Risk-Adjusted Capital Allocation and Misallocation[*]

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Abstract

We develop a theory linking misallocation, i.e., dispersion in static marginal products of capital (MPK), to systematic investment risks. In our setup, firms differ in their exposure to these risks, leading to heterogeneity in firm-level risk premia and thus MPK. The theory predicts that cross-sectional dispersion in MPK (i) depends on cross-sectional dispersion in risk exposures and (ii) fluctuates with the price of risk, and thus is countercyclical. We empirically evaluate these predictions and document strong support for them. We devise a novel empirical strategy to quantify variation in firm-level risk exposures using data on the dispersion of expected stock market returns. Our estimates imply that risk considerations explain about 40% of observed MPK dispersion among US firms and in particular, can rationalize a large persistent component in firm-level MPK deviations. Our framework provides a sharp link between cross-sectional asset pricing, aggregate (e.g., business cycle) volatility and long-run economic performance – MPK dispersion induced by risk premium effects, although not *prima facie* inefficient, lowers the average level of aggregate TFP by as much as 8%.

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1 Introduction

A large and growing body of work has documented the "misallocation" of resources across firms, measured by dispersion in the marginal product of inputs into production. Further, the failure of marginal product equalization has been shown to have potentially sizable negative effects on aggregate outcomes, such as productivity and output. Recent studies have found that even after accounting for a host of leading candidates – for example, adjustment costs, financial frictions, or imperfect information – a substantial portion of observed misallocation seems to stem from other firm-specific factors, specifically, of a type that are orthogonal to firm fundamentals and are extremely persistent (if not permanent) to the firm. Identifying exactly what – if any – underlying economic mechanisms can lead to this type of distortion has proven puzzling.

In this paper, we propose, empirically test and quantitatively evaluate just such a mechanism. Our theory links capital misallocation to systematic investment risks. To the best of our knowledge, we are the first to make the connection between standard notions of the risk-return tradeoff faced by investors and the resulting dispersion in the marginal product of capital (MPK) across firms. Indeed, our framework provides a natural way to translate the findings of the rich literature on cross-sectional asset pricing into the implications for the capital allocation across firms. Further, we are able to quantify the effects of risk considerations – e.g., dispersion in risk premia across firms and the nature of business cycle volatility – on aggregate economic outcomes, such as total factor productivity (TFP). Through the marginal product dispersion they induce, risk premia effects – although not *prima facie* inefficient – influence both the long-run level and higher frequency fluctuations in measured TFP.

Our point of departure is a standard neoclassical model of firm investment in the face of both aggregate and idiosyncratic uncertainty. Firms discount future payoffs using a stochastic discount factor that is also a function of aggregate conditions. Critically, this setup implies that firms optimally equalize not necessarily MPK, but expected, appropriately discounted, MPK.[1] With little more structure than this, the framework gives rise to an asset pricing equation governing the firm's expected MPK – firms with higher exposure to the aggregate risk factors require a higher risk premium on investments, which translates into a higher expected MPK. This firm-specific risk premium appears exactly as what would otherwise be labeled a persistent idiosyncratic "distortion" (in the sense that it shows up as a persistent firm-specific wedge in the Euler equation). In fact, the model implies a beta pricing equation of exactly the same form that is often used to price the cross-section of stock market returns. That equation simply states that a firm's expected MPK should be linked to the exposure of its MPK to systematic

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[1]Importantly, this is a statement only about expected MPK; realized MPKs may differ across firms for additional reasons, i.e., uncertainty over future shocks.

risk (i.e., the firm's "beta"), and the price of that risk.

We begin our analysis by demonstrating that the simple logic of the pricing equation contains substantial empirical content. We state and empirically investigate four key predictions of our general framework – (i) exposure to standard risk factors priced in asset markets is an important determinant of expected MPK, (ii) movements in factor risk prices are linked to fluctuations in the conditional expected MPK, (iii) MPK dispersion is positively related to beta dispersion, and (iv) movements in factor risk prices are linked to fluctuations in MPK dispersion. We use a combination of firm-level production and stock market data to provide empirical support for each of these predictions. For example, (i) high MPK firms tend to offer high expected stock returns, suggesting that higher MPK is linked to higher exposure to systematic risk, (ii) common return predictors such as credit spreads and the aggregate price/dividend ratio predict fluctuations in firm-level MPK, (iii) in the cross-section, industries with higher dispersion in factor exposures, i.e., betas, have higher dispersion in MPK, and (iv) both MPK dispersion and the return on a portfolio of high-minus-low MPK stocks contain predictable, and in fact countercyclical, components, as indicated by standard return and macroeconomic predictors such as credit spreads, excess bond premia, and the price/dividend ratio.

After establishing these empirical results, we interpret them and gauge their magnitudes through the lens of a quantitative model. To that end, we develop a theory of firm investment dynamics in the face of both idiosyncratic and aggregate risk, in the form of shocks to productivity. We add two key elements to this framework – first, an exogenously specified stochastic discount factor designed to match standard asset pricing moments, as has become standard in the cross-sectional asset pricing literature (e.g., Zhang (2005) and Gomes and Schmid (2010)). Second, we allow for *ex-ante* cross-sectional heterogeneity in exposure, that is, beta, with respect to the systematic productivity risk. In other words, the productivity of high beta firms is highly sensitive to the realization of aggregate productivity, low beta firms have low sensitivity, and indeed, the productivity of firms with negative beta may move countercyclically. The investment side of the model is analytically tractable and yields sharp characterizations of firm investment decisions and MPK.

This setup is consistent with the key empirical results described above, namely, firm-level expected MPKs are dependent on exposures to the aggregate shock and are countercyclical, as is the cross-sectional dispersion in expected MPK. Further, we derive an expression for aggregate TFP, which is a strictly decreasing function of MPK dispersion. In other words, by inducing MPK dispersion, greater cross-sectional variation in factor risk exposures or a higher price of risk reduce the average long-run level of TFP in the economy. Moreover, through this channel, the countercyclical nature of factor risk prices adds a predictable, procyclical component to TFP (i.e., causes dispersion to decrease in expansions and increase in downturns, potentially

amplifying the effects of the underlying shock), holding fixed the distribution of firm-level betas. Thus, our model provides a direct quantifiable link between financial market conditions, i.e., the nature of aggregate risk, and long-run economic performance.

In our framework, the strength of this connection relies on three key parameters – the extent of variation in firm-level risk exposures and the magnitude and time-series variation in the price of risk. We devise a novel empirical strategy to pin down these parameters using salient moments from firm-level and aggregate stock market data, specifically, (i) the cross-sectional dispersion in expected stock returns, (ii) the average market equity premium and (iii) the average market Sharpe ratio. We use a linearized version of our model to derive closed-form solutions for these moments and show that they are tightly linked to the structural parameters. The latter two pin down the long-run level and volatility of the price of risk and the first identifies the cross-sectional dispersion in firm-level risk exposures. Indeed, in some simple cases of our model, the dispersion in expected MPK induced by risk premium effects is directly proportional to the dispersion in expected stock returns – intuitively, both of these moments are determined by cross-sectional variation in betas.

Before quantitatively evaluating this mechanism, we explore the effects of adding other investment frictions to our environment. First, we add capital adjustment costs. Although they do not change the main insights from our simpler model, we uncover an important interaction between these costs and risk premia – namely, adjustment costs actually amplify the effects of beta variation on MPK dispersion. Intuitively, beta dispersion leads to persistent differences in firm-level capital choices, even if these firms have the same average level of productivity. Adjustment costs further increase the dispersion in capital, which leads to even larger effects on MPK. On their own, adjustment costs do not lead to any persistent dispersion in MPK, but they augment the effects of other factors that do, such as the variation in risk premia we analyze here. Next, we add a flexible class of other firm-specific "distortions" of the type that have been emphasized in the misallocation literature. These distortions can be correlated or uncorrelated with the idiosyncratic component of firm-productivity and can be fixed or time-varying. To a first-order approximation, we show these additional factors do not affect our results or identification approach. In other words, although observed misallocation may stem from a variety of firm-specific factors, our empirical strategy to measuring risk premium effects yields an accurate estimate of the contribution of this one source alone.

We apply our empirical methodology to data on US firms from Compustat/CRSP and aggregates, e.g., productivity and stock market returns. Our estimates reveal substantial variation in firm-level betas and a sizable price of risk – together, these imply a significant amount of risk-induced MPK dispersion. For example, if this were the only source of MPK dispersion, variation in risk premia would account for about 15% of total MPK dispersion among Compu-

stat firms. In the presence of adjustment costs, this figure is notably higher – in this case, risk premia effects explain 44% of total dispersion in MPK. Importantly, the dispersion from this channel is persistent – in other words, risk effects manifest themselves as persistent MPK deviations at the firm level, exactly of the type that have been shown to compose a large portion of observed misallocation. Indeed, our results can account for as much as 67% of this permanent component in the data. The consequences of these values for the long-run level of aggregate TFP are modest, but significant – cross-sectional variation in risk reduces TFP by as much as 8%. Note that this represents a quantitative estimate of the impact of the the rich set of findings in the cross-sectional asset pricing literature on aggregate performance and further, a new connection between the nature of business cycle volatility and long-run outcomes in the spirit of Lucas (1987). Here, higher aggregate volatility leads to greater aggregate risk, increasing dispersion in required rates of return and MPK and thus reducing TFP.

Our estimates also imply a significant predictable countercyclical element in expected MPK dispersion. For example, our parameterized model produces a correlation between the cross-sectional variance in expected MPK and the state of the business cycle of -0.25. This result provides a risk-based explanation for the observation, made forcefully by Eisfeldt and Rampini (2006), that capital reallocation is procyclical, in spite of apparent countercyclical productivity gains. Further, because aggregate TFP is decreasing in MPK dispersion, the fact that this correlation is negative suggests that variation in the price of risk can amplify the effects of the underlying aggregate productivity shocks by worsening the allocation of capital in downturns and improving it in expansions. Quantitatively, our findings suggest this channel is potentially non-negligible – in response to a negative 1% shock to aggregate productivity, measured aggregate TFP would fall by about 1.2%.

Before concluding, we perform two important additional exercises. First, we provide direct evidence on the extent of beta dispersion. Rather than relying on stock market data, we compute firm-level betas using production-side data by estimating time-series regressions of measures of firm-level productivity on measures of aggregate productivity. The beta is the coefficient from this regression. We show that this exercise yields beta dispersion on par with the dispersion implied by the cross-section of stock market returns. Second, we demonstrate the crucial role of *ex-ante* dispersion in risk exposures in generating a quantitatively realistic dispersion in expected returns. To do so, we examine a model with no beta dispersion, but adjustment costs and potentially heterogeneity in other firm-level parameters, for example, curvature of the production function. We find that adjustment costs alone do not lead to significant expected return dispersion. Further, although heterogeneity in firm-level production parameters can generate non-negligible expected return dispersion, it is still only a relatively small fraction of the wide dispersion observed in the data, suggesting that variation in betas is

a key ingredient in matching this moment.

Related Literature. Our paper relates to several branches of the literature. First is the large body of work investigating and quantifying the effects of resource misallocation across firms, seminal examples of which include Hsieh and Klenow (2009) and Restuccia and Rogerson (2008). A number of papers have explored the role of particular economic forces in leading to misallocation. For example, Asker et al. (2014) study the role of capital adjustment costs, Midrigan and Xu (2014), Moll (2014) and Buera et al. (2011) financial frictions, Peters (2016) markup dispersion and David et al. (2016) information frictions.[2] Gopinath et al. (2017) and Kehrig and Vincent (2017) show that the interactions of adjustment costs and financial frictions are important in determining the recent dynamics of misallocation in Spain and the extent of misallocation across plants within firms, respectively. David and Venkateswaran (2017) provide an empirical methodology to disentangle various sources of capital misallocation and establish a large role for other firm-specific factors, in particular, ones that are essentially permanent to the firm. We build on this literature by exploring the implications of a different dimension of financial markets for marginal product dispersion, namely, the risk-return tradeoff faced by risk-averse investors. Importantly, our theory generates what appears to be a permanent firm-specific "wedge" exactly of the type found by David and Venkateswaran (2017), but which in our framework is a function of each firm's exposure to aggregate risk. The addition of aggregate risk is a key innovation of our analysis - existing work has typically abstracted from this channel.[3] We show that the link between aggregate risk and observed misallocation is quite tight in the presence of heterogeneous exposures to that risk.

Kehrig (2015) documents in detail the countercyclical nature of productivity dispersion. We build on this finding by relating fluctuations in MPK dispersion to time-series variation in the price of risk. A growing literature, starting with Eisfeldt and Rampini (2006), investigates the reasons underlying the observation that capital reallocation is procyclical. This indeed seems puzzling since given higher cross-sectional dispersion in MPK in downturns, one should expect to see capital flowing to highly productive, high MPK firms in recessions. Our results bear on that observation by noting that given a countercyclical price of risk, and a countercyclical premium on the high-minus-low MPK portfolio, from a risk perspective, reallocation to high MPK firms would require capital to flow to the riskiest of firms.

In a related effort, Binsbergen and Opp (2017) also investigate the implications of asset

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[2]Many papers study the role of firm-specific distortions, e.g., Bartelsman et al. (2013). Restuccia and Rogerson (2017) and Hopenhayn (2014) provide recent overviews of this line of work.

[3]Two important exceptions are Gopinath et al. (2017), who analyze the transitional effects of an interest rate shock on misallocation, and Kehrig (2015), who constructs a model of misallocation over the business cycle featuring overhead inputs. Neither of these papers examines risk premium effects, either because there is no aggregate uncertainty or firms are risk-neutral.

market considerations for the real economic decisions of firms. They propose a framework in which distortions in agents' subjective beliefs map to "alphas," i.e., cross-sectional mispricings, and through this channel to real efficiency losses. Our analyses are complementary in that where they focus on the implications of mispricing of financial assets for corporate decisions, we investigate the extent of efficient dispersion in firm-level marginal products induced by variation in firm-level exposures to aggregate risk factors. Our empirical work establishes a connection between observed marginal products and asset market outcomes and our quantitative work uses a workhorse macroeconomic model of firm dynamics augmented to feature risk-sensitive agents and aggregate risk to evaluate the implications of this insight. One of our key messages shares a common theme with the findings in Binsbergen and Opp (2017) – asset market considerations can have sizable effects on real outcomes by affecting capital allocation decisions.[4]

Our work exploits the insight, due to Cochrane (1991) and Restoy and Rockinger (1994), that stock returns and investment returns are closely linked. Indeed, under the assumption of constant returns to scale, stock and investment returns effectively coincide. Crucially, for our purposes, investment returns are intimately linked with the marginal product of capital. Balvers et al. (2015) explore and confirm the close albeit more complicated relationship under deviations from constant returns to scale. In this context, our work is closely related to the growing literature that examines the cross-section of stock returns by viewing them from the perspective of investment returns, e.g., Zhang (2005), Gomes et al. (2006) and Liu et al. (2009), and recently forcefully summarized in Zhang (2017). This literature interprets common risk factors such as the Fama-French factors through firms' investment policies, and most recently, shows that risk factors related to corporate investment patterns themselves capture risks priced in the cross-section of returns, culminating in the recent Q-factor model. Our objective is quite different and in some sense turns that logic on its head, in that we examine investment returns and the marginal product of capital as a manifestation of exposure to systematic risk, most readily measured through stock returns.

2   Motivation

In this section, we lay out a simple version of the standard, frictionless neoclassical theory of investment to motivate our empirical explorations. Section 4 enriches this environment for purposes of our quantitative work.

Firms produce output using capital and labor according to a standard Cobb-Douglas production function. Labor is chosen period-by-period in a spot market at a competitive wage.

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[4]Relatedly, David et al. (2014) find that risk considerations play an important role in determining the allocation of capital across countries, i.e., can explain some portion of the "Lucas Paradox."

At the end of each period, firms choose investment in new capital, which becomes available for production in the following period so that $K_{it+1} = I_{it} + (1 - \delta) K_{it}$, where δ is the rate of depreciation. Let $\Pi_{it} = \Pi_{it} (X_t, Z_{it}, K_{it})$ denote the operating profits of the firm – revenues net of labor costs – where X_t and Z_{it} denote aggregate and idiosyncratic shocks, respectively, and K_{it} the firm's level of capital. The analysis can accommodate a number of interpretations of these shocks, for example, as productivity or demand shifters. Given the Cobb-Douglas technology, the profit function takes a Cobb-Douglas form, is homogeneous in K of degree $\theta < 1$ and is proportional to revenues.[5] The marginal product of capital is equal to $\theta \frac{\Pi_{it}}{K_{it}}$. The payout of the firm in period t is equal to $D_{it} = \Pi_{it} - I_{it}$.

Firms discount future cash flows using a stochastic discount factor (SDF), M_{t+1}, which may be correlated with the aggregate component of firm fundamentals, i.e., with X_t. We can write the firm's problem in recursive form as

$$V (X_t, Z_{it}, K_{it}) = \max_{K_{it+1}} \Pi_{it} (X_t, Z_{it}, K_{it}) - K_{it+1} + (1 - \delta) K_{it} + \mathbb{E}_t [M_{t+1} V (X_{t+1}, Z_{it+1}, K_{it+1})] \; , \tag{1}$$

where $\mathbb{E}_t [\cdot]$ denotes the firm's expectations conditional on time t information. Standard techniques give the Euler equation

$$1 = \mathbb{E}_t [M_{t+1} (MPK_{it+1} + 1 - \delta)] \quad \forall \; i, t \; , \tag{2}$$

where $MPK_{it+1} = \frac{\partial \Pi_{it+1}}{\partial K_{it+1}}$ denotes the marginal product of capital of firm i at time $t + 1$.

