exercise in January 1975 for a total of $F = 438$ monthly forecasts. We consider 3 forecast horizons h equal to 3, 6, and 12 months. In addition, we construct a dataset of 141 macroeconomic variables from the Federal Reserve Bank of Saint Louis FRED data repository that are listed in the Appendix and we follow Stock and Watson (2002c) in transforming the variables to induce stationarity. Of the 141 variables included in the panel, 118 variables have observations starting in January 1960, 126 in January 1970, and all variables are available since January 1980. We estimate the forecasting models discussed in the previous Section on both a recursive (expanding) window and a rolling (fixed) window of $180 - h$ months. In the subsequent discussion and in the Tables we indicate the five forecasting models by QAR (Quantile Auto-Regressive model in Equation 1), FA-QAR (Factor Augmented- QAR in Equation 2 for $K = 3$ and 5), LASSO-QAR (Equation 3), POST-LASSO-QAR (Equation 4), and TFA-QAR (Targeted FA-QAR in Equation 5 for $K = 3$) and attach to the model's name the label REC if the model is estimated on a recursive window or ROLL if a rolling window is used. Since the panel of macroeconomic indicators is unbalanced, we only include those variables that are available since inception when estimating the models on a recursive window (118 variables). However, when we use a rolling scheme we consider all variables with no missing data in the estimation window so that the information set is expanding as new variables are included. Finally, the relative nature of the forecast evaluation requires the specification of a benchmark model. Based on the Great Moderation literature that indicates a decline in volatility for several macro-variables after 1984, we chose the QAR model estimated on a rolling window (QAR ROLL) as the benchmark for the analysis since it allows to (slowly) incorporate the regime changes occurring in the economy.

4.1 Forecast accuracy tests

Tables (1) to (4) report the test statistic of the Quantile Score (QS) test for the null hypothesis of equal forecast accuracy of a model relative to the benchmark QAR ROLL model. Negative values of the test statistic indicate that the alternative model outperforms the QAR ROLL benchmark (in bold are reported the statistics significant at 5% against this one-sided alternative), while positive values suggest that the QAR ROLL forecasts outperform the alternative model forecasts (significance at 5% is denoted in italic).

The comparison of the forecast performance of the QAR ROLL benchmark to the QAR REC model shows that they are equally accurate when forecasting the h-month growth rate of INDPRO at the 3 and 6 month horizons, but not at $h = 12$ where QAR REC outperforms the rolling forecasts for quantile levels not greater than 0.3. Hence, the evidence does not support the hypothesis of a decline of the INDPRO growth rate volatility starting from the mid-1980s since it should have resulted in a better performance of the rolling estimation window relative to the recursive scheme. However, it is also possible that averaging the differential performance of the models over a long forecasting period might hide the higher accuracy of the QAR ROLL forecasts if it occurred in a short time window. The fluctuation analysis in the following Section will help uncover the possible time variation in the relative performance of the two models. Augmenting the QAR REC model with the factors results in forecasts that are more accurate at low quantiles relative to the benchmark at all horizons, in particular when the model is estimated on a recursive window. The alternative approach of selecting a subset of macroeconomic variables using LASSO provides significant improvements relative to the benchmark. At the two shortest horizons, the LASSO-QAR model provides higher accuracy at low and high quantiles, in particular when the model is estimated recursively. Furthermore, we do not find improvements in performance when considering refinements to the baseline LASSO-QAR model, such as re-estimating the model with only the variables selected by LASSO or using targeted factors. In terms of the dependence between the degree of shrinkage and forecast accuracy, our findings indicate that, overall, the intermediate value of $c = 2$ seems to provide higher forecast accuracy across quantiles, in particular for $h = 6$. On the other hand, the largest value of c leads to the selection of a small number of variables which might be excluding some relevant variables and thus negatively affect the performance of the forecasting model.

The results for PAYEMS in Table (2) indicate that the QAR REC and QAR ROLL have similar accuracy at $h = 3$ and 6 (although there is evidence of QAR REC outperforming at quantiles 0.30 to 0.50). Contrary to INDPRO however, for $h = 12$ the positive and significant QS statistics at high quantile levels indicate that the rolling scheme performs better than the recursive one. This suggests that there could have been some structural change in the quantile dynamics related to the Great Moderation. The fact that the rolling scheme is more accurate only at the highest quantiles might be interpreted as a downward shift of the upper tail of the conditional distribution of the PAYEMS growth rate, rather than a symmetric shift in both tails. Using a measure of dispersion that assumes symmetry (e.g., the standard deviation) would not be able to highlight this fact. For PAYEMS we find that macroeconomic variables provide accuracy gains at the shortest horizons of 3 and 6 months, but very limited evidence at the one year horizon. In particular, augmenting the QAR model with the macroeconomic factors and LASSO selected variables improves the forecast accuracy for quantiles below the median and at high quantiles only when using a rolling scheme. Also for this variable the choice of the shrinkage parameter c equal to 2 seems to provide a reasonable compromise between parsimony and performance.

Table (3) reports our findings on the quantile accuracy of the models when forecasting the h-month CPI-AUCSL growth rate. Also for CPI inflation we find that estimating the time series QAR model using a recursive, rather than rolling, window does not affect significantly the performance. Furthermore, the LASSO selection of the macroeconomic variables seems only successful when the models are estimated on a rolling window, a result that is consistent also across the different horizons considered. In this case, it seems that a shrinkage parameter c equal to 1 achieves similar results to $c = 2$, while the largest value of c underperforms the other choices (except for h equal to 12). The findings for core PCE (PCEPILFE) are reported in Table (4) and show that, also for this variable, there is no significant difference in performance between the QAR REC and QAR ROLL, except for two quantiles at $h = 3$. However, the role of macroeconomic variables seems to differ from the earlier results since the LASSO selection outperforms the QAR ROLL model at high quantiles (larger or equal to 0.70), in particular when the models are estimated on a rolling window. However, these findings are mostly confined to the shortest horizons, while for $h = 12$ there is very limited evidence of the benefit of including macroeconomic variables in the analysis.

The QS test discussed so far allows for a very detailed evaluation and comparison of the local performance of the quantile forecasts, although in certain situations a forecaster might be interested in a more general assessment of the accuracy of the alternative forecasts. This can be achieved by using score functions such as the LS, WQS (with uniform, center, left and right tail weights), and the IS. To save space, in Tables (5) and (6) we report results for the LASSO-based forecasts with the constant c equal to 2 which seems to provide a reasonable level of shrinkage across the different variables considered. The results in Table (5) show that the LS test for INDPRO rejects the rolling QAR model forecasts in favor of the recursive model at the 3 and 12-month horizon and also when considering the FA-QAR and LASSO-QAR at the shortest horizon. In addition, the WQS with uniform weight displays rejections for all LASSO-based models at $h = 3$ and for the recursive and rolling LASSO-QAR when $h = 6$. The rejections of the WQS-unif can be attributed to the superior performance of the LASSO forecasts on the left tail of the distribution as established by the WQS that focuses on the left tail, and in some cases due to higher accuracy at the center or right tail of the distribution. At the 6-month horizon we find, consistent with the earlier results for the QS test, that the recursive FA-QAR and LASSO models outperform the benchmark on the left tail of the distribution, while at the 1-year horizon the WQS that focuses on the left tail is significant for the recursive and rolling LASSO models.

Some forecasters might be interested in producing prediction intervals and the IS test allows to compare the accuracy of these forecasts. The Table reports the IS test for θ equal to 0.70 and 0.90, along with the length of the interval and its coverage, which represents the frequency that the variable realizations falls in the forecast intervals. At the 3 and 6-month horizon, most LASSO-based models outperform the rolling QAR interval forecasts for both $\theta = 0.70$ and 0.90. This might be due to the fact that the coverage of the LASSO models is closer to the nominal level relative to QAR ROLL, in addition to the fact that their forecast intervals are shorter in length relative to the benchmark. The results for PAYEMS show that both the FA-QAR and LASSO-QAR models improve, compared to the benchmark, the distribution forecasts on the left tail for $h = 3$ and 6, while at the 3-month horizon the rolling LASSO-QAR model outperforms also at the center of the distribution. In terms of interval forecasts, the ability of macroeconomic variables to improve quantile forecasts at short horizons explains their better performance in the IS test for $\theta = 0.70$ and for $\theta = 0.90$. On the other hand, the irrelevance of macroeconomic indicators in forecasting the distribution of PAYEMS at the 1-year horizon is confirmed by the lack of rejections, although in most cases the t-statistics are negative.

Table (6) provides the results for the CPIAUCSL h-month inflation rate. The rolling LASSO-based models outperform the time series benchmark for $h = 3$ at the center and left part of the distribution, in addition to providing more accurate 70% interval forecasts. The same models also dominate the benchmark for $h = 12$ in all parts of the distribution considered (center, left, and right tail) and the comparison of the 70% and 90% forecast intervals confirm these results. At this horizon, the coverage of the rolling QAR intervals is much smaller than the nominal level which also influences the results for the FA-QAR and LASSO-QAR models. However, the macro-augmented models have typically better coverage and a smaller average length of the interval which can, partly, explain the outperformance in terms of the IS test. The results for PCEPILFE h-month inflation shows that the rolling LASSO-QAR outperforms the rolling QAR benchmark on the right tail of the distribution at the shortest horizons, in addition to provide more accurate 90% interval forecasts. However, at the 12-month horizon there is no evidence of any advantage from including the macroeconomic variables in the analysis.

4.2 Fluctuation tests

A drawback of the analysis in the previous Section is that it relies on averaging the difference in forecasting performance of the competing models over a long period of time that goes from January 1975 to June 2011 (438 months). Giacomini and Rossi (2010) propose to statistically evaluate the hypothesis of time-variation in the performance of two models by testing the hypothesis of equal accuracy of their forecasts at any point of the out-of-sample period. This is implemented by performing a predictability test over a rolling window of m forecasts, rather than over the full out-of-sample period F. However, under this null hypothesis the test statistic follows a non-standard distribution whose critical values have been simulated in Giacomini and Rossi (2010). This testing framework can be applied to any of the tests discussed in Section (3) and we decided to consider the Quantile Score (QS) at three representative quantile values: $\tau = 0.10$, 0.50 and 0.90. The quantile score has the advantage, compared to global measures such as the logarithmic score, to allow the comparison of the (possibly) time-varying performance of the forecasting models at different parts of the distribution (low, center, and high quantiles). As in the previous Section, we consider the QAR ROLL as the benchmark model and compare its quantile forecasts to the QAR REC and the models that include the macroeconomic indicators (either via factors or LASSO selection). More specifically, denote the quantile score of the QAR ROLL model for target date t by $QS^{\text{QAR ROLL}}_{t|t-h}(\tau)$ and by $QS^{i}_{t|t-h}(\tau)$ the quantile score for model i (for $i = QAR\ REC, FA - QAR, \cdots$). The τ-level fluctuation QS test for model i (relative to the benchmark) in period t and for window size m, indicated as $fQS^{i}_{t,m}(\tau)$, is given by

$$fQS^{i}_{t,m}(\tau) = \frac{1}{m} \sum_{s=t-m}^{t} QS^{i}_{s|s-h}(\tau) - QS^{\text{QAR ROLL}}_{s|s-h}(\tau) \tag{11}$$

where $t = T + m, \cdots, T + F$ and T indicates the first forecast month and F the total number of forecasts. In practice, we use a window of 10 years of monthly data ($m = 120$) such that the ratio m/F is approximately 0.30 for which the two-sided critical values are 3.012 (5%), and 2.766 (10%), and the one sided are 2.770 (5%), and 2.482 (10%). We index the fluctuation test $fQS^{i}_{t,m}(\tau)$ with the last observation of the estimation window, e.g., the first value of the test refers to December 1984 which corresponds to the 120-month window from January 1975 to December 1984. The t-statistic for the QS fluctuation test is calculated using Newey-West standard errors as discussed in Section (3) and rejections for negative values indicate that model i outperforms the QAR ROLL benchmark at some point during the out-of-sample period.

Figures (1) to (4) show the fluctuation QS test for the four series we are forecasting at the 3, 6 and 12 month horizons. In each graph we report only the fluctuation test for the QAR REC model, the FA-QAR (for $K = 3$), LASSO-QAR and TFA-QAR estimated both recursively and on a rolling window. In addition to evaluating the time-varying performance of the models considered, the fluctuation test allows to relate these results to the evidence of the decline in macroeconomic volatility in the post-1984 period (see Stock and Watson, 2002b among others). Based on this evidence, we can formulate the following predictions about the performance of the models, as well as offer new insights. First, a decrease in the volatility of macroeconomic variables (in particular for output-related variables such as INDPRO and PAYEMS) that happens during the out-of-sample period would imply that the QAR estimated on a rolling window should outperform the forecasts of the recursive QAR, since the rolling estimation adapts faster to the distributional changes in the series being forecast. We should thus expect the fluctuation test to show the QAR ROLL (at least temporarily) outperforming the QAR REC in the post-1984 period. A second issue in the Great Moderation debate is whether the decrease in volatility can be ascribed to macroeconomic factors or whether it was the result of changes in the distribution of shocks. Findings of better performance for the QAR models augmented by macroeconomic variables (relative to QAR ROLL) in the post-1984 period could indicate that these variables predicted the distributional changes in the variables being forecast, even when compared to the rolling QAR benchmark which incorporates these changes gradually. Furthermore, the evidence for the Great Moderation is typically based on measures (e.g., the standard deviation) which assume that macroeconomic risk behaves symmetrically on the tails of the variable distribution. However, it could be the case that the decrease in standard deviation might have been asymmetric, i.e., the result of a downward shift of the upper tail of the distribution and possibly no (upward) shift of the lower tail. The fluctuation test based on the QS score at low and high quantiles might shed some light on the possibility of an asymmetric decline in macroeconomic volatility.

In relation to the performance of the recursive and rolling estimation schemes for the QAR model, for INDPRO and PAYEMS the two estimation methods perform quite similarly at the low ($\tau = 0.1$) and median ($\tau = 0.5$) quantiles for all horizons, although in some cases the test is outside the one-sided critical values. However, when considering τ equal to 0.9 the evidence suggests that the rolling scheme outperforms the recursive. For INDPRO at the 6 and 12-month horizon the fluctuation test shows large and positive values which indicate the QAR REC performs (significantly) worse compared to the rolling QAR. In particular, for $h = 12$ the rolling model forecasts begin to be more accurate relative to the recursive ones for testing windows ending after 1986 and up to approximately 1996 and also during the latest recessionary period. The evidence for the mid-1980s can be associated to the effect of the Great Moderation which was incorporated faster by the rolling scheme and led to its higher accuracy in the post-1984 period. When considering PAYEMS in Figure (2), the fluctuation test for the top quantile shows significant better performance of the rolling scheme starting from the 10-year window that ends in 1996. The explanation for this finding is that the 90% quantile forecasts of the rolling QAR are systematically lower compared to the recursive quantile forecast. For instance, at the one-year horizon the rolling 90% quantile forecast is smaller than the recursive forecast every month after 1986 with the average difference between quantiles equal to 1.59%. This evidence seems to support the hypothesis discussed above of a distributional change of the PAYEMS h-month growth rate, although we find that the effect seems asymmetric in the sense that it occurs only at the top quantiles.

As concerns the inflation measures, we find that for CPIAUCSL the recursive and rolling QAR forecasts are equally accurate throughout the period considered at the 3 and 6-month horizons, but at the annual horizon there are some episodes of differential performance, which can be associated with the aftermath of the 2001 recession. This episode seems also to play a role in the better performance of QAR ROLL relative to the QAR REC at the 90% quantile for $h = 6$ and 12. Comparing the 90% quantile forecasts of the two models in the period after 1997, we find that the rolling forecasts are smaller than the recursive ones in 91.3% of the months in the period 1997-2011 and the average difference between them is 0.68% (2.32%

vs 3.0%). This thus suggests that the rolling forecasts absorbed faster the structural change in inflation dynamics and is thus more accurate in forecasting the top quantiles of the distribution.

The full-sample test results discussed above for INDPRO and PAYEMS indicate that adding macroeconomic variables to the QAR specification (either via factors or LASSO selection) provides more accurate forecasts, relative to the benchmark, at short horizons ($h = 3$ and 6) and largely at lower quantiles. The fluctuation QS test in Figures (1) and (2) show that these findings are mostly driven by the forecasting results up to the mid-1990s, with the recursive LASSO- and TFA-QAR performing better for INDPRO, and the rolling models providing higher accuracy for PAYEMS. Interestingly, these augmented models prove to be useful in forecasting the median of the variables at the beginning of the sample (1975-1985) and for the 10-year windows that end in the late 1990s, although the results for the full out-of-sample period show some rejections only for PAYEMS at $h = 3$. Also at the one-year horizon the macroeconomic variables seem to provide some predictive power, in particular at the very beginning of the forecasting period. For CPIAUCSL the fluctuation analysis shows that the higher accuracy found in the full-sample results is concentrated on the first half of the out-of-sample period, in particular for the tail quantiles. In the case of PCEPILFE h-month inflation, the fluctuation analysis shows some relevance of the data-rich models in forecasting the upper quantile.

4.3 LASSO variable selection

The overall performance of the LASSO-based models suggests that the method is able to select indicators with predictive power for the variables being forecast. It is thus interesting to examine which indicators, among the many considered, have contributed the most to the performance of LASSO. Tables (7) to (10) show the five most selected variables for the recursive and rolling POST-LASSO-QAR model for the three forecast horizons considered and at five quantile levels ($\tau = 0.1,\ 0.3,\ 0.5,\ 0.7$, and 0.9). In addition to the series ID (see the Data Appendix for the variable description and the transformation), the Tables report the frequency of selection of the variable (out of 438 forecasts), as well as the average coefficient in the months in which the indicator was selected[2]. Since the macroeconomic variables have been standardized to have mean zero and variance one, the coefficient should be interpreted as the effect of a one standard deviation change in the indicator. To save space, we only report the variable selected for the shrinkage parameter $c = 2$.

There is significant overlap in the variables selected as predictors for INDPRO and PAYEMS and those that are selected for the inflation measures (CPIAUCSL and PCEPILFE). The main findings when forecasting INDPRO and PAYEMS are:

- *Price Indexes*: we find that several Producer Price indexes are often selected, in particular "Finished Goods, Capital Equipment" (PPICPE), "Intermediate Materials, Supplies & Components" (PPI-ITM), "Finished Consumer Goods Excluding Food" (PFCGEF), in addition to several Consumer

[2]There are several cases in which, for a given quantile, some variables are selected more often than others, although their average coefficients might be smaller. We decided to report the variables by frequency of selection rather than absolute magnitude of their coefficient to avoid cases of variables that are seldomly selected, although having a large impact when selected.

Price Indexes such as "Medical Care" (CPIMEDSL) and "All items less food and energy" (CPILFESL). For both INDPRO and PAYEMS, we find that the (average) coefficients of these indicators are negative.

- *Spread*: the most selected spread is the 3-month (SPREAD3M), along with other short-term spreads (SPREAD6M). At the one-year forecast horizon, also SPREADAAA and SPREADBAA are selected.

- *Money and banking aggregates*: in several instances savings aggregates are selected, especially "Savings Deposits at Thrift Institutions" (SVGTI), "Savings and small Time Deposits" (SVSTSL), and "Savings deposits - Total" (SAVINGSL), and, in the case of PAYEMS, they are only selected at the top quantiles.

- *Employment*: at long horizons, "Average (Mean) duration of unemployment" (UEMPMEAN) is often selected, together with several sub-aggregates of "All Employees", in particular "Retail Trade" (USTRADE), "Financial Activities" (USFIRE), and "Government" (USGOVT).

- *Consumption (PCE)*: none of the aggregates are among the top five variables to predict INDPRO, but the "Services" sub-aggregate (PCES) is selected as a predictor of PAYEMS for $h = 3$ for the recursive scheme.

- *Housing*: housing-related variables are often selected - at all horizons - to predict PAYEMS, but only in few cases to predict INDPRO. The most selected indicators are "Building permits" (PERMIT), "Building permits - in structures with 1 unit" (PERMIT1), "Building permits in Midwest census region" (PERMITMW), "New one family houses sold: United States" (HSN1F; mostly selected using the rolling window), "Housing starts in Midwest census region" (HOUSTMW), and "Housing starts: 1-Unit structures" (HOUST1F).

- *NAPM indicators*: for both variables the most selected NAPM indicators are "New Orders Index" (NAPMNOI), "Production Index" (NAPMPI), and "Prices Index" (NAPMPRI; negative sign). It is interesting that, for both variables and for the rolling scheme, NAPMNOI is selected exclusively at short forecast horizons, as opposed to NAPMPRI which is selected mostly at the longest horizons.

For the inflation measures, we find the following predictors as the most relevant:

- *Price Indexes*: some Producer Price Indexes, such as "Finished consumer foods" (PPIFCF) AND "Crude materials for further processing" (PPICRM) are selected as predictors of CPIAUCSL at $h = 3$ for the recursive scheme. For PCEPILFE, several of the Producer Price Indexes and CPI sub-aggregates discussed earlier are among the top predictors, in particular at the 3- and 6-month horizons.

- *Spread*: in the case of CPIAUCSL the most relevant spread is SPREADPRIME (Bank prime loan rate minus the federal fund rate) which is selected for both estimation schemes and at all horizons. In addition, the spread between BAA and AAA corporate bond yield (CREDIT) is selected at low quantiles by the recursive LASSO for $h = 3$ and 6. Instead, for PCEPILFE we find that only the

3-year spread (SPREAD3Y) is selected at the top quantiles when $h = 12$. This contrasts with the results for INDPRO and PAYEMS for which we found that the 3-month spread was overwhelmingly selected, along with other short-term spreads. All spreads selected have an inverse relationship with the inflation measures as documented by the negative sign of the (average) coefficient.

- *Money and banking aggregates*: no variable in this group is selected to forecast PCEPILFE, whereas several are highly relevant for CPIAUCSL. For instance, the "Effective federal funds rate" (FEDFUNDS) is often selected at the short horizons, savings aggregates ("Savings and small time deposits at commercial banks", SVSTCBSL), and credit aggregates such as "Commercial and Industrial loans at all commercial banks" (BUSLOANS), "Real estate loans at all commercial banks" (REALLN), "Total nonrevolving credit outstanding" (NONREVSL). In addition, monetary aggregates like "M1 Money stock" (M1SL), "M2 Money stock" (M2SL), "Board of Governors Monetary Base, Adjusted for Changes in Reserve Requirements" (BOGAMBSL) are selected when the model is estimated recursively.

- *Employment*: variables in this group are selected when the models are estimated recursively, but in only one case for the rolling window. The most relevant indicators are "Civilians unemployed for 27 weeks and over" (UEMP27OV; negative sign) for high quantiles at $h = 3$, NDMANEMP, and some PAYEMS subaggregates, such as "Wholesale trade" (USWTRADE) and "Financial activities" (USFIRE).

- *Housing*: no housing-related variable is selected as predictor of PCEPILFE, while for CPIAUCSL the predictors are similar to the ones selected earlier for INDPRO and PAYEMS (PERMIT1, PERMITS, PERMITMW, and HSN1FW).

- *NAPM indicators*: interestingly, the NAPM indicators selected for INDPRO and PAYEMS are not useful in forecasting the inflation measures, but other NAPM indexes are often selected, such as "Supplier deliveries index" (NAPMSDI), "Employment Index" (NAPMEI), as well as the "Price Index" (NAPMPRI) for both the recursive and rolling schemes.

Another fact that emerges from the Tables is the heterogeneity in the estimated coefficients when a variable is selected at different quantile levels. This shows the models proposed offer a flexible specification to forecast the quantiles both in terms of the predictors that are included as well as in terms of the (potentially) heterogeneous effect of these predictors at different parts of the distribution.

In Figure (5) we show the effect of the shrinkage parameter c on the number of variables selected by LASSO estimated on a recursive or rolling scheme for $\tau = 0.1$, 0.5 and 0.9. To save space, we only report the case of INDPRO at $h = 12$ and similar results are also obtained for the other variables and horizons. As expected, increasing the value of c from 1 to 3 leads to selecting less variables: in the recursive case, LASSO selects between 11 and 20 variables for $c = 1$, which reduces to 1 to 11 when c in increased to 2, and further decreases between 0 and 8 variables when the shrinkage parameter is further increased to 3. The graphs seem also to suggest that LASSO selects a larger number of variables for the median relative to the low and high quantiles. When the LASSO-QAR model is estimated on a rolling window the number of variables selected follows a similar pattern of reducing as the penalization is increased. The extreme case is when $\tau = 0.9$ and $c = 3$ where no macroeconomic variable is selected at any point in time so that the LASSO-QAR becomes the QAR model.

LASSO selection can become unreliable when there is high correlation between the candidate variables, as it is the case for a large panel of macroeconomic variables at the monthly frequency. As a way to assess if this is a problem in the current exercise, in Figure (6) we show the time series of the coefficient estimates for the 5 most frequently selected variables for LASSO-QAR REC. We consider the case of the Industrial Production growth rate for three quantile levels (τ = 0.1, 0.5 and 0.9) and horizons (h = 3,6 and 12), and similar results are also obtained for the other variables being forecast. The overall finding is that the coefficient estimates seem to significantly change over time, but these changes happen, in most cases, smoothly rather than in a highly volatile manner as we would have expected in case of an unstable variable selection methodology. In this sense, we believe the selection performed by LASSO provides reliable results which are also confirmed by the forecasting performance comparison.

4.4 Evaluation of recession probability forecasts

The power of the yield spread to forecast changes in business cycle conditions is a long-standing fact since the late 1980s (see Stock and Watson, 1989, and Estrella and Hardouvelis, 1991). In a recent paper, Rudebusch and Williams (2009) compare the accuracy of the yield spread in forecasting recessions to the probability forecasts from the Survey of Professional Forecasters (SPF). Their findings suggest that, at two to four quarter horizons, the yield spread model outperforms the SPF, even for a sample period that starts in the late 1980s when SPF participants knew of the predictive power of the yield spread. Hence, the yield spread model still seems to be a relevant benchmark to compare the recession probability forecasts obtained from the quantile-based models discussed earlier. Below we briefly discuss the probit specification for the yield spread model following Rudebusch and Williams (2009) with the only difference that we estimate the model to monthly rather than quarterly data. This implies that we use INDPRO as a proxy for output, since real GDP is only available at the quarterly frequency. In addition, we also examine PAYEMS which can also be considered as an indicator of the business cycle. Since the aim in this case is to forecast the probability of a recession, we introduce score functions that are targeted to evaluate and compare these probability events (see Rudebusch and Williams, 2009).