MPK dispersion. An immediate consequence of expression (2) is that expected MPK need not be equated across firms; rather, it is only appropriately discounted expected MPK that is equalized. To the extent that firms load differently on the SDF, their expected MPKs will differ. Assuming a single source of aggregate risk for the sake of illustration, Appendix A derives the following factor model for expected MPK:

$$\mathbb{E}_t [MPK_{it+1}] = \alpha_t + \beta_{it} \lambda_t \; . \tag{3}$$

Here, α_t is the "risk-free" MPK, which equals the riskless user cost of capital, $r_{ft} + \delta$, where r_{ft} is the net risk-free rate, $\beta_{it} \equiv -\frac{\text{cov}_t(M_{t+1}, MPK_{it+1})}{\text{var}_t(M_{t+1})}$ measures the exposure, or loading, of the firm's MPK on the SDF, i.e., the riskiness of the firm, and $\lambda_t \equiv \frac{\text{var}_t(M_{t+1})}{\mathbb{E}_t[M_{t+1}]}$ is the market price of that risk. In the language of asset pricing, the Euler equation gives rise to a conditional one-

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[5]This structure follows, for example, if firms are perfectly competitive and the production function features decreasing returns to scale or firms are monopolistically competitive and face CES demand curves. We clarify these assumptions in our more detailed model in Section 4

factor model for expected MPK. Expression (3) highlights that expected MPK is not necessarily common across firms and is a function of the risk-free rate of return, the firm's beta on the SDF, which may vary across firms, and the market price of risk. The cross-sectional variance of date-t conditional expected MPK is then equal to

$$\sigma^2_{\mathbb{E}_t[MPK_{it+1}]} = \sigma^2_{\beta_t}\lambda_t^2 . \tag{4}$$

The extent to which risk considerations lead to dispersion in the expected MPK depends on (1) the cross-sectional dispersion in firm-level betas and (2) the level of the price of risk. Taking unconditional expectations, the theory can clearly generate persistent deviation in firm-level MPK, which is driven by the dispersion in required rates of return across firms:

$$\mathbb{E}[MPK_{it}] = \alpha + \beta_i\lambda + \text{cov}(\beta_{it}, \lambda_t) ,$$

where $\alpha \equiv \mathbb{E}[r_{ft} + \delta]$, $\beta_i \equiv \mathbb{E}[\beta_{it}]$ and $\lambda \equiv \mathbb{E}[\lambda_t]$ denote the unconditional expectations of the risk-free MPK, conditonal MPK factor betas and factor prices, respectively. So long as the relationship between mean betas and the time-series correlation of those betas with the price of risk is weak, we can write the variance of mean MPK approximately as[6]

$$\sigma^2_{\mathbb{E}[MPK]} \approx \sigma^2_\beta\lambda^2 , \tag{5}$$

where σ^2_β denotes the cross-sectional variance of mean β's. We note that this observation generalizes in a straightforward manner to environments more recently considered in the cross-sectional asset pricing literature emphasizing the presence of multiple aggregate risk factors. Most prominently, beyond excess returns on the market portfolio and innovations to aggregate consumption growth as considered in the classical CAPM and Breeden-Lucas Consumption CAPM, these risk factors have been linked to excess returns on size, as well as book-to-market sorted portfolios (Fama-French factors), or investment returns or profitability (the Q-factor model of Hou et al. (2015) and Zhang (2017)).

The strength of the mechanism linking persistent dispersion in MPK to exposure to aggregate risk can be understood by inspection of expression (5) - predicted MPK dispersion is increasing in the dispersion in betas and also in the price of risk, λ. A key observation underlying our analysis is that asset pricing data suggest that risk prices are rather high. A lower bound is given by the Sharpe ratio on the market portfolio, estimated to be around 0.5. However,

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[6]Specifically, as long as $\text{cov}(\beta_i, \text{cov}(\beta_{it}, \lambda_t))$ is small. In line with the results in Lewellen and Nagel (2006), we find the time-series variation in β's to be quite small, suggesting the validity of the approximation. In the case of that the β of a firm is constant, for example, which we assume in our quantitative model, the expression is exact.

even easily implementable trading strategies such as those based on value-growth portfolios, or momentum, suggest numbers closer to 0.8, while hedge fund strategies report Sharpe ratios in excess of one. Taken at face value, these numbers suggest the possibility for substantial MPK dispersion - even in frictionless models - after taking risk exposure in account. In Sections 4 and 5 we develop a model and empirical approach to quantify this link using data on risk prices and cross-sectional variation in expected stock market returns.

Empirical Predictions. Even under the general structure we have outlined thus far, the theory has a good deal of empirical content. Specifically, the expressions laid out above contain a number of both cross-sectional and time-series predictions:

1. Exposure to standard risk factors is a determinant of expected MPK. Expression (3) shows that the same factors that determine the cross-section of stock returns - namely, exposure to the SDF - determine the cross-section of MPK. Firms with a higher loading on the SDF, i.e., higher β's, should have higher conditional expected MPK.

2. Variation in the price of risk, λ_t, leads to predictable variation in mean expected MPK. In particular, the mean conditional expected MPK should increase when the price of risk does. This is the time-series implication of expression (3) - holding fixed the distribution of β's, movements in λ_t should positively affect the mean expected MPK. Since the price of risk is known to be countercyclical, this implies that the mean expected MPK is as well.

3. MPK dispersion is related to β dispersion. Expression (5) shows that unconditional variation in the cross-section of MPK is proportional to the variation in β. Segments of the economy, for example, industries, with higher dispersion in β should display higher dispersion in MPK.

4. MPK dispersion is positively correlated with the price of risk. Expression (4) has a time-series prediction linking MPK dispersion to time variation in the price of risk. For a given degree of cross-sectional dispersion in β, when required compensation for bearing risk increases, MPK dispersion should increase as well.

Illustrative examples. Section 3 investigates each of these predictions in detail. Before doing so, however, it is useful to consider a number of more concrete illustrative examples (derivations for this section are in Appendix A).

Example 1: no aggregate risk (or risk neutrality). In the case of no aggregate risk, we have

$\beta_{it} = 0 \; \forall \; i, t$, i.e., all shocks are idiosyncratic to the firm. Expressions (3) and (4) show that there will be no dispersion in expected MPK and for each firm, $\mathbb{E}_t[MPK_{it+1}] = r_f + \delta$, which is simply the riskless user cost of capital (which is constant in the absence of aggregate shocks). This is the standard result from the stationary models widely used in the misallocation litera- ture where without additional frictions, expected MPK should be equalized across firms.[7] It is straightforward to show this expression also holds in an environment with aggregate shocks but risk neutral preferences, which implies M_{t+1} is simply a constant (equal to the time discount factor).

Example 2: CAPM. In the CAPM, the SDF is linearly related to the market return, i.e., $M_{t+1} = a - bR_{mt+1}$ for some constants a and b. Because the market portfolio is itself an asset with $\beta = 1$, it is straightforward to derive

$$\mathbb{E}_t[MPK_{it+1}] = \alpha_t + \underbrace{\frac{\text{cov}_t(R_{mt+1}, MPK_{it+1})}{\text{var}_t(R_{mt+1})}}_{\beta_{it}} \underbrace{\mathbb{E}_t[R_{mt+1} - R_{ft}]}_{\lambda_t},$$

i.e., expected MPK is determined by the covariance of the firm's MPK with the market return (its market β), which is the the risk factor in this environment. The price of risk is equal to the expected excess return on the market portfolio, i.e., the equity premium (R_{ft} is the risk-free rate of return from period t to $t+1$).

Example 3: CCAPM. In the case that the utility function is CRRA with coefficient of rel- ative risk aversion γ, standard approximation techniques give the pricing equation from the consumption capital asset pricing model:

$$\mathbb{E}_t[MPK_{it+1}] = \alpha_t + \underbrace{\frac{\text{cov}_t(\Delta c_{t+1}, MPK_{it+1})}{\text{var}_t(\Delta c_{t+1})}}_{\beta_{it}} \underbrace{\gamma \text{var}_t(\Delta c_{t+1})}_{\lambda_t},$$

where Δc_{t+1} denotes log consumption growth. Expected MPK is determined by the covariance of the firm's MPK with consumption growth (its consumption β), which is now the risk factor. The market price of risk is the product of the coefficient of relative risk aversion and the conditional volatility of consumption growth.

In Sections 4 and 5, we follow the recent literature on production-based asset pricing and explicitly model the sources of uncertainty as arising from technology shocks, both at the firm

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[7]With time-to build for capital and uncertainty over upcoming shocks there may still be dispersion in *realized* MPK, but not in *expected* terms, and so these forces do not lead to persistent deviations from MPK equalization for a particular firm.

and aggregate level, and quantify the implications of those shocks for MPK dispersion.

3   Empirical Results

In this section, we investigate the empirical predictions outlined in Section 2.

Data. Our data come primarily from the Center for Research in Security Prices (CRSP) and Compustat. We use data on nonfinancial firms with common equities listed on the NYSE, NASDAQ, or AMEX over the period 1962 to 2014. We supplement this panel with time-series data on market factors and aggregate conditions related to the price of risk. The risk factors we consider are the Fama and French (1992) factors, Hou, Xue, and Zhang (2015) investment-CAPM factors, as well as the growth rate of non-durable and services consumption from the Bureau of Economic Analysis (BEA). We also use data on aggregate macroeconomic and financial market variables from the BEA and the Gilchrist and Zakrajsek (2012) (GZ) credit spread.[8] We measure the firm's capital stock, K_{it}, as the (net of depreciation) value of property, plant and equipment (Compustat series PPENT) and firm revenue, Y_{it}, as reported sales (series SALE). Ignoring constant terms, which will play no role in our analysis, we measure the marginal product of capital (in logs) as $mpk_{it} = y_{it} - k_{it}$.[9]

We can now revisit the main predictions from Section 2.

1. Exposure to standard risk factors is a determinant of expected MPK. To investigate this implication of our framework, Table 1 assesses the relationship between MPK and both contemporaneous and future excess stock returns. We sort firms into 10 portfolios based on their year t MPK, where portfolio 1 contains low MPK firms and portfolio 10 high MPK ones. We then compute the contemporaneous and one-period ahead equal-weighted excess stock return to each portfolio. Following Fama and French (1992), we use the MPK reported by firms in their fiscal-year-end filing in date t-1 with firm returns from July of year t to June of year t+1 when computing future returns. We additionally compute excess returns on on a high-minus-low portfolio (MPK-HML), which is an annually rebalanced portfolio that is long on stocks in the highest MPK portfolio and short on stocks in the lowest.

Table 1 reveals a strong relationship between MPK and stock returns - portfolios with higher MPK earn higher excess returns. Panel A shows that the difference in contemporaneous returns between high and low MPK firms, i.e., the excess return on the MPK-HML portfolio

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[8]We obtain measures of the GZ spread from Simon Gilchrist's website.

[9]Recall that in our setup, operating profits are proportional to revenues, making this a valid measure of the *mpk*.

Table 1: Excess Returns on MPK Sorted Portfolios

	Low	2	3	4	5	6	7	8	9	High	MPK-HML
						Portfolio					
					Panel A: Not Industry-Adjusted						
r_t^e	6.026*	9.288**	9.258**	10.26***	10.65***	12.21***	12.86***	14.57***	15.20***	17.69***	11.11***
	(1.68)	(2.44)	(2.42)	(2.72)	(2.86)	(3.13)	(3.11)	(3.36)	(3.39)	(3.74)	(4.05)
r_{t+1}^e	6.632*	10.48***	11.19***	12.68***	12.59***	13.25***	13.46***	13.01***	13.22***	13.58***	6.583**
	(1.87)	(2.83)	(2.99)	(3.44)	(3.45)	(3.36)	(3.30)	(3.12)	(3.03)	(3.00)	(2.46)
					Panel B: Industry-Adjusted						
r_t^e	8.909*	8.208*	9.408**	9.386**	10.06***	11.58***	11.05***	13.80***	16.03***	17.67***	8.870***
	(1.71)	(1.95)	(2.41)	(2.55)	(2.76)	(3.05)	(2.86)	(3.28)	(3.45)	(3.60)	(5.17)
r_{t+1}^e	10.11*	11.48***	10.88***	11.73***	11.15***	11.95***	11.39***	13.33***	13.08***	13.20***	3.286**
	(1.96)	(2.79)	(2.85)	(3.15)	(3.11)	(3.21)	(2.98)	(3.35)	(3.03)	(2.81)	(1.99)

Notes: $r_t^{\mathbb{E}}$ denotes equal-weighted contemporaneous annual excess stock returns (over the risk-free rate) measured in the year of the portfolio formation from January to December of year t. r_{t+1}^e denotes the analogous future returns, measured in the year following the portfolio formation, from July of year $t + 1$ to June of year $t + 2$. Industry adjustment is done by de-meaning returns by industry-year, where an industry is defined as a 4 digit SIC code. t-statistics in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

is about 11% annually and remains high, about 6.5%, for one-period ahead returns. Both contemporaneous and future spreads are statistically different from zero at the 95% level. Firms that offer high stock returns tend to also have MPKs, both in a realized and an expected sense.

The focus in the misallocation literature is generally on within-industry variation in the MPK. Panel B of Table 1 reports within-industry results, defined at the 4-digit SIC level. To compute these values, from each return observation we subtract the mean return within that industry-year.[10] Although the magnitudes fall somewhat, the relationship between MPK and stock returns remains strong even when comparing across firms within a particular industry, both in an economic and statistical sense - the MPK-HML contemporaneous excess return is almost 9% annually and the future excess return almost 3.5%. Both are statistically significant at the 95% level.

2. Variation in the price of risk λ_t leads to predictable variation in expected MPK. Expression (3) implies that the market price of risk, λ_t, is positively related to the level of expected MPK in the following period. To test this, we estimate regressions of firm mpk on three lagged (by one year) measures related to the price of risk: 1) the aggregate price/dividend ratio; 2) the Gilchrist and Zakrajsek (2012) (GZ) spread, a high-information and duration-adjusted measure of the mean credit spread; and 3) the Excess Bond Premium, which measures the portion of the GZ spread not attributable to default risk. We control for the changing composition of firms in the following way: using only those firms where our measure of mpk is observed for the firm in consecutive quarters, we compute changes in mean mpk for every pair of years. We then use those changes to construct a synthetic composition-adjusted mean mpk which is unaffected by

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[10]We define an industry as a 4-digit SIC code and examine industry-year pairs with at least 10 observations.

new additions or deletions from the dataset. Table 2 reports the results of these regressions. In line with the theory, columns (2) and (3) show that the GZ spread and the excess bond premium (which are likely positively correlated with the market price of risk) predict higher future mpk, while column (1) shows that the price-dividend ratio (likely negatively correlated with the market price of risk) predicts lower future mpk.

Table 2: Predictability of $\mathbb{E}_t\left[MPK_{it}\right]$

	(1)	(2)	(3)
PD Ratio	-0.0114***		
	(-4.61)		
GZ Spread		0.0452***	
		(3.01)	
Excess Bond Premium			0.0603**
			(2.59)
Constant	-0.00653	-0.0803***	-0.00595
	(-0.41)	(-2.73)	(-0.36)
Observations	148	148	148
R^2	0.142	0.094	0.056

Notes: Table reports time-series regressions of average composition-adjusted mpk on lagged (by one year) measures of the price of risk. Long-term trends in mpk and the price/dividend ratio are removed using a one-sided hp filter. t-statistics are in parentheses. t-statistics in parentheses, which are computed using Newey-West standard errors. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. All observations are observed at the quarterly frequency.

3. MPK dispersion is related to β dispersion. Expression (5) implies that for particular groups of firms, dispersion in expected mpk should be positively related to the dispersion in β. In particular, this suggests that dispersion of mpk within an industry, a common measure of misallocation, is positively correlated with dispersion in expected stock returns and β's. We investigate this prediction using variation in the dispersion of firm-level β's across industries. For each industry in each year, we compute the standard deviation of mpk, $\sigma\left(mpk\right)$, expected returns, $\sigma\left(\mathbb{E}\left[ret\right]\right)$ and β's, $\sigma\left(\beta\right)$ and estimate a pooled regression of industry-level mpk dispersion on the dispersion in stock returns and β's. To avoid potential biases from the realization of shocks, we lag the independent variables (dispersion in expected stock returns and β's) by a year. We detail our computation of firm-level measures of β and excess returns in Appendix D.

Table 3 reports the results of these regressions and demonstrates that indeed, industries with higher dispersion in expected stock returns and β's exhibit greater dispersion in mpk. Column (1) reveals this fact using expected returns calculated from the Fama-French 3 factor model. Column (2) shows this relationship continues to hold using expected returns predicted

using β's only. The Fama-French model explains between about 25% and 30% of the variation in MPK dispersion across industry-years. Column (3) estimates a multiple regression of mpk dispersion on each of the three individual factors - dispersion in each is significantly related to mpk dispersion. In column (4) we take a slightly different approach - we estimate more direct measures of "mpk β's" by regressing firm-level mpk directly on the Fama-French factors (rather than stock returns). For each industry-year, we compute the standard deviation of these β's. The results in column (4) show that dispersion in these alternative measures of β are also significantly related dispersion in mpk. The relationships are highly statistically significant and the R^2 remains close to 25%. In Table 9 in Appendix D, we report results from related regressions where we average our dispersion measures across years for each industry. The findings there are broadly similar (indeed, slightly stronger).[11]

4. MPK dispersion is positively correlated with the price of risk. Expression (3) implies that the price of risk is positively related to mpk dispersion. We investigate this prediction in two ways. First, we show that the measures of the market price of risk considered before (the PD ratio, GZ spread, and excess bond premium) predict time series variation in measures of MPK dispersion. Second, we show that the future expected return on a long-short MPK portfolio are also predicted by these measures of the market price of risk.

We show that both the unconditional dispersion in mpk, and the dispersion of mpk within industries are positively correlated with the lagged price of risk. We control for the changing composition of firms in the following way: using only those firms where our measure of mpk is observed for the firm in consecutive quarters, we compute changes in the standard deviation of mpk (or of industry-demeaned mpk for the within-industry dispersion) for every pair of years. We then use those changes to construct a synthetic composition-adjusted measure of the dispersion of mpk which is unaffected by new additions or deletions from the dataset. Table 4 displays a regression of the standard deviation of mpk (both within industries and unconditional) on lagged (by one year) measures of the price/dividend ratio, GZ spread, and excess bond premium. All three measures of the business cycle and the market price of risk significantly predict mpk dispersion, and in the direction our theory would suggest: The GZ Spread and excess bond premium predict greater mpk dispersion, while the PD ratio predicts lower mpk dispersion.