The recession indicator in month $t+h$, R_{t+h}, is defined as $R_{t+h} = I(Y_{t+h} < 0)$, where $I(\cdot)$ denotes the indicator function that takes value 1 if the condition is satisfied and 0 otherwise, and Y_{t+h} represents the h-month growth rate of monthly Industrial Production Index (INDPRO) or Total Non-farm Employees (PAYEMS). The yield spread in month t, S_t, is given by the difference between a long and short maturity rate[3] and it is typically used in a probit specification for R_{t+h} as follows:

$$R^{\text{PROBIT}}_{t+h|t} = P_t\left(R_{t+h} = 1\right) = N\left[\psi_0 + \psi_1 S_t\right]$$

[3] Typically the 10-year US Treasury note minus the 3-month T-bill rate. Notice that this definition of the spread differs from the one used in the previous Section where the benchmark short rate was the FEDFUNDS rate instead of the 3-month T-bill rate.

where $N(\cdot)$ represents the standard normal CDF and the ψ_0 and ψ_1 are parameters to be estimated. We also consider a dynamic specification where we augment the probit model with the lagged value of the business cycle indicator which could be useful to improve the forecasting performance of the model, in particular at short forecasting horizons. The dynamic probit model is thus given by

$$R_{t+h|t}^{\text{DYN PROBIT}} = P_t\left(R_{t+h} = 1\right) = N\left[\psi_0 + \psi_1 S_t + \psi_2 Y_t\right]$$

The probit specifications are estimated on a recursive window that includes only information available up to the month the forecast is produced and expands as the target date moves forward. We denote the recession probability obtained from the quantile model i ($i = QAR, FA - QAR$ and $LASSO - QAR$) by $R_{t+h|t}^i$ and is obtained as $R_{t+h|t}^i = \hat{F}_{t+h|t}^i(0)$, where $\hat{F}_{t+h|t}^i(\cdot)$ represents the distribution forecast obtained by interpolating the quantile estimates of model i. To evaluate and compare the probability forecasts, we follow Rudebusch and Williams (2009) and consider the following score functions for a generic recession probability forecast $R_{t+h|t}^j$ (where $j = \text{PROBIT}, \text{DYN PROBIT}, i$):

- Mean Absolute Error (MAE): $MAE(j,h) = (1/F)\sum_{t=T-h}^{T+F-h} |R_{t+h|t}^j - R_{t+h}|$
- Mean Square Error (MSE): $MSE(j,h) = (1/F)\sum_{t=T-f}^{T+F-h} \left(R_{t+h|t}^j - R_{t+h}\right)^2$
- Logarithmic Probability Score (LPS): $LPS(j,h) = -(1/F)\sum_{t=T-h}^{T+F-h} (1 - R_{t+h}) \ln\left(1 - R_{t+h|t}^j\right) + R_{t+h}\ln\left(R_{t+h|t}^j\right)$

Similarly to the approach discussed in Section (3), we evaluate the recession probability forecasts relative to a benchmark, which we choose to be the QAR ROLL and the dynamic probit specification. The test statistic is based on the difference of the score for model j and the score of the benchmark b ($b = \text{QAR ROLL}, \text{DYN PROBIT}$), that is,

$$\Delta S(j,b,h) = S(j,h) - S(b,h)$$

where $S(j,h)$ can either represent the MAE, MSE, and LPS scores discussed above. The null hypothesis of the test is the equal predictive accuracy of the benchmark b and the alternative model j and it can be evaluated using a t-type statistic with Newey-West standard error. Rejections that occur for negative values indicate that model j outperforms the benchmark, while the opposite is true for rejections on the positive side.

The results of the recession probability comparison are presented in Table (11) for INDPRO and Table (12) for PAYEMS that report the difference in MAE, MSE, and LPS scores along with the t-statistic for the null hypothesis of equal predictive accuracy when the benchmark model is QAR ROLL or DYN PROBIT. For INDPRO, we find that the QAR ROLL is outperformed by the DYN PROBIT at all horizons considered. Augmenting the QAR with macroeconomic factors or with variables selected by LASSO significantly improves the performance (compared to the QAR ROLL) at all horizons, in particular when the score function is MAE and MSE. However, when considering the dynamic probit model as benchmark there is no evidence of better performance of the quantile regression models, thus indicating that the yield spread model performs as well as more sophisticated specifications that incorporate information about a large number of variables. The results for PAYEMS (see Table 12) suggest the recession probability forecasts of the QAR ROLL and DYN PROBIT are equally accurate based on the MAE score for $h = 3$ and 6, but the other two score functions point to the higher accuracy of the DYN PROBIT probability forecasts. For $h = 12$ all scores indicate the DYN PROBIT as the most accurate model. As for INDPRO, adding information about a large of macroeconomic variables provides more accurate forecasts relative to the QAR ROLL, but outperform the DYN PROBIT forecasts in only a few cases for $h = 3$.

In Figures (7) and (8) we show the probability forecasts of negative h-month growth for INDPRO and PAYEMS, respectively, produced by the two probit specifications and the LASSO-QAR estimated on a recursive and rolling window. Considering INDPRO first, for $h = 3$ the four probability forecasts have similar overall dynamics, in particular up to the beginning of the 1990s. In the mid-1990s the yield spread seems to signal an increased probability of negative growth, which is not the case however for the LASSO-QAR models that forecast a probability close to zero. This occurs also at $h = 6$ and 12, although the LASSO-QAR models continue to forecast low probabilities even during and after the March-November 2001 recession. Also during the 2008-2009 recession the yield spread seems to provide a more reliable signal (at the one-year horizon) compared to the LASSO-QAR approach. As concerns PAYEMS, Figure (8) shows that at short horizons ($h = 3, 6$) the quantile models deliver more timely and responsive forecasts of decreases in payroll employment, although for $h = 12$ they missed to signal the two recessionary periods in the 2000s compared to the yield spread.

5 Forecasting before and during the 2008-2009 recession

An interesting exercise is to examine the forecast distributions produced by these models at a particular point in time and evaluate, in a qualitative manner, their performance and characteristics in light of the (future) realizations of the variable being forecast. We consider as forecast bases the end of January 2007, 2008, and 2009 which represent three crucial times for monetary policy-making leading to and during the recent recession that started in December 2007 and ended in June 2009 (as dated by the NBER dating committee). The aim of this exercise is to produce distribution forecasts based on the information that was available at the time the Federal Open Market Committee (FOMC) meeting took place and compare them with the outlook for the economy provided in the FOMC press releases[4]. In particular, following the January 2007 meeting the FOMC decided to keep the federal fund rate at 5.25% since the "economy seems likely to expand at a moderate pace over coming quarters" and "inflation pressures seem likely to moderate over time", although "some inflation risks remain". In January 21, 2008 the FOMC cuts the fund rate by 75 basis points to 3.5% due to "increasing downside risks to growth" while it "expects inflation to moderate in coming quarters". This decision was followed the week after by an additional cut of the fund rate to 3% on concerns about the tightening of the credit market and the "deepening of the housing contraction". Later, the NBER Business Cycle Dating Committee declared the business cycle peaked in December 2007. The third forecast date that we consider coincides with the end of January 2009 meeting, in which the FOMC decided to maintain the federal funds rate in the interval between 0 and 0.25% due to

[4]The press releases of the FOMC meeting can be found at `http://www.federalreserve.gov/monetarypolicy/fomccalendars.htm`.

the expectation that "economic conditions are likely to warrant exceptionally low levels of the federal funds rate for some time". In addition, "The Committee anticipates that a gradual recovery in economic activity will begin later this year, but the downside risks to that outlook are significant ... the Committee expects that inflation pressures will remain subdued in coming quarters". Furthermore, the FOMC seemed also concerned with the possibility of price deflation since "the Committee sees some risk that inflation could persist for a time below rates that best foster economic growth and price stability in the longer term."

We thus generate forecasts from the QAR, FA-QAR and LASSO-QAR models (for both recursive and rolling estimation) and we only report the results for $K = 3$ (FA) and $c = 2$ (LASSO). In generating the forecasts, we use only information available up to the December ahead of the FOMC meeting since the January data is released in mid-February. For each date, we forecast the annualized growth rate of the variables from 1 up to 16 months ahead and Figures (9) to (12) show the time series of the quantile forecasts. For FA-QAR and LASSO-QAR we plot the 0.05, 0.25, 0.45, 0.55, 0.75, and 0.95 quantiles and use increasingly darker shades for the 90%, the 50%, and the 10% intervals, while the median is represented by the darkest line. In addition, in each graph we also plot the forecasts of the QAR model for the 0.05, 0.50, and 0.95 quantiles (dashed lines) estimated on the same window as the FA- and LASSO-QAR models. This allows to visually evaluate the effect on the quantile forecasts of augmenting the time series model with the macroeconomic indicators via factors or LASSO selection. For instance, we expect the FA-QAR and the LASSO-QAR quantile forecasts to be equal (or very close) to those of the QAR model in case the factors or the variables selected by LASSO are irrelevant predictors. On the other hand, if the macroeconomic indicators are relevant the two quantile forecasts might deviate significantly based on macroeconomic conditions.

Figure (9) shows the quantiles forecasts for the annualized growth rate of INDPRO for the three dates (columns) and the four models considered (rows). The forecasts generated in January 2007 show that the median, for all models, predicted moderate growth, with most differences among models occurring at the outer quantiles. A first characteristic of these forecasts is represented by the smaller interval forecasts for the QAR ROLL compared to QAR REC. In addition, the 25% quantile forecast of the recursive FA-QAR and LASSO-QAR oscillates around zero, as opposed to the case of the rolling window for which it is positive for $h \geq 6$. Augmenting the QAR model with the macroeconomic variables contributes to shift the outer quantiles in the recursive case, while in the rolling case they mostly affect the lower quantile, in particular when considering FA-QAR. In January 2008 indications of a slowing economy were emerging and this is reflected particularly well by the recursive LASSO-QAR. The median of its distribution is very close to zero, and remarkably different from the QAR REC median forecast which still predicted positive growth. In this sense, the macroeconomic variables predicted an increase of the probability of negative INDPRO growth to over 50%, in addition to significantly reducing also the top quantiles to lower values compared to the time series models. The recursive FA-QAR provides similar results, although it seems to predict a moderate recovery at the end of 2008 and beginning of 2009. On the other hand, the medians of the rolling FA- and LASSO-QAR models are quite close to the QAR ROLL median, but the lower part of the FA-QAR forecast distributions seems to shift downward with the 5% quantile becoming more negative relative to the QAR quantile. As it is clear from the realization of the (annualized) INDPRO growth, the contraction in output was significantly more severe than the models predicted, in particular in the second half of 2008. By January 2009 all major economic indicators pointed to a deterioration of the macroeconomic outlook, driven by declines in consumer demand and by a weak housing market. The distribution forecasts are all heavily shifted in negative territory at short horizons, but differ on the speed at which they predict a recovery. The median forecasts of the recursive LASSO-QAR model crosses zero in June 2009 and continues increasing afterwards in a marked difference with the median of the QAR model that becomes positive in January 2010. In addition, the 5 and 25% quantiles for the recursive LASSO-QAR predicted a recovery (in the sense of a small probability of negative INDPRO growth) in the first quarter of 2010, again a quite different forecast compared to QAR. The recursive FA-QAR displays a similar pattern, although it tracks more closely the QAR distribution and forecasts a slower recovery. Instead, both rolling models display significant downside risk in the short-run and a very slow recovery. In particular, the FA-QAR model predicts a close to 50% probability of negative output well into 2010.

The forecasts for PAYEMS in Figure (10) produced in January 2007 pointed to moderate growth in employment, with only the recursive forecasts suggesting a small probability of negative growth. Also for this variable we find that the 90% interval for the rolling estimation is smaller than the recursive at all horizons. In January 2008, the LASSO-QAR suggested a deterioration of the labor market conditions, with a shift downward of the distribution, in particular in the recursive case for which the median approaches zero starting in May 2008. The shift was particularly large (compared to the time series QAR) at the top and center of the distribution, but less pronounced at the bottom quantiles. Instead, the FA-QAR models indicated an increased risk of negative employment growth, in particular in the rolling case, although the center of the distribution was still forecasting moderate growth in PAYEMS. The decline in employment that emerged in the following months was much deeper than predicted by the models, in particular at the longest horizons. The forecasts generated in January 2009 predicted a slow recovery of PAYEMS from the recession, in particular when estimating the models on a rolling window.

Figure (11) and (12) show the forecasts for the CPIAUCSL and PCEPILFE inflation rate. When forecasting headline CPI inflation, the difference between rolling and recursive schemes are less pronounced, compared to the previous discussion, in terms of width of the 90% interval. In both cases and for both LASSO- and FA-QAR, the quantile forecasts in January 2007 predicted that inflation would be stable around 2%, with only the factors indicating some higher upside risk. The forecasts in January 2008 have similar characteristics to the 2007 forecasts, with the major difference that the distributions for both LASSO and FA-QAR estimated recursively seem to edge down toward the end of 2008 and into 2009. However, the forecasts produced in January 2009 when the contraction was underway show a quite different outlook compared to the previous dates. The recursive LASSO and FA-QAR forecast a remarkably different path for CPI inflation compared to the recursive QAR model. While the QAR quantiles predicted a high likelihood of negative inflation up to August 2009, followed by positive inflation in the first quarter of 2010, the other models' quantile forecasts departed significantly from the QAR by forecasting negative inflation in the out-of-sample period even at the top quantile level. The rolling window estimation provides a less extreme perspective on future CPI inflation, although the factors seem to have the effect of shifting the distribution (compared to QAR) downward. The interpretation of these results is that the macroeconomic conditions in January 2009 were so severe to shift the quantile forecasts (compared to QAR forecasts) in negative territory, except in the case of the rolling LASSO-QAR whose forecasts largely overlap with the time series forecasts.

When considering core PCE inflation, the main inflation indicator followed by the Federal Reserve, the recursive distribution forecasts are typically wider compared to the rolling window ones which are characterized by very narrow intervals, in particular at the longest horizons. Overall, the outlook that emerges from the forecasts produced in January 2007 and 2008 is that of a stable core PCE inflation with the median forecasts at all horizons around 2%, although the factors predicted more upside risk compared to the time series models at the beginning of 2007. For the January 2009 forecasts, only the recursive FA-QAR assigns a high likelihood of a decrease in the PCEPILFE price index, while the other models anticipated a moderation in inflation to lower, but still positive, levels. Also in this case, the severity of the downturn is evident in shifting the time series quantiles toward lower levels.

6 Conclusion

In this paper we provide evidence that macroeconomic variables are indeed useful to forecast business cycle and inflation indicators. The results suggest that their predictive power occurs primarily in the tails of the distribution and at the 3-6 month horizon for the output and employment growth rates, but at the 6-12 month horizon for the inflation measures. These conclusions were obtained considering a large number of macroeconomic variables and comparing different approaches to isolate the relevant information in the panel. We find that augmenting a time series model using a small number of predictors selected by the LASSO method provides comparable and often more accurate forecasts compared to the alternative approach of constructing principal components. A possible reason for this result is that the LASSO selection of the predictors is specifically targeted to forecast the indicator of interest, as opposed to the factors that have no relation to the variable being forecast. We also find that, among the variables most often selected, there are several that are rarely considered in the forecasting literature such as the Producer Price Indices and the National Association of Purchasing Management (NAPM) indices. In both cases, the forecasting power of these indices relates to their inherently forward looking nature as indicators of the business cycle. We also show the practical relevance of forecasting the distribution of economic variables by producing multi-step ahead (real time) forecasts for the indicators of economic activity and inflation measures at three crucial moments before and during the last recession. The analysis suggests that the forecasts are, to a large extent, consistent with the qualitative view of the state of the economy expressed by the FOMC at the time the forecast was made. In addition, the comparison of the time series and the multivariate forecasts illustrates effectively the role of the indicators in shifting the distribution forecasts, at times asymmetrically, to reflect the changing macroeconomic outlook.

Data Appendix: Forecasting the distribution of inflation in a data-rich environment

We created a list of 141 macroeconomic variables in the FRED economic database maintained by the Federal Reserve Bank of Saint Louis. Of the 141 series, 118 start in January 1960 or earlier, 126 in January 1970 or earlier, and 141 in January 1980 or earlier. Since we are considering both recursive and rolling forecasting schemes, it should be taken into account that the dataset is expanding for the rolling scheme, but it is kept constant for the recursive scheme to include the 118 variables available since the beginning of the sample. The Table below shows in column T the transformations that have been applied to the series which are: 0 (no transformation), 1 (log-transformation), 2 (first difference), 3 (annualized first log-difference). `AC` denotes the variables that have been created by the author.

Series ID:	Title:	T	Start date:
AHECONS	Average Hourly Earnings Of Production And Nonsupervisory Employees: Construction	3	1959M1
AHEMAN	Average Hourly Earnings Of Production And Nonsupervisory Employees: Manufacturing	3	1959M1
AHETPI	Average Hourly Earnings of Production and Nonsupervisory Employees: Total Private	3	1964M1
ALTSALES	Light Weight Vehicle Sales: Autos & Light Trucks	3	1976M1
AWHI	Index of Aggregate Weekly Hours: Production and Nonsupervisory Employees: Total Private Industries	2	1964M1
AWHMAN	Average Weekly Hours of Production and Nonsupervisory Employees: Manufacturing	2	1959M1
AWHNONAG	Average Weekly Hours Of Production And Nonsupervisory Employees: Total private	2	1964M1
AWOTMAN	Average Weekly Overtime Hours of Production and Nonsupervisory Employees: Manufacturing	2	1959M1
BOGAMBSL	Board of Governors Monetary Base, Adjusted for Changes in Reserve Requirements	3	1959M1
BUSLOANS	Commercial and Industrial Loans at All Commercial Banks	3	1959M1
CONSUMER	Consumer (Individual) Loans at All Commercial Banks	3	1959M1
CPIAPPSL	Consumer Price Index for All Urban Consumers: Apparel	3	1959M1
CPIAUCSL	Consumer Price Index for All Urban Consumers: All Items	3	1959M1
CPIENGSL	Consumer Price Index for All Urban Consumers: Energy	3	1959M1
CPILFESL	Consumer Price Index for All Urban Consumers: All Items Less Food & Energy	3	1959M1
CPIMEDSL	Consumer Price Index for All Urban Consumers: Medical Care	3	1959M1
CPITRNSL	Consumer Price Index for All Urban Consumers: Transportation	3	1959M1
CPIUFDSL	Consumer Price Index for All Urban Consumers: Food	3	1959M1
CREDIT	Moody's Seasoned Aaa minus Baa Corporate Bond Yield (`AC`)	0	1959M1
CURRSL	Currency Component of M1	3	1959M1
DEMDEPSL	Demand Deposits at Commercial Banks	3	1959M1
DMANEMP	All Employees: Durable goods	3	1959M1
DSPIC96	Real Disposable Personal Income	3	1959M1
FEDFUNDS	Effective Federal Funds Rate	2	1959M1
HOUST	Housing Starts: Total: New Privately Owned Housing Units Started	1	1959M1
HOUST1F	Privately Owned Housing Starts: 1-Unit Structures	1	1959M1
HOUST2F	Housing Starts: 2-4 Units	1	1963M8
HOUST5F	Privately Owned Housing Starts: 5-Unit Structures or More	1	1959M1
HOUSTMW	Housing Starts in Midwest Census Region	1	1959M1
HOUSTNE	Housing Starts in Northeast Census Region	1	1959M1
HOUSTS	Housing Starts in South Census Region	1	1959M1
HOUSTW	Housing Starts in West Census Region	1	1959M1
HSN1F	New One Family Houses Sold: United States	1	1963M1
HSN1FMW	New One Family Houses Sold in Midwest Census Region	1	1973M1
HSN1FNE	New One Family Houses Sold in Northeast Census Region	1	1973M1
HSN1FS	New One Family Houses Sold in South Census Region	1	1973M1
HSN1FW	New One Family Houses Sold in West Census Region	1	1973M1
INDPRO	Industrial Production Index	3	1959M1
INVEST	Total Investments at All Commercial Banks	3	1959M1
IPBUSEQ	Industrial Production: Business Equipment	3	1959M1
IPCONGD	Industrial Production: Consumer Goods	3	1959M1
IPDCONGD	Industrial Production: Durable Consumer Goods	3	1959M1
IPDMAT	Industrial Production: Durable Materials	3	1959M1
IPFINAL	Industrial Production: Final Products	3	1959M1
IPMAT	Industrial Production: Materials	3	1959M1
IPMINE	Industrial Production: Mining	3	1972M1
IPNCONGD	Industrial Production: Nondurable Consumer Goods	3	1959M1
IPNMAT	Industrial Production: nondurable Materials	3	1959M1
IPUTIL	Industrial Production: Electric and Gas Utilities	3	1972M1
LOANINV	Total Loans and Investments at All Commercial Banks	3	1959M1
LOANS	Total Loans and Leases at Commercial Banks	3	1959M1
M1SL	M1 Money Stock	3	1959M1
M2SL	M2 Money Stock	3	1959M1
MANEMP	All Employees: Manufacturing	3	1959M1
MICH	University of Michigan Inflation Expectation	2	1978M1
MORTG	30-Year Conventional Mortgage Rate	2	1971M4

NAPM	ISM Manufacturing: PMI Composite Index	0	1959M1
NAPMEI	ISM Manufacturing: Employment Index	0	1959M1
NAPMII	ISM Manufacturing: Inventories Index	0	1959M1
NAPMNOI	ISM Manufacturing: New Orders Index	0	1959M1
NAPMPI	ISM Manufacturing: Production Index	0	1959M1
NAPMPRI	ISM Manufacturing: Prices Index	0	1959M1
NAPMSDI	ISM Manufacturing: Supplier Deliveries Index	0	1959M1
NDMANEMP	All Employees: Nondurable goods	3	1959M1
NONREVSL	Total Nonrevolving Credit Outstanding	3	1959M1
OILPRICE	Spot Oil Price: West Texas Intermediate	3	1959M1
OTHSEC	Other Securities at All Commercial Banks	3	1959M1
PAYEMS	All Employees: Total nonfarm	3	1959M1
PCE	Personal Consumption Expenditures	3	1959M1
PCEDG	Personal Consumption Expenditures: Durable Goods	3	1959M1
PCEND	Personal Consumption Expenditures: Nondurable Goods	3	1959M1
PCEPI	Personal Consumption Expenditures: Chain-type Price Index	3	1959M1
PCEPILFE	Personal Consumption Expenditures: Chain-Type Price Index Less Food and Energy	3	1959M1
PCES	Personal Consumption Expenditures: Services	3	1959M1
PERMIT	New Private Housing Units Authorized by Building Permits	1	1960M1
PERMIT1	New Private Housing Units Authorized by Building Permits - In Structures with 1 unit	1	1960M1
PERMIT24	New Private Housing Units Authorized by Building Permits - In structures with 2 to 4 units	1	1960M1
PERMIT5	New Private Housing Units Authorized by Building Permits - In structures with 5 units	1	1960M1
PERMITMW	New Private Housing Units Authorized by Building Permits in the Midwest census region	1	1960M1
PERMITNE	New Private Housing Units Authorized by Building Permits in the Northeast Census Region	1	1960M1
PERMITS	New Private Housing Units Authorized by Building Permits in the South Census Region	1	1960M1
PERMITW	New Private Housing Units Authorized by Building Permits in the West Census Region	1	1960M1
PFCGEF	Producer Price Index: Finished Consumer Goods Excluding Foods	3	1959M1
PPIACO	Producer Price Index: All Commodities	3	1959M1
PPICPE	Producer Price Index: Finished Goods: Capital Equipment	3	1959M1
PPICRM	Producer Price Index: Crude Materials for Further Processing	3	1959M1
PPIFCF	Producer Price Index: Finished Consumer Foods	3	1959M1
PPIFCG	Producer Price Index: Finished Consumer Goods	3	1959M1
PPIFGS	Producer Price Index: Finished Goods	3	1959M1
PPIITM	Producer Price Index: Intermediate Materials: Supplies & Components	3	1959M1
PSAVERT	Personal Saving Rate	2	1959M1
REALLN	Real Estate Loans at All Commercial Banks	3	1959M1
REVOLSL	Total Revolving Credit Outstanding	3	1968M2
RMFSL	Retail Money Funds	3	1973M11
SAVINGSL	Savings Deposits - Total	3	1959M1
SPREAD10Y	10-Year Treasury Constant Maturity Rate minus FEDFUNDS (AC)	0	1959M1
SPREAD1Y	1-Year Treasury Constant Maturity Rate minus FEDFUNDS (AC)	0	1959M1
SPREAD2Y	2-Year Treasury Constant Maturity Rate minus FEDFUNDS (AC)	0	1976M6
SPREAD3M	3-Month Treasury Bill minus FEDFUNDS (AC)	0	1959M1
SPREAD3Y	3-Year Treasury Constant Maturity Rate minus FEDFUNDS (AC)	0	1959M1
SPREAD5Y	5-Year Treasury Constant Maturity Rate minus FEDFUNDS (AC)	0	1959M1
SPREAD6M	6-Month Treasury Bill minus FEDFUNDS (AC)	0	1959M1
SPREAD7Y	7-Year Treasury Constant Maturity Rate minus FEDFUNDS (AC)	0	1969M7
SPREADAAA	Moody's Seasoned Aaa Corporate Bond Yield minus FEDFUNDS (AC)	0	1959M1
SPREADBAA	Moody's Seasoned Baa Corporate Bond Yield minus FEDFUNDS (AC)	0	1959M1
SPREADPRIME	Bank Prime Loan Rate minus FEDFUNDS (AC)	0	1959M1
SRVPRD	All Employees: Service-Providing Industries	3	1959M1
SVGCBSL	Savings Deposits at Commercial Banks	3	1959M1
SVGTI	Savings Deposits at Thrift Institutions	3	1959M1
SVSTCBSL	Savings and Small Time Deposits at Commercial Banks	3	1959M1
SVSTSL	Savings and Small Time Deposits - Total	3	1959M1
TCU	Capacity Utilization: Total Industry	2	1967M1
TOTALSL	Total Consumer Credit Outstanding	3	1959M1
TWEXBMTH	Trade Weighted Exchange Index: Broad	3	1973M1
TWEXMMTH	Trade Weighted Exchange Index: Major Currencies	3	1973M1
TWEXOMTH	Trade Weighted Exchange Index: Other Important Trading Partners	3	1973M1
UEMP15OV	Civilians Unemployed - 15 Weeks and Over	3	1959M1
UEMP15T26	Civilians Unemployed for 15-26 Weeks	3	1959M1
UEMP27OV	Civilians Unemployed for 27 Weeks and Over	3	1959M1
UEMP5TO14	Civilians Unemployed for 5-14 Weeks	3	1959M1
UEMPLT5	Civilians Unemployed - Less Than 5 Weeks	3	1959M1
UEMPMEAN	Average (Mean) Duration of Unemployment	2	1959M1
UMCSENT	University of Michigan: Consumer Sentiment	2	1978M1
UNEMPLOY	Unemployed	3	1959M1
UNRATE	Civilian Unemployment Rate	2	1959M1
USCONS	All Employees: Construction	3	1959M1
USEHS	All Employees: Education & Health Services	3	1959M1
USFIRE	All Employees: Financial Activities	3	1959M1
USGOOD	All Employees: Goods-Producing Industries	3	1959M1
USGOVT	All Employees: Government	3	1959M1
USGSEC	U.S. Government Securities at All Commercial Banks	3	1959M1
USINFO	All Employees: Information Services	3	1959M1
USLAH	All Employees: Leisure & Hospitality	3	1959M1
USMINE	All Employees: Mining and logging	3	1959M1
USPBS	All Employees: Professional & Business Services	3	1959M1
USPRIV	All Employees: Total Private Industries	3	1959M1
USSERV	All Employees: Other Services	3	1959M1
USTPU	All Employees: Trade, Transportation & Utilities	3	1959M1
USTRADE	All Employees: Retail Trade	3	1959M1
USWTRADE	All Employees: Wholesale Trade	3	1959M1
W875RX1	Personal income excluding current transfer receipts (Billions of Chained 2005 Dollars)	3	1959M1

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Table 1: **INDPRO - QS TEST**