As a final test of this prediction, we construct a long-short MPK portfolio and investigate

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[11]Our results are also robust to using a number of different asset pricing models to compute measures of β and expected returns, including CAPM, the Hou et al. (2015) Investment-CAPM, and the Consumption-CAPM models. This relationship is robust to a variety of different controls and industry definitions as well. Table 10 in Appendix D displays the same regression as in Table 3, but with year fixed-effects (reporting within-year R^2), which generates similar results as well.

Table 3: Industry-level Dispersion in mpk, Stock Returns and β

	(1)	(2)	(3)	(4)
$\sigma(E[ret])$	2.542***			
	(34.14)			
$\sigma(E_\beta[ret])$		11.63***		
		(31.43)		
$\sigma(\beta_{MKT})$			0.244***	
			(12.22)	
$\sigma(\beta_{HML})$			0.120***	
			(10.63)	
$\sigma(\beta_{SMB})$			0.116***	
			(8.54)	
$\sigma(\beta_{CAPM,MPK})$				0.137***
				(9.30)
$\sigma(\beta_{HML,MPK})$				0.0412***
				(3.90)
$\sigma(\beta_{SMB,MPK})$				0.0549***
				(7.87)
Observations	2721	2746	2734	1427
R^2	0.300	0.265	0.306	0.219

Notes: $\mathbb{E}[ret]$ is the expected return computed from a Fama-Macbeth regression. $\mathbb{E}[ret(\beta)]$ is the expected return predicted from the β's of that regression alone. β' denotes the stock return β on the FF factors and β_{MPK} the mpk β on the same factors. t-statistics are in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

its relation with market price of risk. The portfolio is long the top decile of MPK firms and short the bottom decile, re-balancing every every June based on MPK from the previous year. Table 5 reports a regression of the cumulative twelve month returns on the long-short MPK portfolio on the Pd ratio, GZ spread, and excess bond premium. The GZ spread and excess bond premium rate predict higher future returns on the MPK portfolio, while the PD ratio predicts lower future returns.

4 The Model

In the next two sections, we use a more detailed version of the investment model laid out above to quantitatively investigate the contribution of heterogeneous risk premia to observed MPK dispersion. The model is kept deliberately simple in order to isolate the impact of our basic mechanism, namely dispersion in exposure to systematic risk. The theory consists of two main building blocks: (1) a stochastic discount factor, which we directly parameterize to be consistent with salient patterns in financial markets, i.e., high and countercyclical prices of

Table 4: Predictability of MPK Dispersion

	Within Industry			Unconditional		
PD Ratio	-0.00188***			-0.00502***		
	(-3.01)			(-7.24)		
GZ Spread		0.0123***			0.0221***	
		(3.69)			(3.53)	
EB Premium			0.0271***			0.0472***
			(4.41)			(4.72)
Constant	0.000700	-0.0198***	-0.000349	0.0000136	-0.0362***	-0.00110
	(0.16)	(-2.96)	(-0.08)	(0.00)	(-2.86)	(-0.20)
Observations	148	148	148	148	148	148
R^2	0.054	0.095	0.156	0.200	0.161	0.244

Notes: We regress our measure of composition-adjusted MPK Dispersion (both within industry dispersion or unconditional) on time-series factors. Long-term trends in mpk dispersion and the price/dividend ratio are removed using a one-sided hp filter. t-statistics are in parentheses, computed using Newey-West standard errors. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. All observations are quarterly.

risk, and (2) a cross-section of heterogeneous firms, which make optimal investment decisions in the presence of firm-level and aggregate risk, given the stochastic discount factor. Specifying the stochastic discount factor exogenously allows us to sidetrack challenges with generating empirically relevant risk prices in general equilibrium, and focus on gauging the quantitative strength of our mechanism. To hone in on the effects of risk premia, we begin with simplified version in which we abstract from adjustment costs. In this case, our framework yields exact closed form solutions for firm investment decisions. In Section 4.3, we extend the model to include capital adjustment costs. Our theoretical results there reveal an important amplification effect of these costs on the impact of risk premia.

4.1 The Environment

Heterogeneity in risk exposures. The setup is a fleshed-out version of that in Section 2. We consider a discrete time, infinite-horizon economy. A continuum of firms of fixed measure one, indexed by i, produce a homogeneous good using capital and labor according to:

$$Y_{it} = X_t^{\hat{\beta}_i} \hat{Z}_{it} K_{it}^{\theta_1} N_{it}^{\theta_2}, \quad \theta_1 + \theta_2 < 1$$

Firm productivity (in logs) is equal to $\hat{\beta}_i x_t + \hat{z}_{it}$, where x_t denotes an aggregate component that is common across firms and $\hat{\beta}_i$ captures the exposure of the productivity of firm i to

Table 5: Predictability of MPK-HML Portfolio Returns

	(1)	(2)	(3)
PD Ratio	-0.000456*		
	(-1.70)		
GZ Spread		0.00384**	
		(2.31)	
Excess Bond Premium			0.00580**
			(2.36)
Constant	0.00921***	0.00256	0.00892***
	(8.05)	(1.01)	(8.42)
Observations	152	152	152
R^2	0.048	0.128	0.097

Notes: The dependent variable is the equal-weighted returns from going long firms in the top decile of MPK (after demeaning by sic4) and short the bottom decile, for the following twelve months. Long-term trends in the price-dividend ratio are removed using a one-sided HP filter. t-statistics are in parentheses, computed using Newey-West standard errors. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

aggregate conditions.[12] We assume that $\hat{\beta}_i$ is distributed as $\hat{\beta}_i \sim \mathcal{N}\left(\bar{\hat{\beta}}, \sigma_{\hat{\beta}}^2\right)$ across firms. Heterogeneity in this exposure is a key ingredient of our framework – cross-sectional variation in $\hat{\beta}_i$ will lead directly to dispersion in expected mpk. The term $\hat{z}_{it}$ denotes a firm-specific, idiosyncratic component of productivity.

The two productivity components follow AR(1) processes (in logs):

$$
\begin{aligned}
x_{t+1} &= \rho_x x_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right) \\
\hat{z}_{it+1} &= \rho_z \hat{z}_{it} + \hat{\varepsilon}_{it+1}, \quad \hat{\varepsilon}_{it+1} \sim \mathcal{N}\left(0, \sigma_{\hat{\varepsilon}}^2\right)
\end{aligned}
\tag{6}
$$

There are two sources of uncertainty at the firm level – aggregate uncertainty, with conditional variance σ_ε^2, and idiosyncratic uncertainty, with variance $\hat{\sigma}_{\hat{\varepsilon}}^2$.

Stochastic discount factor. In line with the large literature on cross-sectional asset pricing in production economies, we parameterize directly the pricing kernel without explicitly modeling

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[12]We use lower-case to denote natural logs, a convention we follow throughout, so that, e.g., $x_{it} = \log X_{it}$.

the consumer's problem. In particular, we specify the SDF as

$$\log M_{t+1} \equiv m_{t+1} = \log \rho - \gamma_t \varepsilon_{t+1} - \frac{1}{2}\gamma_t^2 \sigma_\varepsilon^2 \tag{7}$$

$$\gamma_t = \gamma_0 + \gamma_1 x_t$$

where ρ, $\gamma_0 > 0$ and $\gamma_1 \leq 0$ are constant parameters.[13] The SDF is determined by shocks to aggregate productivity. The conditional volatility of the SDF, given by $\sigma_m = \gamma_t \sigma_\varepsilon$, varies through time as determined by γ_t. This formulation allows us to capture in a simple manner a high, time-varying and countercyclical price of risk, as observed in the data (since $\gamma_1 < 0$, γ_t will be higher following economic contractions, i.e., when x_t is negative). Additionally, directly parameterizing γ_0 and γ_1 enables the model to be quantitatively consistent with key moments of asset returns, which are important for our analysis. The risk free rate is constant and equal to $-\log \rho$. Thus, γ_0 and γ_1 only affect the properties of equity returns., easing the interpretation of these parameters. The maximum attainable Sharpe ratio is equal to the conditional standard deviation of the SDF, i.e., $SR_t = \gamma_t \sigma_\varepsilon$ and the market price of risk is equal to the square of the Sharpe ratio, $\gamma_t^2 \sigma_\varepsilon^2$.

Input choices. Firms hire labor period-by-period after the realization of shocks at a competitive wage W_t. To keep the labor market simple, we assume that the equilibrium wage is given by

$$W_t = X_t^\omega$$

that is, the wage is a constant elasticity and increasing function of aggregate productivity, where $\omega \in [0, 1]$ determines the sensitivity of wages to aggregate conditions.[14] Maximizing over the static labor decision gives operating profits, i.e., revenues less labor costs, as

$$\Pi_{it} = G X_t^{\beta_i} Z_{it} K_{it}^\theta \tag{8}$$

where $G \equiv (1 - \theta_2)\theta_2^{\frac{\theta_2}{1-\theta_2}}$, $\beta_i \equiv \frac{1}{1-\theta_2}\left(\hat{\beta}_i - \omega\theta_2\right)$, $Z_{it} \equiv \hat{Z}_{it}^{\frac{1}{1-\theta_2}}$ and $\theta \equiv \frac{\theta_1}{1-\theta_2}$. The exposure of firm profits to aggregate conditions is captured by β_i, which is a simple transformation of the underlying exposure of firm productivity to the aggregate component, $\hat{\beta}_i$, and the sensitivity of wages, ω.[15] The idiosyncratic component of productivity is similarly scaled, by $\frac{1}{1-\theta_2}$. The curvature of the profit function is equal to θ, which depends on the relative elasticities of capital and labor in production. These scalings reflect the leverage effects of labor liabilities on profits.

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[13]This specification builds closely on those in, for example, Zhang (2005) and Jones and Tuzel (2013).

[14]This setup follows, for example, Belo et al. (2014) and İmrohoroğlu and Tüzel (2014).

[15]The adjustment term for labor supply, $\omega\theta_2$, has a small effect on the mean of the β distribution, but otherwise does not affect our analysis.

From here on, we will primarily work with z_{it}, which has the same persistence as $\hat{z}_{it}$, i.e., ρ_z, and innovations $\varepsilon_{it+1} = \frac{1}{1-\theta_2}\hat{\varepsilon}_{t+1}$ with variance $\sigma_{\varepsilon}^2 = \left(\frac{1}{1-\theta_2}\right)^2 \sigma_{\hat{\varepsilon}}^2$. We will also use the fact that $\sigma_{\beta}^2 = \left(\frac{1}{1-\theta_2}\right)^2 \sigma_{\hat{\beta}}^2$. Notice that the profit function takes precisely the form assumed in Section 2. Thus, the firm's dynamic investment problem is given by expression (1).

Optimal investment. The simplicity of this setting leads to exact analytical expressions for the firm's investment decision. Specifically, we show in Appendix A.2.1 that the firm's optimal investment policy is given by:

$$k_{it+1} = \frac{1}{1-\theta}\left(\tilde{\alpha} + \beta_i \rho_x x_t + \rho_z z_{it} - \beta_i \gamma_t \sigma_{\varepsilon}^2\right) \tag{9}$$

where $\tilde{\alpha} \equiv \log\theta + \log G - \alpha$, $\alpha \equiv r_f + \log\left(1-(1-\delta)\rho\right)$ is a constant.[16] The firm's choice of capital is increasing in x_t and z_{it} due to their direct effect on expected future productivity (i.e., $\beta_i \rho_x x_t + \rho_z z_{it} = \mathbb{E}_t\left[\beta_i x_{t+1} + z_{it+1}\right]$), but, *ceteris paribus*, firms with higher betas choose a lower level of capital. The magnitude of this effect is larger when γ_t is large, i.e., in economic downturns. Clearly, with risk neutrality, i.e., $\gamma_0 = \gamma_1 = 0$, the last term is zero and investment is purely determined by expected productivity.

For a slightly different intuition, we substitute for γ_t and write the expression as

$$k_{it+1} = \frac{1}{1-\theta}\left(\tilde{\alpha} + \beta_i\left(\rho_x - \gamma_1 \sigma_{\varepsilon}^2\right)x_t + \rho_z z_{it} - \beta_i \gamma_0 \sigma_{\varepsilon}^2\right) \tag{10}$$

The risk premium affects the investment choice through both the time-varying and constant components of the price of risk: first, a more negative γ_1 increases the responsiveness of firms to aggregate conditions. Intuitively, a high (low) realization of x_t has two effects – first, since x_t is persistent, it signals that productivity is likely to be high (low) in the future, increasing (decreasing) investment (this force is captured by the ρ_x term). Moreover, a high (low) realization of x_t implies a low (high) price of risk, which further increases (decreases) investment. Second, the constant component of the risk premium, γ_0, adds a firm-specific constant – i.e., a firm fixed-effect – to the choice of capital, which leads to permanent dispersion in firm-level capital choices.

MPK dispersion. By definition, the realized *mpk* is given by $mpk_{it+1} = \log\theta + \pi_{it+1} - k_{it+1}$. Substituting for k_{it+1},

$$mpk_{it+1} = \alpha + \varepsilon_{it+1} + \beta_i \varepsilon_{t+1} + \beta_i \gamma_t \sigma_{\varepsilon}^2 \tag{11}$$

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[16]More precisely, there are also terms that reflect the variance of shocks. Because these terms are negligible and play no role in our analysis (they are independent of the risk premium effects we measure), we view them as nuisance terms and ignore them here. The full expressions are given in Appendix A.2.1.

and taking conditional expectations,

$$Empk_{it+1} \equiv \mathbb{E}_t\left[mpk_{it+1}\right] = \alpha + \beta_i\gamma_t\sigma_\varepsilon^2 \tag{12}$$

where α is as defined in equation (9) and reflects the risk-free user cost of capital. Expression (11) shows that dispersion in the realized mpk is due to both uncertainty over the realization of shocks, as well as the risk premium term, which is persistent at the firm level and depends on (1) the firm's exposure to the aggregate shock β_i (and is increasing in β_i) and (2) the time-t price of risk, which is reflected in the term $\gamma_t\sigma_\varepsilon^2$. Intuitively, firm-level mpk deviations are composed of both a transitory component due to uncertainty and a persistent component due to the risk premium. The transitory components, however, are i.i.d. over time and thus lead to purely temporary deviations in mpk (this is true even though the underlying productivity processes are autocorrelated); the risk premium, on the other hand, leads to persistent deviations, in which firms that are more exposed to aggregate shocks, and so are riskier, will feature persistently high mpk deviations.

Expression (12) hones in on this second force and shows the persistent effects of risk premia on the conditional expectation of time-$t + 1$ mpk, denoted $Empkt$. Indeed, in this simple case, the ranking of firms' mpk will be constant in expectation as determined by the risk premium – high beta firms will have permanently high $Empkt$ and low beta firms the opposite. Importantly, the *value* of $Empkt$ will fluctuate with γ_t, but the ordering across firms will be preserved. This is the sense that we call this component persistent/permanent. Expression (11) shows that this ordering will not be preserved period-by-period in terms of realized mpk – due to the realization of shocks, the ranking of firms' mpk deviations will fluctuate, but the firm-specific risk premium adds a persistent component.[17] Because the uncertainty portion of the realized mpk is always additively separable (in logs) and is independent of our mechanism, from here on we will primarily work with $Empkt$.

Expression (13) presents the cross-sectional variance of $Empkt$:

$$\sigma_{Empkt}^2 \equiv \sigma_{\mathbb{E}_t[mpk_{it+1}]}^2 = \sigma_\beta^2\left(\gamma_t\sigma_\varepsilon^2\right)^2 \tag{13}$$

Cross-sectional variation in $Empkt$ depends on the dispersion in β's and the price of risk. Dispersion will be greater when risk prices, reflected by $\gamma_t\sigma_\varepsilon^2$, are higher and so will be countercyclical. The average long-run level of $Empk$ dispersion is given by

$$E\sigma_{Empk}^2 \equiv \mathbb{E}\left[\sigma_{Empkt}^2\right] = \sigma_\beta^2\left(\gamma_0^2 + \gamma_1^2\sigma_x^2\right)\left(\sigma_\varepsilon^2\right)^2 \tag{14}$$

---

[17]With adjustment costs, there will be another factor confounding the relationship between β and the realized and expected mpk.

where $\sigma_x^2 = \frac{\sigma_\varepsilon^2}{1-\rho_x^2}$.

An examination of expressions (12) and (13) confirms that the richer model here is consistent with the four key implications from Section 2, namely – (1) exposure to risk factors is a determinant of $Empkt$; (2) variation in the price of risk leads to predictable variation in mean $Empkt$ across firms; (3) mpk dispersion is related to β dispersion; and (4) mpk dispersion is increasing in the market price of risk, and so naturally contains a countercyclical element.

Aggregate outcomes.   What are the implications of this dispersion in $Empk$ for the aggregate economy? Our framework provides a natural way to quantify this by computing measures of aggregate productivity (TFP) and output. Appendix A.3 shows that aggregate output can be expressed as

$$\log Y_{t+1} \equiv y_{t+1} = a_{t+1} + \theta_1 k_{t+1} + \theta_2 n_{t+1}$$

where k_{t+1} denotes the aggregate capital stock, n_{t+1} aggregate labor and a_{t+1} the level of aggregate TFP, given by

$$a_{t+1} = a_{t+1}^* - \frac{1}{2}\frac{\theta_1(1-\theta_2)}{1-\theta_1-\theta_2}\sigma_{mpk,t+1}^2 \tag{15}$$

where $\sigma_{mpk,t+1}^2$ is realized mpk dispersion in period $t+1$. The term a_{t+1}^* is the first-best level of TFP in the absence of any frictions (i.e., where marginal products are equalized). Thus, aggregate TFP monotonically decreases in the extent of capital misallocation, captured by σ_{mpk}^2. The effect of misallocation on aggregate TFP depends on the overall curvature in the production function, $\theta_1 + \theta_2$ and the relative shares of capital and labor. The higher is $\theta_1 + \theta_2$, that is, the closer we are to constant returns to scale, the more severe the losses from misallocated resources. Similarly, fixing the degree of overall returns to scale, for a larger capital share, θ_1, a given degree of misallocation has larger effects on aggregate outcomes.

Using equation (13), the time-t conditional expectation of one period ahead TFP is given by

$$\mathbb{E}_t[a_{t+1}] = \mathbb{E}_t[a_{t+1}^*] - \frac{1}{2}\frac{\theta_1(1-\theta_2)}{1-\theta_1+\theta_2}\sigma_\beta^2\left(\gamma_t\sigma_\varepsilon^2\right)^2 \tag{16}$$

Since γ_t is countercyclical, the expression shows that heterogeneity in risk premia lead to relatively lower (higher) levels of TFP during business cycle downturns (expansions). Taking expectations gives the effects on the average long-run level of productivity in the economy:

$$\bar{a} \equiv \mathbb{E}[\mathbb{E}_t[a_{t+1}]] = a^* - \frac{1}{2}\frac{\theta_1(1-\theta_2)}{1-\theta_1+\theta_2}\sigma_\beta^2\left(\gamma_0^2 + \gamma_1^2\sigma_x^2\right)\left(\sigma_\varepsilon^2\right)^2 \tag{17}$$

The expression directly links the extent of cross-sectional dispersion in required rates of return (which are in turn determined by the prices of risk and volatility of aggregate shocks) to the long-

run level of aggregate productivity and gives a natural way to quantify the implications of these effects. In Sections 6.1 and 6.2, we show that our model can be extended to include multiple risk factors and to allow γ_t to depend on additional factors beyond the state of technology and so expressions (16) and (17) provide a more general connection between financial conditions (that may be less than perfectly correlated with the real economy) and aggregate productivity. Thus, these expressions provide one way to link the rich findings of the literature on cross-sectional asset pricing to real allocations and measures of aggregate performance. Further, they provide a new connection between aggregate volatility, e.g., the properties of the business cycle, and long-run outcomes.

4.2   The Cross-Section of Expected Stock Returns and MPK

In this section, we derive a sharp link between a firm's beta – and so its expected *mpk* – and its expected stock market return. This connection points to a novel empirical strategy to measure the dispersion in betas and so to quantify the *mpk* dispersion that arises from risk considerations. Our key finding in this section is that, to a first-order approximation, the firm's expected stock return is a linear (and increasing) function of its beta.[18] Indeed, in the simple version of our model outlined thus far, the firm's expected *mpk* is proportional to its expected stock return. This link, first, justifies our use of data on expected stock returns and stock market betas as a proxy for expected *mpk* in Section 3 and second, shows that the dispersion in expected stock returns puts tight empirical discipline on the dispersion in betas and so expected *mpk* arising from risk channels - indeed, under some circumstances, they are proportional to one another. We use this connection to provide transparent intuition for our empirical approach to estimating the degree of *mpk* dispersion due to cross-sectional variation in risk premia in Section 5.

We obtain an analytic approximation for expected stock market returns by log-linearizing around the non-stochastic steady state where $X_t = Z_t = 1$. Our main result in this section is that, to a first-order, the (log of the) expected excess stock return of firm i (over the risk-free rate) is equal to (derivations in Appendix A.4)

$$Er_{it+1} \equiv \log \mathbb{E}_t \left[R_{it+1}^e \right] = \psi \beta_i \gamma_t \sigma_\varepsilon^2 \tag{18}$$

where

$$\psi = \frac{\frac{1}{\rho} + \delta - 1}{\frac{1}{\rho} + \delta \left(1 - \theta \right) - 1} \frac{1 - \rho}{1 - \rho \rho_x + \rho \gamma_1 \sigma_\varepsilon^2}$$

---

[18]It is well known that a first-order approximation may not be sufficient to capture risk premia. In our quantitative work in Section 5, we work with numerical higher order approximations.

The expected excess return depends on the firm's beta (indeed, is linear and increasing in beta) and the price of risk. The risk premium is increasing in γ_0, and as γ_1 becomes more negative. Further, because the price of risk is countercyclical, risk premia increase during downturns for all firms and fall during expansions.[19] The time-t cross-sectional dispersion in expected returns is given by

$$\sigma_{Ert}^2 \equiv \sigma_{\log \mathbb{E}_t[R_{it+1}]}^2 = \psi^2 \sigma_\beta^2 \left(\gamma_t \sigma_\varepsilon^2\right)^2 \tag{19}$$

Similar to our findings for expected mpk, the expression reveals a tight link between beta dispersion and expected stock return dispersion. Indeed, were firms homogeneous with respect to their loadings on aggregate conditions, dispersion in expected returns would be zero. Moreover, as with expected mpk dispersion, expected stock return dispersion is increasing in the price of risk and so is countercyclical.

Comparing equations (12) and (18) shows that, in this setting, expected returns, Er_{it+1}, are proportional to expected mpk, $Empk_{it+1}$ and equations (13) and (19) show that σ_{Ert}^2 is exactly proportional to σ_{Empkt}^2. Thus, the expressions reveal a tight connection between cross-sectional variation in expected stock returns and expected mpk – both are dependent on the variation in betas. Although the direct proportionality will not hold exactly in the full non-linear model, we will use this intuition to quantify the role of risk considerations in generating dispersion in expected mpk.

Our results in this section point to an empirical strategy to estimate the three key structural parameters of our mechanism using readily available stock market data. First, equation (19) reveals that dispersion in beta is tightly linked to dispersion in expected returns. Next, expressing equation (18) for the market index gives

$$Er_{mt+1} = \psi \bar{\beta} \gamma_t \sigma_\varepsilon^2, \qquad Erm \equiv \mathbb{E}\left[Er_{mt+1}\right] = \psi \bar{\beta} \gamma_0 \sigma_\varepsilon^2 \tag{20}$$

which shows that the market equity premium is increasing as γ_1 becomes more negative through its effects on ψ ($\bar{\beta}$ denotes the mean beta across firms). Lastly, the Sharpe ratio for an individual firm is

$$SR_{it} = \frac{\beta_i \gamma_t \sigma_\varepsilon^2}{\sqrt{\left(\frac{1-\rho\rho_x+\rho\gamma_1\sigma_\varepsilon^2}{1-\rho\rho_z}\right)^2 \sigma_{\tilde\varepsilon}^2 + \beta_i^2 \sigma_\varepsilon^2}} \qquad \mathbb{E}\left[SR_{it}\right] = \frac{\beta_i \gamma_0 \sigma_\varepsilon^2}{\sqrt{\left(\frac{1-\rho\rho_x+\rho\gamma_1\sigma_\varepsilon^2}{1-\rho\rho_z}\right)^2 \sigma_{\tilde\varepsilon}^2 + \beta_i^2 \sigma_\varepsilon^2}} \tag{21}$$

Due to the presence of idiosyncratic risk, individual firms do not attain the maximum Sharpe ratio. However, for a perfectly diversified market portfolio, i.e., one with no idiosyncratic risk,

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[19]Strictly speaking, these results hold in the approximation so long as $1 - \rho\rho_x + \rho\gamma_1\sigma_{\tilde\varepsilon}^2 > 0$. This condition does not play a role in the numerical solution.

the expressions collapse to

$$SR_{mt} = \gamma_t \sigma_\varepsilon, \qquad ESRm \equiv \mathbb{E}\left[SR_{mt}\right] = \gamma_0 \sigma_\varepsilon \tag{22}$$

which is the maximum achievable Sharpe ratio. Although the market may not attain this value due to the nonlinear effects of idiosyncratic shocks, the expression highlights that the Sharpe ratio on the market portfolio is informative about γ_0. Together, equations (19), (20) and (22) link three readily observable moments of asset market data to the three parameters, γ_0, γ_1 and σ_β^2.

4.3   Adjustment Costs

In this section, we extend our framework to include capital adjustment costs. Although the main insights from the previous sections go through, we illustrate an important interaction effect between these costs and the effects of risk premia, namely that adjustment frictions can amplify the impact of these systematic risk exposures on expected mpk dispersion.

We assume that capital investment is subject to quadratic adjustment costs, given by

$$\Phi\left(I_{it}, K_{it}\right) = \frac{\xi}{2}\left(\frac{I_{it}}{K_{it}} - \delta\right)^2 K_{it}$$

The firm's investment problem then takes the form

$$V\left(X_t, Z_{it}, K_{it}\right) = \max_{K_{it+1}} \quad GX_t^{\beta_i} Z_t K_{it}^\theta - K_{it+1} + \left(1 - \delta\right) K_{it} - \Phi\left(I_{it}, K_{it}\right)$$
$$+ \quad \mathbb{E}_t\left[M_{t+1} V\left(X_{t+1}, Z_{it+1}, K_{it+1}\right)\right]$$

In the presence of adjustment costs, exact solutions to the firm's problem are are no longer available. Appendix A.2.2 shows that to a first-order approximation, the firm's policy function is

$$k_{it+1} = \phi_{00} + \phi_1 \beta_i x_t + \phi_2 z_{it} + \phi_3 k_{it} - \phi_{01} \beta_i \tag{23}$$

where

$$0 = \left((\theta - 1) - \hat{\xi}(1 + \rho)\right)\phi_3 + \rho\hat{\xi}\phi_3^2 + \hat{\xi}$$

$$\phi_1 = \frac{(\rho_x - \gamma_1\sigma_\varepsilon^2)\phi_3}{\hat{\xi}(1 - \rho\rho_x\phi_3 + \rho\gamma_1\sigma_\varepsilon^2\phi_3)}, \qquad \phi_2 = \frac{\rho_z\phi_3}{\hat{\xi}(1 - \rho\rho_z\phi_3)}$$

$$\phi_{01} = \frac{\phi_3}{\hat{\xi}(1 - \rho\phi_3)}\frac{\gamma_0\sigma_\varepsilon^2}{1 - \rho\rho_x\phi_3 + \rho\gamma_1\sigma_\varepsilon^2\phi_3}$$

We define the constant ϕ_{00} in the Appendix.[20] The term $\hat{\xi}$ is a composite parameter that captures the severity of adjustment costs, defined by $\hat{\xi} \equiv \frac{\xi}{1 - \rho(1 - \delta)}$.

Now, the current level of capital affects the new chosen level. The coefficient ϕ_3 captures the strength of this relationship. It lies between zero and one and is increasing in the adjustment cost, $\hat{\xi}$. It is independent of the risk premium. The other coefficients each have a counterpart in equation (10), but are modified to reflect the influence of adjustment costs. The coefficients ϕ_1 and ϕ_2 are both decreasing in the adjustment cost – intuitively, adjustment costs reduce the responsiveness to shocks. As adjustment costs tend to infinity, ϕ_3 approaches one and the latter two coefficients go to zero. As adjustment costs become large, the firm's choice of capital becomes more autocorrelated and eventually unresponsive to shocks. Importantly, ϕ_{01} is increasing in these costs, showing that additional adjustment frictions increase the importance of the firm's beta in determining its choice of capital.[21]

The expression for ϕ_{01} reveals an interaction between adjustment costs and time-varying risk – the denominator contains the product of ϕ_3 and γ_1, which implies that a more negative γ_1 leads to higher values of ϕ_{01} as long as adjustment costs are non-zero. Clearly this term disappears if adjustment costs are zero. In a moment, we will relate the value of ϕ_{01} to $Empk$ dispersion. Thus, this interaction effect will increase the impact of risk premia on that dispersion.

In this setting, both risk premium effects and adjustment costs lead to $Empk$ dispersion (realized mpk dispersion also depends on uncertainty, as above). Closed-form solutions are not available for period-by-period dispersion. However, to gain some intuition, we are able to characterize the mean of firm-level expected mpk (which is also the mean of realized mpk, since the shocks are mean-zero) at the firm-level and thus the dispersion in this mean component:

$$\mathbb{E}[Empk_{it+1}] = -\frac{1 - \theta}{1 - \phi_3}(\phi_{00} - \phi_{01}\beta_i) \quad \Rightarrow \quad \sigma_{\mathbb{E}[Empk_{it+1}]}^2 = \left(\frac{1 - \theta}{1 - \phi_3}\right)^2\phi_{01}^2\sigma_\beta^2 \qquad (24)$$

---

[20]As above, we ignore terms reflecting variance adjustments that are close to zero.

[21]Strictly speaking, this is true so long as $1 - \rho\rho_x\phi_3 + \rho\gamma_1\sigma_\varepsilon^2\phi_3 > 0$. This condition holds for any reasonable level of adjustment costs, i.e., given our estimates of the other parameters, ξ must be less than approximately 2,180.

Loosely speaking, the measure is the variance of the mean (i.e., permanent) component of firm-level mpk deviations. Recall that on their own, heterogeneous risk exposures only lead to persistent mpk deviations (in terms of the ordering across firms). These are exactly the effects we are picking up in (24).[22] Further, we are particularly interested in this component, since the data show an important role for a highly persistent (if not permanent) component in firm-level mpk deviations. Notice also that ϕ_{01} is multiplicative in γ_0; in the absence of risk effects, there is no persistent $Empk$ dispersion, even in the presence of adjustment costs.

Thus, expression (24) shows that the extended model continues to give rise to mpk deviations that are persistent at the firm-level. Moreover, the expression reveals a second amplification effect of adjustment costs through the $1 - \phi_3$ term in the denominator. Recall that ϕ_3 is increasing in these costs, as is ϕ_{01}, so that holding fixed the other parameters, higher adjustment costs unambiguously increase risk effects on dispersion in $Empk$. An interesting implication of this result is that, perhaps surprisingly, adjustment frictions do not only affect transitory dispersion in the mpk. While this is true on their own, in conjunction with a fixed component in the mpk, which we have here, these frictions can serve to amplify the effects of that component.

Finally, how do adjustment costs change the relationship between expected mpk, beta and expected stock returns? Appendix A.4 shows that under a first-order approximation, expected returns are not affected by adjustment costs and so all the results from Section 4.2) continue to hold.[23] Thus, the arguments made in that section linking the key parameters of our model to moments of asset returns go through unchanged.

4.4 Other Distortions

Section 4.2 demonstrated a tight connection between dispersion in expected mpk and stock returns, pointing to a natural approach to quantify our mechanism, namely, using the properties of stock market data. Before implementing that approach numerically in the next section, we demonstrate here a key feature of that strategy – namely, it is robust to the presence of other idiosyncratic factors that may affect the firm's investment decisions and lead to mpk dispersion. Recent work has pointed to a number of such factors, including financial frictions, variable markups or policy-induced distortions. Moreover, it has been pointed out that attempts to

---

[22]Due to the interaction with adjustment costs, it is possible that these exposures can add to/subtract from the transitory dispersion created by those costs. We have derived an alternative measure of $Empk$ dispersion, namely, the mean of the variance, defined as $\mathbb{E}\left[\sigma_{Empk}^2\right]$ (in contrast, equation (24) computes the variance of the mean). Quantitatively, we find that the contribution of risk exposures under this alternative is almost exactly that in expression (24). Because of nonlinearities, for that exercise, we calculate the portion due to risk premia effects as the total minus the implied dispersion when $\gamma_0 = \gamma_1 = 0$.

[23]Although this is only exactly true under our first-order approximation, Table 7 verifies numerically that adjustment costs have modest effects on moments of returns.

identify one of these forces - while abstracting from others - may yield misleading conclusions. This section demonstrates that our strategy of using asset market data is robust to this critique. In other words, our approach yields accurate results even in the presence of other, un-modeled, distortions.

We follow the broad literature, e.g., Hsieh and Klenow (2009) and Restuccia and Rogerson (2008), and introduce these distortions as purely idiosyncratic "taxes" or "wedges" on firm revenues, $1 - e^{\tau_{it+1}}$ (so that the firm keeps a portion $e^{\tau_{it+1}}$). We work with the following specification for the wedge:

$$\tau_{it+1} = -\nu z_{it+1} - \eta_{it+1} \tag{25}$$

The wedge is composed of two pieces. The first component is correlated with the firm's idiosyncratic productivity, where the strength of the relationship is captured by ν. If $\nu > 0$, the wedge discourages (encourages) investment by high (low) productivity firms. If $\nu < 0$, the opposite is true. The second component is uncorrelated with firm characteristics and can be either time-varying or fixed. Low values of η spur greater investment by firms irrespective of their underlying characteristics. We assume the firm knows the uncorrelated piece, η_{it+1}, when it chooses period t investment, i.e., k_{it+1}. Further, we assume that both components of the wedge are uncorrelated with the firm's beta. David and Venkateswaran (2017) show that this formulation can capture, for example, certain forms of financial frictions (due, e.g., to liquidity costs) and markups, in addition to policy-related distortions. We loosely refer to the wedge as a "distortion," although we do not take a stand on whether it stems from efficient factors or not - simply that there are other frictions in the allocation process. Appendix A.5 derives the following expression for the realized mpk:

$$
\begin{aligned}
mpk_{it+1} &= \alpha + \varepsilon_{it+1} + \beta_i \varepsilon_{t+1} + \nu \rho_z z_{it} + \eta_{it+1} + \beta_i \gamma_t \sigma_\varepsilon^2 \\
&= \alpha + \varepsilon_{it+1} + \beta_i \varepsilon_{t+1} + \nu \mathbb{E}_t [z_{it+1}] + \eta_{it+1} + \beta_i \gamma_t \sigma_\varepsilon^2
\end{aligned}
\tag{26}
$$

The distortion has several effects on the realized mpk. After the constant, the first two terms capture the effects of uncertainty over shocks and are identical to those in the baseline case. Next, the mpk includes a component that reflects the severity of the correlated distortion, ν, and depends on the firm's expectations of its idiosyncratic productivity - for example, if $\nu > 0$ (the empirically relevant case), the distortion discourages (encourages) investment by high (low) productivity firms, leading to mpk deviations that are correlated with idiosyncratic productivity. Next, the mpk also depends on the uncorrelated component of distortions, η: firms with a high realization of η_{it+1} will invest more than their fundamentals would dictate, again leading to mpk deviations. Finally, the last term reflects the risk premium, which, importantly, is independent of the distortions.

From expression (26), we can derive $Empk$ as

$$Empk_{it+1} = \alpha + \nu\rho_z z_{it} + \eta_{it+1} + \beta_i\gamma_t\sigma_\varepsilon^2$$

and its cross-sectional variance:

$$\sigma_{Empk}^2 = (\nu\rho_z)^2\sigma_z^2 + \sigma_\eta^2 + \sigma_\beta^2\left(\gamma_t\sigma_\varepsilon^2\right)^2 \tag{27}$$

Dispersion in $Empk$ is generated by three forces - first, the correlated component of the distortion, ν (its contribution to mpk dispersion also depends on the cross-sectional variance of expected idiosyncratic productivity, which is the term in parentheses); second, the variance of the uncorrelated component; and third, the variation in the risk premium.