Model	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
						h=3					
QAR REC	0.00	-0.23	-0.83	-0.65	-0.38	-0.95	-0.83	-1.05	-0.51	-0.44	-0.45
FA-QAR REC K=3	**-2.45**	**-2.02**	**-2.08**	-1.53	-0.88	-0.26	-0.27	-0.47	-0.64	-1.62	**-2.06**
FA-QAR REC K=5	**-2.16**	**-2.05**	**-2.04**	-1.49	-1.08	-0.95	-0.81	-1.25	-1.42	**-2.73**	**-2.40**
FA-QAR ROLL K=3	0.20	-0.60	-1.26	-0.89	-0.41	-0.24	-0.20	-0.74	-0.66	-0.87	-0.44
FA-QAR ROLL K=5	-0.14	-0.81	-1.62	-1.57	-1.21	-0.53	-0.21	-0.66	-0.81	-0.24	0.76
						$c = 1$					
LASSO-QAR REC	**-3.93**	**-3.54**	**-3.56**	**-2.51**	**-2.08**	**-1.84**	**-1.76**	**-2.43**	**-2.71**	**-2.33**	-1.54
POST-LASSO-QAR REC	**-2.44**	**-2.15**	-1.55	-1.18	-1.18	-1.06	-1.30	**-1.96**	**-2.49**	**-2.48**	**-1.75**
TFA-QAR REC	**-3.53**	**-2.98**	**-2.70**	**-1.77**	-1.47	-1.41	-1.39	**-1.74**	**-1.74**	**-1.96**	-1.47
LASSO-QAR ROLL	-1.33	**-3.24**	**-3.47**	**-3.19**	**-2.46**	-1.61	-1.34	-1.44	-1.33	-1.33	0.06
POST-LASSO-QAR ROLL	-0.97	**-2.08**	**-2.00**	-1.45	-1.28	-1.27	-0.72	-0.88	-0.55	-1.10	-0.44
TFA-QAR ROLL	-1.50	**-2.26**	**-2.19**	-1.57	-0.99	-0.85	-0.77	-0.90	-0.90	**-1.79**	-1.41
						$c = 2$					
LASSO-QAR REC	**-3.57**	**-2.89**	**-2.37**	**-2.14**	-1.17	**-1.68**	**-1.81**	**-2.23**	**-1.83**	-1.59	-0.89
POST-LASSO-QAR REC	**-3.02**	**-2.46**	-1.44	-0.93	-0.76	-0.67	-0.81	**-1.69**	**-1.94**	**-1.75**	-0.98
TFA-QAR REC	**-3.42**	**-2.77**	**-1.82**	-1.36	-0.97	-0.91	-1.18	**-1.98**	**-2.14**	**-2.02**	-0.97
LASSO-QAR ROLL	**-3.64**	**-4.17**	**-3.67**	**-3.41**	**-2.69**	**-1.99**	-1.61	**-1.96**	-1.48	-1.34	0.38
POST-LASSO-QAR ROLL	**-2.52**	**-3.02**	**-2.07**	-1.11	-0.68	-1.05	-0.90	-0.94	-0.61	-0.99	-0.38
TFA-QAR ROLL	**-2.45**	**-3.21**	**-2.23**	-1.16	-0.67	-0.97	-0.92	-1.03	-0.90	-1.06	0.05
						$c = 3$					
LASSO-QAR REC	-1.06	-0.98	-1.56	-1.18	-0.36	-1.16	-1.12	-1.14	-0.74	-0.68	-0.46
POST-LASSO-QAR REC	-1.57	-0.88	**-1.85**	-1.23	-0.51	-0.64	-0.63	-0.34	-0.47	-0.82	-0.36
TFA-QAR REC	**-1.66**	-0.99	**-1.86**	-1.25	-0.49	-0.66	-0.63	-0.34	-0.53	-0.81	-0.36
LASSO-QAR ROLL	1.49	-1.32	**-2.17**	-1.11	0.81	-0.22	-1.20	-0.52	1.29	-0.91	0.84
POST-LASSO-QAR ROLL	-0.92	-1.31	**-2.54**	**-2.13**	-0.74	-0.65	-0.01	-1.04	-0.62	-0.94	0.00
TFA-QAR ROLL	-0.92	-1.31	**-2.54**	**-2.13**	-0.76	-0.65	-0.03	-1.06	-0.62	-0.94	0.00
						h=6					
QAR REC	-0.75	-0.92	-1.13	-1.42	**-1.64**	-0.82	-1.22	-1.14	-0.89	-0.21	-0.38
FA-QAR REC K=3	**-2.35**	**-2.47**	**-1.85**	-1.55	-1.37	-1.12	-1.25	-1.23	-1.11	-0.98	-1.16
FA-QAR REC K=5	**-2.04**	**-1.91**	**-1.72**	-1.41	-1.23	-1.15	-1.49	**-1.64**	**-1.80**	-1.45	-1.15
FA-QAR ROLL K=3	0.23	0.06	-0.08	-0.10	0.28	0.30	-0.29	-0.37	-0.34	0.56	0.85
FA-QAR ROLL K=5	-0.15	0.09	-0.07	-0.23	-0.36	-0.51	-1.10	-1.22	**-1.71**	-0.57	0.24
						$c = 1$					
LASSO-QAR REC	**-3.27**	**-3.30**	**-2.36**	**-1.78**	-1.47	-1.21	**-1.75**	**-1.95**	**-1.95**	-1.55	**-1.96**
POST-LASSO-QAR REC	**-2.88**	**-2.41**	-1.58	-0.91	-0.71	-0.62	-1.04	-1.25	-1.52	-1.07	-1.31
TFA-QAR REC	**-2.27**	**-2.32**	-1.61	-1.07	-0.98	-0.92	-1.42	-1.58	**-1.71**	-1.52	**-1.76**
LASSO-QAR ROLL	-0.88	-0.49	-0.62	-0.86	-0.85	-0.75	-1.47	**-1.86**	**-2.34**	-0.95	0.18
POST-LASSO-QAR ROLL	-0.49	-0.28	-0.26	-0.35	-0.34	-0.23	-0.57	-0.65	-0.48	0.37	0.53
TFA-QAR ROLL	-0.62	-0.68	-0.76	-0.84	-0.79	-0.55	-0.90	-1.16	**-1.75**	-1.19	-0.71
						$c = 2$					
LASSO-QAR REC	**-3.19**	**-3.04**	**-3.05**	**-2.97**	**-2.90**	**-1.99**	**-2.21**	**-2.07**	**-2.02**	**-1.65**	-1.07
POST-LASSO-QAR REC	**-2.68**	**-1.83**	-1.07	-0.92	-0.49	-0.35	-0.90	-1.09	**-1.66**	**-1.80**	-1.43
TFA-QAR REC	**-1.79**	-1.34	-1.04	-0.83	-0.66	-0.71	-1.22	-1.55	**-1.91**	**-1.79**	-1.30
LASSO-QAR ROLL	**-1.82**	**-2.33**	**-1.83**	**-1.78**	**-1.69**	-0.95	-1.62	**-1.76**	**-1.98**	**-1.64**	-0.20
POST-LASSO-QAR ROLL	**-2.01**	-0.95	-0.30	-0.23	-0.24	-0.04	-0.65	-1.14	**-1.64**	-1.07	**-1.78**
TFA-QAR ROLL	**-2.13**	-1.22	-0.59	-0.33	-0.47	-0.11	-0.63	-1.04	-1.52	-1.36	-1.18
						$c = 3$					
LASSO-QAR REC	**-2.71**	**-2.87**	**-3.56**	**-3.94**	**-3.44**	**-2.30**	**-2.34**	**-1.72**	-1.31	-0.79	-0.56
POST-LASSO-QAR REC	**-2.09**	-0.81	-0.58	-0.95	-1.08	-1.04	-1.54	-1.36	-1.02	-0.64	-0.82
TFA-QAR REC	**-2.43**	-1.16	-0.78	-1.00	-1.10	-1.01	-1.50	-1.32	-1.10	-0.65	-0.85
LASSO-QAR ROLL	**-2.81**	**-3.24**	**-3.58**	**-2.93**	**-1.78**	-1.31	**-1.70**	**-1.85**	-1.23	-0.85	-0.50
POST-LASSO-QAR ROLL	**-2.45**	**-2.33**	**-1.72**	-1.32	-1.16	-1.12	-1.50	-1.57	**-1.81**	-1.19	-1.24
TFA-QAR ROLL	**-2.45**	**-2.39**	**-1.83**	-1.36	-1.20	-1.12	-1.55	-1.60	**-1.87**	-1.13	-1.25
						h=12					
QAR REC	**-1.87**	**-2.55**	**-1.99**	**-2.02**	-1.58	-0.75	-0.46	-0.25	0.04	-0.18	-0.79
FA-QAR REC K=3	**-1.71**	**-1.92**	-1.46	-1.18	-0.93	-0.80	-0.82	-0.73	-0.66	-0.74	-1.12
FA-QAR REC K=5	**-2.12**	**-2.33**	**-1.69**	-1.23	-1.04	-0.90	-0.92	-0.77	-0.32	0.10	0.01
FA-QAR ROLL K=3	-0.57	-0.86	-0.25	0.10	0.06	0.13	0.13	0.17	0.42	0.71	0.76
FA-QAR ROLL K=5	-0.56	-0.96	-0.70	-0.44	-0.36	-0.26	-0.39	-0.51	-0.44	0.32	0.14
						$c = 1$					
LASSO-QAR REC	-1.42	-1.56	-1.14	-0.85	-0.79	-0.70	-0.83	-0.94	-1.09	-1.42	-1.61
POST-LASSO-QAR REC	-1.12	-1.28	-0.76	-0.47	-0.43	-0.40	-0.45	-0.59	-0.87	-1.19	-1.55
TFA-QAR REC	-1.61	**-1.81**	-1.33	-0.99	-0.81	-0.56	-0.52	-0.61	-0.67	-1.03	-1.63
LASSO-QAR ROLL	-0.15	-0.66	-0.60	-0.67	-0.79	-0.83	-1.10	-1.27	-1.01	-0.56	-0.53
POST-LASSO-QAR ROLL	-0.07	-0.37	-0.21	-0.16	-0.29	-0.35	-0.56	-0.56	-0.48	-0.52	-1.14
TFA-QAR ROLL	-0.46	-0.62	-0.33	-0.30	-0.44	-0.54	-0.82	-0.92	-0.80	-0.66	-1.15
						$c = 2$					
LASSO-QAR REC	**-2.63**	**-2.55**	**-1.77**	-1.38	-1.18	-1.04	-1.05	-1.11	-1.05	-0.88	-1.05
POST-LASSO-QAR REC	-1.56	-1.49	-0.83	-0.57	-0.53	-0.45	-0.49	-0.70	-0.80	-1.11	-1.18
TFA-QAR REC	**-2.04**	**-1.65**	-1.11	-0.92	-0.78	-0.65	-0.70	-0.87	-0.89	-1.16	-1.21
LASSO-QAR ROLL	-1.10	-1.56	-1.55	-1.22	-1.31	-1.26	-1.46	**-1.64**	-1.52	-1.40	-1.00
POST-LASSO-QAR ROLL	-1.22	-1.05	-0.78	-0.60	-0.66	-0.71	-1.02	-1.34	-1.52	**-1.64**	-1.36
TFA-QAR ROLL	-1.32	-1.04	-0.80	-0.65	-0.66	-0.65	-0.91	-1.20	-1.36	**-1.79**	-1.48
						$c = 3$					
LASSO-QAR REC	**-2.67**	**-2.98**	**-2.42**	**-1.90**	-1.62	-1.36	-1.09	-0.75	-0.36	-0.44	-0.80
POST-LASSO-QAR REC	**-2.22**	**-1.97**	-1.32	-0.78	-0.64	-0.56	-0.58	-0.62	-0.58	-0.70	-0.81
TFA-QAR REC	**-2.29**	**-2.08**	-1.55	-1.05	-0.76	-0.66	-0.71	-0.74	-0.62	-0.73	-0.81
LASSO-QAR ROLL	-1.50	**-2.32**	**-1.99**	-1.25	-1.13	-1.31	-1.62	**-2.01**	**-1.97**	-1.32	-1.16
POST-LASSO-QAR ROLL	**-1.81**	-1.63	-1.44	-1.19	-0.84	-0.86	-1.13	-1.50	-1.50	-1.00	-1.25
TFA-QAR ROLL	**-1.79**	-1.62	-1.40	-1.17	-0.93	-0.90	-1.19	-1.57	-1.60	-1.04	-1.26

The values represent the t statistic for the null hypothesis of equal accuracy of the quantile forecasts from the models indicated relative to the QAR ROLL benchmark. The values of τ that we consider for the QS test are indicated at the top row of the Table, while h denotes the forecasting horizon, and κ the SIC penalty used in the LASSO selection. A negative value of the test statistic indicates that the alternative model outperforms the benchmark (QAR ROLL) and in bold are denoted the rejections of equal accuracy at 5% against the one-sided al ernative that the benchmark is outperformed.

Table 2: **PAYEMS - QS TEST**

Model	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
						h=3					
QAR REC	-0.92	-1.16	-1.01	-0.71	-0.45	-0.59	-0.22	-0.05	1.10	1.40	0.83
FA-QAR REC K=3	**-2.18**	**-2.27**	**-2.98**	**-3.03**	**-2.32**	-1.30	-0.65	-0.05	0.86	1.33	1.31
FA-QAR REC K=5	**-2.35**	**-2.42**	**-3.14**	**-3.42**	**-2.79**	**-2.03**	-1.42	-0.97	-0.86	-0.66	-0.01
FA-QAR ROLL K=3	**-2.82**	**-2.85**	**-2.96**	**-2.58**	-1.45	-0.98	-0.90	-0.68	-0.26	0.01	0.63
FA-QAR ROLL K=5	**-3.01**	**-2.96**	**-3.26**	**-2.71**	**-1.81**	-1.43	-0.94	-0.66	-0.28	0.11	0.50
						$c = 1$					
LASSO-QAR REC	**-5.42**	**-4.74**	**-4.49**	**-4.58**	**-3.86**	**-2.98**	**-2.17**	**-1.81**	-0.90	-0.73	-0.41
POST-LASSO-QAR REC	**-3.60**	**-3.60**	**-3.74**	**-3.71**	**-2.90**	**-2.18**	-1.56	-1.35	-0.80	-0.71	-0.59
TFA-QAR REC	**-4.13**	**-4.13**	**-4.05**	**-3.98**	**-3.25**	**-2.47**	**-1.81**	-1.41	-1.02	-0.92	-0.59
LASSO-QAR ROLL	**-2.87**	**-3.97**	**-3.85**	**-3.31**	**-2.88**	**-2.48**	**-2.34**	**-2.30**	**-2.02**	**-1.92**	-0.40
POST-LASSO-QAR ROLL	**-2.58**	**-2.79**	**-2.23**	**-1.85**	**-1.78**	**-1.76**	**-1.88**	-1.60	-0.86	-0.84	-0.25
TFA-QAR ROLL	**-3.30**	**-3.72**	**-3.58**	**-3.13**	**-2.83**	**-2.65**	**-2.65**	**-2.29**	-1.58	-1.22	-0.59
						$c = 2$					
LASSO-QAR REC	**-3.73**	**-4.42**	**-3.05**	**-2.62**	**-2.24**	**-1.87**	-1.26	-0.64	0.11	0.30	0.32
POST-LASSO-QAR REC	**-3.42**	**-2.72**	**-2.88**	**-3.07**	**-2.54**	-1.58	-1.07	-0.89	-0.24	-0.30	-0.16
TFA-QAR REC	**-3.26**	**-2.77**	**-2.95**	**-3.29**	**-2.74**	**-1.68**	-1.12	-0.95	-0.07	-0.24	-0.18
LASSO-QAR ROLL	**-2.32**	**-3.24**	**-4.48**	**-3.57**	**-3.02**	**-1.78**	**-1.91**	**-2.46**	**-2.60**	**-2.66**	-0.92
POST-LASSO-QAR ROLL	**-3.23**	**-3.32**	**-2.66**	**-2.18**	**-1.71**	-1.15	-1.48	**-1.90**	**-1.70**	**-1.65**	-0.36
TFA-QAR ROLL	**-3.11**	**-3.11**	**-2.81**	**-2.37**	**-1.76**	-1.21	-1.63	**-2.30**	**-2.05**	**-2.00**	-0.52
						$c = 3$					
LASSO-QAR REC	-1.48	**-1.79**	**-1.82**	-1.42	-0.85	-0.80	-0.30	-0.17	0.73	0.95	0.67
POST-LASSO-QAR REC	**-2.28**	**-1.96**	-1.30	-1.29	-1.20	-0.75	0.06	0.50	0.71	0.65	0.49
TFA-QAR REC	**-2.24**	**-1.86**	-1.32	-1.32	-1.22	-0.80	0.02	0.47	0.70	0.65	0.49
LASSO-QAR ROLL	0.41	-1.36	**-3.60**	**-3.14**	**-3.33**	-1.33	-1.16	-0.92	-1.52	**-3.29**	-0.71
POST-LASSO-QAR ROLL	-0.06	-1.38	**-3.45**	**-3.03**	**-2.66**	**-2.10**	**-1.82**	**-1.88**	**-1.98**	**-2.11**	0.22
TFA-QAR ROLL	-0.06	-1.38	**-3.46**	**-3.02**	**-2.68**	**-2.08**	**-1.81**	**-1.84**	**-1.98**	**-2.11**	0.22
						h=6					
QAR REC	-1.48	-1.18	-1.19	**-1.81**	**-2.16**	**-1.67**	-0.78	0.28	1.12	1.57	0.98
FA-QAR REC K=3	**-2.42**	**-2.82**	**-3.27**	**-3.54**	**-2.72**	**-1.74**	-0.97	-0.25	0.49	1.19	1.34
FA-QAR REC K=5	**-2.07**	**-2.56**	**-3.25**	**-3.86**	**-3.17**	**-2.29**	**-1.67**	-0.98	-0.34	0.22	0.29
FA-QAR ROLL K=3	**-2.03**	**-2.58**	**-2.88**	**-2.83**	**-1.79**	-1.03	-0.69	-0.41	-0.63	-0.47	0.29
FA-QAR ROLL K=5	**-1.76**	**-2.36**	**-2.76**	**-2.73**	**-1.80**	-0.89	-0.43	-0.12	0.15	0.41	0.68
						$c = 1$					
LASSO-QAR REC	**-2.06**	**-2.74**	**-3.23**	**-3.75**	**-3.01**	**-1.91**	-1.25	-1.03	-0.72	-0.59	-0.18
POST-LASSO-QAR REC	**-2.08**	**-2.73**	**-3.13**	**-3.39**	**-2.74**	**-1.76**	-1.09	-0.65	-0.72	-0.59	-0.16
TFA-QAR REC	**-2.06**	**-2.75**	**-2.99**	**-3.24**	**-2.67**	**-1.67**	-1.11	-0.73	-0.44	-0.12	0.03
LASSO-QAR ROLL	**-2.79**	**-2.97**	**-2.66**	**-2.94**	**-2.40**	**-1.75**	-1.59	-1.56	-1.36	-0.90	0.06
POST-LASSO-QAR ROLL	**-2.38**	**-2.57**	**-2.22**	**-2.48**	**-1.91**	-1.11	-0.68	-0.59	-0.53	-0.19	0.34
TFA-QAR ROLL	**-2.62**	**-3.12**	**-3.28**	**-3.79**	**-2.70**	**-1.73**	-1.21	-0.78	-0.61	-0.68	0.03
						$c = 2$					
LASSO-QAR REC	**-2.06**	**-2.45**	**-2.77**	**-3.56**	**-3.03**	**-1.64**	-0.94	-0.44	-0.01	0.19	0.33
POST-LASSO-QAR REC	**-2.00**	**-2.27**	**-2.46**	**-2.84**	**-2.25**	-1.15	-0.67	-0.44	-0.07	-0.04	0.03
TFA-QAR REC	**-1.90**	**-2.14**	**-2.28**	**-2.73**	**-2.20**	-1.19	-0.81	-0.59	-0.36	-0.05	0.15
LASSO-QAR ROLL	-1.42	**-3.08**	**-3.51**	**-3.53**	**-2.45**	-1.33	-1.32	**-1.76**	-1.34	-1.63	0.06
POST-LASSO-QAR ROLL	**-2.12**	**-2.71**	**-1.73**	**-1.79**	-1.23	-0.77	-0.90	-1.38	**-1.72**	**-1.90**	**-1.74**
TFA-QAR ROLL	**-2.69**	**-3.21**	**-2.56**	**-2.59**	**-1.73**	-1.36	-1.42	**-1.87**	**-2.10**	**-2.49**	-1.32
						$c = 3$					
LASSO-QAR REC	**-1.97**	**-2.02**	**-2.31**	**-3.12**	**-2.87**	-1.51	-0.83	-0.08	0.56	0.92	0.78
POST-LASSO-QAR REC	**-1.90**	**-2.14**	**-2.27**	**-2.46**	**-1.65**	-0.51	0.09	0.51	0.70	0.43	0.61
TFA-QAR REC	**-1.86**	**-2.09**	**-2.35**	**-2.64**	**-1.87**	-0.70	-0.06	0.41	0.65	0.42	0.61
LASSO-QAR ROLL	-0.27	**-1.92**	**-3.30**	**-3.65**	**-3.02**	-1.33	-0.83	-0.79	-1.04	-2.84	-1.19
POST-LASSO-QAR ROLL	-1.60	**-3.32**	**-3.61**	**-3.35**	**-2.52**	-1.24	-1.08	-1.11	-1.39	**-2.51**	-1.26
TFA-QAR ROLL	-1.61	**-3.33**	**-3.63**	**-3.37**	**-2.57**	-1.29	-1.18	-1.19	-1.38	**-2.41**	-1.32
						h=12					
QAR REC	-1.60	**-1.77**	-1.63	-1.47	-0.24	0.49	0.83	*1.64*	*1.92*	*1.64*	0.42
FA-QAR REC K=3	**-1.87**	**-1.76**	-1.63	-1.34	-0.83	-0.39	-0.07	0.24	0.46	0.52	0.05
FA-QAR REC K=5	-1.62	**-1.82**	**-1.96**	**-1.88**	-1.41	-0.93	-0.57	-0.28	-0.02	0.40	0.51
FA-QAR ROLL K=3	-0.80	-1.18	-1.41	-1.18	-0.64	-0.21	0.07	0.31	0.43	0.74	0.83
FA-QAR ROLL K=5	-0.74	-1.11	-1.53	-1.51	-0.95	-0.37	0.05	0.54	0.83	1.14	1.01
						$c = 1$					
LASSO-QAR REC	-1.23	-1.32	-1.51	-1.39	-0.91	-0.44	-0.11	0.03	0.11	0.40	0.07
POST-LASSO-QAR REC	-1.30	-1.46	-1.52	-1.33	-0.80	-0.32	0.02	0.27	0.32	0.42	-0.18
TFA-QAR REC	-1.34	-1.48	-1.53	-1.40	-1.00	-0.53	-0.13	0.06	0.08	0.30	-0.15
LASSO-QAR ROLL	-0.45	-0.74	-1.40	-1.39	-0.54	-0.12	0.02	0.05	-0.13	0.10	0.54
POST-LASSO-QAR ROLL	-0.49	-0.47	-0.77	-0.68	-0.12	0.25	0.38	0.36	0.25	0.54	0.19
TFA-QAR ROLL	-0.74	-0.85	-1.15	-0.96	-0.29	0.19	0.40	0.51	0.40	0.53	0.59
						$c = 2$					
LASSO-QAR REC	-1.61	-1.34	-1.42	-1.25	-0.64	-0.12	0.12	0.37	0.72	0.81	0.15
POST-LASSO-QAR REC	-1.28	-1.30	-1.36	-1.23	-0.72	-0.28	0.10	0.26	0.17	0.49	-0.03
TFA-QAR REC	-1.21	-1.07	-1.20	-1.20	-0.87	-0.46	-0.06	0.18	0.12	0.49	-0.01
LASSO-QAR ROLL	-0.91	**-1.99**	**-2.52**	**-2.26**	-1.40	-0.68	-0.42	-0.49	-0.95	-0.89	-0.29
POST-LASSO-QAR ROLL	-1.15	-1.30	-1.63	-1.34	-0.64	-0.05	0.17	0.03	-0.41	-0.23	-0.02
TFA-QAR ROLL	-1.11	-1.50	**-1.90**	**-1.74**	-0.90	-0.23	0.05	-0.07	-0.56	-0.37	0.01
						$c = 3$					
LASSO-QAR REC	**-1.70**	**-1.67**	-1.49	-1.20	-0.47	0.24	0.62	1.11	1.45	1.43	0.40
POST-LASSO-QAR REC	-1.42	-1.02	-0.91	-0.74	-0.37	0.05	0.35	0.69	1.01	1.08	0.35
TFA-QAR REC	-1.49	-1.00	-0.92	-0.78	-0.41	-0.00	0.31	0.70	1.07	1.12	0.37
LASSO-QAR ROLL	**-1.80**	**-2.48**	**-2.87**	**-2.75**	**-1.75**	-0.50	-0.20	-0.28	-0.73	-0.06	0.58
POST-LASSO-QAR ROLL	-1.36	**-2.04**	**-1.95**	-1.54	-0.98	-0.56	-0.38	-0.40	-0.38	0.11	0.72
TFA-QAR ROLL	-1.37	**-2.04**	**-1.97**	-1.57	-1.01	-0.58	-0.40	-0.41	-0.44	0.10	0.68

The values represent the t statistic for the null hypothesis of equal accuracy of the quantile forecasts from the models indicated relative to the QAR ROLL benchmark. The values of τ that we consider for the QS test are indicated at the top row of the Table, while h denotes the forecasting horizon, and κ the SIC penalty used in the LASSO selection. A negative value of the test statistic indicates that the alternative model outperforms the benchmark (QAR ROLL) and in bold are denoted the rejections of equal accuracy at 5% against the one-sided alternative that the benchmark is outperformed.