Turning to stock market returns on the other hand, Appendix A.5 proves that equation (18) still holds. In other words, expected stock returns are independent of idiosyncratic distortions. These results imply that the mapping from expected returns to beta is, to a first-order, unaffected by these other distortions, as is the mapping from beta dispersion to its effects on $Empk$. Thus, even in the richer environment here, featuring the additional sources of misallocation revealed in expressions (26) and (27), using stock market data continues to yield accurate estimates of the effects of heterogeneous risk exposures alone.

Aggregate wedges. In principle, we can allow the wedge to also be correlated with aggregate productivity, x_t. In that case, it turns out the effects depend on the precise specification. For example, consider the following formulation:

$$\tau_{it+1} = -\nu_z z_{it+1} - \nu_x x_{t+1} - \eta_{it+1}$$

Here, the parameter ν_x captures the correlation of the distortion with the state of the business cycle. All firms are equally distorted by the aggregate component of the wedge. In this case, we can prove a similar result as with only idiosyncratic wedges – the distortion does not affect the cross-sectional dispersion in expected stock returns and so that moment still accurately pins down the relevant risk exposures (the wedge also does not affect the dispersion in $Empk$ coming from risk premium effects).[24]

As a second example, consider the following specification:

$$\tau_{it+1} = -\nu_z z_{it+1} - \nu_x \beta_i x_{t+1} - \eta_{it+1}$$

---

[24]The proofs for this section are in Appendix A.5.

Here, high beta firms are also disproportionately affected by the aggregate distortion. In this case, we can prove that expected return dispersion gives a lower bound on risk premium effects if the wedge worsens in downturns, i.e., if $\gamma_x < 0$, which may be a plausible conjecture. On the other hand, we could be at risk of overstating these effects if the wedge worsens in expansions, i.e., $\gamma_x > 0$. It turns out that our empirical results suggest a tight upper bound on the extent of the potential bias – specifically, the fact that $Empk$ is countercyclical from prediction (2). To see this, Appendix A.5 derives the following expression for $Empk$:

$$\mathbb{E}_t \left[mpk_{it+1} \right] = \alpha + \nu_z \rho_z z_{it} + \nu_x \beta_i \rho_x x_t + (1 - \nu_x) \beta_i \gamma_t \sigma_\varepsilon^2 + \eta_{it+1} \ .$$

Rearranging and substituting for γ_t,

$$\mathbb{E}_t \left[mpk_{it+1} \right] = \alpha + \nu_z \rho_z z_{it} + \left(\nu_x \rho_x + (1 - \nu_x) \gamma_1 \sigma_\varepsilon^2 \right) \beta_i x_t + (1 - \nu_x) \beta_i \gamma_0 \sigma_\varepsilon^2 + \eta_{it+1} \ .$$

The fact that $Empk$ is countercyclical implies that the term in parentheses multiplying x_t is negative, which puts the following bound on ν_x:

$$\frac{\nu_x}{1 - \nu_x} < -\frac{\gamma_1 \sigma_\varepsilon^2}{\rho_x}$$

Intuitively, a positive value of ν_x adds a procyclical element to expected mpk. That $Empk$ is actually countercyclical then puts a sharp bound on how large a positive value of ν_x can be. Using the parameter estimates from the next section, this calculation yields a maximum value of ν_x of about 0.065. In Appendix A.5 we derive an equation characterizing the potential bias from a positive value of ν_x – even at this upper bound, the bias is quantitatively negligible.

These results show that our empirical strategy yields accurate, and possibly conservative, estimates in the presence of other, possibly aggregate, distortions in the firm's investment problem. However, precisely pinning down the properties of such an aggregate wedge would involve taking a stand on its sources, e.g., time-varying financial frictions or markups, or policy-related distortions, and estimating its cyclical properties.

5   Quantitative Analysis

The analytical results in the previous section showed a tight relationship between salient moments of asset market data and the effects of heterogeneous risk premia on mpk dispersion – specifically, between the cross-sectional dispersion of expected returns, σ_{Er}^2, the equity premium, Erm and the Sharpe ratio, $ESRm$ on the one hand, and the structural parameters γ_0, γ_1 and σ_β^2 on the other. In this section, we use this insight to develop an empirical strategy to

quantify the extent of mpk dispersion arising from our mechanism.

5.1 Parameterization

We begin by assigning values to the more standard production parameters of our model. Following Atkeson and Kehoe (2005), we set the overall returns to scale in production $\theta_1 + \theta_2$ to 0.85. We assume standard shares for capital and labor of 0.33 and 0.67, respectively, which gives $\theta_1 = 0.28$ and $\theta_2 = 0.57$. These values imply $\theta = 0.65$.[25] We assume a period length of one year and accordingly set the rate of depreciation to $\delta = 0.08$. We estimate the adjustment cost parameter, ξ, in order to match the autocorrelation of investment, denoted corr $(\Delta k_t, \Delta k_{t-1})$, which is 0.38 in our data. Equation (30) in Appendix A.6 provides a closed-form expression for this moment, which reveals a tight connection with the severity of adjustment frictions.[26]

To estimate the parameters governing the aggregate shock process, we build a long sample of Solow residuals for the US economy using data from the Bureau of Economic Analysis on real GDP and aggregate labor and capital. The construction of this series is standard (details in Appendix B.2). With these data, we use a standard autoregression to estimate the parameters ρ_x and σ_ε^2. This procedure gives values of 0.94 and 0.0247 for ρ_x and σ_ε, respectively.[27] Under our assumptions, firm-level productivity (including the aggregate component) can be measured directly (up to an additive constant) as $va_{it} - \theta k_{it}$ where va_{it} denotes the log of value added. After controlling for the level of aggregate productivity, a similar autoregression on the residual (firm-specific) component yields values for ρ_z and $\sigma_{\tilde\varepsilon}$ of 0.93 and 0.28, respectively.

Turning to the parameters of the SDF, we set $\rho = 0.988$ to match an average annual risk-free rate of 1.2%. Following the arguments in Section 4.2, we estimate the values of γ_0 and γ_1 to match the post-war (1947-2017) average annual excess return on the market index of 8% and Sharpe ratio of 0.53.[28] This strategy is equivalent to matching both the mean and volatility of market excess returns (the standard deviation is 14.6%). To be comparable to the data, stock returns in the model need to be adjusted for financial leverage. To do so, the mean and

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[25]This is close to the values generally used in the literature. For example, Cooper and Haltiwanger (2006) estimate a value of 0.59 for US manufacturing firms. David and Venkateswaran (2017) use a value of 0.62.

[26]The expression also reveals that for ρ_x close to ρ_z, which we find in the data, described next, the autocorrelation of within-firm investment is almost invariant to the firm's beta (indeed, the invariance is exact if $\rho_x = \rho_z$). Thus, even with dispersion in betas, we may not see large variation in this moment across firms.

[27]The autoregression does not reject the presence of a unit root at standard confidence levels. We have also worked with the annual TFP series developed by John Fernald, available at `https://www.frbsf.org/economic-research/indicators-data/total-factor-productivity-tfp/`. These data are only available for the more recent post-war period, but also show that the series is close to a random walk (i.e., the autocorrelation of growth rates is essentially zero). To address possible concerns about aggregation in these series, we have also constructed an alternative series directly from the firm-level data by averaging across the firms in each year. This gives results quite similar to the baseline, $\rho_x = 0.92$ and $\sigma_\varepsilon = 0.0245$. Details are in Appendix B.2.

[28]We calculate these values using annualized monthly excess returns obtained from Kenneth French's website, `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`.

standard deviation of the model-implied returns need to be scaled by a factor of $1 + \frac{D}{E}$ where $\frac{D}{E}$ is the debt-to-equity ratio. We follow, e.g., Barro (2006) and assume an average debt-to-equity ratio of 0.5. Because both the numerator and denominator are scaled by the same constant, the Sharpe ratio is unaffected. For ease of interpretation, in what follows we report the properties of levered returns. To compute the return on the market, we must also take a stand on the mean beta across firms. Assuming that the mean of $\hat{\beta}_i$ (the underlying productivity beta) is one, and using the value of ω (the sensitivity of wages to aggregate shocks) suggested by İmrohoroğlu and Tüzel (2014) of 0.20, we can compute the mean beta to be 1.99.[29] This is simply the mean productivity beta adjusted for the leverage effects of labor liabilities. This procedure yields values of $\gamma_0 = 35$ and $\gamma_1 = -120$.

Finally, again following the insights in Section 4.2, we estimate the dispersion in betas to match the cross-sectional dispersion in expected stock returns. Because expected returns are not directly observable, we must choose an asset pricing model with which to estimate them. To be consistent with the broad literature, we use the Fama and French (1992) 3-factor model. This procedure yields an average annual variance of expected returns of 0.0169 (standard deviation of 0.13). The corresponding estimate of σ_β is 12 and adjusting for the scaling $1 - \theta_2$ gives the dispersion in underlying productivity betas, $\hat{\beta}_i$ to be 5.00.

To accurately capture the properties of the time-varying risk premium, we solve for returns numerically. Specifically, we implement a fourth-order approximation using Dynare++. We describe the details of the numerical procedure in Appendix C. In brief, for a given set of parameters, we use the model solution to simulate time series of excess returns for a large panel of firms that differ in their betas. Averaging returns across these firms in each time period yields a series for the market return. We can then compute the mean and standard deviation (i.e., Sharpe ratio) of the market return. For each beta-type in each time period, we compute the expected return directly as the conditional expecation $\mathbb{E}_t [R_{it+1}]$ and then average over the time periods to obtain the average expected return for a firm of that beta-type. For this set of parameters, we also compute the autocorrelation of investment by applying equation (30). We then estimate the four parameters γ_0, γ_1, σ_β^2 and ξ so that the results of this procedure leads to values of (1) market excess returns, (2) market Sharpe ratio, (3) cross-sectional dispersion in expected returns and (4) the autocorrelation of investment that match the empirical values.

Table 6 summarizes our empirical approach/results.

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[29]İmrohoroğlu and Tüzel (2014) estimate this value to match the cyclicality of wages.

Table 6: Parameterization - Summary

Parameter	Description	Value
Production		
θ_1	Capital share	0.28
θ_2	Labor share	0.57
δ	Depreciation rate	0.08
ξ	Adjustment cost	0.04
$\sigma_{\hat{\beta}}$	Std. dev of risk exposures	5.00
Stochastic Processes		
ρ_x	Persistence of agg. shock	0.94
σ_ε	Std. dev. of agg. shock	0.0247
ρ_z	Persistence of idiosyncratic shock	0.93
$\sigma_{\tilde{\varepsilon}}$	Std. dev. of idiosyncratic shock	0.28
ω	Wage elasticity	0.20
Stochastic Discount Factor		
ρ	Time discount rate	0.988
γ_0	SDF – constant component	35
γ_1	SDF – time-varying component	-120

5.2 Risk-Based Dispersion in MPK

Table 7 presents our main results. We report four variants of our framework. The first column ("Baseline") corresponds to our full model with time-varying risk and adjustment costs. In the second column ("Only RIsk"), we report the effects of risk premia without adjustment costs (i.e., ignoring the interaction effects demonstrated above). The third column ("Constant Risk") examines a version with adjustment costs but a constant price of risk (i.e., $\gamma_1 = 0$). The last column ("Only Constant Risk") has a constant price of risk and no adjustment costs. Our goal in showing these different permutations is to understand the role that each element of our model plays in leading to various patterns in mpk dispersion.

Long-run effects. The first row of the table shows the average long-run level of mpk dispersion that stems from heterogeneous risk exposures. Once we have the estimated parameters, this is a simple application of expression (24), or without adjustment costs, the special case in (14). The second and third rows show the percentage of total observed misallocation that this value accounts for. In our sample, overall σ^2_{mpk} is 0.45. This is the denominator in the second row. Next, we compute the dispersion stemming from only the permanent component of observed misallocation. For each firm, we regress the time-series of its mpk deviations on a firm-level fixed effect. The fixed-effect is the permanent component of firm-level mpk and the residuals transitory components. We then compute the variance of the permanent component,

Table 7: Risk Premium Effects and Misallocation

	Baseline (1)	Only Risk (2)	Constant Risk (3)	Only Constant Risk (4)
MPK Implications				
$E\sigma^2_{Empk}$	0.20	0.07	0.18	0.07
$\frac{E\sigma^2_{Empk}}{\sigma^2_{mpk}}$	0.44	0.15	0.40	0.15
$\frac{E\sigma^2_{Empk}}{\sigma^2_{\overline{mpk}}}$	0.67	0.23	0.60	0.22
$\Delta\overline{a}$	0.08	0.03	0.07	0.03
corr $\left(\sigma^2_{Empkt}, x_t\right)$	-0.25	-0.98	0.48	0.08
Moments				
Erm	0.08	0.10	0.05	0.06
$ESRm$	0.53	0.67	0.55	0.72
corr $\left(\Delta k_t, \Delta k_{t-1}\right)$	0.38	-0.03	0.38	-0.03

which yields a value of $\sigma^2_{\overline{mpk}} = 0.30$, about two-thirds of the total. This is the denominator in the third row of the table. That row compares the mpk dispersion generated by risk effects, which is essentially all persistent in nature, to the permanent piece in the data. The next row quantifies the aggregate implications of the estimated dispersion for the long-run level of aggregate TFP. It reports the gains in TFP from eliminating this source of mpk dispersion, denoted $\Delta\overline{a}$.[30] This is just an application of expression (17).

Column (1) shows that in the full model, risk premium effects lead to mpk dispersion of 0.2. This accounts for about 44% of overall mpk dispersion in our data and about two-thirds of the permanent component. These values lead to TFP losses of about 8% (compared to an environment without risk premia, i.e., where $\gamma_0 = \gamma_1 = 0$). These results suggest that (1) variation in firm-level risk exposures can generate significant misallocation, particularly when compared to the permanent component in the data, and (2) the consequences for measures of aggregate performance such as TFP can be substantial. Column (2) shows that on their own, these exposures generate mpk dispersion of 0.07, which accounts for 15% of total σ^2_{mpk} in the data and for 23% of the permanent component. In other words, though the impact of risk premia remain significant on their own, they are less than half of those in column (1). These results highlight the important interaction effects with adjustment costs revealed in expression (24) – in the first column, these effects are taken into account; in the second column, they are not. The associated TFP losses are modest, but significant – the results show that the long-run level of TFP would be 3% higher without these effects.

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[30]Note that this calculation does not mean that policies eliminating this source of mpk dispersion here would be desirable. We merely see this as a useful way to quantify the implications of our findings.

Columns (3) and (4) show that the majority of these effects stems from the presence of a high permanent component in the price of risk, i.e., γ_0. Setting $\gamma_1 = 0$ only modestly reduces the size of these effects in the presence of adjustment costs (compare columns (1) and (3)) and has a negligible effect on the results without them (columns (2) vs. (4)). The implication is that time-varying prices of risk do not add much to the average long-run level of mpk dispersion.

Time-variation. The last row in the top panel examines the second main implication of the theory, namely, the countercyclicality of mpk dispersion, which we measure as the correlation of σ^2_{Empkt} with the state of the business cycle, i.e., x_t.[31] Column (1) shows that the full model generates significantly countercyclical dispersion in $Empk$ – the correlation of σ^2_{Empkt} with the state of the cycle is -0.25. Column (2) shows that as the only factor behind $Empk$ dispersion, the time-varying nature of risk premia would lead to an almost perfectly negative correlation with the business cycle. This is a clear implication of equation (13). The presence of adjustment costs in the first column breaks this relationship and leads to a smaller correlation (in absolute value). Finally, the last two columns show that time-varying risk is key to generating counter-cyclical dispersion. Without this element, $Empk$ dispersion is actually positive (significantly so with adjustment costs and only mildly so without). Thus, it seems that the interaction of a countercyclical price of risk with adjustment frictions is crucial in yielding a negative (though far from negative one) correlation between $Empk$ dispersion and the state of the business cycle.

Our findings suggest that time-varying risk adds a notably countercyclical element to mpk dispersion. To highlight the potential implications of this result, consider the connection with the empirical results in Eisfeldt and Rampini (2006), who show that firm-level dispersion measures tend to be countercyclical, yet most capital reallocation is procyclical. In light of our results, it may not be as surprising that countercyclical dispersion obtains, even in a completely frictionless model.

How quantitatively important is this countercyclicality, e.g., for measures of aggregate TFP? To answer this question, consider the effects of a negative 2% productivity shock (beginning from the long-run mean). At the estimated values of γ_0 and γ_1, γ_t increases by 7%. Then (putting aside adjustment costs for this calculation), expression (13) implies that σ^2_{Empk} goes up about 14%, from 0.065 to 0.075. Recalling that the average total mpk dispersion in our sample is 0.45, this is a rather mild change, i.e., total dispersion increases about 2%. The effect on aggregate TFP, though modest, is non-negligible - in response to this underlying negative 2% shock, the increase in $Empk$ dispersion stemming from a higher price of risk leads to an additional 0.4% fall in measured aggregate TFP. The effects are symmetric in response to a

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[31]As noted above, with adjustment costs, we do not have analytic expressions for period-by-period $Empk$ dispersion. We compute these values using simulation. Without adjustment costs, we can use expression (13) directly.

positive 2% shock to productivity.

The approximation in expression (18) implies that the negative 2% shock raises the equity premium by about 7% for all firms. Consider the market index. By construction, the average expected return is about 8% in the estimated model. The results imply that this value rises to about 8.5% in response to the shock (and symmetrically, falls to about 7.5% in response to a positive 2% shock). Again, this is a noticeable, though not extremely large increase.[32]

Moments. In the bottom panel of Table 7, we investigate the role that each element plays in matching the salient moments. Our full model in column (1) is directly parameterized to match the three target moments, i.e., the equity premium, Sharpe ratio and autocorrelation of investment. In the second column, we show these moments from the version of our model without adjustment costs (i.e., setting $\xi = 0$ and the holding the other parameters at their estimated value). As implied by the approximation in Section 4.3, adjustment costs have a modest effect on the properties of returns (eliminating them raises the equity premium somewhat and the Sharpe ratio accordingly). However, the autocorrelation of investment falls dramatically without any adjustment friction, indeed, becoming slightly negative (due to the mean-reverting nature of shocks). Thus, some degree of adjustment costs are necessary to match this latter moment. Comparing columns (1) and (3) shows that without time-varying risk, the model struggles to match the equity premium, which falls almost by half, from 8% to 5%. As implied by expressions (22) and (30), time-varying risk has only modest effects on the average Sharpe ratio and the autocorreltion of investment. A similar pattern emerges from columns (2) and (4) – in the absence of adjustment costs, removing time-varying risk significantly reduces the equity premium but has little effect on the other two moments.

In sum, our results in Table 7 show first, ex-ante firm-level variation in risk exposures lead to quantitatively important dispersion in mpk; moreover, the dispersion from this source is persistent and can account for a significant portion of what seems to be a puzzling pattern in the data, namely, persistent mpk deviations at the firm-level. Second, these exposures add a notably countercyclical element to mpk dispersion, going some way towards reconciling the empirical observation (e.g., Eisfeldt and Rampini (2006)) that firm-level dispersion measures tend to be countercyclical.

5.3   Directly Measuring Productivity β's

Our baseline approach to measuring firm-level risk exposures used the tight link between a firm's beta and its expected stock return laid out in Section 4.2. In this section, we use an alternative

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[32]We have also numerically simulated the effect of a similar shock on returns. The results are close to those from the approximation.

strategy to estimate the dispersion in these exposures using only production-side data. In one sense, this approach is more direct – there is no need to employ firm-level stock market data to measure risk exposures. On the other hand, computing β's directly from production-side data has its drawbacks - the data are of a lower frequency (quarterly at best) and the time dimension of the panel is shorter. Further, it may be difficult to apply this method to firms in developing countries (where measured misallocation tends to be larger), since most firm-level datasets there have relatively short panels and are at the annual frequency. For those reasons, we view our results here as an informative check on our baseline findings above.

The approach is as follows. For each firm, we regress measured productivity growth, i.e., $\Delta z_{it} + \beta_i \Delta x_t$ on aggregate productivity growth Δx_t. It is straightforward to verify that the coefficient from this regression is exactly equal to β_i. Using these estimates, we calculate the cross-sectional dispersion in β's. This procedure gives a values of $\sigma_{\hat{\beta}}$ between about 5 and 11. Thus, our estimate using stock market data, i.e., 5, is well within this range and indeed, corresponds to the lower end, making our results likely conservative.

5.4   Other Forms of Heterogeneity

Variation in betas across firms is an essential ingredient in our theory. Our empirical approach links these betas to dispersion in firm-level expected returns. Here, we explore whether other forms of firm-level heterogeneity can quantitatively generate the significant return dispersion observed in the data. In other words, we ask whether our estimates of beta dispersion are picking up meaningful dispersion from some other potential sources.

First, we examine whether adjustments costs alone can generate substantial dispersion in conditional expected returns. To do so, we simulate a large panel of firms of a single beta-type (we set this to $\beta = 1.99$, which corresponds to the mean productivity beta of $\hat{\beta} = 1$). Although the firms are all of a single type, heterogeneity in conditional expected returns can arise from the presence of adjustment costs in combination with different histories of idiosyncratic shocks. The first column of Table 8 reports the results using the estimated value of ξ. The top row shows the minimum of the average of firm-level expected returns (i.e., we simulate a time series of expected returns for each firm, compute the long-run average for each firm and report the minimum), the second row the mean and the third row the maximum. The last row reports the spread between the minimum and the maximum. The estimated adjustment costs lead to very little dispersion in mean expected returns, e.g., the spread between the low and the high firm is only about 0.2%.[33] To verify the robustness of this finding, column (2) repeats this analysis

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[33]This result should not be overly surprising - in the long run, the firms are identical, so mean expected returns should essentially be the same. We have also examined whether adjustment costs can lead to significant *transitory* dispersion in expected returns. To do so, we again simulate a large panel of firms with $\beta = 1.99$

with a higher level of adjustment costs, namely, $\xi = 3$. The larger level of costs increases the level of expected returns slightly (recall that to a first-order, these costs should have no effects on the properties of expected returns) and has virtually no effect on the spread. Thus, it is unlikely that our estimates of β are reflecting the effects of adjustment costs.

Table 8: Expected Return Dispersion - Other Forms of Heterogeneity

| | Adj. Costs | Large Adj. Costs | θ | δ | ρ_z | $\sigma_{\tilde{\varepsilon}}^2$ |
	(1)	(2)	(3)	(4)	(5)	(6)
Min. Er	0.036	0.038	0.029	0.031	0.032	0.023
Mean Er	0.037	0.039	0.037	0.037	0.037	0.037
Max. Er	0.038	0.040	0.056	0.042	0.042	0.040
Spread	0.002	0.002	0.027	0.012	0.010	0.017

The remaining columns of Table 8 allow for variation in technological parameters across firms. Expression (18) provides some guidance as to the effects of some of these parameters on expected returns – taking derivatives, the expression implies that expected returns should be increasing in θ and δ. We also allow firms to differ in the properties of the stochastic process of idiosyncratic shocks, ρ_z and $\sigma_{\tilde{\varepsilon}}^2$. Although these do not influence expected returns under a first-order approximation, there may be some effects due to the nonlinearities in the numerical model.

Column (3) examines heterogeneity in θ, the curvature of the production function. Although there is little guidance on the extent of this heterogeneity (recall that all our estimations are within a single industry), we compute expected returns for three values of θ, namely 0.85 (our baseline), 0.95 and 0.75.[34] In line with the predictions of expression (18), expected returns are increasing in θ. The first row reports the average expected return for a firm with low θ (0.75), the second row is the baseline and the third row is a high θ firm (0.95). The difference in mean expected returns between the highest and lowest θ firms is almost 3%. This is an economically significant spread, suggesting that large differences in this parameter can result in meaningful differences in firm-level risk premia. On the other hand, even this substantial

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and then compute period-by-period dispersion in expected returns. The mean of the cross-sectional standard deviation, i.e., $E\sigma_{Ert}$ is 0.008 (and the maximum 0.038). This is relatively small compared to the observed standard deviation of expected returns, i.e., 0.13. Finally, we have also explored the effects of adjustment costs on the dispersion in realized returns. At the estimated level of costs, the mean standard deviation of realized stock returns for firms with the mean beta is 0.139. This figure increases only mildly with the higher adjustments costs, to 0.149, again suggesting that adjustment costs alone, while economically significant, do not generate substantial amounts of dispersion in these measures.

[34]To put these values in context, they loosely correspond to the range of values found in the literature. For example, İmrohoroğlu and Tüzel (2014) use a value of 0.95. Clementi and Palazzo (2016) use 0.80.

degree of heterogeneity cannot account for the even larger differences observed in the data.

The last three columns show similar results for the remaining three parameters, namely, the depreciation rate and persistence and volatility of idiosyncratic shocks. Expected returns are increasing in the first (as suggested by (18)) and decreasing in the other two. We examine values of ρ_z ranging from 0.50 to 0.95. For the other two parameters, the values in the table report the average expected return when doubling or halving their baseline values (the second row always reports the baseline).[35] Even for these large differences in parameter values, the predicted spread in expected returns only ranges from about 1.0% to 1.7%. Thus, a consistent message emerges across the experiments – unobserved variation in technological parameters seems unlikely to account for the large spreads in expected returns observed in the data.

6   Extensions

The framework we have outlined highlights the link between financial market conditions and the allocation of capital across heterogeneous firms. Indeed, through this channel, the theory provides a natural way to quantify the effects of risk considerations on "real" outcomes, e.g., aggregate TFP. Thus far, the strength of that connection has been tightly disciplined by the single aggregate risk factor in the economy – i.e., the state of technology – which determined both the common component of firm-level productivities and the price of risk. In this section, we generalize that setup to allow for more flexible formulations of the determinants of financial conditions. Although empirically disciplining the additional factors added here may be challenging, we demonstrate that the same insights from our baseline setup go through.

6.1   A Multifactor Model

In principle, it is straightforward to extend our baseline model to include multiple aggregate risk factors. In Appendix A.7, we lay out a simple extension along these lines and show that similar insights go through. There are J factors. The profits of each firm has a vector of heterogeneous loadings on these factors, β_i, where the j-th element of β_i is the loading of firm i on factor j. The exposure of the SDF to the factors is captured by a vector of exposures γ, where element γ_j captures the exposure of the SDF to the j-th factor. For purposes of illustration, we assume γ is constant through time. The covariance matrix of factor innovations is given by Σ_f. The realized mpk is given by

$$mpk_{it+1} = \alpha + \varepsilon_{it+1} + \beta_i \varepsilon_{t+1} + \beta_i \Sigma_f \gamma'$$

---

[35]For $\sigma_{\tilde{\varepsilon}}^2$, we double or half the standard deviation, $\sigma_{\tilde{\varepsilon}}$, so the variance is scaled by a factor of four.

where ε_{t+1} the vector of shocks to these factors. The expected *mpk* and its cross-sectional dispersion are given by

$$\mathbb{E}_t\left[mpk_{it+1}\right] = \alpha + \beta_i \Sigma_f \gamma', \quad \sigma^2_{\mathbb{E}_t[mpk_{it+1}]} = \gamma \Sigma'_f \Sigma_\beta \Sigma_f \gamma'$$

where Σ_β is the covariance matrix of factor loadings across firms. This is the natural analog of expression (13): (1) expected *mpk* is determined by the firm's exposure to (all) the aggregate risk factors in the economy and the prices of those risks, and (2) *mpk* dispersion is a function of the dispersion in those exposures across firms as captured by Σ_β.

Next, we can derive the following approximations for expected stock market returns and the cross-sectional dispersion in expected returns:

$$\log \mathbb{E}_t\left[R^e_{it+1}\right] = \beta_i \psi \Sigma_f \gamma', \quad \sigma^2_{\log \mathbb{E}_t[R_{it+1}]} = \gamma \Sigma'_f \psi' \Sigma_\beta \psi \Sigma_f \gamma'$$

where ψ is a diagonal matrix with

$$\psi_{jj} = \frac{\frac{1}{\rho} + \delta - 1}{\frac{1}{\rho} + (1-\theta)\,\delta - 1} \frac{1-\rho}{1-\rho\rho_j}$$

where ρ_j denotes the persistence of factor j. These are the analogs of expressions (18) and (19) – expected returns depend on factor exposures and the risk prices of those factors. Expected return dispersion depends on the dispersion in those exposures, here captured by Σ_β.

To quantify the impact of these factors on *mpk* dispersion, we would need to know all the primitives of the model governing the dynamics of the factors and exposures, i.e., the exposures of the SDF, γ, the covariance matrix Σ_f, and the vectors of firm loadings, Σ_β. This would likely entail taking a stand on the nature of each factor, computing their properties from the data and calibrating/estimating the γ vector and the covariance matrix of firm exposures, Σ_β.

6.2   Financial Shocks

Our baseline model tightly links financial conditions, for example, the price of risk, to "real" conditions, i.e., the state of aggregate technology. However, financial conditions may not comove one-for-one with the business cycle. It is straightforward to extend our setup to include pure financial shocks. Consider the following extension. The stochastic discount factor takes the form

$$
\begin{aligned}
m_{t+1} &= \log \rho - \gamma_t \varepsilon_{t+1} - \frac{1}{2}\gamma_t^2 \sigma_\varepsilon^2 \\
\gamma_t &= \gamma_0 + \gamma_f f_t,
\end{aligned}
\tag{28}
$$

where

$$f_{t+1} = \rho_f f_t + \varepsilon_f, \quad \varepsilon_f \sim \mathcal{N}\left(0, \sigma_{\varepsilon_f}^2\right)$$

In this formulation, f_t denotes the time-varying state of financial conditions, which is now disconnected from the state of aggregate technology. These financial factors may be correlated with real conditions, x_t, but need not be perfectly so. Thus, there is scope for changes in financial conditions, independent of those in real conditions, to affect the price of risk and through this channel, the allocation of capital.[36] Note also the difference between this setup and the one in Section 6.1 – here, the financial factor, f_t does not directly enter the profit function of the firm, it only affects the price of risk. Thus, it is a shock purely to financial market conditions. In contrast, the factors considered in Section 6.1 directly affected firm profitability.

Keeping the remainder of the environment the same as Section 4, we can derive exactly the same expressions for expected mpk and its cross-sectional variance, i.e.,

$$\mathbb{E}_t\left[mpk_{it+1}\right] = \alpha + \beta_i \gamma_t \sigma_\varepsilon^2, \quad \sigma_{\mathbb{E}_t[mpk_{it+1}]}^2 = \sigma_\beta^2 \left(\gamma_t \sigma_\varepsilon^2\right)^2$$

where now γ_t is a function of financial market conditions. Higher volatility in financial conditions, i.e., $\sigma_{\varepsilon_f}^2$, results in increased dispersion in $Empk$. When credit market conditions tighten (i.e., when f_t is small/negative since $\gamma_f < 0$), γ_t is high and mpk dispersion will rise.

7 Conclusion

In this paper, we have revisited the notion of "misallocation" from the perspective of a risk-sensitive, or risk-adjusted, version of the stochastic growth model with heterogeneous firms. The standard first order condition for investment in that framework suggests that expected firm-level marginal products should reflect exposure to factor risks, and their pricing. To the extent that firms are differentially exposed to these risks, as the literature on cross-sectional asset pricing suggests, the implication is that cross-sectional dispersion in mpk may not only reflect true misallocation, but also risk-adjusted capital allocation. We provide empirical support for this proposition and demonstrate that a suitably calibrated model of firm-level investment behavior suggests that, indeed, risk-adjusted capital allocation accounts for a substantial fraction of observed mpk dispersion among US firms. Importantly, the majority of this dispersion is persistent in nature, which speaks to the large portion of observed mpk dispersion that arises from seemingly persistent/permanent factors at the firm-level.

There are several promising directions for future research. Our framework points to a

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[36]Our baseline model is the nested case where $\gamma_f = \gamma_1$ and f_t and x_t are perfectly correlated.

new connection between business cycle dynamics and the cross-sectional allocation of inputs. Further investigation of this link, for example, a deeper exploration of the source of beta variation across firms, would lead to a better understanding of the underlying causes of observed misallocation across firms. The tractability of our setup allowed us to quantify the effects of financial market considerations, e.g., cross-sectional variation in required rates of return, on measures of economic performance, i.e., aggregate TFP. This link should be useful beyond the misallocation literature and provides a new way to evaluate the implications of the rich set of empirical findings in cross-sectional asset pricing. For example, pursuing multifactor extensions of our analysis (e.g., along the lines laid out in Sections 6.1 and 6.2) to incorporate the many risk factors pointed out in that literature would be fruitful to measure the implications of those factors for allocative efficiency and further assess the role of risk considerations in leading to misallocation. Of particular interest would be whether those factors are efficient or not, e.g., to what extent do capital allocations reflect the "mispricing" of assets.

References

ASKER, J., A. COLLARD-WEXLER, AND J. DE LOECKER (2014): "Dynamic inputs and resource (mis) allocation," *Journal of Political Economy*, 122, 1013–1063.

ATKESON, A. AND P. J. KEHOE (2005): "Modeling and measuring organization capital," *Journal of Political Economy*, 113, 1026–1053.

BALVERS, R. J., L. GU, D. HUANG, M. LEE-CHIN, ET AL. (2015): "Profitability, value and stock returns in production-based asset pricing without frictions," *Journal of Money, Credit, and Banking*.

BARRO, R. J. (2006): "Rare disasters and asset markets in the twentieth century," *The Quarterly Journal of Economics*, 121, 823–866.

BARTELSMAN, E., J. HALTIWANGER, AND S. SCARPETTA (2013): "Cross Country Differences in Productivity: The Role of Allocative Efficiency," *American Economic Review*, 103, 305–334.

BELO, F., X. LIN, AND S. BAZDRESCH (2014): "Labor hiring, investment, and stock return predictability in the cross section," *Journal of Political Economy*, 122, 129–177.

BINSBERGEN, J. V. AND C. OPP (2017): "Real Anomalies," Wharton Working Paper.

BUERA, F. J., J. P. KABOSKI, AND Y. SHIN (2011): "Finance and Development: A Tale of Two Sectors," *American Economic Review*, 101, 1964–2002.

CLEMENTI, G. L. AND B. PALAZZO (2016): "Entry, exit, firm dynamics, and aggregate fluctuations," *American Economic Journal: Macroeconomics*, 8, 1–41.

COCHRANE, J. (1991): "Production-Based Asset Pricing and the Link Between Stock Returns and Economic Fluctuations," *Journal of Finance*, 46, 207–234.

COOPER, R. W. AND J. C. HALTIWANGER (2006): "On the nature of capital adjustment costs," *The Review of Economic Studies*, 73, 611–633.

DAVID, J. M., E. HENRIKSEN, AND I. SIMONOVSKA (2014): "The risky capital of emerging markets," Tech. rep., National Bureau of Economic Research.

DAVID, J. M., H. A. HOPENHAYN, AND V. VENKATESWARAN (2016): "Information, Misallocation and Aggregate Productivity," *The Quarterly Journal of Economics*, 131, 943–1005.

DAVID, J. M. AND V. VENKATESWARAN (2017): "The Sources of Capital Misallocation," Tech. rep., National Bureau of Economic Research.

EISFELDT, A. AND A. RAMPINI (2006): "Capital reallocation and liquidity," *Journal of Monetary Economics*, 53, 369–399.

FAMA, E. F. AND K. R. FRENCH (1992): "Cross-Section of Expected Stock Returns," *The Journal of Finance*, 47, 3247–3265.

GILCHRIST, S. AND E. ZAKRAJSEK (2012): "Credit Spreads and Business Cycle Fluctuations," *American Economic Review*, 102, 1692–1720.

GOMES, J. AND L. SCHMID (2010): "Levered Returns," *Journal of Finance*, 65, 467–494.

GOMES, J., A. YARON, AND L. ZHANG (2006): "Asset pricing implications of firms financing constraints," *Review of Financial Studies*, 19, 1321–1356.

GOPINATH, G., Ş. KALEMLI-ÖZCAN, L. KARABARBOUNIS, AND C. VILLEGAS-SANCHEZ (2017): "Capital Allocation and Productivity in South Europe," *The Quarterly Journal of Economics*, 132, 1915–1967.

HOPENHAYN, H. A. (2014): "Firms, misallocation, and aggregate productivity: A review," *Annu. Rev. Econ.*, 6, 735–770.