Table 3: **CPIAUCSL - QS TEST**

Model	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
						h=3					
QAR REC	0.74	0.83	-0.26	-0.41	-0.34	-0.07	0.21	0.31	0.13	-0.09	-0.34
FA-QAR REC K=3	-1.41	-1.29	-1.41	-1.02	-0.82	-0.51	-0.12	0.12	0.09	-0.06	0.26
FA-QAR REC K=5	-1.05	-0.62	-1.05	-0.83	-0.81	-0.67	-0.37	-0.13	-0.03	-0.01	0.35
FA-QAR ROLL K=3	**-1.97**	**-2.07**	-1.29	-1.18	-0.46	0.20	0.72	1.51	*1.82*	1.44	1.24
FA-QAR ROLL K=5	-1.48	-1.45	-1.32	-1.49	-1.39	-1.05	-0.84	-0.37	0.05	-0.18	0.29
						$c = 1$					
LASSO-QAR REC	-0.70	-0.99	-1.60	-1.57	-1.40	-1.15	-0.84	-0.23	-0.04	0.16	0.50
POST-LASSO-QAR REC	-0.98	-0.55	-1.14	-1.17	-0.88	-0.61	-0.32	-0.04	0.06	0.14	0.45
TFA-QAR REC	-1.08	-0.95	-1.47	-1.33	-1.08	-0.55	-0.16	0.23	0.27	0.24	0.33
LASSO-QAR ROLL	**-2.13**	**-2.26**	**-2.53**	**-3.26**	**-3.21**	**-3.35**	**-3.70**	**-2.43**	-0.99	-0.15	1.10
POST-LASSO-QAR ROLL	**-2.27**	**-2.13**	**-2.11**	**-2.48**	**-2.52**	**-2.37**	**-2.55**	**-1.86**	-0.74	-0.35	-0.17
TFA-QAR ROLL	**-2.95**	**-2.53**	**-2.42**	**-2.58**	**-2.44**	**-2.24**	**-1.94**	-1.28	-0.35	-0.32	-0.09
						$c = 2$					
LASSO-QAR REC	0.06	-0.36	-1.43	-1.31	-0.91	-0.55	-0.02	0.29	0.15	-0.03	-0.31
POST-LASSO-QAR REC	-0.29	-0.50	-1.04	-0.98	-0.67	-0.35	-0.02	0.47	0.27	-0.12	-0.00
TFA-QAR REC	-0.05	-0.58	-1.12	-1.04	-0.66	-0.37	-0.06	0.34	0.23	-0.09	0.00
LASSO-QAR ROLL	-1.12	**-1.95**	**-2.87**	**-3.01**	**-2.19**	-1.58	-1.53	-1.17	-0.59	0.28	1.10
POST-LASSO-QAR ROLL	-1.04	**-1.97**	**-2.10**	**-2.51**	**-2.28**	**-2.08**	-1.56	-1.06	-0.07	-0.47	0.63
TFA-QAR ROLL	-1.01	**-1.97**	**-2.10**	**-2.42**	**-2.28**	**-2.07**	-1.34	-0.94	-0.07	-0.51	0.69
						$c = 3$					
LASSO-QAR REC	0.50	0.68	-0.64	-0.71	-0.38	-0.04	0.26	0.37	0.13	-0.05	-0.23
POST-LASSO-QAR REC	-0.21	-0.10	-0.76	-0.68	-0.31	0.17	0.41	0.52	0.25	-0.02	-0.18
TFA-QAR REC	-0.25	-0.12	-0.80	-0.72	-0.33	0.19	0.45	0.52	0.24	-0.02	-0.16
LASSO-QAR ROLL	-1.32	-1.41	**-1.99**	-1.56	-0.61	-0.82	-0.86	0.48	-0.49	-0.81	-1.52
POST-LASSO-QAR ROLL	-0.93	**-1.74**	**-2.54**	**-1.77**	0.42	1.20	0.32	-0.74	-0.96	0.53	-0.49
TFA-QAR ROLL	-0.93	**-1.75**	**-2.54**	**-1.74**	0.38	1.16	0.35	-0.71	-0.96	0.60	-0.55
						h=6					
QAR REC	0.34	0.36	0.34	-0.34	-0.42	-0.17	-0.18	0.08	0.46	0.45	0.41
FA-QAR REC K=3	-1.50	**-1.92**	-1.09	-1.06	-0.80	-0.27	0.09	0.34	0.44	0.60	0.65
FA-QAR REC K=5	-0.85	-1.43	-0.88	-0.87	-0.73	-0.34	-0.05	0.17	0.39	0.67	0.77
FA-QAR ROLL K=3	**-1.85**	**-2.33**	**-1.87**	-1.54	-1.01	-0.36	0.30	0.93	1.24	1.31	1.35
FA-QAR ROLL K=5	-0.89	**-1.99**	**-1.74**	**-1.76**	-1.57	-1.32	-0.92	-0.49	-0.01	0.70	0.97
						$c = 1$					
LASSO-QAR REC	-0.48	-1.22	-0.88	-1.02	-0.81	-0.30	0.07	0.40	0.56	0.70	0.66
POST-LASSO-QAR REC	-0.89	-1.04	-0.46	-0.55	-0.27	0.20	0.49	0.68	0.91	0.97	0.84
TFA-QAR REC	-0.73	-1.26	-0.86	-0.99	-0.85	-0.45	0.01	0.43	0.64	0.61	0.67
LASSO-QAR ROLL	-1.12	**-2.24**	**-2.85**	**-2.14**	**-1.99**	**-2.35**	**-2.22**	**-1.64**	-1.01	-0.74	-0.92
POST-LASSO-QAR ROLL	**-1.68**	**-2.53**	**-2.30**	**-1.89**	**-1.81**	**-1.77**	-1.50	-1.05	-0.41	0.20	0.09
TFA-QAR ROLL	-1.24	**-2.57**	**-2.43**	**-1.81**	**-1.79**	**-1.85**	**-1.79**	-1.43	-1.53	-1.42	-1.63
						$c = 2$					
LASSO-QAR REC	-0.14	-0.85	-0.44	-0.55	-0.49	-0.27	-0.03	0.30	0.53	0.57	0.60
POST-LASSO-QAR REC	-1.06	**-1.68**	-0.94	-0.84	-0.63	-0.11	0.33	0.65	0.76	0.74	0.67
TFA-QAR REC	-0.91	**-1.76**	-1.05	-0.96	-0.63	-0.16	0.27	0.55	0.64	0.61	0.59
LASSO-QAR ROLL	**-1.86**	**-2.03**	**-1.96**	**-1.76**	**-2.01**	**-3.10**	**-3.03**	**-1.75**	-1.29	-1.36	-1.08
POST-LASSO-QAR ROLL	-1.11	**-1.87**	**-1.68**	**-1.64**	-1.63	**-1.94**	**-2.16**	**-1.67**	-0.96	-1.35	**-2.18**
TFA-QAR ROLL	-1.29	**-2.12**	**-1.89**	**-1.73**	**-1.71**	**-2.09**	**-2.32**	-1.63	-0.96	-1.42	**-2.19**
						$c = 3$					
LASSO-QAR REC	0.21	0.16	0.32	-0.22	-0.32	-0.09	-0.05	0.22	0.52	0.50	0.56
POST-LASSO-QAR REC	-0.34	-0.80	0.15	-0.01	-0.04	0.27	0.62	0.84	0.77	0.62	0.50
TFA-QAR REC	-0.29	-0.92	0.02	-0.12	-0.05	0.25	0.59	0.84	0.80	0.62	0.51
LASSO-QAR ROLL	0.60	-0.25	-0.91	-1.38	**-1.95**	-1.55	-1.21	-0.58	-0.62	1.22	-0.05
POST-LASSO-QAR ROLL	-1.30	-0.39	-0.08	-1.31	**-1.79**	**-1.76**	-1.18	0.12	0.97	0.60	-1.24
TFA-QAR ROLL	-1.30	-0.35	-0.08	-1.30	**-1.79**	**-1.76**	-1.17	0.11	0.97	0.59	-1.25
						h=12					
QAR REC	-0.54	-0.60	-0.77	-0.78	-0.74	-0.56	-0.40	0.15	0.31	0.60	0.36
FA-QAR REC K=3	**-1.82**	**-1.70**	-1.55	-1.53	-1.32	-1.16	-1.15	-0.93	-0.67	-0.17	0.10
FA-QAR REC K=5	-1.47	-1.27	-1.07	-0.94	-0.70	-0.55	-0.47	-0.20	0.17	0.58	0.81
FA-QAR ROLL K=3	**-1.98**	**-2.09**	**-1.82**	-1.37	-1.18	-1.46	**-2.04**	**-2.37**	**-2.35**	**-1.74**	-1.20
FA-QAR ROLL K=5	-0.47	-0.71	-0.88	-0.61	-0.55	-0.80	-1.20	-1.18	-0.77	-0.29	0.22
						$c = 1$					
LASSO-QAR REC	-1.55	-1.51	-1.29	-1.23	-1.03	-0.75	-0.60	-0.37	-0.10	0.23	0.36
POST-LASSO-QAR REC	**-1.82**	-1.51	-1.17	-0.94	-0.53	-0.20	-0.10	0.04	0.23	0.45	0.59
TFA-QAR REC	**-1.89**	**-1.85**	-1.57	-1.36	-0.98	-0.74	-0.69	-0.47	-0.19	0.20	0.45
LASSO-QAR ROLL	**-1.81**	**-1.77**	**-1.85**	**-2.11**	**-2.36**	**-2.41**	**-2.69**	**-2.73**	**-2.50**	**-1.79**	-1.20
POST-LASSO-QAR ROLL	**-1.83**	-1.19	-1.53	**-1.67**	**-1.73**	**-1.82**	**-2.13**	**-2.13**	**-1.78**	-1.13	-0.85
TFA-QAR ROLL	**-2.86**	**-3.03**	**-2.47**	**-2.27**	**-2.19**	**-2.09**	**-2.25**	**-2.37**	**-2.36**	**-1.97**	**-1.71**
						$c = 2$					
LASSO-QAR REC	-1.22	-1.32	-1.35	-1.26	-1.16	-1.05	-0.99	-0.62	-0.25	0.15	0.05
POST-LASSO-QAR REC	**-1.90**	**-1.84**	-1.35	-0.99	-0.79	-0.52	-0.50	-0.33	-0.09	0.26	0.35
TFA-QAR REC	**-1.76**	**-1.80**	-1.59	-1.45	-1.19	-0.97	-0.94	-0.70	-0.40	0.04	0.27
LASSO-QAR ROLL	**-1.87**	**-2.23**	**-2.16**	**-2.23**	**-2.43**	**-2.68**	**-3.03**	**-2.97**	**-2.56**	**-1.72**	-1.09
POST-LASSO-QAR ROLL	**-2.34**	**-2.60**	**-2.20**	**-1.89**	**-1.72**	**-1.83**	**-2.21**	**-2.40**	**-2.24**	**-1.83**	-1.36
TFA-QAR ROLL	**-2.19**	**-2.38**	**-2.19**	**-1.99**	**-1.86**	**-2.05**	**-2.29**	**-2.47**	**-2.28**	**-1.86**	-1.39
						$c = 3$					
LASSO-QAR REC	-0.84	-0.97	-1.10	-0.99	-0.92	-0.79	-0.82	-0.44	-0.29	-0.17	-0.18
POST-LASSO-QAR REC	-1.47	-1.56	-1.25	-0.90	-0.51	-0.33	-0.29	-0.10	0.10	0.23	0.17
TFA-QAR REC	-1.46	-1.52	-1.29	-1.02	-0.77	-0.55	-0.57	-0.39	-0.12	0.14	0.12
LASSO-QAR ROLL	-0.47	-1.34	-1.32	**-1.64**	**-1.94**	**-2.02**	**-2.28**	**-2.37**	**-1.84**	-1.14	-0.57
POST-LASSO-QAR ROLL	**-1.77**	-1.46	**-1.87**	**-2.26**	**-2.42**	**-2.44**	**-2.60**	**-2.29**	-1.43	-0.98	-0.89
TFA-QAR ROLL	**-1.79**	-1.43	**-1.89**	**-2.31**	**-2.44**	-2.43	-2.62	**-2.30**	-1.44	-0.98	-0.89

The values represent the t statistic for the null hypothesis of equal accuracy of the quantile forecasts from the models indicated relative to the QAR ROLL benchmark. The values of τ that we consider for the QS test are indicated at the top row of the Table, while h denotes the forecasting horizon, and κ the SIC penalty used in the LASSO selection. A negative value of the test statistic indicates that the alternative model outperforms the benchmark (QAR ROLL) and in bold are denoted the rejections of equal accuracy at 5% against the one-sided alternative that the benchmark is outperformed.

Table 4: **PCEPILFE - QS TEST**

Model	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
						h=3					
QAR REC	-1.12	-1.34	-1.17	-0.63	-0.86	-0.82	-1.27	-1.56	**-1.80**	**-1.71**	-1.04
FA-QAR REC K=3	-0.35	0.09	-0.02	-0.01	-0.31	-0.39	-0.75	-0.67	-0.52	-0.68	-0.83
FA-QAR REC K=5	0.55	0.67	0.42	0.25	-0.18	-0.28	-0.42	-0.62	-0.74	-0.71	-0.67
FA-QAR ROLL K=3	*1.71*	*1.86*	*1.84*	*1.92*	*1.66*	1.50	0.93	0.50	0.34	-0.28	-0.88
FA-QAR ROLL K=5	1.59	1.62	1.27	1.27	1.08	0.84	0.37	-0.01	-0.40	-1.19	**-1.67**
						$c = 1$					
LASSO-QAR REC	-0.89	-0.32	-0.48	-0.18	-0.33	-0.38	-1.01	**-1.70**	**-1.92**	**-2.33**	**-2.07**
POST-LASSO-QAR REC	0.08	0.44	0.38	0.92	0.87	0.82	0.39	-0.31	-0.66	-1.17	-1.17
TFA-QAR REC	-0.45	0.02	0.03	0.41	0.53	0.49	0.02	-0.46	-0.85	-1.10	-1.34
LASSO-QAR ROLL	0.62	1.32	0.33	-0.08	-0.48	-0.45	-1.31	**-1.82**	**-1.85**	**-2.30**	**-2.14**
POST-LASSO-QAR ROLL	0.75	*1.74*	1.25	1.34	0.93	0.80	0.22	-0.34	-0.67	-0.95	-1.67
TFA-QAR ROLL	0.57	1.52	0.95	0.71	0.46	0.14	-0.29	-0.83	-0.97	**-1.75**	**-2.31**
						$c = 2$					
LASSO-QAR REC	-1.11	-1.05	-1.00	-0.32	-0.80	-0.91	-1.34	**-1.83**	**-1.96**	**-2.08**	**-1.75**
POST-LASSO-QAR REC	-0.49	0.35	0.13	0.81	0.63	0.40	-0.23	-0.99	-1.20	**-1.64**	**-1.80**
TFA-QAR REC	-0.49	0.29	0.13	0.73	0.57	0.33	-0.22	-0.89	-1.15	-1.56	**-1.66**
LASSO-QAR ROLL	*1.74*	1.46	-0.54	-0.18	-0.58	-1.48	**-2.44**	**-2.25**	**-2.96**	**-2.34**	-0.69
POST-LASSO-QAR ROLL	-0.95	0.71	0.21	0.29	0.27	0.48	-0.29	-1.36	**-2.16**	**-2.15**	-1.46
TFA-QAR ROLL	-0.88	0.48	0.27	0.27	0.20	0.35	-0.41	-1.42	**-2.19**	**-2.18**	-1.44
						$c = 3$					
LASSO-QAR REC	-1.20	-1.35	-1.19	-0.65	-0.85	-0.84	-1.23	-1.58	**-1.88**	**-1.73**	-1.08
POST-LASSO-QAR REC	-1.22	-1.35	-1.11	-0.53	-0.88	-0.86	-1.44	**-1.91**	-1.63	-1.27	-0.75
TFA-QAR REC	-1.22	-1.35	-1.11	-0.53	-0.88	-0.86	-1.44	**-1.91**	-1.63	-1.27	-0.75
LASSO-QAR ROLL	0.51	1.25	1.02	0.21	0.30	0.67	0.52	-1.15	-0.92	0.25	0.97
POST-LASSO-QAR ROLL	*1.67*	-0.32	-0.08	0.90	0.00	0.53	-1.45	**-1.80**	**-2.19**	-0.21	0.72
TFA-QAR ROLL	*1.67*	-0.32	-0.08	0.90	0.00	0.53	-1.45	**-1.80**	**-2.19**	-0.21	0.72
						h=6					
QAR REC	-1.25	-1.24	-0.82	-0.90	-0.65	-0.90	-0.94	-1.06	-0.58	0.29	0.34
FA-QAR REC K=3	-0.82	-0.97	-1.31	-1.11	-0.72	-0.43	-0.04	0.12	0.16	0.36	0.23
FA-QAR REC K=5	-0.66	-0.64	-0.82	-0.52	-0.08	0.02	0.17	0.37	0.18	0.14	0.07
FA-QAR ROLL K=3	1.16	0.84	0.37	0.72	1.07	1.10	0.94	0.47	0.21	0.04	-0.21
FA-QAR ROLL K=5	0.41	0.36	0.01	0.33	0.57	0.61	0.54	0.39	-0.29	-0.46	-0.75
						$c = 1$					
LASSO-QAR REC	-1.28	-1.48	**-1.84**	-1.37	-0.84	-0.61	-0.59	-0.60	-0.78	-0.95	-1.04
POST-LASSO-QAR REC	-1.19	-1.35	-1.16	-0.69	-0.05	0.30	0.70	0.71	0.20	-0.08	-0.45
TFA-QAR REC	-1.15	-1.42	-1.50	-1.16	-0.74	-0.47	-0.12	0.02	-0.36	-0.61	-0.96
LASSO-QAR ROLL	-0.31	-0.34	-1.10	-0.79	-0.57	-0.55	-0.85	-1.28	-1.54	**-1.89**	**-2.68**
POST-LASSO-QAR ROLL	-0.09	-0.20	-0.67	-0.18	0.49	0.69	0.77	0.54	-0.55	-1.17	-1.58
TFA-QAR ROLL	-0.04	-0.39	-0.95	-0.64	-0.34	-0.29	-0.36	-0.76	-1.24	-1.40	-1.55
						$c = 2$					
LASSO-QAR REC	-1.19	-1.20	-1.03	-0.97	-0.64	-0.98	-1.05	-1.10	-0.93	-0.82	-0.39
POST-LASSO-QAR REC	-0.51	-0.40	-0.87	-0.54	-0.04	0.07	0.16	-0.06	-0.55	-0.58	-0.55
TFA-QAR REC	-0.56	-0.51	-0.95	-0.69	-0.15	-0.08	0.02	-0.21	-0.59	-0.61	-0.68
LASSO-QAR ROLL	0.52	-0.01	-0.29	-0.39	-0.93	-1.11	**-2.01**	**-2.57**	**-2.37**	**-2.90**	**-3.30**
POST-LASSO-QAR ROLL	-0.04	0.21	-0.17	0.13	0.42	0.43	-0.44	-1.42	**-1.90**	**-2.23**	**-2.55**
TFA-QAR ROLL	-0.07	0.15	-0.21	0.15	0.34	0.33	-0.41	-1.40	**-1.96**	**-2.28**	**-2.63**
						$c = 3$					
LASSO-QAR REC	-1.25	-1.24	-0.82	-0.91	-0.68	-0.95	-1.02	-1.06	-0.65	0.07	0.36
POST-LASSO-QAR REC	-0.94	-0.76	-0.19	-0.16	0.13	-0.25	-0.31	-0.38	-0.11	-0.31	0.06
TFA-QAR REC	-0.94	-0.76	-0.20	-0.18	0.12	-0.25	-0.32	-0.32	-0.09	-0.26	0.06
LASSO-QAR ROLL	0.45	1.21	0.37	-1.15	0.07	-0.20	-0.81	**-1.72**	**-2.24**	**-2.28**	**-2.15**
POST-LASSO-QAR ROLL	1.24	0.48	0.89	-0.50	-1.20	-1.41	-1.33	-1.02	-1.55	**-1.85**	-1.49
TFA-QAR ROLL	1.30	0.44	0.90	-0.44	-1.31	-1.56	-1.42	-1.14	-1.51	**-1.85**	-1.52
						h=12					
QAR REC	-1.23	-0.65	-0.02	-0.87	-1.10	-1.22	-1.31	-1.16	-0.40	1.05	2.03
FA-QAR REC K=3	0.70	0.39	-0.31	-1.02	-1.09	-1.06	-0.87	-0.44	0.22	0.69	0.91
FA-QAR REC K=5	1.28	1.07	0.25	-0.63	-0.77	-0.65	-0.50	-0.09	0.50	0.95	1.09
FA-QAR ROLL K=3	*2.13*	*2.09*	1.37	0.01	-0.54	-0.92	-0.98	-0.66	-0.08	0.33	0.58
FA-QAR ROLL K=5	*1.99*	*1.87*	1.38	0.17	-0.38	-0.60	-0.59	-0.29	0.17	0.57	0.78
						$c = 1$					
LASSO-QAR REC	0.22	-0.20	-1.24	**-2.03**	**-2.08**	**-1.88**	-1.61	-1.16	-0.30	0.03	0.16
POST-LASSO-QAR REC	1.28	0.97	0.26	-0.43	-0.44	-0.39	-0.31	-0.05	0.37	0.68	0.80
TFA-QAR REC	1.11	0.49	-0.57	-1.37	-1.50	-1.46	-1.28	-0.95	-0.31	0.09	0.22
LASSO-QAR ROLL	0.18	-0.04	-0.48	-1.32	-1.58	-1.63	-1.51	-1.01	-0.02	0.54	0.68
POST-LASSO-QAR ROLL	*1.83*	*1.69*	1.13	0.36	0.11	-0.08	-0.05	0.17	0.53	0.89	1.02
TFA-QAR ROLL	0.42	0.27	-0.55	-1.25	-1.53	**-1.68**	**-1.73**	-1.54	-0.87	-0.24	0.06
						$c = 2$					
LASSO-QAR REC	-0.57	-0.28	-1.09	**-2.11**	**-1.89**	**-1.90**	**-1.90**	**-1.77**	-1.21	-0.80	-0.27
POST-LASSO-QAR REC	1.32	0.66	-0.38	-1.32	-1.43	-1.36	-1.30	-1.04	-0.40	-0.32	-0.14
TFA-QAR REC	1.26	0.56	-0.58	**-1.67**	**-1.78**	**-1.68**	-1.50	-1.18	-0.58	-0.61	-0.41
LASSO-QAR ROLL	0.52	-0.38	-0.92	**-1.64**	-1.29	-1.33	-1.47	-1.50	-1.16	-0.59	-0.26
POST-LASSO-QAR ROLL	1.34	1.14	0.35	-0.75	-0.95	-1.03	-1.15	-1.05	-0.27	-0.07	0.12
TFA-QAR ROLL	1.26	0.96	0.05	-0.91	-1.06	-1.19	-1.32	-1.23	-0.45	-0.38	-0.23
						$c = 3$					
LASSO-QAR REC	-0.98	-0.46	0.05	-1.16	-1.40	-1.57	**-1.66**	-1.57	-1.10	-0.26	0.76
POST-LASSO-QAR REC	0.22	0.10	0.03	-0.64	-0.97	-1.45	-1.62	-1.48	-0.80	-0.12	-0.03
TFA-QAR REC	0.29	0.11	0.01	-0.78	-1.14	-1.62	**-1.78**	**-1.65**	-1.01	-0.32	-0.23
LASSO-QAR ROLL	0.92	1.32	0.69	-0.46	-0.62	-0.78	-0.90	-0.63	0.25	0.71	0.51
POST-LASSO-QAR ROLL	1.08	0.30	-0.47	-0.78	-0.30	-0.58	-0.59	-0.29	0.06	0.68	0.02
TFA-QAR ROLL	1.07	0.28	-0.47	-0.78	-0.30	-0.58	-0.60	-0.30	0.06	0.68	0.00

The values represent the t statistic for the null hypothesis of equal accuracy of the quantile forecasts from the models indicated relative to the QAR ROLL benchmark. The values of τ that we consider for the QS test are indicated at the top row of the Table, while h denotes the forecasting horizon, and κ the SIC penalty used in the LASSO selection. A negative value of the test statistic indicates that the alternative model outperforms the benchmark (QAR ROLL) and in bold are denoted the rejections of equal accuracy at 5% against the one-sided alternative that the benchmark is outperformed.