HOU, K., C. XUE, AND L. ZHANG (2015): "Digesting anomalies: An investment approach," *Review of Financial Studies*.

HSIEH, C. AND P. KLENOW (2009): "Misallocation and Manufacturing TFP in China and India," *Quarterly Journal of Economics*, 124, 1403–1448.

İMROHOROĞLU, A. AND Ş. TÜZEL (2014): "Firm-level productivity, risk, and return," *Management Science*, 60, 2073–2090.

JONES, C. S. AND S. TUZEL (2013): "Inventory Investment and The Cost of Capital," *Journal of Financial Economics*, 107, 557–579.

KEHRIG, M. (2015): "The Cyclical Nature of the Productivity Distribution," *Working paper*.

KEHRIG, M. AND N. VINCENT (2017): "Do Firms Mitigate or Magnify Capital Misallocation? Evidence from Planet-Level Data," *Working Paper*.

LEWELLEN, J. AND S. NAGEL (2006): "The conditional CAPM does not explain asset-pricing anomalies," *Journal of Financial Economics*, 82, 289–314.

LIU, L. X., T. M. WHITED, AND L. ZHANG (2009): "Investment-based expected stock returns," *Journal of Political Economy*, 117, 1105–1139.

LUCAS, R. E. (1987): *Models of Business Cycles*, vol. 26, Basil Blackwell Oxford.

MIDRIGAN, V. AND D. Y. XU (2014): "Finance and misallocation: Evidence from plant-level data," *The American Economic Review*, 104, 422–458.

MOLL, B. (2014): "Productivity losses from financial frictions: can self-financing undo capital misallocation?" *The American Economic Review*, 104, 3186–3221.

PETERS, M. (2016): "Heterogeneous Mark-Ups, Growth and Endogenous Misallocation," *Working Paper*.

RESTOY, F. AND M. ROCKINGER (1994): "On Stock Market Returns and Returns on Investment," *Journal of Finance*, 49, 543–556.

RESTUCCIA, D. AND R. ROGERSON (2008): "Policy Distortions and Aggregate Productivity with Heterogeneous Establishments," *Review of Economic Dynamics*, 11, 707–720.

——— (2017): "The causes and costs of misallocation," *Journal of Economic Perspectives*, 31, 151–74.

ZHANG, L. (2005): "The Value Premium," *Journal of Finance*, 60, 67–103.

——— (2017): "The Investment CAPM," *European Financial Management*.

Appendix

A   Derivations and Proofs

This appendix provides detailed derivations for the expressions in the text.

A.1   Motivation

Derivation of equation (3).

$$
\begin{aligned}
1 &= \mathbb{E}_t\left[M_{t+1}\left(MPK_{it+1}+1-\delta\right)\right] \\
&= \mathbb{E}_t\left[M_{t+1}\right]\mathbb{E}_t\left[MPK_{it+1}+1-\delta\right]+\mathrm{cov}\left(M_{t+1},MPK_{it+1}\right)
\end{aligned}
$$

Consider the MPK of a 'risk-free' firm defined by $\mathrm{cov}\left(M_{t+1},MPK_{ft+1}\right)=0$. We have

$$
1 = \mathbb{E}_t\left[M_{t+1}\right]\left(MPK_{ft+1}+1-\delta\right)
$$

and combining,

$$
\begin{aligned}
\mathbb{E}_t\left[MPK_{it+1}\right] &= MPK_{ft+1}-\frac{\mathrm{cov}\left(M_{t+1},MPK_{it+1}\right)}{\mathbb{E}_t\left[M_{t+1}\right]} \\
&= \alpha_t+\beta_{it}\lambda_t
\end{aligned}
$$

where α_t, β_{it} and λ_t are as defined in the text.. By a no-arbitrage condition, it must be the case that $\frac{1}{\mathbb{E}_t[M_{t+1}]}=MPK_{ft+1}+1-\delta=R_{ft}$ where R_{ft} is the gross risk-free interest rate.

No aggregate risk.   With no aggregate risk, $M_{t+1}=\rho\ \forall\ t$ where ρ is the rate of time discount. The Euler equation gives

$$
1 = \rho\left(\mathbb{E}_t\left[MPK_{it+1}\right]+1-\delta\right)\ \forall\ i,t \quad\Rightarrow\quad \mathbb{E}_t\left[MPK_{it+1}\right]=\frac{1}{\rho}-\left(1-\delta\right)=r_f+\delta
$$

CAPM.   Clearly, $-\mathrm{cov}\left(M_{t+1},MPK_{it+1}\right)=b\mathrm{cov}\left(R_{mt+1},MPK_{it+1}\right)$ and $\mathrm{var}\left(M_{t+1}\right)=b^2\mathrm{var}\left(R_{mt+1}\right)$. Since the market return is an asset, it must satisfy $\mathbb{E}_t\left[R_{mt+1}\right]=R_{ft}+\frac{\lambda_t}{b}$ so that $\lambda_t=b\left(\mathbb{E}_t\left[R_{mt+1}\right]-R_{ft}\right)$. Substituting into expression (3) gives the CAPM expression in the text.

CCAPM.   A log-linear approximation to the SDF around its unconditional mean gives $M_{t+1}\approx\mathbb{E}\left[M_{t+1}\right]\left(1+m_{t+1}-\mathbb{E}\left[m_{t+1}\right]\right)$ and in the case of $CRRA$ utility, $m_{t+1}=-\gamma\Delta c_{t+1}$ where Δc_{t+1}

is log consumption growth. Substituting for M_{t+1} into expression (3) gives the CCAPM expression in the text.

A.2 Model Solution

A.2.1 Baseline Environment

The static labor choice solves

$$\max e^{\hat{z}_{it}+\hat{\beta}_i x_t} K_{it}^{\theta_1} N_{it}^{\theta_2} - W_t N_{it}$$

with the associated first order condition

$$N_{it} = \left(\frac{\theta_2 e^{\hat{z}_{it}+\hat{\beta}_i x_t} K_{it}^{\theta_1}}{W_t} \right)^{\frac{1}{1-\theta_2}}$$

Substituting for the wage with $W_t = X_t^\omega$ and rearranging gives operating profits

$$\Pi_{it} = G e^{\beta_i x_t + z_{it}} K_{it}^{\theta}$$

where $G \equiv (1-\theta_2)\theta_2^{\frac{\theta_2}{1-\theta_2}}$, $\beta_i = \frac{1}{1-\theta_2}\left(\hat{\beta}_i - \omega\theta_2\right)$, $z_{it} = \frac{1}{1-\theta_2}\hat{z}_{it}$ and $\theta = \frac{\theta_1}{1-\theta_2}$, which is equation (8) in the text.

The first order and envelope conditions associated with (1) give the Euler equation:

$$
\begin{aligned}
1 &= \mathbb{E}_t\left[M_{t+1}\left(\theta e^{z_{it+1}+\beta_i x_{t+1}} G K_{it+1}^{\theta-1} + 1 - \delta\right)\right]\\
&= (1-\delta)\mathbb{E}_t\left[M_{t+1}\right] + \theta G K_{it+1}^{\theta-1}\mathbb{E}_t\left[e^{m_{t+1}+z_{it+1}+\beta_i x_{t+1}}\right]
\end{aligned}
$$

Substituting for m_{t+1} and rearranging,

$$
\begin{aligned}
\mathbb{E}_t\left[e^{m_{t+1}+z_{it+1}+\beta_i x_{t+1}}\right] &= \mathbb{E}_t\left[e^{\log\rho - \gamma_t \varepsilon_{t+1} - \frac{1}{2}\gamma_t^2\sigma_\varepsilon^2 + z_{it+1}+\beta_i x_{t+1}}\right]\\
&= \mathbb{E}_t\left[e^{\log\rho + \rho_z z_{it} + \varepsilon_{it+1} + \beta_i\rho_x x_t + (\beta_i - \gamma_t)\varepsilon_{t+1} - \frac{1}{2}\gamma_t^2\sigma_\varepsilon^2}\right]\\
&= e^{\log\rho + \rho_z z_{it} + \beta_i\rho_x x_t + \frac{1}{2}\sigma_{\tilde{\varepsilon}}^2 + \frac{1}{2}\beta_i^2\sigma_\varepsilon^2 - \beta_i\gamma_t\sigma_\varepsilon^2}
\end{aligned}
$$

and

$$\mathbb{E}_t\left[M_{t+1}\right] = \mathbb{E}_t\left[e^{\log\rho - \gamma_t\varepsilon_{t+1} - \frac{1}{2}\gamma_t^2\sigma_\varepsilon^2}\right] = e^{\log\rho + \frac{1}{2}\gamma_t^2\sigma_\varepsilon^2 - \frac{1}{2}\gamma_t^2\sigma_\varepsilon^2} = \rho$$

so that

$$\theta G K_{it+1}^{\theta-1} = \frac{1 - (1-\delta)\rho}{e^{\log\rho + \rho_z z_{it} + \beta_i\rho_x x_t + \frac{1}{2}\sigma_{\tilde{\varepsilon}}^2 + \frac{1}{2}\beta_i^2\sigma_\varepsilon^2 - \beta_i\gamma_t\sigma_\varepsilon^2}}$$

and rearranging and taking logs,

$$k_{it+1} = \frac{1}{1-\theta}\left(\tilde{\alpha} + \frac{1}{2}\sigma_{\tilde{\varepsilon}}^2 + \frac{1}{2}\beta_i^2\sigma_{\varepsilon}^2 + \rho_z z_{it} + \beta_i\rho_x x_t - \beta_i\gamma_t\sigma_{\varepsilon}^2\right)$$

where

$$
\begin{aligned}
\tilde{\alpha} &= \log\theta + \log G - \alpha \\
\alpha &= -\log\rho + \log\left(1 - (1-\delta)\rho\right) = r_f + \log\left(1 - (1-\delta)\rho\right)
\end{aligned}
$$

Ignoring the variance terms gives equation (9).

The realized mpk is given by

$$
\begin{aligned}
mpk_{it+1} &= \log\theta + \pi_{it+1} - k_{it+1} \\
&= \log\theta + \log G + z_{it+1} + \beta_i x_{t+1} - (1-\theta)k_{it+1} \\
&= \log\theta + \log G + z_{it+1} + \beta_i x_{t+1} - \tilde{\alpha} - \rho_z z_{it} - \beta_i\rho_x x_t + \beta_i\gamma_t\sigma_{\varepsilon}^2 \\
&= \alpha + \varepsilon_{it+1} + \beta_i\varepsilon_{t+1} + \beta_i\gamma_t\sigma_{\varepsilon}^2
\end{aligned}
$$

The time-t conditional expected mpk is

$$\mathbb{E}_t\left[mpk_{it+1}\right] = \alpha + \beta_i\gamma_t\sigma_{\varepsilon}^2$$

and the time-t and mean cross-sectional variances are, respectively,

$$
\begin{aligned}
\sigma_{\mathbb{E}_t[mpk_{it+1}]}^2 &= \sigma_{\beta}^2\left(\gamma_t\sigma_{\varepsilon}^2\right)^2 \\
\mathbb{E}\left[\sigma_{\mathbb{E}_t[mpk_{it+1}]}^2\right] &= \mathbb{E}\left[\sigma_{\beta}^2\left(\gamma_0 + \gamma_1 x_t\right)^2\left(\sigma_{\varepsilon}^2\right)^2\right] = \sigma_{\beta}^2\left(\gamma_0^2 + \gamma_1^2\sigma_x^2\right)\left(\sigma_{\varepsilon}^2\right)^2
\end{aligned}
$$

A.2.2 Adjustment Costs

Policy function. The first order and envelope conditions associated with (23) give the Euler equation:

$$
\begin{aligned}
1 + \xi\left(\frac{K_{it+1}}{K_{it}} - 1\right) &= \mathbb{E}_t\left[M_{t+1}\left(G\theta e^{z_{it+1}+\beta_i x_{t+1}}K_{it+1}^{\theta-1} + 1 - \delta - \frac{\xi}{2}\left(\frac{K_{it+2}}{K_{it+1}} - 1\right)^2 + \xi\left(\frac{K_{it+2}}{K_{it+1}} - 1\right)\frac{K_{it+2}}{K_{it+1}}\right)\right] \\
&= \mathbb{E}_t\left[M_{t+1}\left(G\theta e^{z_{it+1}+\beta_i x_{t+1}}K_{it+1}^{\theta-1} + 1 - \delta + \frac{\xi}{2}\left(\frac{K_{it+2}}{K_{it+1}}\right)^2 - \frac{\xi}{2}\right)\right]
\end{aligned}
$$

In the non-stochastic steady state,

$$
\begin{aligned}
MPK &= G\theta K^{\theta-1} = \frac{1}{\rho} + \delta - 1 \quad \Rightarrow \quad K = \left[\frac{1}{G\theta}\left(\frac{1}{\rho} + \delta - 1\right)\right]^{\frac{1}{\theta-1}} \\
\Pi &= GK^{\theta} \quad \Rightarrow D = GK^{\theta} - \delta K \\
P &= \frac{\rho}{1-\rho}D \\
R &= 1 + \frac{D}{P} = \frac{1}{\rho} \quad \Rightarrow r_f = -\log \rho
\end{aligned}
$$

Define the investment return:

$$
R^I_{it+1} = \frac{G\theta e^{z_{it+1}+\beta_i x_{t+1}} K_{it+1}^{\theta-1} + 1 - \delta + \frac{\xi}{2}\left(\frac{K_{it+2}}{K_{it+1}}\right)^2 - \frac{\xi}{2}}{1 + \xi\left(\frac{K_{it+1}}{K_{it}} - 1\right)}
$$

and log-linearizing,

$$
\begin{aligned}
r^I_{it+1} &= \rho G\theta K^{\theta-1}\left(z_{it+1} + \beta_i x_{t+1}\right) + \left(\rho G\theta\left(\theta - 1\right)K^{\theta-1} - \xi\left(1+\rho\right)\right)k_{it+1} + \rho\xi k_{it+2} + \xi k_{it} \\
&\quad - \log \rho - \rho G\theta\left(\theta - 1\right)K^{\theta-1}k
\end{aligned}
$$

where $k = \log K$.

To derive the investment policy function, conjecture it takes the form

$$
k_{it+1} = \phi_{0i} + \phi_1 \beta_i x_t + \phi_2 z_{it} + \phi_3 k_{it}
$$

Then,

$$
k_{it+2} = \phi_{0i}\left(1 + \phi_3\right) + \phi_1 \beta_i\left(\rho_x + \phi_3\right)x_t + \phi_2\left(\rho_z + \phi_3\right)z_{it} + \phi_3^2 k_{it} + \phi_1 \beta_i \varepsilon_{t+1} + \phi_2 \varepsilon_{it+1}
$$

Substituting into the investment return,

$$
\begin{aligned}
r^I_{it+1} &= \left(\rho G\theta\left(\theta - 1\right)K^{\theta-1} - \xi\left(1 - \rho\phi_3\right)\right)\phi_{0i} - \log \rho - \rho G\theta\left(\theta - 1\right)K^{\theta-1}k \\
&\quad + \left(\rho G\theta K^{\theta-1}\rho_z + \left(\rho G\theta\left(\theta - 1\right)K^{\theta-1} - \xi\left(1+\rho\right)\right)\phi_2 + \rho\xi\left(\rho_z + \phi_3\right)\phi_2\right)z_{it} \\
&\quad + \left(\rho G\theta K^{\theta-1}\rho_x + \left(\rho G\theta\left(\theta - 1\right)K^{\theta-1} - \xi\left(1+\rho\right)\right)\phi_1 + \rho\xi\left(\rho_x + \phi_3\right)\phi_1\right)\beta_i x_t \\
&\quad + \left(\left(\rho G\theta\left(\theta - 1\right)K^{\theta-1} - \xi\left(1+\rho\right)\right)\phi_3 + \rho\xi\phi_3^2 + \xi\right)k_{it} \\
&\quad + \left(\rho G\theta K^{\theta-1} + \rho\xi\phi_2\right)\varepsilon_{it+1} + \left(\rho G\theta K^{\theta-1} + \rho\xi\phi_1\right)\beta_i \varepsilon_{t+1}
\end{aligned}
$$

and

$$
\begin{aligned}
r^I_{it+1} + m_{it+1} &= \left(\rho G\theta\,(\theta-1)\,K^{\theta-1} - \xi\,(1-\rho\phi_3)\right)\phi_{0i} - \rho G\theta\,(\theta-1)\,K^{\theta-1}k - \frac{1}{2}\gamma_0^2\sigma_\varepsilon^2 - \frac{1}{2}\gamma_1^2\sigma_\varepsilon^2 x_t^2 \\
&+ \left(\rho G\theta K^{\theta-1}\rho_z + \left(\rho G\theta\,(\theta-1)\,K^{\theta-1} - \xi\,(1+\rho)\right)\phi_2 + \rho\xi\,(\rho_z+\phi_3)\,\phi_2\right) z_{it} \\
&+ \left(\left(\rho G\theta K^{\theta-1}\rho_x + \left(\rho G\theta\,(\theta-1)\,K^{\theta-1} - \xi\,(1+\rho)\right)\phi_1 + \rho\xi\,(\rho_x+\phi_3)\,\phi_1\right)\beta_i - \gamma_0\gamma_1\sigma_\varepsilon^2\right) x_t \\
&+ \left(\left(\rho G\theta\,(\theta-1)\,K^{\theta-1} - \xi\,(1+\rho)\right)\phi_3 + \rho\xi\phi_3^2 + \xi\right) k_{it} \\
&+ \left(\rho G\theta K^{\theta-1} + \rho\xi\phi_2\right)\varepsilon_{it+1} + \left(\left(\rho G\theta K^{\theta-1} + \rho\xi\phi_1\right)\beta_i - \gamma_0 - \gamma_1 x_t\right)\varepsilon_{t+1}
\end{aligned}
$$