Table 5: **INDPRO & PAYEMS - LS, WQS, and IS TEST**

Model	LS	WQS					IS(0.70)			IS(0.90)		
		unif	center	tails	left	right	Test	Length	Cover	Test	Length	Cover
INDPRO							*h*=3					
QAR ROLL								9.89	0.67		17.74	0.86
QAR REC	**-2.44**	-0.96	-0.98	-0.74	-0.40	-0.57	-0.38	12.50	0.80	-0.42	21.23	0.94
FA-QAR REC K=3	**-2.54**	-1.33	-0.99	**-2.23**	**-2.33**	-1.39	**-1.65**	10.83	0.72	**-2.92**	17.64	0.91
FA-QAR REC K=5	**-3.55**	**-1.82**	-1.51	**-2.57**	**-2.16**	**-2.18**	**-1.98**	9.86	0.68	**-2.92**	16.58	0.87
FA-QAR ROLL K=3	0.04	-0.65	-0.62	-0.71	-0.55	-0.69	-0.92	9.10	0.62	-0.12	15.47	0.82
FA-QAR ROLL K=5	0.48	-0.91	-0.97	-0.71	-0.95	-0.09	-1.08	9.06	0.62	0.37	14.99	0.78
LASSO-QAR REC	**-2.68**	**-2.58**	**-2.33**	**-2.96**	**-3.89**	-1.59	**-2.50**	10.60	0.74	**-2.33**	19.07	0.94
POST-LASSO-QAR REC	-0.75	**-1.67**	-1.35	**-2.50**	**-2.38**	**-1.70**	**-1.96**	10.03	0.71	**-2.24**	18.16	0.92
TFA-QAR REC	**-1.65**	**-2.04**	**-1.70**	**-2.85**	**-2.90**	**-1.86**	**-2.05**	10.10	0.71	**-2.35**	18.17	0.92
LASSO-QAR ROLL	-0.91	**-3.93**	**-3.57**	**-3.96**	**-4.85**	-1.42	**-3.22**	9.22	0.65	**-2.86**	17.01	0.85
POST-LASSO-QAR ROLL	0.19	**-1.71**	-1.37	**-2.54**	**-2.78**	-0.96	**-1.88**	9.40	0.67	**-2.48**	16.88	0.87
TFA-QAR ROLL	-0.11	**-1.82**	-1.46	**-2.67**	**-2.88**	-1.03	**-2.19**	9.31	0.66	**-2.31**	16.85	0.87
							h=6					
QAR ROLL								9.43	0.66		15.46	0.81
QAR REC	-1.29	-1.29	-1.36	-1.05	-1.06	-0.55	-0.85	11.29	0.79	-0.79	19.47	0.94
FA-QAR REC K=3	-1.22	**-1.66**	-1.49	**-2.03**	**-2.30**	-1.17	**-1.79**	9.72	0.73	**-2.18**	14.40	0.86
FA-QAR REC K=5	-1.00	**-1.84**	**-1.66**	**-2.17**	**-1.97**	-1.60	**-2.10**	8.86	0.67	**-2.02**	13.19	0.85
FA-QAR ROLL K=3	1.57	0.05	-0.00	0.20	0.09	0.30	0.02	8.45	0.61	0.56	12.36	0.76
FA-QAR ROLL K=5	1.34	-0.56	-0.64	-0.32	-0.07	-0.76	-0.39	8.12	0.59	-0.05	11.65	0.73
LASSO-QAR REC	**-1.64**	**-2.84**	**-2.66**	**-3.04**	**-3.62**	-1.70	**-3.06**	8.61	0.68	**-2.50**	14.94	0.89
POST-LASSO-QAR REC	0.19	-1.34	-1.03	**-2.16**	**-1.96**	-1.66	**-1.72**	8.32	0.64	**-2.60**	14.13	0.87
TFA-QAR REC	-0.31	-1.42	-1.22	**-1.92**	-1.46	**-1.74**	**-1.64**	8.04	0.62	**-1.99**	13.76	0.86
LASSO-QAR ROLL	0.70	**-2.07**	**-1.88**	**-2.48**	**-2.27**	**-1.90**	**-2.16**	8.01	0.59	**-1.83**	13.81	0.81
POST-LASSO-QAR ROLL	0.82	-0.80	-0.59	-1.41	-1.04	-1.60	-0.79	7.74	0.56	**-2.27**	13.11	0.82
TFA-QAR ROLL	1.24	-0.92	-0.70	-1.58	-1.29	-1.50	-1.01	7.69	0.57	**-2.23**	13.03	0.82
							h=12					
QAR ROLL								7.79	0.63		11.98	0.74
QAR REC	**-2.72**	-1.35	-1.12	-1.77	**-2.29**	-0.35	-1.56	9.74	0.79	**-2.01**	16.18	0.92
FA-QAR REC K=3	-1.24	-1.18	-1.04	-1.51	**-1.67**	-0.86	-1.48	6.59	0.62	**-1.77**	10.05	0.79
FA-QAR REC K=5	-1.26	-1.33	-1.17	**-1.75**	**-2.03**	-0.18	**-1.78**	6.09	0.57	**-1.97**	9.61	0.75
FA-QAR ROLL K=3	1.09	0.00	0.06	-0.14	-0.51	0.58	-0.25	5.74	0.52	-0.15	8.46	0.65
FA-QAR ROLL K=5	0.99	-0.51	-0.46	-0.61	-0.72	-0.05	-0.83	5.24	0.46	-0.46	8.08	0.60
LASSO-QAR REC	**-2.13**	-1.58	-1.36	**-2.17**	**-2.43**	-1.07	**-2.00**	6.62	0.64	**-2.80**	11.27	0.85
POST-LASSO-QAR REC	-0.75	-0.86	-0.70	-1.31	-1.27	-1.01	-1.22	6.12	0.59	**-1.77**	10.51	0.80
TFA-QAR REC	-1.14	-1.10	-0.93	-1.58	-1.59	-1.09	-1.43	5.97	0.57	**-2.21**	10.38	0.81
LASSO-QAR ROLL	-0.07	**-1.71**	-1.57	**-1.84**	**-1.64**	-1.55	**-2.02**	5.50	0.55	-1.29	10.38	0.73
POST-LASSO-QAR ROLL	-0.06	-1.04	-0.93	-1.30	-1.04	-1.56	-1.40	5.10	0.50	-1.50	9.42	0.72
TFA-QAR ROLL	0.00	-1.03	-0.90	-1.32	-1.06	-1.59	-1.32	5.22	0.50	**-1.64**	9.49	0.73
PAYEMS							*h*=3					
QAR ROLL								2.75	0.73		4.45	0.86
QAR REC	-0.08	0.05	-0.14	0.40	-1.18	1.01	0.53	3.23	0.80	0.28	5.28	0.92
FA-QAR REC K=3	-0.46	-1.28	-1.37	-0.91	**-2.73**	1.05	-0.86	2.95	0.79	-0.62	4.88	0.91
FA-QAR REC K=5	**-1.71**	**-2.25**	**-2.22**	**-2.11**	**-3.00**	-0.57	**-2.17**	2.58	0.75	**-1.92**	4.20	0.90
FA-QAR ROLL K=3	-0.51	-1.46	-1.37	-1.57	**-3.12**	0.04	-1.59	2.38	0.68	-1.56	3.86	0.86
FA-QAR ROLL K=5	-0.73	-1.56	-1.56	-1.46	**-3.33**	0.06	-1.44	2.25	0.68	-1.25	3.72	0.84
LASSO-QAR REC	-1.53	**-1.74**	**-1.85**	-1.33	**-4.71**	0.16	-1.07	2.81	0.76	-0.99	4.73	0.91
POST-LASSO-QAR REC	-1.32	**-2.13**	**-2.05**	**-2.03**	**-3.30**	-0.37	-1.55	2.62	0.73	**-1.68**	4.44	0.90
TFA-QAR REC	-1.52	**-2.19**	**-2.15**	**-1.99**	**-3.35**	-0.32	-1.49	2.67	0.75	-1.60	4.49	0.89
LASSO-QAR ROLL	**-2.44**	**-3.47**	**-3.21**	**-3.79**	**-3.68**	**-2.54**	**-3.54**	2.54	0.71	**-1.88**	4.23	0.86
POST-LASSO-QAR ROLL	-0.53	**-2.30**	**-2.03**	**-3.01**	**-3.40**	-1.62	**-2.78**	2.43	0.67	**-2.66**	4.18	0.87
TFA-QAR ROLL	-0.70	**-2.58**	**-2.30**	**-3.21**	**-3.37**	**-1.94**	**-2.98**	2.41	0.67	**-2.64**	4.15	0.87
							h=6					
QAR ROLL								2.98	0.71		4.63	0.85
QAR REC	-0.55	-0.74	-0.95	-0.30	-1.46	1.13	0.15	3.41	0.77	-0.50	5.55	0.89
FA-QAR REC K=3	-1.24	**-1.89**	**-1.87**	**-1.78**	**-2.99**	0.76	-1.47	2.99	0.75	-1.57	4.63	0.88
FA-QAR REC K=5	**-2.06**	**-2.66**	**-2.57**	**-2.58**	**-2.77**	-0.14	**-2.29**	2.62	0.73	**-2.03**	4.02	0.88
FA-QAR ROLL K=3	-0.65	**-1.91**	-1.63	**-2.38**	**-2.71**	-0.30	**-2.41**	2.47	0.69	**-1.79**	3.69	0.81
FA-QAR ROLL K=5	-0.68	-1.56	-1.34	**-2.01**	**-2.45**	0.33	**-1.81**	2.35	0.66	**-1.64**	3.49	0.80
LASSO-QAR REC	**-2.06**	**-1.86**	**-1.96**	-1.51	**-2.63**	0.09	-1.40	2.74	0.72	-1.16	4.64	0.88
POST-LASSO-QAR REC	-1.00	-1.62	-1.56	-1.56	**-2.44**	-0.11	-1.41	2.68	0.67	-1.30	4.44	0.88
TFA-QAR REC	-1.33	-1.62	-1.58	-1.50	**-2.32**	-0.17	-1.39	2.66	0.69	-1.17	4.49	0.89
LASSO-QAR ROLL	**-2.20**	**-2.95**	**-2.69**	**-3.25**	**-3.20**	-1.38	**-3.36**	2.55	0.67	-1.21	4.18	0.84
POST-LASSO-QAR ROLL	-0.32	**-1.87**	-1.55	**-2.84**	**-2.46**	**-1.80**	**-2.79**	2.44	0.67	**-2.40**	3.98	0.85
TFA-QAR ROLL	-0.99	**-2.58**	**-2.21**	**-3.54**	**-3.17**	**-2.24**	**-3.37**	2.42	0.67	**-2.75**	3.98	0.84
							h=12					
QAR ROLL								3.02	0.68		4.81	0.80
QAR REC	-0.95	-0.14	0.13	-0.64	**-1.70**	1.30	-0.18	3.67	0.72	-1.20	5.87	0.86
FA-QAR REC K=3	-1.03	-0.84	-0.69	-1.18	**-1.74**	0.32	-1.04	2.86	0.64	-1.54	4.24	0.79
FA-QAR REC K=5	-0.98	-1.29	-1.23	-1.40	**-1.78**	0.23	-1.49	2.46	0.65	-1.35	3.76	0.78
FA-QAR ROLL K=3	0.24	-0.45	-0.44	-0.44	-1.09	0.68	-0.69	2.18	0.58	-0.19	3.22	0.73
FA-QAR ROLL K=5	0.67	-0.53	-0.56	-0.42	-1.08	1.00	-0.63	2.00	0.55	-0.17	3.01	0.69
LASSO-QAR REC	-0.80	-0.71	-0.55	-1.04	-1.50	0.52	-0.84	2.81	0.65	-1.43	4.77	0.81
POST-LASSO-QAR REC	-0.47	-0.76	-0.63	-1.02	-1.30	0.22	-1.01	2.59	0.63	-1.17	4.32	0.78
TFA-QAR REC	-0.66	-0.75	-0.67	-0.91	-1.17	0.19	-0.85	2.63	0.63	-1.11	4.37	0.78
LASSO-QAR ROLL	-1.02	**-1.64**	-1.45	**-1.98**	**-1.86**	-0.76	**-2.71**	2.30	0.60	-1.03	4.21	0.78
POST-LASSO-QAR ROLL	-0.37	-0.86	-0.68	-1.23	-1.35	-0.20	-1.43	2.13	0.59	-1.14	4.01	0.79
TFA-QAR ROLL	-0.69	-1.10	-0.94	-1.39	-1.48	-0.32	**-1.74**	2.19	0.61	-1.10	4.01	0.79

The values represent the t statistic for the null hypothesis of equal accuracy of the model forecasts relative to the QAR ROLL benchmark based on the Logarithmic Score (LS), Weighted Quantile Score (WQS), and Interval Score (IS). The values of θ for the IS test are 0.7 and 0.9 and h denotes the forecasting horizon. A negative value of the test statistic indicates that the alternative model outperforms the benchmark (QAR ROLL) with the values in bold denoting rejections of equal accuracy at 5% against the one-sided alternative that the benchmark is outperformed. The shrinkage parameter c for the LASSO selection is set equal to 2.

Table 6: **CPIAUCSL & PCEPILFE - LS, WQS, and IS TEST**

Model	LS	WQS					IS(0.70)			IS(0.90)		
		unif	center	tails	left	right	Test	Length	Cover	Test	Length	Cover
CPIAUCSL					h=3							
QAR ROLL								3.48	0.57		5.90	0.79
QAR REC	**-2.72**	0.05	0.02	0.11	0.49	-0.09	-0.01	3.44	0.62	0.01	5.77	0.83
FA-QAR REC K=3	**-2.96**	-0.56	-0.53	-0.61	-1.44	0.10	-0.91	3.70	0.66	-0.55	6.04	0.84
FA-QAR REC K=5	**-3.41**	-0.51	-0.56	-0.37	-0.97	0.11	-0.65	3.46	0.64	-0.23	5.69	0.81
FA-QAR ROLL K=3	0.26	-0.15	-0.03	-0.47	**-1.89**	1.56	-0.64	3.53	0.56	-0.50	5.65	0.77
FA-QAR ROLL K=5	-0.84	-1.20	-1.16	-1.18	-1.56	0.01	-1.19	3.37	0.59	-0.91	5.37	0.77
LASSO-QAR REC	**-2.96**	-0.43	-0.46	-0.33	-0.73	-0.06	-0.48	3.34	0.63	-0.27	5.71	0.84
POST-LASSO-QAR REC	**-2.52**	-0.29	-0.30	-0.26	-0.71	0.06	-0.47	3.40	0.63	-0.14	5.99	0.84
TFA-QAR REC	**-2.79**	-0.33	-0.35	-0.26	-0.69	0.05	-0.52	3.44	0.65	-0.02	5.95	0.84
LASSO-QAR ROLL	**-1.64**	**-2.81**	**-2.71**	**-2.54**	**-2.38**	-0.06	**-2.88**	3.40	0.58	-0.85	5.90	0.78
POST-LASSO-QAR ROLL	-0.72	**-2.42**	**-2.39**	**-2.18**	**-2.05**	-0.24	**-1.96**	3.50	0.61	-0.81	6.11	0.80
TFA-QAR ROLL	-0.77	**-2.37**	**-2.34**	**-2.15**	**-2.03**	-0.19	**-1.90**	3.50	0.61	-0.73	6.11	0.80
					h=6							
QAR ROLL								3.12	0.57		4.88	0.78
QAR REC	**-2.17**	0.11	-0.01	0.41	0.29	0.40	0.55	3.07	0.64	0.47	5.09	0.81
FA-QAR REC K=3	**-2.20**	-0.23	-0.28	-0.13	-1.62	0.57	-0.12	3.06	0.64	-0.01	4.75	0.83
FA-QAR REC K=5	-1.46	-0.17	-0.27	0.05	-1.12	0.61	-0.11	3.00	0.63	0.30	4.73	0.82
FA-QAR ROLL K=3	0.60	-0.55	-0.56	-0.49	**-2.21**	1.32	-0.55	2.98	0.52	-0.09	4.46	0.74
FA-QAR ROLL K=5	-0.44	-1.24	-1.35	-0.84	**-1.66**	0.49	-1.06	2.87	0.57	-0.18	4.29	0.74
LASSO-QAR REC	**-1.83**	0.03	-0.07	0.24	-0.50	0.54	0.22	2.92	0.60	0.39	4.72	0.80
POST-LASSO-QAR REC	-1.43	0.07	0.02	0.19	-1.34	0.71	0.28	2.95	0.61	0.21	4.84	0.82
TFA-QAR REC	-1.58	-0.03	-0.08	0.08	-1.35	0.60	0.11	2.99	0.62	0.18	4.80	0.83
LASSO-QAR ROLL	-1.11	**-3.10**	**-2.95**	**-2.75**	**-2.41**	-1.61	**-2.06**	2.95	0.56	**-1.95**	4.79	0.77
POST-LASSO-QAR ROLL	**-1.67**	**-2.31**	**-2.16**	**-2.49**	**-1.98**	**-1.68**	**-2.09**	2.90	0.57	**-1.78**	4.89	0.80
TFA-QAR ROLL	**-1.77**	**-2.47**	**-2.29**	**-2.74**	**-2.23**	**-1.71**	**-2.39**	2.92	0.58	**-1.96**	4.89	0.80
					h=12							
QAR ROLL								3.17	0.57		4.52	0.71
QAR REC	-1.36	-0.47	-0.51	-0.34	-0.65	0.41	-0.36	3.46	0.65	-0.29	5.53	0.81
FA-QAR REC K=3	**-2.35**	-1.14	-1.21	-0.93	**-1.70**	-0.25	-1.03	2.88	0.63	-0.66	4.39	0.81
FA-QAR REC K=5	-0.61	-0.36	-0.48	-0.08	-1.25	0.52	-0.20	2.60	0.56	0.24	4.05	0.74
FA-QAR ROLL K=3	-1.28	**-2.10**	**-1.95**	**-2.42**	**-1.99**	**-1.85**	**-2.50**	2.51	0.53	**-2.34**	3.83	0.73
FA-QAR ROLL K=5	0.19	-0.77	-0.86	-0.53	-0.73	-0.31	-0.73	2.34	0.46	-0.04	3.59	0.67
LASSO-QAR REC	**-2.05**	-0.92	-0.98	-0.75	-1.30	-0.07	-0.70	2.90	0.60	-0.75	4.57	0.80
POST-LASSO-QAR REC	-1.50	-0.56	-0.58	-0.49	**-1.69**	0.15	-0.46	2.69	0.58	-0.50	4.46	0.79
TFA-QAR REC	**-2.25**	-0.94	-1.01	-0.77	**-1.73**	-0.05	-0.83	2.64	0.60	-0.53	4.34	0.80
LASSO-QAR ROLL	-1.55	**-2.80**	**-2.79**	**-2.66**	**-2.48**	**-2.07**	**-2.54**	2.68	0.54	**-2.08**	4.30	0.71
POST-LASSO-QAR ROLL	**-2.31**	**-2.36**	**-2.23**	**-2.67**	**-2.64**	**-1.91**	**-2.54**	2.57	0.57	**-2.64**	4.41	0.76
TFA-QAR ROLL	**-2.50**	**-2.45**	**-2.34**	**-2.69**	**-2.51**	**-1.96**	**-2.58**	2.59	0.57	**-2.65**	4.38	0.76
PCEPILFE					h=3							
QAR ROLL								1.84	0.66		3.06	0.88
QAR REC	**-2.06**	-1.59	-1.42	**-1.78**	-1.26	-1.55	**-2.26**	1.86	0.70	-1.30	3.25	0.92
FA-QAR REC K=3	-0.90	-0.56	-0.50	-0.66	-0.14	-0.73	-0.60	1.98	0.72	-0.87	3.20	0.91
FA-QAR REC K=5	-0.49	-0.34	-0.32	-0.36	0.57	-0.71	-0.44	1.97	0.73	-0.36	3.20	0.89
FA-QAR ROLL K=3	2.76	1.30	1.32	1.13	1.90	-0.23	1.20	1.80	0.64	1.02	2.80	0.82
FA-QAR ROLL K=5	1.52	0.58	0.67	0.32	1.59	-1.09	0.30	1.81	0.66	0.02	2.80	0.84
LASSO-QAR REC	**-2.01**	**-1.78**	-1.52	**-2.14**	-1.08	**-2.06**	**-2.29**	1.86	0.72	**-1.85**	3.15	0.92
POST-LASSO-QAR REC	-0.41	-0.46	-0.15	-1.31	0.15	**-1.64**	-1.23	1.99	0.74	**-1.72**	3.18	0.92
TFA-QAR REC	-0.37	-0.47	-0.17	-1.25	0.11	-1.53	-1.14	1.99	0.75	-1.61	3.17	0.92
LASSO-QAR ROLL	-0.52	**-2.35**	**-2.40**	**-1.91**	1.39	**-2.29**	**-2.22**	1.77	0.66	-0.21	2.99	0.87
POST-LASSO-QAR ROLL	0.28	-1.04	-0.66	**-1.84**	0.14	**-2.02**	**-1.87**	1.77	0.66	**-2.07**	2.93	0.88
TFA-QAR ROLL	0.23	-1.12	-0.75	**-1.88**	0.13	**-2.04**	**-1.92**	1.77	0.66	**-2.04**	2.93	0.88
					h=6							
QAR ROLL								1.66	0.69		2.66	0.90
QAR REC	-1.08	-0.86	-0.94	-0.61	-1.15	-0.01	-0.69	1.88	0.76	-0.29	3.11	0.94
FA-QAR REC K=3	-0.27	-0.40	-0.44	-0.28	-1.21	0.25	-0.45	1.77	0.76	-0.07	2.75	0.90
FA-QAR REC K=5	-0.32	-0.13	-0.09	-0.23	-0.74	0.16	-0.39	1.70	0.73	-0.22	2.69	0.89
FA-QAR ROLL K=3	1.53	0.73	0.80	0.51	0.88	0.05	0.23	1.60	0.66	0.44	2.45	0.84
FA-QAR ROLL K=5	0.64	0.23	0.35	-0.09	0.30	-0.43	-0.25	1.64	0.66	-0.32	2.47	0.85
LASSO-QAR REC	-1.27	-1.22	-1.17	-1.22	-1.17	-0.80	-1.23	1.78	0.74	-0.83	2.89	0.93
POST-LASSO-QAR REC	-0.59	-0.42	-0.29	-0.72	-0.61	-0.54	-0.92	1.76	0.75	-0.62	2.83	0.93
TFA-QAR REC	-0.68	-0.57	-0.43	-0.85	-0.71	-0.62	-0.99	1.77	0.75	-0.74	2.83	0.93
LASSO-QAR ROLL	-1.24	**-2.44**	**-2.18**	**-2.82**	-0.14	**-3.02**	**-2.52**	1.57	0.69	**-3.01**	2.53	0.90
POST-LASSO-QAR ROLL	-0.15	-0.88	-0.59	**-1.74**	0.05	**-2.24**	**-1.64**	1.59	0.71	**-2.18**	2.54	0.90
TFA-QAR ROLL	-0.30	-0.90	-0.62	**-1.77**	0.00	**-2.31**	**-1.76**	1.59	0.70	**-2.21**	2.54	0.90
					h=12							
QAR ROLL								1.79	0.65		2.75	0.88
QAR REC	-0.77	-0.84	-1.04	0.10	-0.75	0.43	0.09	2.06	0.79	1.74	3.47	0.96
FA-QAR REC K=3	0.38	-0.36	-0.63	0.40	-0.08	0.49	0.40	1.67	0.70	1.00	2.63	0.87
FA-QAR REC K=5	0.77	0.12	-0.19	0.90	0.70	0.78	0.85	1.60	0.68	1.39	2.52	0.83
FA-QAR ROLL K=3	1.14	0.01	-0.34	0.79	1.74	0.23	0.80	1.51	0.63	1.25	2.32	0.78
FA-QAR ROLL K=5	0.74	0.22	-0.06	0.78	1.54	0.49	0.76	1.48	0.60	1.07	2.30	0.79
LASSO-QAR REC	-1.44	**-1.78**	**-1.89**	-1.30	-1.15	-1.00	-1.14	1.69	0.76	-0.44	2.94	0.95
POST-LASSO-QAR REC	-0.74	-1.03	-1.19	-0.40	-0.00	-0.44	-0.26	1.70	0.75	0.27	2.87	0.93
TFA-QAR REC	-1.01	-1.31	-1.47	-0.68	-0.19	-0.68	-0.53	1.69	0.75	-0.01	2.84	0.94
LASSO-QAR ROLL	**-1.95**	-1.43	-1.47	-1.13	-1.03	-0.95	-0.96	1.55	0.62	-0.10	2.63	0.88
POST-LASSO-QAR ROLL	-0.81	-0.76	-0.92	-0.08	0.60	-0.27	0.14	1.56	0.64	0.58	2.66	0.88
TFA-QAR ROLL	-1.32	-1.00	-1.12	-0.41	0.33	-0.55	-0.20	1.55	0.64	0.27	2.65	0.88

The values represent the t statistic for the null hypothesis of equal accuracy of the model forecasts relative to the QAR ROLL benchmark based on the Logarithmic Score (LS), Weighted Quantile Score (WQS), and Interval Score (IS). The values of θ for the IS test are 0.7 and 0.9 and h denotes the forecasting horizon. A negative value of the test statistic indicates that the alternative model outperforms the benchmark (QAR ROLL) with the values in bold denoting rejections of equal accuracy at 5% against the one-sided alternative that the benchmark is outperformed. The shrinkage parameter c for the LASSO selection is set equal to 2.

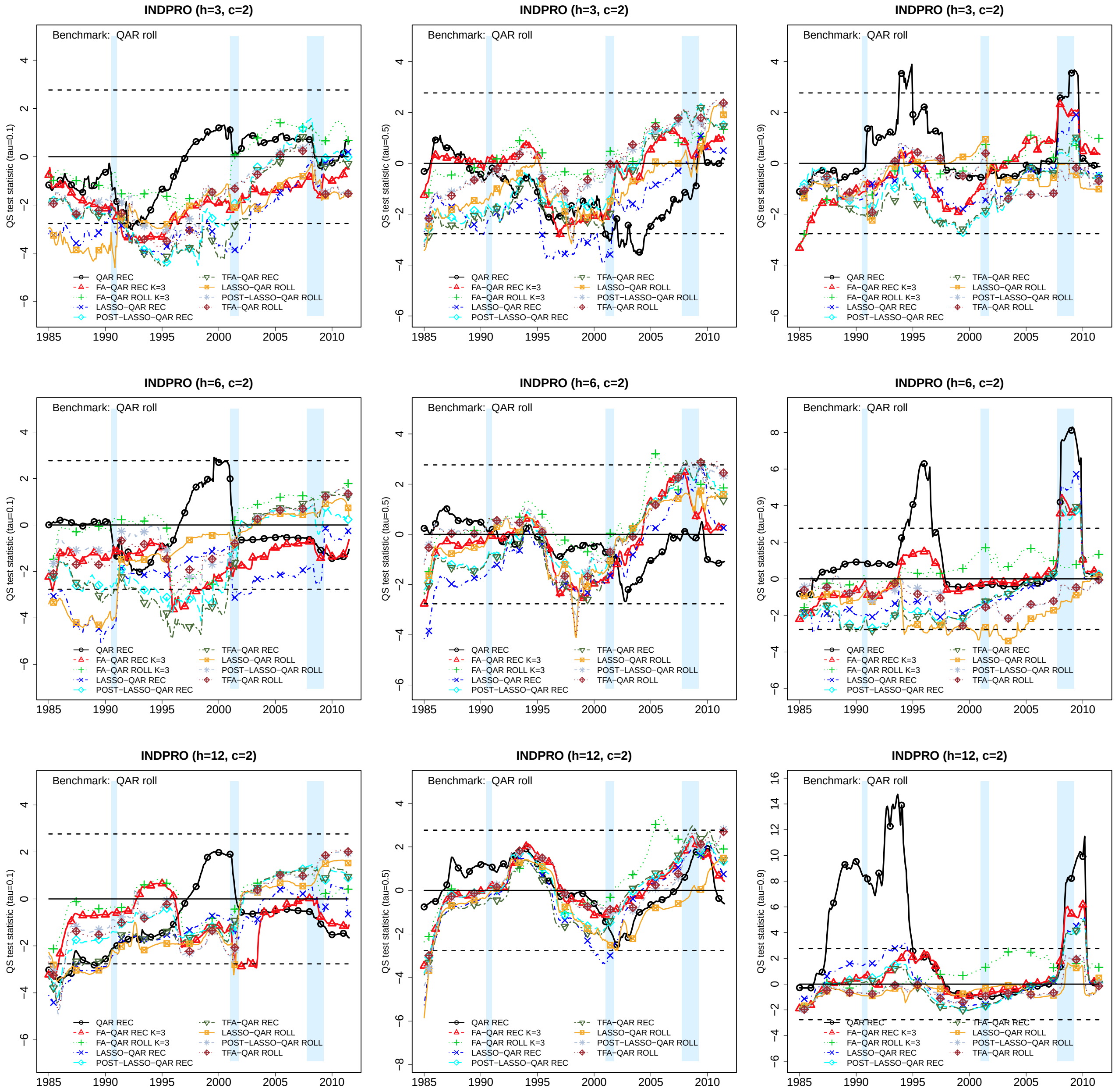

Figure 1: Fluctuation QS test for INDPRO at $\tau = 0.1$, 0.5 and 0.9 (columns) and the forecast horizon h equal to 3, 6, and 12 (rows). The vertical axis represent the value of the t-statistic of the $fQS^{i}_{t,m}(\tau)$ with the rolling window m equal to 120 months. The horizontal dashed lines represent the 5% one-sided critical values from Giacomini and Rossi (2010) and the vertical bars denote the NBER recession periods. The t values reported in the caption of the sub-figures represent the t statistic for the QS test at level τ of the QAR ROLL model against QAR REC from Table (1). The value of c is set to 2.

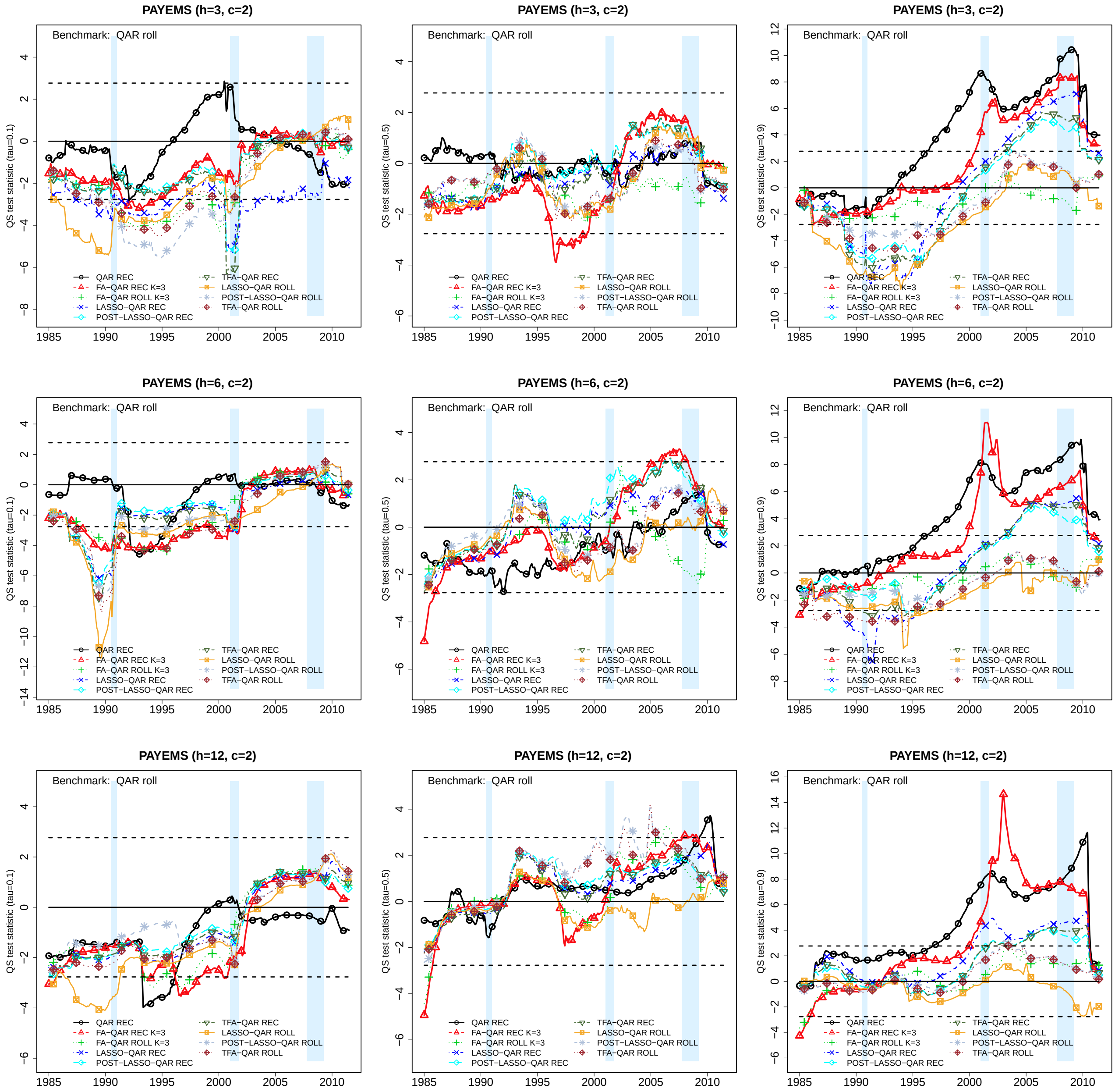

Figure 2: Fluctuation QS test for PAYEMS at $\tau = 0.1$, 0.5 and 0.9 (columns) and the forecast horizon h equal to 3, 6, and 12 (rows). The vertical axis represent the value of the t-statistic of the $fQS^{i}_{t,m}(\tau)$ with the rolling window m equal to 120 months. The horizontal dashed lines represent the 5% one-sided critical values from Giacomini and Rossi (2010) and the vertical bars denote the NBER recession periods. The t values reported in the caption of the sub-figures represent the t statistic for the QS test at level τ of the QAR ROLL model against QAR REC from Table (2). The value of c is set to 2.

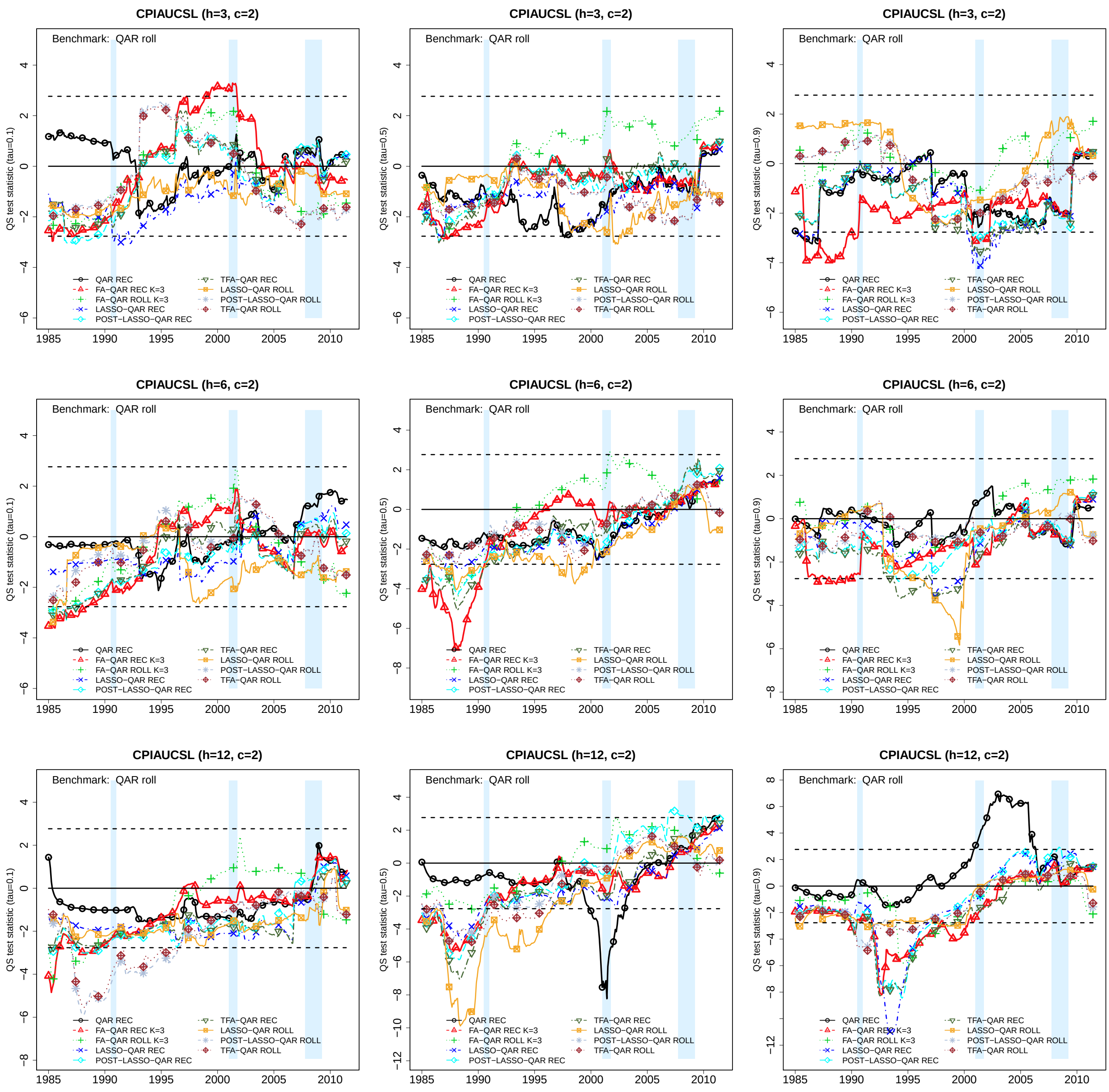

Figure 3: Fluctuation QS test for CPIAUCSL at $\tau = 0.1$, 0.5 and 0.9 (columns) and the forecast horizon h equal to 3, 6, and 12 (rows). The vertical axis represent the value of the t-statistic of the $fQS^{i}_{t,m}(\tau)$ with the rolling window m equal to 120 months. The horizontal dashed lines represent the 5% one-sided critical values from Giacomini and Rossi (2010) and the vertical bars denote the NBER recession periods. The t values reported in the caption of the sub-figures represent the t statistic for the QS test at level τ of the QAR ROLL model against QAR REC from Table (3). The value of c is set to 2.

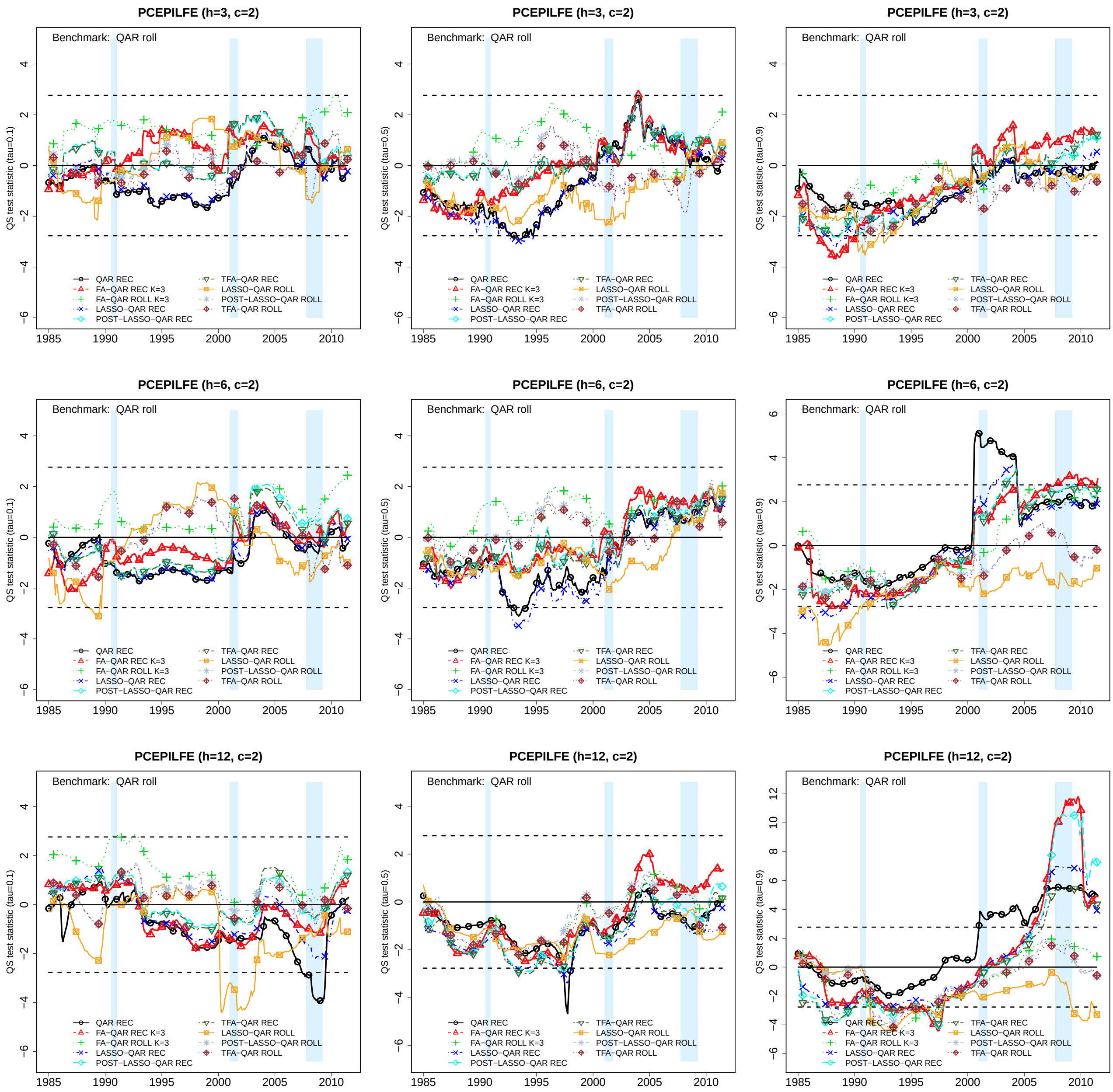

Figure 4: Fluctuation QS test for PCEPILFE at $\tau = 0.1$, 0.5 and 0.9 (columns) and the forecast horizon h equal to 3, 6, and 12 (rows). The vertical axis represent the value of the t-statistic of the $fQS^{i}_{t,m}(\tau)$ with the rolling window m equal to 120 months. The horizontal dashed lines represent the 5% one-sided critical values from Giacomini and Rossi (2010) and the vertical bars denote the NBER recession periods. The t values reported in the caption of the sub-figures represent the t statistic for the QS test at level τ of the QAR ROLL model against QAR REC from Table (4). The value of c is set to 2.

Table 7: **INDPRO**

h	τ	1^{st}			2^{nd}			3^{rd}			4^{th}			5^{th}		
Recursive																
3	0.1	SPREAD3M	0.993	1.757	PPICPE	0.897	-1.876	PFCGEF	0.518	-0.388	PERMIT1	0.349	0.349	SPREAD10Y	0.297	0.135
	0.3	SPREAD3M	0.995	1.505	PPICPE	0.801	-0.747	NAPMNOI	0.575	1.177	SVGTI	0.466	0.307	CPIAPPSL	0.333	-0.123
	0.5	SPREAD3M	0.993	1.67	NAPMNOI	0.731	1.669	CPIMEDSL	0.553	-0.562	SVGTI	0.548	0.337	SAVINGSL	0.429	0.266
	0.7	SPREAD3M	1	1.36	NAPMNOI	0.767	1.939	SVGTI	0.676	0.638	SVSTSL	0.6	0.36	CPIMEDSL	0.509	-0.294
	0.9	SPREAD3M	0.733	1.268	SVSTSL	0.674	0.492	NAPMNOI	0.493	1.135	CPIMEDSL	0.409	-0.393	SVGTI	0.315	0.335
6	0.1	SPREAD3M	0.995	1.766	PPICPE	0.982	-1.28	PERMITMW	0.662	0.576	PPIITM	0.541	-0.526	NAPMNOI	0.527	0.595
	0.3	SPREAD3M	0.961	1.914	CPIMEDSL	0.788	-0.581	SVGTI	0.785	0.536	NAPMNOI	0.598	1.182	HOUST1F	0.358	0.142
	0.5	SPREAD3M	0.991	1.47	CPIMEDSL	0.852	-0.753	NAPMNOI	0.642	1.2	SVGTI	0.61	0.585	SPREAD6M	0.523	0.253
	0.7	SVSTSL	0.936	0.715	SPREAD3M	0.836	0.885	CPIMEDSL	0.79	-0.596	NAPMNOI	0.733	1.057	SVGTI	0.594	0.447
	0.9	SVSTSL	0.861	0.816	CPIMEDSL	0.685	-0.432	NAPMNOI	0.642	1.042	SVGTI	0.587	0.54	PPICPE	0.534	-0.37
12	0.1	SPREADAAA	0.724	1.078	CPIMEDSL	0.715	-0.331	PPIITM	0.655	-0.376	SPREAD3M	0.55	0.655	SPREAD6M	0.466	0.545
	0.3	SPREAD3M	0.934	0.91	CPIMEDSL	0.918	-0.466	SPREAD6M	0.749	0.526	PFCGEF	0.737	-0.473	SPREADAAA	0.712	0.835
	0.5	CPIMEDSL	0.804	-0.336	CPILFESL	0.795	-0.275	SPREAD6M	0.735	0.92	SVSTSL	0.733	0.521	PPIITM	0.664	-0.287
	0.7	CPILFESL	0.824	-0.329	SVSTSL	0.801	0.617	CPIMEDSL	0.744	-0.448	PCEPILFE	0.658	-0.453	SPREADAAA	0.648	0.612
	0.9	SVGTI	0.66	0.574	SVSTSL	0.632	0.402	PCEPILFE	0.621	-0.527	CPILFESL	0.584	-0.236	CPIAUCSL	0.541	-0.194
Rolling																
3	0.1	SPREAD3M	0.345	0.868	PPICPE	0.317	-0.849	HSN1FW	0.16	0.273	SAVINGSL	0.13	0.223	PPIITM	0.096	-0.037
	0.3	SPREAD3M	0.365	0.909	SVGTI	0.249	0.312	HSN1FW	0.203	0.272	SVGCBSL	0.176	0.187	PPICPE	0.153	-0.226
	0.5	SVGTI	0.45	0.534	SPREAD3M	0.313	0.759	HSN1FW	0.112	0.134	SVSTCBSL	0.094	0.085	NAPMPRI	0.089	-0.062
	0.7	SVGTI	0.358	0.45	SPREAD3M	0.308	0.438	SVSTSL	0.274	0.227	NAPMPRI	0.139	-0.112	CPIMEDSL	0.123	-0.121
	0.9	CPIMEDSL	0.155	-0.142	SVGTI	0.144	0.211	SPREADAAA	0.128	0.162	SVSTSL	0.082	0.076	SPREAD3M	0.068	0.167
6	0.1	SPREAD3M	0.377	0.896	PPICPE	0.363	-0.796	CPIMEDSL	0.356	-0.368	UEMPMEAN	0.221	0.268	SPREADBAA	0.21	0.232
	0.3	SPREAD3M	0.402	0.869	SVGTI	0.333	0.299	SAVINGSL	0.272	0.316	CPIMEDSL	0.265	-0.215	NAPMPRI	0.237	-0.239
	0.5	SVGTI	0.573	0.684	CPIMEDSL	0.393	-0.423	SPREAD3M	0.37	0.638	NAPMPRI	0.34	-0.289	UEMPMEAN	0.258	0.236
	0.7	CPIMEDSL	0.432	-0.399	SVGTI	0.358	0.34	SVSTSL	0.304	0.247	NAPMPRI	0.272	-0.304	SPREAD3M	0.235	0.383
	0.9	CPIAPPSL	0.292	-0.152	SVSTSL	0.249	0.289	PPICPE	0.21	-0.188	SVGTI	0.167	0.195	SVGCBSL	0.128	0.128
12	0.1	UEMPMEAN	0.299	0.363	SPREADAAA	0.247	0.407	NAPMPRI	0.217	-0.443	PFCGEF	0.217	-0.182	SPREADBAA	0.217	0.363
	0.3	NAPMPRI	0.447	-0.637	SPREADAAA	0.37	0.623	CPIMEDSL	0.354	-0.249	PFCGEF	0.292	-0.237	UEMPMEAN	0.263	0.245
	0.5	SPREADAAA	0.422	0.705	NAPMPRI	0.384	-0.555	CPILFESL	0.304	-0.123	CPIMEDSL	0.269	-0.222	UEMPMEAN	0.242	0.23
	0.7	NAPMPRI	0.386	-0.426	SPREADAAA	0.349	0.574	CPIMEDSL	0.281	-0.184	SVGTI	0.26	0.185	UEMPMEAN	0.258	0.22
	0.9	SVGTI	0.324	0.337	NAPMPRI	0.148	-0.128	UEMPMEAN	0.114	0.064	HSN1FMW	0.112	0.112	CPIAPPSL	0.098	-0.034

Top 5 variables selected by LASSO at quantiles $\tau = 0.1, 0.3, 0.5, 0.7$, and 0.9 and for each forecast horizon h considered. The indicators are denoted by their FRED series ID and more details can be found in the Appendix and on the FRED website. The columns next to the variable ID report the frequency of selection in the out-of-sample period (438 months) and the average value of the parameter in the months in which the variable was selected (in interpreting the coefficients notice that the RHS variables have been standardize to have mean zero and variance one). The value of c is set to 2.

Table 8: **PAYEMS**

h	τ	1^{st}			2^{nd}			3^{rd}			4^{th}			5^{th}		
Recursive																
3	0.1	PPICPE	0.696	-0.259	SPREAD3M	0.696	0.249	PCEDG	0.603	0.174	CPILFESL	0.518	-0.17	PERMIT1	0.413	0.164
	0.3	SPREAD3M	1	0.393	NAPMNOI	0.427	0.153	SVGTI	0.384	0.085	SAVINGSL	0.354	0.067	PERMIT1	0.283	0.073
	0.5	SPREAD3M	0.998	0.487	NAPMNOI	0.678	0.286	SAVINGSL	0.461	0.093	M2SL	0.29	0.058	PCES	0.276	0.049
	0.7	SPREAD3M	0.838	0.416	PCES	0.543	0.135	SVGTI	0.418	0.106	PERMITMW	0.402	0.084	SVSTSL	0.393	0.098
	0.9	SVGTI	0.527	0.131	SVSTSL	0.486	0.137	SPREAD6M	0.276	0.067	SPREAD1Y	0.276	0.105	PERMIT24	0.249	0.041
6	0.1	PPICPE	0.92	-0.387	SPREAD3M	0.518	0.284	CPIMEDSL	0.489	-0.105	NAPMNOI	0.374	0.144	SVGTI	0.372	0.081
	0.3	SPREAD3M	0.991	0.337	NAPMNOI	0.776	0.277	PPICPE	0.539	-0.113	SVGTI	0.473	0.112	SPREAD6M	0.406	0.087
	0.5	SPREAD6M	0.781	0.271	NAPMNOI	0.66	0.239	SAVINGSL	0.47	0.133	SPREAD3M	0.443	0.218	PERMIT1	0.363	0.101
	0.7	SPREAD6M	0.637	0.31	SVSTSL	0.457	0.131	PERMITMW	0.447	0.126	SAVINGSL	0.434	0.136	NAPMNOI	0.384	0.091
	0.9	SVSTSL	0.591	0.167	SPREAD6M	0.582	0.203	SVGTI	0.578	0.199	IPDCONGD	0.4	0.065	SAVINGSL	0.297	0.106
12	0.1	SPREAD6M	0.628	0.258	NAPMPI	0.623	0.279	PERMITMW	0.584	0.239	SPREADAAA	0.539	0.219	SPREAD3M	0.486	0.261
	0.3	PERMITMW	0.811	0.279	SPREAD6M	0.74	0.24	NAPMNOI	0.726	0.328	SPREAD3M	0.653	0.231	PPIITM	0.562	-0.119
	0.5	SPREAD6M	0.813	0.333	PERMITMW	0.655	0.179	PPIITM	0.591	-0.12	SPREADAAA	0.518	0.241	SVSTSL	0.441	0.145
	0.7	SPREAD6M	0.696	0.322	SVSTSL	0.443	0.133	SAVINGSL	0.416	0.127	SPREAD10Y	0.416	0.2	HOUSTMW	0.386	0.066
	0.9	SVGTI	0.674	0.238	SVSTSL	0.587	0.136	CONSUMER	0.55	0.1	SPREAD6M	0.491	0.212	USGOVT	0.235	0.043
Rolling																
3	0.1	PPICPE	0.322	-0.139	SPREAD3M	0.279	0.151	CPILFESL	0.196	-0.05	CPIMEDSL	0.196	-0.063	UEMPMEAN	0.139	0.031
	0.3	SPREAD3M	0.381	0.232	HSN1F	0.308	0.136	SVGTI	0.178	0.049	CPIMEDSL	0.16	-0.051	SPREADBAA	0.114	0.027
	0.5	SPREAD3M	0.397	0.25	SVGTI	0.267	0.088	SVSTCBSL	0.123	0.037	NAPMPRI	0.103	-0.016	HSN1F	0.094	0.066
	0.7	SPREAD3M	0.308	0.162	SVGTI	0.228	0.083	HSN1F	0.221	0.11	AWOTMAN	0.18	0.048	SAVINGSL	0.139	0.037
	0.9	AWOTMAN	0.205	0.053	SVGTI	0.155	0.052	SPREAD3M	0.146	0.069	HSN1F	0.116	0.033	CPIMEDSL	0.11	-0.025
6	0.1	CPIMEDSL	0.388	-0.149	PPICPE	0.326	-0.153	SPREAD3M	0.301	0.258	SPREADBAA	0.155	0.058	HSN1F	0.1	0.044
	0.3	CPIMEDSL	0.4	-0.148	SVGTI	0.381	0.096	HSN1F	0.342	0.203	SPREAD3M	0.324	0.172	NAPMPRI	0.203	-0.072
	0.5	HSN1F	0.452	0.275	SVGTI	0.349	0.101	NAPMPRI	0.285	-0.105	SPREAD3M	0.285	0.123	SPREAD6M	0.217	0.102
	0.7	HSN1F	0.267	0.151	SVGCBSL	0.178	0.046	SVSTSL	0.171	0.043	NAPMPRI	0.169	-0.072	SVGTI	0.167	0.054
	0.9	SVSTSL	0.288	0.061	SVGTI	0.158	0.055	SAVINGSL	0.151	0.046	SPREAD6M	0.126	0.061	NAPMPRI	0.11	-0.04
12	0.1	SPREADBAA	0.247	0.132	SPREAD6M	0.231	0.109	CPIMEDSL	0.228	-0.08	PERMIT1	0.18	0.164	SPREADAAA	0.18	0.134
	0.3	NAPMPRI	0.39	-0.209	SPREADAAA	0.363	0.278	HSN1F	0.317	0.135	CPIMEDSL	0.279	-0.075	SPREAD3M	0.187	0.051
	0.5	NAPMPRI	0.411	-0.186	SPREADAAA	0.324	0.274	HSN1F	0.251	0.13	SPREAD6M	0.201	0.1	HOUST1F	0.192	0.072
	0.7	NAPMPRI	0.406	-0.136	SPREADAAA	0.192	0.151	USFIRE	0.189	0.054	SPREAD10Y	0.171	0.13	SAVINGSL	0.151	0.049
	0.9	SPREAD10Y	0.171	0.161	SVGTI	0.153	0.046	UEMPMEAN	0.114	0.033	HSN1FMW	0.096	0.08	USFIRE	0.096	0.018

Top 5 variables selected by LASSO at quantiles $\tau = 0.1, 0.3, 0.5, 0.7$, and 0.9 and for each forecast horizon h considered. The indicators are denoted by their FRED series ID and more details can be found in the Appendix and on the FRED website. The columns next to the variable ID report the frequency of selection in the out-of-sample period (438 months) and the average value of the parameter in the months in which the variable was selected (in interpreting the coefficients notice that the RHS variables have been standardize to have mean zero and variance one). The value of c is set to 2.

Table 9: **CPIAUCSL**

h	τ	1^{st}			2^{nd}			3^{rd}			4^{th}			5^{th}		
Recursive																
3	0.1	NAPMEI	0.767	0.344	FEDFUNDS	0.66	0.127	SPREADPRIME	0.457	-0.18	PERMIT1	0.452	0.154	CREDIT	0.269	-0.119
	0.3	NAPMEI	1	0.446	SPREADPRIME	0.699	-0.187	USFIRE	0.482	0.148	PPIITM	0.436	0.186	PPICRM	0.418	0.131
	0.5	NAPMEI	0.979	0.541	SPREADPRIME	0.571	-0.141	PPIITM	0.559	0.289	USFIRE	0.393	0.093	PERMIT1	0.283	0.107
	0.7	NAPMEI	0.603	0.269	NAPMSDI	0.425	0.13	SPREADPRIME	0.393	-0.094	SPREAD5Y	0.331	-0.11	NONREVSL	0.249	0.059
	0.9	PPIFCF	0.502	0.112	PERMITW	0.5	0.129	PPICRM	0.322	0.072	CPIUFDSL	0.208	0.054	M1SL	0.205	0.04
6	0.1	NAPMEI	0.767	0.252	SPREADPRIME	0.438	-0.154	PERMIT1	0.427	0.172	PERMITMW	0.416	0.088	FEDFUNDS	0.397	0.078
	0.3	FEDFUNDS	0.966	0.188	NAPMEI	0.838	0.212	SPREADPRIME	0.758	-0.23	M1SL	0.687	0.15	NAPMSDI	0.575	0.112
	0.5	NAPMEI	0.811	0.2	SPREADPRIME	0.779	-0.271	FEDFUNDS	0.614	0.127	PERMIT1	0.573	0.305	BOGAMBSL	0.55	0.109
	0.7	M1SL	0.669	0.163	HOUSTMW	0.642	0.134	NAPMEI	0.635	0.166	SPREADPRIME	0.482	-0.137	PERMIT1	0.47	0.205
	0.9	PERMIT	0.667	0.386	SPREAD5Y	0.39	-0.139	SPREADPRIME	0.331	-0.134	PERMITS	0.299	0.145	REALLN	0.203	0.047
12	0.1	NAPMEI	0.918	0.315	HOUSTMW	0.539	0.12	PERMITMW	0.395	0.083	PERMIT1	0.377	0.111	NAPM	0.301	0.09
	0.3	NAPMSDI	0.813	0.216	NAPMEI	0.808	0.245	PERMITMW	0.655	0.195	PERMIT1	0.582	0.372	SPREADPRIME	0.562	-0.149
	0.5	NAPMEI	0.792	0.29	PERMITMW	0.751	0.244	M1SL	0.658	0.148	USFIRE	0.648	0.142	PERMIT1	0.637	0.461
	0.7	NAPMEI	0.975	0.336	NAPMSDI	0.756	0.258	M1SL	0.724	0.172	PERMIT1	0.628	0.432	PERMITMW	0.596	0.187
	0.9	NAPMSDI	0.61	0.286	PERMITS	0.564	0.336	USFIRE	0.432	0.127	PERMITMW	0.422	0.147	PERMIT1	0.413	0.22
Rolling																
3	0.1	USWTRADE	0.265	0.113	FEDFUNDS	0.185	0.071	SPREADPRIME	0.185	-0.131	NAPMEI	0.164	0.086	MANEMP	0.155	0.047
	0.3	NAPMEI	0.379	0.234	SPREADPRIME	0.301	-0.144	USFIRE	0.196	0.026	OILPRICE	0.155	0.054	PPICRM	0.126	0.051
	0.5	SPREADPRIME	0.354	-0.266	OILPRICE	0.263	0.085	NAPMEI	0.167	0.188	USFIRE	0.084	-0.013	MANEMP	0.043	0.02
	0.7	SPREADPRIME	0.242	-0.155	NAPMEI	0.155	0.121	NAPMSDI	0.068	0.038	SPREADBAA	0.066	-0.073	RMFSL	0.05	0.069
	0.9	SPREADPRIME	0.148	-0.091	NAPMSDI	0.068	0.017	BUSLOANS	0.05	0.025	PERMITW	0.041	0.018	NAPMEI	0.037	0.015
6	0.1	NAPMEI	0.324	0.176	SPREADPRIME	0.135	-0.084	PERMITMW	0.128	0.035	FEDFUNDS	0.123	0.032	USTRADE	0.098	0.022
	0.3	SPREADPRIME	0.395	-0.184	NAPMEI	0.393	0.175	FEDFUNDS	0.26	0.085	NAPMSDI	0.18	0.11	HSN1FW	0.155	0.141
	0.5	NAPMEI	0.363	0.192	SPREADPRIME	0.345	-0.183	FEDFUNDS	0.281	0.081	HOUSTMW	0.128	0.035	HSN1FW	0.121	0.101
	0.7	SPREADPRIME	0.279	-0.16	NAPMEI	0.272	0.156	HOUSTMW	0.146	0.036	M1SL	0.11	0.032	PERMIT1	0.094	0.043
	0.9	PERMITS	0.174	0.086	SPREADPRIME	0.126	-0.071	BUSLOANS	0.112	0.033	RMFSL	0.084	0.066	HSN1F	0.062	0.045
12	0.1	NAPMEI	0.281	0.155	OTHSEC	0.146	-0.029	NAPMNOI	0.098	0.036	NAPM	0.091	0.031	HSN1FS	0.078	-0.015
	0.3	NAPMEI	0.402	0.207	PERMIT1	0.295	0.204	SPREADPRIME	0.269	-0.099	USFIRE	0.24	-0.013	NAPMPRI	0.212	0.092
	0.5	PERMIT1	0.39	0.324	NAPMEI	0.34	0.199	SPREADPRIME	0.283	-0.112	USFIRE	0.187	-0.027	HSN1FW	0.169	0.134
	0.7	PERMIT1	0.425	0.349	NAPMEI	0.347	0.219	SPREADPRIME	0.242	-0.108	PERMITS	0.196	0.114	HSN1FW	0.116	0.049
	0.9	PERMIT1	0.192	0.116	PERMITS	0.171	0.111	HSN1FW	0.137	0.144	SPREADPRIME	0.121	-0.05	HSN1F	0.105	0.066

Top 5 variables selected by LASSO at quantiles $\tau = 0.1, 0.3, 0.5, 0.7$, and 0.9 and for each forecast horizon h considered. The indicators are denoted by their FRED series ID and more details can be found in the Appendix and on the FRED website. The columns next to the variable ID report the frequency of selection in the out-of-sample period (438 months) and the average value of the parameter in the months in which the variable was selected (in interpreting the coefficients notice that the RHS variables have been standardize to have mean zero and variance one). The value of c is set to 2.

Table 10: **PCEPILFE**

h	τ	1^{st}			2^{nd}			3^{rd}			4^{th}			5^{th}		
Recursive																
3	0.1	NAPMPRI	0.372	0.072	UEMPMEAN	0.205	-0.052	SPREADBAA	0.066	-0.018	USWTRADE	0.057	0.013	PPIFCG	0.025	0.004
	0.3	USFIRE	0.701	0.109	NAPMPRI	0.491	0.12	SPREADPRIME	0.425	-0.06	SVGTI	0.046	-0.009	CPIUFDSL	0.03	0.004
	0.5	NAPMPRI	0.644	0.138	CPIUFDSL	0.47	0.063	PPIITM	0.313	0.072	SPREADPRIME	0.046	-0.012	PFCGEF	0.043	0.012
	0.7	PPIITM	0.847	0.219	CPIUFDSL	0.648	0.098	NAPMSDI	0.311	0.037	NAPMPRI	0.256	0.058	PFCGEF	0.066	0.023
	0.9	NAPMSDI	0.872	0.196	PPIITM	0.678	0.144	PFCGEF	0.639	0.193	CPIAUCSL	0.619	0.126	CPIUFDSL	0.514	0.074
6	0.1	NAPMSDI	0.217	0.046	USINFO	0.203	0.029	SPREADBAA	0.144	-0.026	NAPMPRI	0.137	0.022	BOGAMBSL	0.046	0.006
	0.3	USFIRE	0.897	0.161	NAPMPRI	0.621	0.163	NAPMSDI	0.502	0.087	PERMITMW	0.064	0.016	M1SL	0.062	0.01
	0.5	NAPMPRI	0.911	0.232	USFIRE	0.55	0.084	PPIITM	0.285	0.074	CPIUFDSL	0.194	0.028	M1SL	0.053	0.007
	0.7	NAPMSDI	0.993	0.184	PPIITM	0.938	0.218	CPIUFDSL	0.482	0.075	UEMPMEAN	0.324	-0.051	NAPMPRI	0.281	0.063
	0.9	NAPMSDI	0.902	0.232	PPIITM	0.822	0.164	UEMPMEAN	0.553	-0.148	SPREAD10Y	0.445	-0.101	NAPMPRI	0.436	0.144
12	0.1	NAPMSDI	0.452	0.081	NAPMPRI	0.269	0.048	SVGTI	0.121	-0.032	SVSTSL	0.096	-0.028	SPREADBAA	0.084	-0.025
	0.3	NAPMSDI	0.774	0.185	USFIRE	0.651	0.128	UEMPMEAN	0.578	-0.092	REALLN	0.338	0.072	NAPMPRI	0.308	0.09
	0.5	NAPMSDI	0.829	0.266	USFIRE	0.594	0.085	UEMPMEAN	0.527	-0.103	NAPMPRI	0.45	0.175	REALLN	0.427	0.085
	0.7	NAPMSDI	0.984	0.396	PPIITM	0.605	0.127	UEMPMEAN	0.53	-0.134	NAPMPRI	0.45	0.162	USFIRE	0.361	0.056
	0.9	NAPMSDI	0.925	0.379	NAPMPRI	0.884	0.273	UEMPMEAN	0.626	-0.179	SPREAD3Y	0.571	-0.138	SPREAD5Y	0.32	-0.075
Rolling																
3	0.1	SPREADBAA	0.023	-0.006	PFCGEF	0.021	0.009	HSN1FS	0.018	0.005	CPIENGSL	0.014	0.005	UEMP27OV	0.014	-0.002
	0.3	SVGTI	0.105	-0.017	SPREADBAA	0.103	-0.035	PERMITNE	0.078	0.012	USFIRE	0.046	0.01	HOUSTNE	0.041	0.008
	0.5	CPIUFDSL	0.135	0.031	SPREADBAA	0.098	-0.028	PPIITM	0.078	0.028	NAPMPRI	0.073	0.015	SVGTI	0.064	-0.011
	0.7	PFCGEF	0.244	0.095	PPIITM	0.233	0.066	NAPMPRI	0.137	0.036	NAPMNOI	0.112	0.019	CPIAUCSL	0.068	0.02
	0.9	PFCGEF	0.315	0.147	PPIITM	0.274	0.077	NAPMPRI	0.103	0.034	TWEXOMTH	0.08	0.034	CPIUFDSL	0.075	0.019
6	0.1	SVSTSL	0.05	-0.007	NAPMPRI	0.034	0.01	UEMP15OV	0.032	-0.007	PERMIT5	0.025	0.006	USFIRE	0.025	0.007
	0.3	USFIRE	0.155	0.026	SPREADBAA	0.13	-0.028	PERMITNE	0.114	0.022	NAPMPRI	0.094	0.034	CPIUFDSL	0.084	0.019
	0.5	NAPMPRI	0.167	0.048	HOUSTNE	0.13	0.023	UEMPMEAN	0.11	-0.021	PPIITM	0.096	0.026	NAPMSDI	0.094	0.026
	0.7	PPIITM	0.221	0.061	NAPMPRI	0.212	0.058	PFCGEF	0.189	0.079	NAPMSDI	0.174	0.056	PERMITNE	0.148	0.026
	0.9	PFCGEF	0.306	0.147	PPIITM	0.263	0.067	NAPMPRI	0.199	0.065	UEMP5TO14	0.187	-0.054	NAPMSDI	0.155	0.048
12	0.1	UEMP15OV	0.153	-0.038	SVGTI	0.105	-0.028	SVSTSL	0.098	-0.007	SPREADBAA	0.059	-0.018	NAPMPRI	0.048	0.011
	0.3	NAPMPRI	0.249	0.102	PERMITNE	0.169	0.022	UEMP15OV	0.148	-0.036	UEMPMEAN	0.128	-0.048	HSN1F	0.114	0.027
	0.5	NAPMPRI	0.322	0.135	UEMPMEAN	0.189	-0.053	NAPMSDI	0.148	0.076	PERMITNE	0.132	0.025	UEMP15OV	0.11	-0.029
	0.7	NAPMPRI	0.4	0.155	NAPMSDI	0.21	0.109	CPIENGSL	0.208	0.063	UEMP5TO14	0.176	-0.042	PFCGEF	0.174	0.036
	0.9	NAPMPRI	0.308	0.134	NAPMSDI	0.208	0.094	UEMP5TO14	0.203	-0.056	SPREAD3M	0.1	-0.04	SPREAD5Y	0.1	-0.03

Top 5 variables selected by LASSO at quantiles $\tau = 0.1, 0.3, 0.5, 0.7,$ and 0.9 and for each forecast horizon h considered. The indicators are denoted by their FRED series ID and more details can be found in the Appendix and on the FRED website. The columns next to the variable ID report the frequency of selection in the out-of-sample period (438 months) and the average value of the parameter in the months in which the variable was selected (in interpreting the coefficients notice that the RHS variables have been standardize to have mean zero and variance one). The value of c is set to 2.

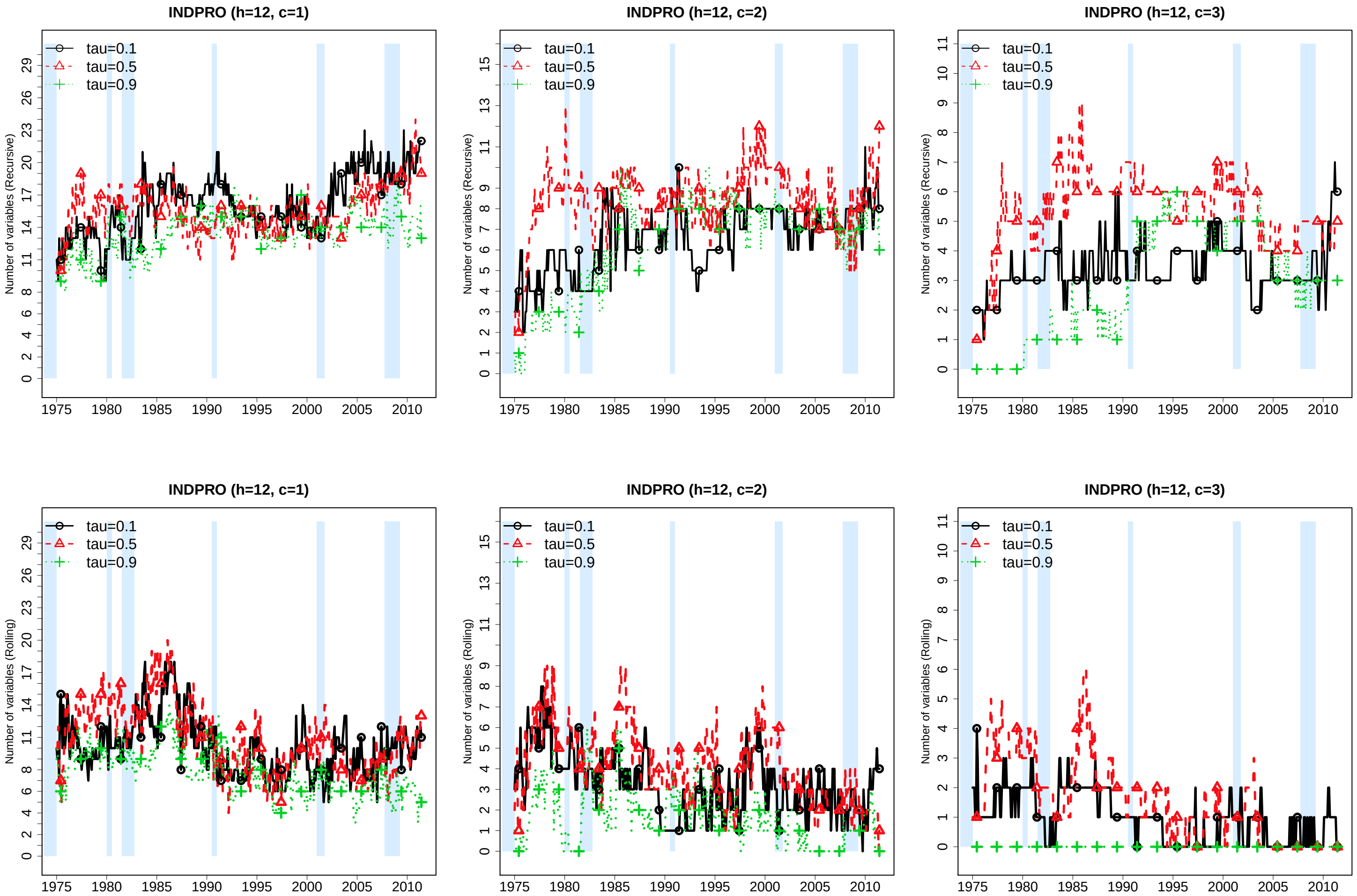

Figure 5: Number of variables selected by LASSO for INDPRO at quantiles $\tau = 0.1, 0.5$, and 0.9 for $h = 12$ and for values of c equal to 1, 2, and 3 (columns). The top row refers to the recursive window estimation and the bottom one to the rolling case.

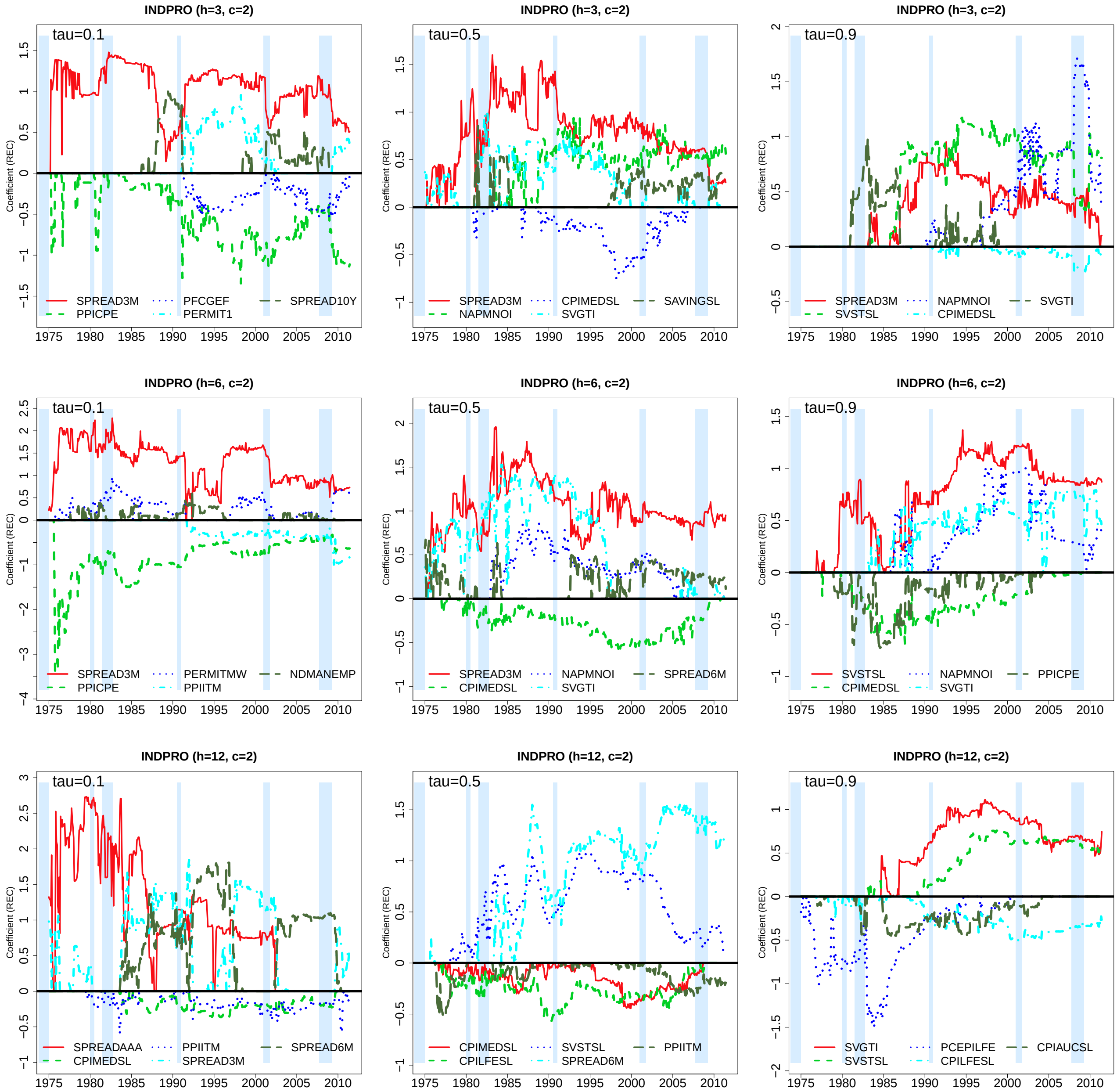

Figure 6: Time series of the coefficient estimates for the top 5 variables selected by LASSO to forecast INDPRO for $h = 3$, 6, and 12 (rows) and the quantile level $\tau = 0.1$, 0.5 and 0.9 (columns). The value of the shrinkage parameter c is equal to 2.

Table 11: **INDPRO - forecasting negative h-period change**

	h	MAE	t-stat	t-stat	MSE	t-stat	t-stat	LPS	t-stat	t-stat
QAR ROLL	3	0.30		*1.96*	0.13		*1.96*	0.42		*2.01*
DYN PROBIT		0.26	**-1.96**		0.11	**-1.96**		0.35	**-2.01**	
PROBIT		0.27	-1.07	1.34	0.12	-0.80	1.03	0.39	-0.63	1.56
QAR REC		0.31	1.04	*3.07*	0.13	-1.31	*1.88*	0.41	-0.72	*2.34*
FA-QAR REC K=3		0.25	**-3.00**	-0.63	0.10	**-3.08**	-0.56	0.34	**-2.98**	-0.40
FA-QAR REC K=5		0.26	**-2.38**	-0.29	0.11	**-2.13**	-0.02	0.36	**-1.98**	0.12
FA-QAR ROLL K=3		0.25	**-2.46**	-0.35	0.12	-1.34	0.58	0.40	-0.68	0.71
FA-QAR ROLL K=5		0.26	**-2.20**	-0.24	0.12	-0.93	0.86	0.40	-0.58	0.86
LASSO-QAR REC		0.27	**-3.39**	0.51	0.11	**-3.57**	0.06	0.36	**-3.20**	0.15
POST-LASSO-QAR REC		0.25	**-3.17**	-0.61	0.10	**-2.91**	-0.57	0.34	**-3.02**	-0.59
TFA-QAR REC		0.25	**-3.47**	-0.88	0.10	**-3.24**	-0.83	0.33	**-3.37**	-0.89
LASSO-QAR ROLL		0.29	-3.14	1.29	0.12	**-2.38**	1.40	0.40	**-2.76**	1.40
POST-LASSO-QAR ROLL		0.28	**-1.64**	0.89	0.12	-1.50	0.93	0.39	**-1.65**	0.99
TFA-QAR ROLL		0.28	**-1.69**	0.90	0.12	-1.49	0.99	0.39	**-1.64**	1.03
QAR ROLL	6	0.31		*2.92*	0.15		*2.77*	0.47		*2.73*
DYN PROBIT		0.23	**-2.92**		0.11	**-2.77**		0.34	**-2.73**	
PROBIT		0.23	**-2.35**	0.36	0.10	**-2.53**	-0.52	0.35	**-2.28**	0.17
QAR REC		0.30	-0.71	*3.45*	0.13	**-1.98**	*1.91*	0.42	-1.46	*2.27*
FA-QAR REC K=3		0.24	**-3.12**	0.51	0.10	**-2.57**	-0.18	0.37	**-1.71**	0.47
FA-QAR REC K=5		0.25	**-1.93**	1.03	0.12	-1.60	0.39	0.50	0.22	1.23
FA-QAR ROLL K=3		0.27	-1.35	1.66	0.15	-0.06	*1.99*	0.51	0.73	2.05
FA-QAR ROLL K=5		0.27	-1.40	1.34	0.14	-0.24	1*1.89*	0.54	0.70	1.99
LASSO-QAR REC		0.25	**-3.80**	1.05	0.11	**-3.24**	0.06	0.35	**-3.14**	0.12
POST-LASSO-QAR REC		0.25	**-2.29**	1.06	0.12	**-1.72**	0.63	0.37	**-1.89**	0.60
TFA-QAR REC		0.24	**-2.79**	0.61	0.11	**-2.27**	0.22	0.35	**-2.40**	0.16
LASSO-QAR ROLL		0.28	**-2.82**	*2.27*	0.13	**-2.37**	*1.75*	0.42	**-2.28**	*1.70*
POST-LASSO-QAR ROLL		0.27	**-2.03**	*1.83*	0.14	-1.42	*1.87*	0.42	-1.51	*1.78*
TFA-QAR ROLL		0.27	**-1.85**	*1.88*	0.13	-1.37	*1.82*	0.42	-1.45	*1.75*
QAR ROLL	12	0.31		*2.88*	0.16		*1.74*	0.55		0.70
DYN PROBIT		0.19	**-2.88**		0.11	**-1.74**		0.44	-0.70	
PROBIT		0.18	-3.08	-0.93	0.09	-2.60	-1.22	0.33	**-2.22**	-1.54
QAR REC		0.29	-1.03	*3.42*	0.13	**-1.94**	0.97	0.43	**-1.89**	-0.10
FA-QAR REC K=3		0.21	**-2.98**	1.00	0.12	-1.29	1.15	0.53	-0.10	0.68
FA-QAR REC K=5		0.23	**-2.19**	1.81	0.12	-1.16	1.06	0.50	-0.30	0.95
FA-QAR ROLL K=3		0.25	**-1.88**	1.92	0.15	-0.47	*2.26*	0.58	0.24	1.14
FA-QAR ROLL K=5		0.23	**-2.35**	1.56	0.14	-0.74	*2.40*	0.77	0.97	*2.17*
LASSO-QAR REC		0.20	**-4.08**	0.42	0.10	**-2.83**	-0.82	0.31	**-3.27**	-1.17
POST-LASSO-QAR REC		0.20	**-3.77**	0.47	0.10	**-2.29**	-0.41	0.33	**-2.70**	-1.04
TFA-QAR REC		0.20	**-4.05**	0.31	0.10	**-2.46**	-0.68	0.32	**-2.89**	-1.12
LASSO-QAR ROLL		0.25	**-3.22**	*1.85*	0.14	**-2.58**	1.00	0.46	**-2.89**	0.10
POST-LASSO-QAR ROLL		0.23	**-3.17**	1.42	0.13	**-2.18**	0.82	0.41	**-2.52**	-0.28
TFA-QAR ROLL		0.23	**-3.08**	1.47	0.13	**-2.16**	0.84	0.42	**-2.41**	-0.21

The values represent the MAE, MSE, and LPS score functions that evaluate the probability forecasts that the h-period growth rate of INDPRO is negative (for h = 3, 6 and 12), while the two adjacent columns report the t-statistics for the null hypothesis of equal accuracy of the models to the benchmarks which are QAR ROLL or DYN PROBIT. A negative value of the test statistic indicates that the alternative model outperforms the benchmark (QAR ROLL or DYN PROBIT) and in bold the rejections of equal accuracy at 5% against the one-sided alternative that the benchmark is outperformed. The value of c is set to 2.

Table 12: **PAYEMS - forecasting negative h-period change**

	h	MAE	t-stat	t-stat	MSE	t-stat	t-stat	LPS	t-stat	t-stat
QAR ROLL	3	0.23		0.43	0.14		*2.13*	0.50		1.62
DYN PROBIT		0.22	-0.43		0.11	**-2.13**		0.37	-1.62	
PROBIT		0.26	0.73	1.39	0.12	-0.70	0.39	0.41	-0.65	0.71
QAR REC		0.23	-0.18	0.61	0.12	**-2.24**	1.39	0.38	-1.36	0.22
FA-QAR REC K=3		0.19	**-2.76**	**-2.75**	0.10	-3.07	-1.02	0.33	**-2.41**	**-1.98**
FA-QAR REC K=5		0.20	**-2.79**	**-2.33**	0.11	-2.73	-0.28	0.38	**-2.71**	0.10
FA-QAR ROLL K=3		0.21	**-2.90**	-1.24	0.13	**-1.90**	1.24	0.46	**-2.04**	1.18
FA-QAR ROLL K=5		0.20	**-2.78**	-1.27	0.13	-1.61	0.87	0.77	1.04	1.23
LASSO-QAR REC		0.21	**-2.67**	**-2.00**	0.12	**-3.08**	0.17	0.35	**-1.78**	-1.01
POST-LASSO-QAR REC		0.19	**-3.17**	**-3.45**	0.10	**-3.12**	-1.15	0.33	**-2.31**	**-1.75**
TFA-QAR REC		0.19	**-3.10**	**-3.17**	0.10	**-3.12**	-0.98	0.33	**-2.25**	-1.56
LASSO-QAR ROLL		0.22	**-3.72**	-0.35	0.14	**-2.64**	1.71	0.48	**-2.66**	1.39
POST-LASSO-QAR ROLL		0.20	**-3.53**	-1.62	0.12	**-2.63**	0.62	0.45	**-2.97**	0.94
TFA-QAR ROLL		0.20	**-3.71**	-1.49	0.12	**-2.62**	0.73	0.45	**-2.86**	1.00
QAR ROLL	6	0.26		1.38	0.17		*2.49*	0.62		*1.98*
DYN PROBIT		0.23	-1.38		0.12	**-2.49**		0.39	**-1.98**	
PROBIT		0.22	-0.99	-0.26	0.11	**-2.10**	-0.85	0.36	-1.47	-0.43
QAR REC		0.25	-1.09	1.23	0.14	**-2.04**	1.30	0.43	-1.27	0.88
FA-QAR REC K=3		0.21	**-2.77**	-1.23	0.13	**-2.47**	0.38	0.37	**-1.88**	-0.61
FA-QAR REC K=5		0.22	**-2.22**	-0.66	0.14	**-1.80**	0.80	0.40	**-1.64**	0.25
FA-QAR ROLL K=3		0.24	-1.60	0.55	0.16	-0.49	*2.04*	0.58	-1.11	*1.80*
FA-QAR ROLL K=5		0.24	-1.21	0.43	0.17	-0.05	*1.66*	0.67	0.72	*1.97*
LASSO-QAR REC		0.22	**-3.08**	-0.70	0.13	**-2.54**	0.81	0.41	-1.44	0.50
POST-LASSO-QAR REC		0.21	**-3.04**	-1.48	0.13	**-2.55**	0.22	0.40	-1.55	0.12
TFA-QAR REC		0.21	**-3.08**	-1.61	0.13	**-2.61**	0.25	0.40	-1.59	0.20
LASSO-QAR ROLL		0.25	**-2.23**	0.80	0.16	-1.17	*2.16*	0.61	-0.95	*1.82*
POST-LASSO-QAR ROLL		0.24	-1.39	0.77	0.16	-0.43	*2.05*	0.61	-0.21	*1.85*
TFA-QAR ROLL		0.24	-1.57	0.70	0.16	-0.72	*1.94*	0.60	-0.58	*1.78*
QAR ROLL	12	0.26		*2.17*	0.16		*2.31*	0.63		*2.00*
DYN PROBIT		0.20	**-2.17**		0.11	**-2.31**		0.35	**-2.00**	
PROBIT		0.18	**-2.18**	-1.16	0.10	**-2.38**	-1.10	0.31	**-2.05**	-1.21
QAR REC		0.25	-0.59	*2.85*	0.13	**-1.75**	*1.87*	0.44	-1.52	*2.08*
FA-QAR REC K=3		0.21	**-2.21**	0.31	0.14	-1.20	*2.27*	0.42	-1.62	*1.70*
FA-QAR REC K=5		0.22	-1.17	0.74	0.15	-0.37	1.97	0.48	-0.98	2.01
FA-QAR ROLL K=3		0.22	**-1.84**	0.75	0.15	-0.74	*2.23*	0.78	0.61	1.61
FA-QAR ROLL K=5		0.23	-1.04	1.07	0.16	-0.05	*2.42*	0.67	0.29	*2.14*
LASSO-QAR REC		0.20	**-3.59**	-0.12	0.12	**-2.81**	0.81	0.36	*-2.04*	0.31
POST-LASSO-QAR REC		0.20	**-3.21**	-0.31	0.12	**-2.35**	0.90	0.37	**-1.89**	0.47
TFA-QAR REC		0.20	**-3.31**	-0.29	0.12	**-2.44**	0.90	0.37	**-1.96**	0.44
LASSO-QAR ROLL		0.24	**-1.84**	*1.66*	0.15	-0.62	*2.18*	0.64	0.23	*2.01*
POST-LASSO-QAR ROLL		0.24	-1.12	*1.95*	0.15	-0.69	*2.13*	0.56	-1.13	1.60
TFA-QAR ROLL		0.24	-1.18	*1.94*	0.15	-0.87	*2.07*	0.55	-1.23	1.57

The values represent the MAE, MSE, and LPS score functions that evaluate the probability forecasts that the h-period growth rate of PAYEMS is negative (for $h = 3$, 6 and 12), while the two adjacent columns report the t-statistics for the null hypothesis of equal accuracy of the models to the benchmarks which are QAR ROLL or DYN PROBIT. A negative value of the test statistic indicates that the alternative model outperforms the benchmark (QAR ROLL or DYN PROBIT) and in bold the rejections of equal accuracy at 5% against the one-sided alternative that the benchmark is outperformed. The value of c is set to 2.

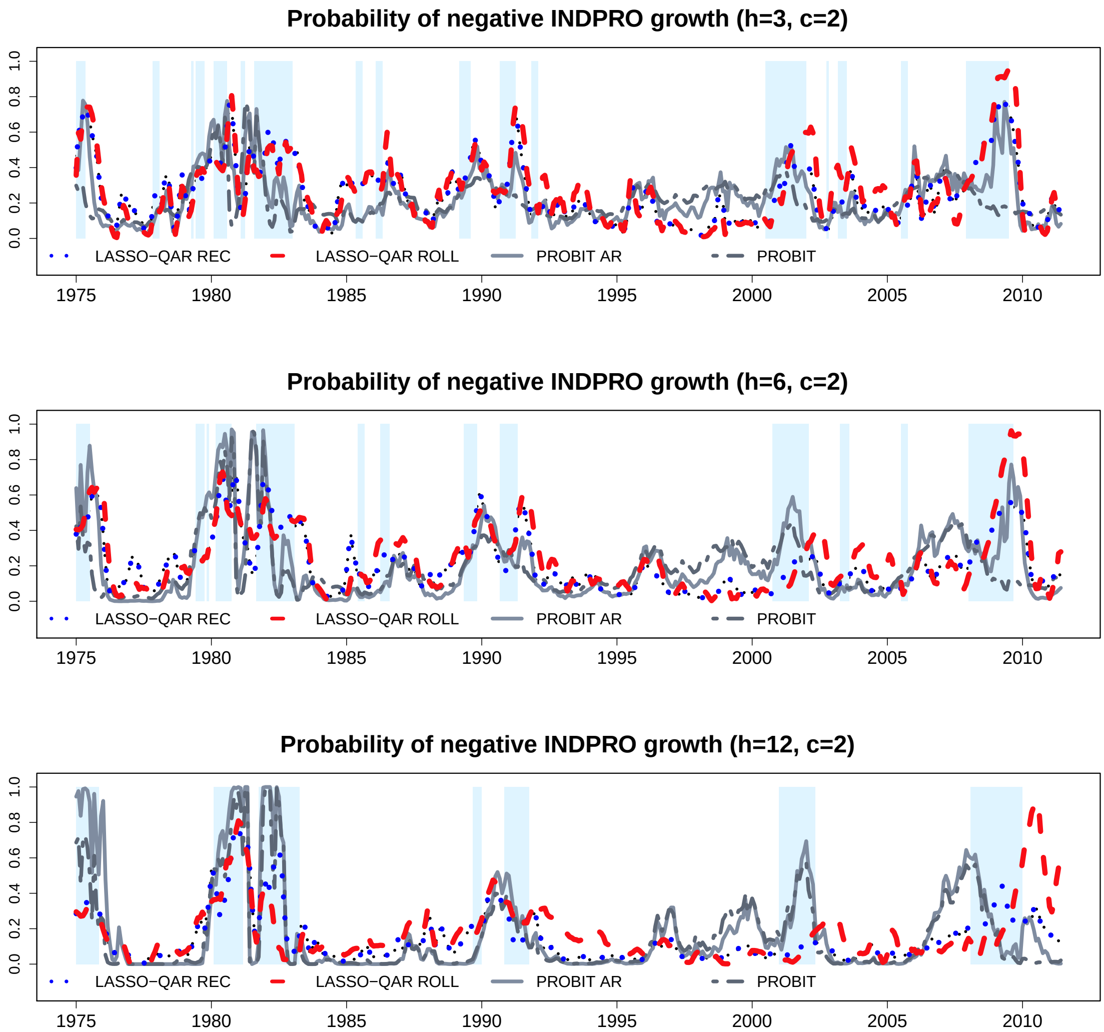

Figure 7: Probability forecasts of negative PAYEMS h-month growth for $h = 3$ (top), $h = 6$ (middle), and $h = 12$ (bottom) for the LASSO quantile model estimated on a recursive or rolling window and the yield spread probit models. The shaded area represent the months in which the realization of the variable is negative.

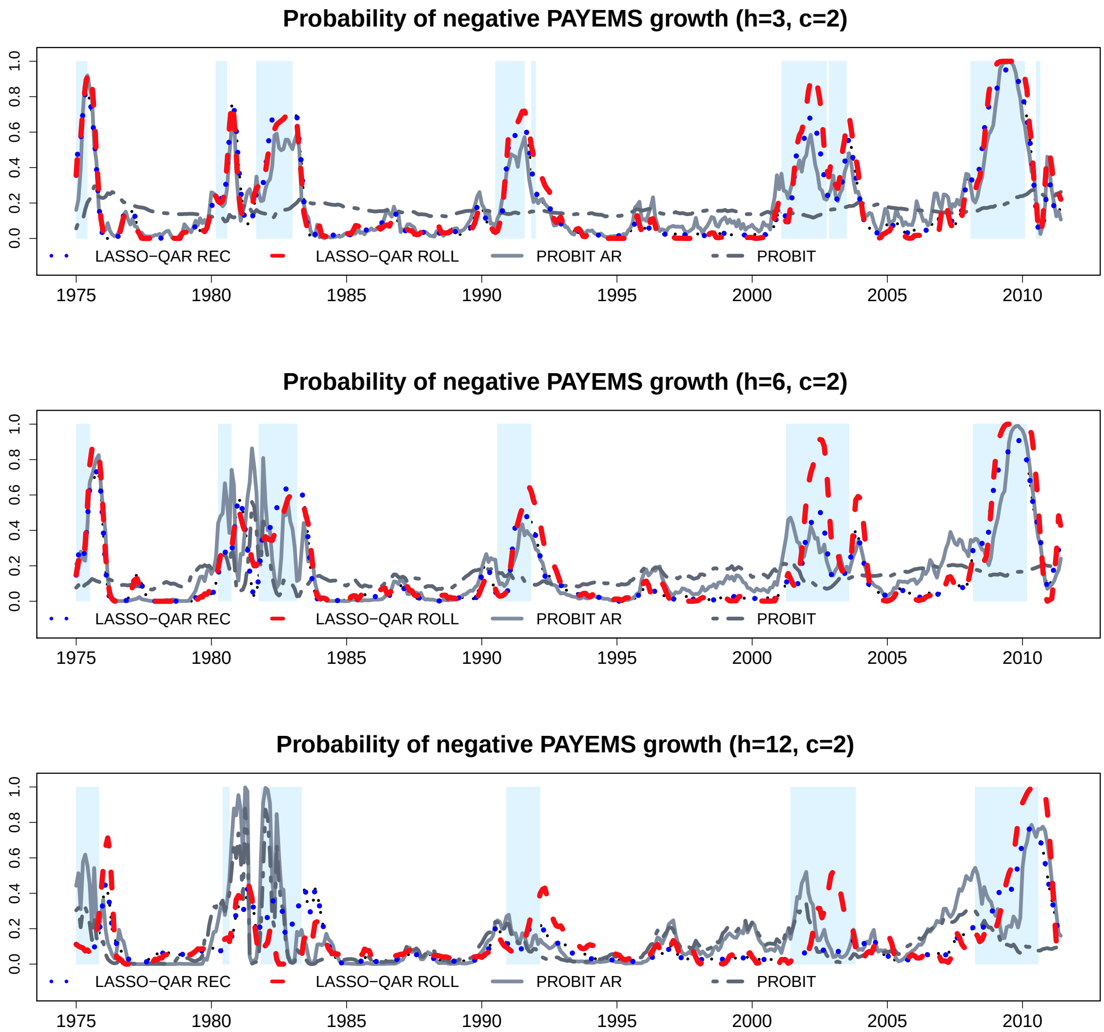

Figure 8: Probability forecasts of negative PAYEMS h-month growth for $h = 3$ (top), $h = 6$ (middle), and $h = 12$ (bottom) for the LASSO quantile model estimated on a recursive or rolling window and the yield spread probit models. The shaded area represent the months in which the realization of the variable is negative.

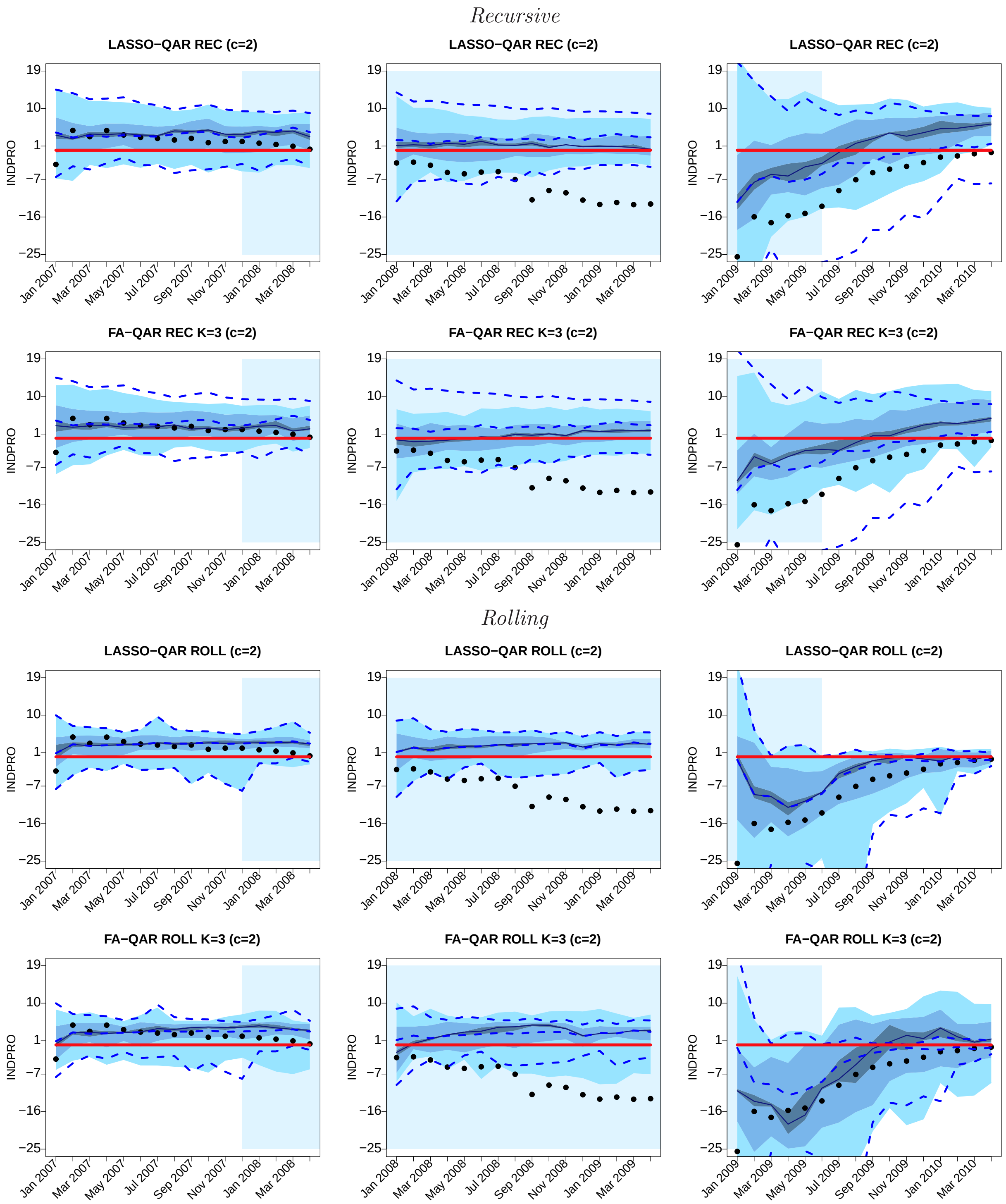

Figure 9: Quantile forecasts for the h-month ($h = 1, \cdots, 16$) growth of INDPRO made at the end of January 2007 (left column), January 2008 (center column), and January 2009 (right column). To save space, we report the quantile forecasts for only four models: LASSO-QAR REC (first row), LASSO-QAR ROLL (second row), FA-QAR REC (third row), and FA-QAR ROLL (last row) where for both factor models we used $K = 3$. The predictive quantiles reported are for $\tau = 0.05, 0.25, 0.45, 0.55, 0.75, 0.95$. The dashed lines denote the forecasts of the QAR model estimated recursively or rolling based on the model they are compared to, and the dots denote the realization of the variable. The shaded area starting in December 2007 denotes the start of the recession as determined by the NBER Business Cycle Dating Committee.

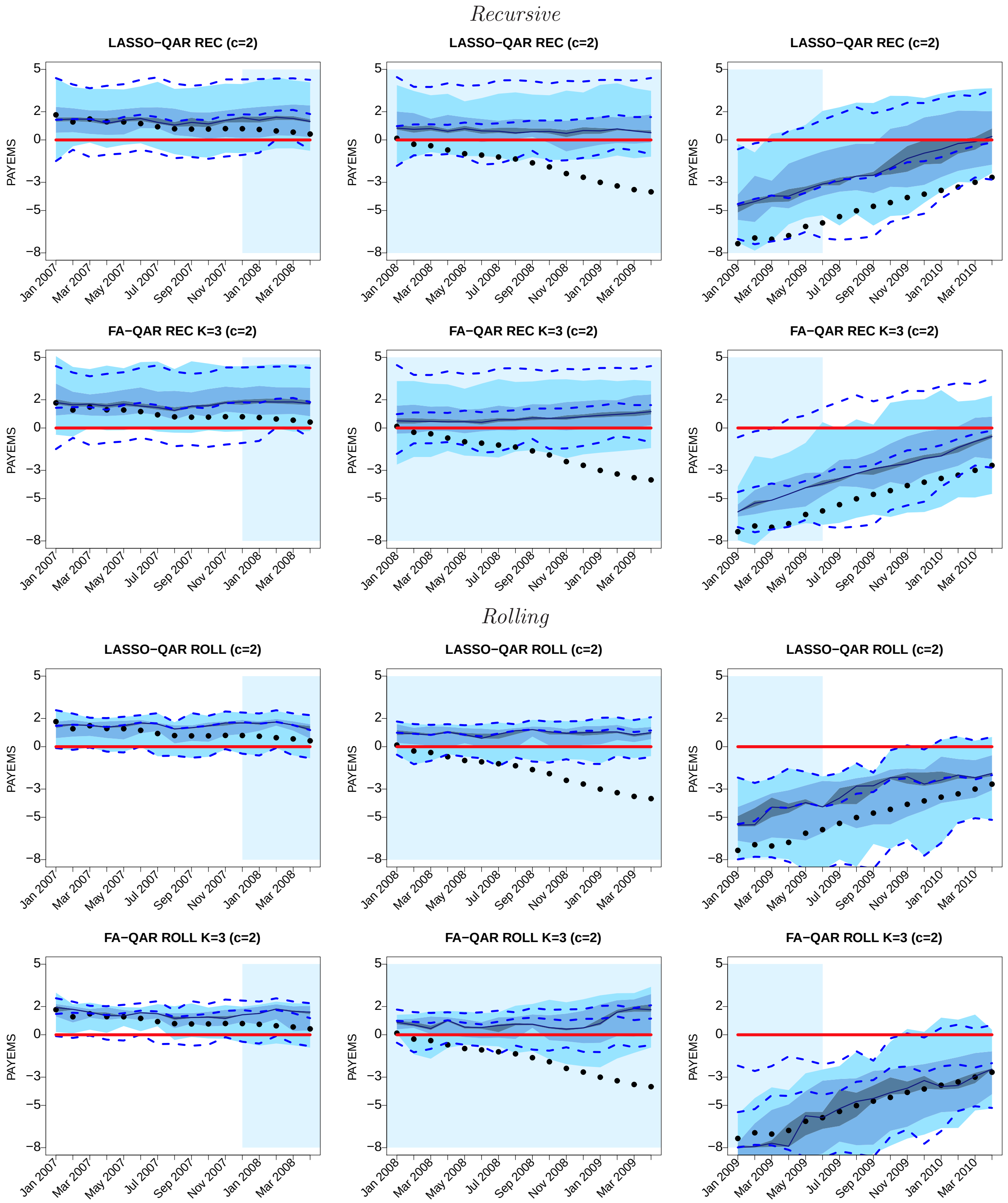

Figure 10: Quantile forecasts for the h-month ($h = 1, \cdots, 16$) growth of PAYEMS made at the end of January 2007 (left column), January 2008 (center column), and January 2009 (right column). To save space, we report the quantile forecasts for only four models: LASSO-QAR REC (first row), LASSO-QAR ROLL (second row), FA-QAR REC (third row), and FA-QAR ROLL (last row) where for both factor models we used $K = 3$. The predictive quantiles reported are for $\tau = 0.05, 0.25, 0.45, 0.55, 0.75, 0.95$. The dashed lines denote the forecasts of the QAR model estimated recursively or rolling based on the model they are compared to, and the dots denote the realization of the variable. The shaded area starting in December 2007 denotes the start of the recession as determined by the NBER Business Cycle Dating Committee.

Recursive

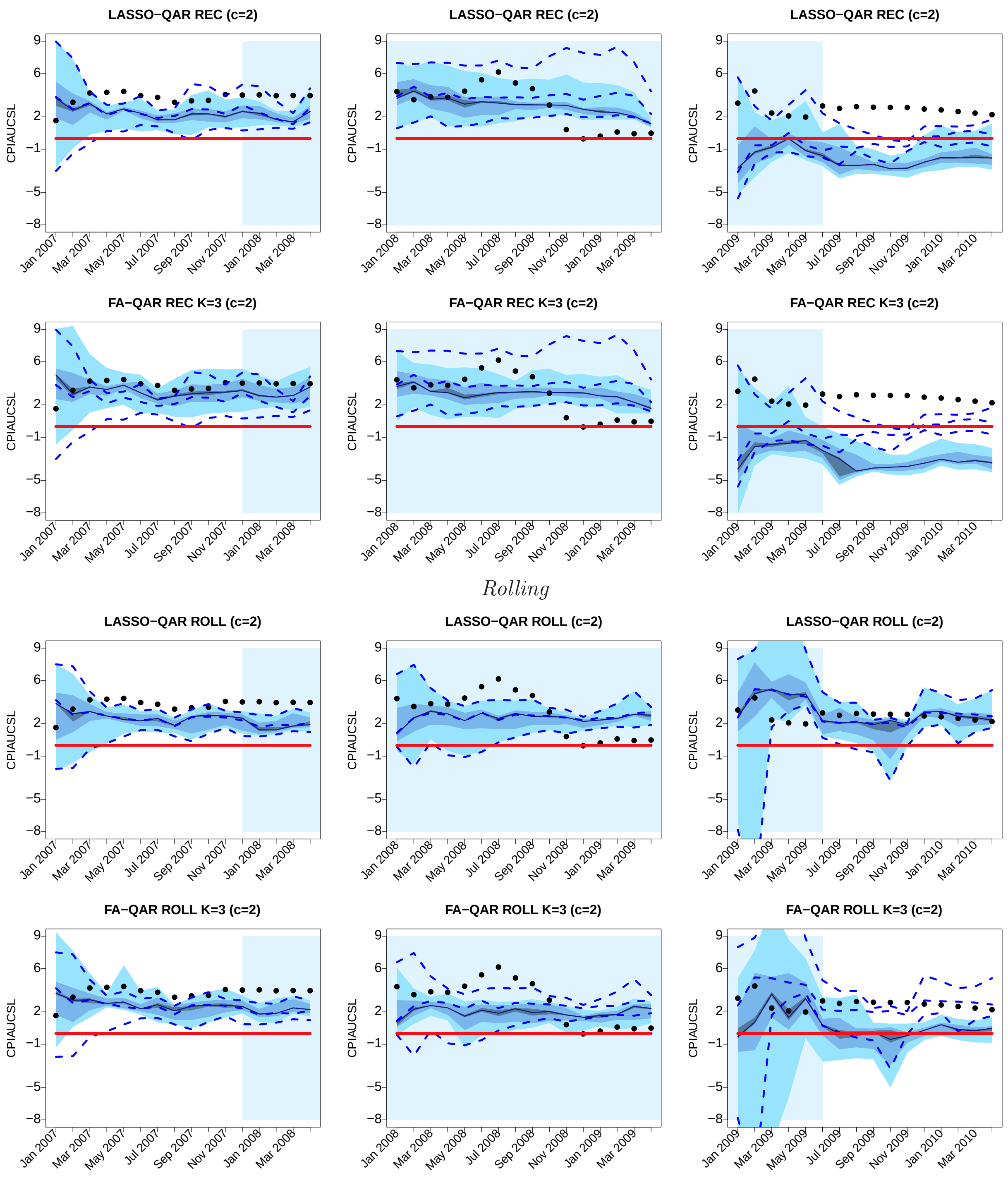

Figure 11: Quantile forecasts for the h-month ($h = 1, \cdots, 16$) growth of CPIAUCSL made at the end of January 2007 (left column), January 2008 (center column), and January 2009 (right column). To save space, we report the quantile forecasts for only four models: LASSO-QAR REC (first row), LASSO-QAR ROLL (second row), FA-QAR REC (third row), and FA-QAR ROLL (last row) where for both factor models we used $K = 3$. The predictive quantiles reported are for $\tau = 0.05, 0.25, 0.45, 0.55, 0.75, 0.95$. The dashed lines denote the forecasts of the QAR model estimated recursively or rolling based on the model they are compared to, and the dots denote the realization of the variable. The shaded area starting in December 2007 denotes the start of the recession as determined by the NBER Business Cycle Dating Committee.

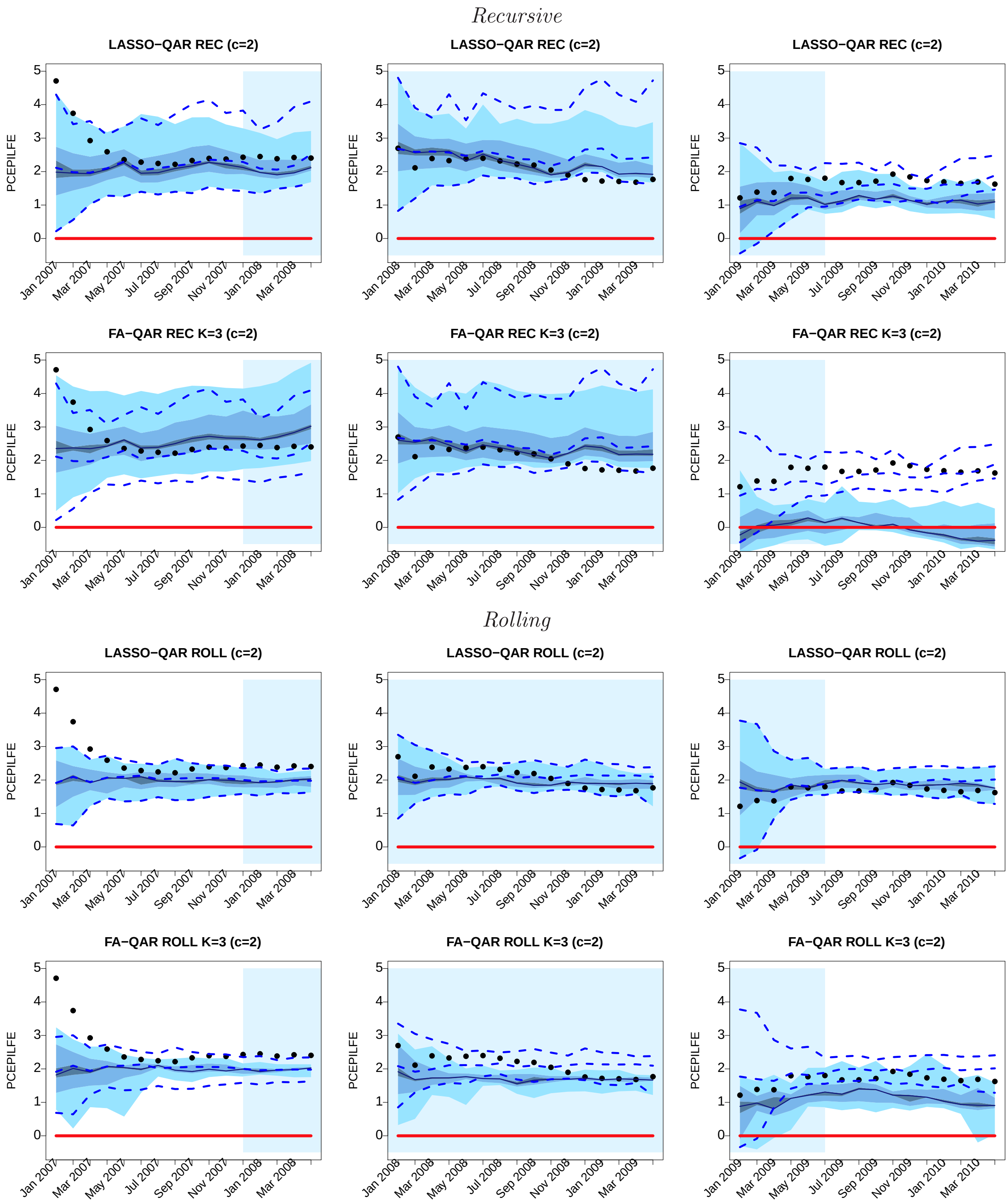

Figure 12: Quantile forecasts for the h-month ($h = 1, \cdots, 16$) growth of PCEPILFE made at the end of January 2007 (left column), January 2008 (center column), and January 2009 (right column). To save space, we report the quantile forecasts for only four models: LASSO-QAR REC (first row), LASSO-QAR ROLL (second row), FA-QAR REC (third row), and FA-QAR ROLL (last row) where for both factor models we used $K = 3$. The predictive quantiles reported are for $\tau = 0.05, 0.25, 0.45, 0.55, 0.75, 0.95$. The dashed lines denote the forecasts of the QAR model estimated recursively or rolling based on the model they are compared to, and the dots denote the realization of the variable. The shaded area starting in December 2007 denotes the start of the recession as determined by the NBER Business Cycle Dating Committee.