The Euler equation governing the investment return implies

$$
\begin{aligned}
0 &= \mathbb{E}_t\left[r^I_{it+1} + m_{t+1}\right] + \frac{1}{2}\mathrm{var}\left(r^I_{it+1} + m_{it+1}\right) \\
&= \left(\rho G\theta\,(\theta-1)\,K^{\theta-1} - \xi\,(1-\rho\phi_3)\right)\phi_{0i} - \rho G\theta\,(\theta-1)\,K^{\theta-1}k \\
&+ \left(\rho G\theta K^{\theta-1}\rho_z + \left(\rho G\theta\,(\theta-1)\,K^{\theta-1} - \xi\,(1+\rho)\right)\phi_2 + \rho\xi\,(\rho_z+\phi_3)\,\phi_2\right) z_{it} \\
&+ \left(\rho G\theta K^{\theta-1}\rho_x + \left(\rho G\theta\,(\theta-1)\,K^{\theta-1} - \xi\,(1+\rho)\right)\phi_1 + \rho\xi\,(\rho_x+\phi_3)\,\phi_1 - \left(\rho G\theta K^{\theta-1} + \rho\xi\phi_1\right)\gamma_1\sigma_\varepsilon^2\right)\beta_i x_t \\
&+ \left(\left(\rho G\theta\,(\theta-1)\,K^{\theta-1} - \xi\,(1+\rho)\right)\phi_3 + \rho\xi\phi_3^2 + \xi\right) k_{it} \\
&+ \frac{1}{2}\left(\rho G\theta K^{\theta-1} + \rho\xi\phi_2\right)^2\sigma_{\tilde\varepsilon}^2 \\
&+ \frac{1}{2}\left(\rho G\theta K^{\theta-1} + \rho\xi\phi_1\right)^2\beta_i^2\sigma_\varepsilon^2 - \left(\rho G\theta K^{\theta-1} + \rho\xi\phi_1\right)\beta_i\gamma_0\sigma_\varepsilon^2
\end{aligned}
$$

and we can solve for the coefficients from:

$$
\begin{aligned}
0 &= \left(\rho G\theta\,(\theta-1)\,K^{\theta-1} - \xi\,(1-\rho\phi_3)\right)\phi_{0i} - \rho G\theta\,(\theta-1)\,K^{\theta-1}k \\
&+ \frac{1}{2}\left(\rho G\theta K^{\theta-1} + \rho\xi\phi_2\right)^2\sigma_{\tilde\varepsilon}^2 \\
&+ \frac{1}{2}\left(\rho G\theta K^{\theta-1} + \rho\xi\phi_1\right)^2\beta_i^2\sigma_\varepsilon^2 - \left(\rho G\theta K^{\theta-1} + \rho\xi\phi_1\right)\beta_i\gamma_0\sigma_\varepsilon^2 \\
&= \rho G\theta K^{\theta-1}\rho_z + \left(\rho G\theta\,(\theta-1)\,K^{\theta-1} - \xi\,(1+\rho)\right)\phi_2 + \rho\xi\,(\rho_z+\phi_3)\,\phi_2 \\
&= \rho G\theta K^{\theta-1}\rho_x + \left(\rho G\theta\,(\theta-1)\,K^{\theta-1} - \xi\,(1+\rho)\right)\phi_1 + \rho\xi\,(\rho_x+\phi_3)\,\phi_1 - \left(\rho G\theta K^{\theta-1} + \rho\xi\phi_1\right)\gamma_1\sigma_\varepsilon^2 \\
&= \left(\rho G\theta\,(\theta-1)\,K^{\theta-1} - \xi\,(1+\rho)\right)\phi_3 + \rho\xi\phi_3^2 + \xi
\end{aligned}
$$

Define $\hat{\xi} = \frac{\xi}{\rho G \theta K^{\theta-1}} = \frac{\xi}{1-\rho(1-\delta)}$. Then,

$$0 = \left((\theta - 1) - \hat{\xi}(1+\rho)\right)\phi_3 + \rho\hat{\xi}\phi_3^2 + \hat{\xi}$$

$$\phi_1 = \frac{(\rho_x - \gamma_1\sigma_\varepsilon^2)\phi_3}{\hat{\xi}(1 - \rho\rho_x\phi_3 + \rho\gamma_1\sigma_\varepsilon^2\phi_3)}$$

$$\phi_2 = \frac{\rho_z\phi_3}{\hat{\xi}(1 - \rho\rho_z\phi_3)}$$

$$\phi_{0i} = \phi_{00} - \phi_{01}\beta_i + \phi_{02}\beta_i^2$$

where

$$\phi_{00} = \frac{\rho G\theta(1-\theta)K^{\theta-1}k + \frac{1}{2}\left(\rho G\theta K^{\theta-1} + \rho\xi\phi_2\right)^2\sigma_{\tilde{\varepsilon}}^2}{\rho G\theta(1-\theta)K^{\theta-1} + \xi(1 - \rho\phi_3)}$$

$$\phi_{01} = \frac{\phi_3}{\hat{\xi}(1 - \rho\phi_3)}\frac{\gamma_0\sigma_\varepsilon^2}{1 - \rho\rho_x\phi_3 + \rho\gamma_1\sigma_\varepsilon^2\phi_3}$$

$$\phi_{02} = \frac{\rho G\theta K^{\theta-1}\rho\xi\phi_1 + \frac{1}{2}(\rho\xi\phi_1)^2 + \frac{1}{2}\left(\rho G\theta K^{\theta-1}\right)^2}{\rho G\theta(1-\theta)K^{\theta-1} + \xi(1 - \rho\phi_3)}\sigma_\varepsilon^2$$

Note that $\frac{\phi_3}{\hat{\xi}}$ goes to $\frac{1}{1-\theta}$ as $\hat{\xi}$ goes to zero and zero as $\hat{\xi}$ goes to infinity. Again ignoring variance terms, the policy function is

$$k_{it+1} = \phi_{00} + \phi_1\beta_i x_t + \phi_2 z_{it} + \phi_3 k_{it} - \phi_{01}\beta_i$$

which is equation (23) in the text.

MPK **Dispersion**   The expected mpk is given by

$$\mathbb{E}_t[mpk_{it+1}] = \log\theta + \log G + \beta_i\rho_x x_t + \rho_z z_{it} - (1-\theta)k_{it+1}$$

and the mean of this is

$$\mathbb{E}[\mathbb{E}_t[mpk_{it+1}]] = \log\theta + \log G - (1-\theta)\mathbb{E}[k_{it+1}]$$

From the policy function,
$$\mathbb{E}[k_{it+1}] = \frac{\phi_{00} - \phi_{01}\beta_i}{1 - \phi_3}$$
so that
$$\mathbb{E}[\mathbb{E}_t[mpk_{it+1}]] = \log\theta + \log G - \frac{1-\theta}{1-\phi_3}(\phi_{00} - \phi_{01}\beta_i)$$

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and the variance of this permanent component is

$$\sigma^2_{\mathbb{E}[\mathbb{E}_t[mpk_{it+1}]]} = \left(\frac{1-\theta}{1-\phi_3}\right)^2 \phi_{01}^2 \sigma_\beta^2$$

which is equation (24) in the text.

A.3 Aggregation

The first order condition on labor gives

$$N_{it} = \left(\frac{\theta_2 e^{\hat{\beta}_i x_t + \hat{z}_{it}} K_{it}^{\theta_1}}{W_t}\right)^{\frac{1}{1-\theta_2}}$$

and substituting for the wage,

$$N_{it} = \left(\theta_2 e^{(\hat{\beta}_i - \omega)x_t + \hat{z}_{it}} K_{it}^{\theta_1}\right)^{\frac{1}{1-\theta_2}}$$

Labor market clearing gives:

$$N_t = \int N_{it} di = \theta_2^{\frac{1}{1-\theta_2}} e^{-\frac{1}{1-\theta_2}\omega x_t} \int e^{\frac{1}{1-\theta_2}\hat{\beta}_i x_t + z_{it}} K_{it}^{\theta} di$$

so that

$$\theta_2^{\frac{\theta_2}{1-\theta_2}} e^{-\frac{\theta_2}{1-\theta_2}\omega x_t} = \left(\frac{N_t}{\int e^{\frac{1}{1-\theta_2}\hat{\beta}_i x_t + z_{it}} K_{it}^{\theta} di}\right)^{\theta_2}$$

Then,

$$\begin{aligned}
Y_{it} &= e^{\hat{\beta}_i x_t + \hat{z}_{it}} K_{it}^{\theta_1} N_{it}^{\theta_2} = \theta_2^{\frac{\theta_2}{1-\theta_2}} e^{-\frac{\theta_2}{1-\theta_2}\omega x_t} e^{\frac{1}{1-\theta_2}\hat{\beta}_i x_t + z_{it}} K_{it}^{\theta} \\
&= \frac{e^{\frac{1}{1-\theta_2}\hat{\beta}_i x_t + z_{it}} K_{it}^{\theta}}{\left(\int e^{\frac{1}{1-\theta_2}\hat{\beta}_i x_t + z_{it}} K_{it}^{\theta} di\right)^{\theta_2}} N_t^{\theta_2}
\end{aligned}$$

By definition,

$$MPK_{it} = \frac{\theta e^{\frac{1}{1-\theta_2}\hat{\beta}_i x_t + z_{it}} K_{it}^{\theta-1}}{\left(\int e^{\frac{1}{1-\theta_2}\hat{\beta}_i x_t + z_{it}} K_{it}^{\theta} di\right)^{\theta_2}} N_t^{\theta_2}$$

and rearranging,

$$K_{it} = \left(\frac{\theta e^{\frac{1}{1-\theta_2}\hat{\beta}_i x_t + z_{it}}}{MPK_{it}} \right)^{\frac{1}{1-\theta}} \left(\frac{N_t}{\int e^{\frac{1}{1-\theta_2}\hat{\beta}_i x_t + z_{it}} K_{it}^\theta di} \right)^{\frac{\theta_2}{1-\theta}}$$

Capital market clearing gives

$$K_t = \int K_{it} di = \theta^{\frac{1}{1-\theta}} \left(\frac{N_t}{\int e^{\frac{1}{1-\theta_2}\hat{\beta}_i x_t + z_{it}} K_{it}^\theta di} \right)^{\frac{\theta_2}{1-\theta}} \int e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t + \frac{1}{1-\theta} z_{it}} MPK_{it}^{-\frac{1}{1-\theta}} di$$

so that

$$K_{it}^\theta = \left(\frac{e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t + \frac{1}{1-\theta} z_{it}} MPK_{it}^{-\frac{1}{1-\theta}}}{\int e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t + \frac{1}{1-\theta} z_{it}} MPK_{it}^{-\frac{1}{1-\theta}} di} K_t \right)^\theta$$

and substituting into the expression for Y_{it},

$$
\begin{aligned}
Y_{it} &= \frac{e^{\frac{1}{1-\theta_2}\hat{\beta}_i x_t + z_{it}} \left(\frac{e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t + \frac{1}{1-\theta} z_{it}} MPK_{it}^{-\frac{1}{1-\theta}}}{\int e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t + \frac{1}{1-\theta} z_{it}} MPK_{it}^{-\frac{1}{1-\theta}} di} K_t \right)^\theta}{\left(\int e^{\frac{1}{1-\theta_2}\hat{\beta}_i x_t + z_{it}} \left(\frac{e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t + \frac{1}{1-\theta} z_{it}} MPK_{it}^{-\frac{1}{1-\theta}}}{\int e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t + \frac{1}{1-\theta} z_{it}} MPK_{it}^{-\frac{1}{1-\theta}} di} K_t \right)^\theta di \right)^{\theta_2}} N_t^{\theta_2} \\
&= \frac{\frac{e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta_2}\hat{\beta}_i x_t + \frac{1}{1-\theta} z_{it}} MPK_{it}^{-\frac{\theta}{1-\theta}}}{\left(\int e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t + \frac{1}{1-\theta} z_{it}} MPK_{it}^{-\frac{1}{1-\theta}} di \right)^\theta}}{\left(\frac{\int e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t + \frac{1}{1-\theta} z_{it}} MPK_{it}^{-\frac{\theta}{1-\theta}} di}{\left(\int e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t + \frac{1}{1-\theta} z_{it}} MPK_{it}^{-\frac{1}{1-\theta}} di \right)^\theta} \right)^{\theta_2}} K_t^{\theta_1} N_t^{\theta_2} \\
&= A_t K_t^{\theta_1} N_t^{\theta_2}
\end{aligned}
$$

where

$$A_t = \left(\frac{\int e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t + \frac{1}{1-\theta} z_{it}} MPK_{it}^{-\frac{\theta}{1-\theta}} di}{\left(\int e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t + \frac{1}{1-\theta} z_{it}} MPK_{it}^{-\frac{1}{1-\theta}} di \right)^\theta} \right)^{1-\theta_2}$$

Taking logs,

$$a_t = (1-\theta_2) \left(\log \int e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t + \frac{1}{1-\theta} z_{it}} MPK_{it}^{-\frac{\theta}{1-\theta}} di - \theta \log \int e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t + \frac{1}{1-\theta} z_{it}} MPK_{it}^{-\frac{1}{1-\theta}} di \right)$$

The first expression in braces is equal to

$$\frac{1}{1-\theta}\frac{1}{1-\theta_2}\bar{\hat{\beta}}x_t - \frac{\theta}{1-\theta}\bar{m}pk + \frac{1}{2}\left(\frac{1}{1-\theta}\right)^2\left(\left(\frac{1}{1-\theta_2}\right)^2 x_t^2\sigma_{\hat{\beta}}^2 + \sigma_z^2\right) + \frac{1}{2}\left(\frac{\theta}{1-\theta}\right)^2\sigma_{mpk}^2$$

$$- \frac{\theta}{(1-\theta)^2}\frac{1}{1-\theta_2}\sigma_{mpk,\hat{\beta}_ix_t+z_{it}}$$

and the second to

$$\frac{\theta}{1-\theta}\frac{1}{1-\theta_2}\bar{\hat{\beta}}x_t - \frac{\theta}{1-\theta}\bar{m}pk + \frac{1}{2}\theta\left(\frac{1}{1-\theta}\right)^2\left(\left(\frac{1}{1-\theta_2}\right)^2 x_t^2\sigma_{\hat{\beta}}^2 + \sigma_z^2\right) + \frac{1}{2}\theta\left(\frac{1}{1-\theta}\right)^2\sigma_{mpk}^2$$

$$- \frac{\theta}{(1-\theta)^2}\frac{1}{1-\theta_2}\sigma_{mpk,\hat{\beta}_ix_t+z_{it}}$$

and combining (and using $\sigma_\beta = \frac{1}{1-\theta_2}\sigma_{\hat{\beta}}$) gives

$$\begin{aligned}
a_t &= \bar{\hat{\beta}}x_t + (1-\theta_2)\left(\frac{1}{2}\frac{1}{1-\theta}\left(x_t^2\sigma_\beta^2 + \sigma_z^2\right) - \frac{1}{2}\frac{\theta}{1-\theta}\sigma_{mpk}^2\right) \\
&= a_t^* - \frac{1}{2}(1-\theta_2)\frac{\theta}{1-\theta}\sigma_{mpk}^2 \\
&= a_t^* - \frac{1}{2}\frac{\theta_1(1-\theta_2)}{1-\theta_1-\theta_2}\sigma_{mpk}^2
\end{aligned}$$

A.4   Stock Market Returns

We derive stock market returns in the environment with adjustment costs. This nests the simpler case without them when $\xi = 0$.

Dividends are equal to

$$D_{it+1} = e^{z_{it+1}+\beta_ix_{t+1}}K_{it+1}^\theta - K_{it+2} + (1-\delta)K_{it+1} - \frac{\xi}{2}\left(\frac{K_{it+2}}{K_{it+1}} - 1\right)^2 K_{it+1}$$

and log-linearizing,

$$d_{it+1} = \frac{\Pi}{D}\left(z_{it+1} + \beta_ix_{t+1}\right) + \left(\theta\frac{\Pi}{D} + (1-\delta)\frac{K}{D}\right)k_{it+1} - \frac{K}{D}k_{it+2} + \log D - \left(\theta\frac{\Pi}{D} - \delta\frac{K}{D}\right)k$$

where $k = \log K$. Substituting for k_{it+1} and k_{it+2} from Appendix A.2.2 and rearranging,

$$d_{it+1} = A_{0i} + \tilde{A}_1z_{it} + A_1\beta_ix_t + \tilde{A}_2\varepsilon_{it+1} + A_2\beta_i\varepsilon_{t+1} + A_3k_{it}$$

where

$$A_{0i} = \log D - \left(\theta \frac{\Pi}{D} - \delta \frac{K}{D}\right)(k - \phi_{0i}) - \frac{K}{D}\phi_{0i}\phi_3$$

$$A_1 = \frac{\Pi}{D}\rho_x + \left(\theta \frac{\Pi}{D} + \frac{K}{D}(1 - \delta - \rho_x - \phi_3)\right)\phi_1$$

$$\tilde{A}_1 = \frac{\Pi}{D}\rho_z + \left(\theta \frac{\Pi}{D} + \frac{K}{D}(1 - \delta - \rho_z - \phi_3)\right)\phi_2$$

$$A_2 = \frac{\Pi}{D} - \frac{K}{D}\phi_1$$

$$\tilde{A}_2 = \frac{\Pi}{D} - \frac{K}{D}\phi_2$$

$$A_3 = \left(\theta \frac{\Pi}{D} + \frac{K}{D}(1 - \delta - \phi_3)\right)\phi_3$$

By definition, returns are equal to

$$R_{it+1} = \frac{D_{it+1} + P_{it+1}}{P_{it}}$$

and log-linearizing,

$$r_{it+1} = \rho p_{it+1} + (1 - \rho)d_{it+1} - p_{it} - \log \rho + (1 - \rho)\log \frac{P}{D}$$

Conjecture the stock price takes the form

$$p_{it} = c_{0i} + c_1 \beta_i x_t + c_2 z_{it} + c_3 k_{it}$$

Then,

$$
\begin{aligned}
r_{it+1} = {} & -\log \rho + (1 - \rho)\left(\log \frac{P}{D} + A_{0i} - c_{0i}\right) + \rho c_3 \phi_{0i} \\
& + \left((\rho\rho_z - 1)c_2 + \rho c_3 \phi_2 + (1 - \rho)\tilde{A}_1\right)z_{it} \\
& + ((\rho\rho_x - 1)c_1 + \rho c_3 \phi_1 + (1 - \rho)A_1)\beta_i x_t \\
& + ((\rho\phi_3 - 1)c_3 + (1 - \rho)A_3)k_{it} \\
& + \left(\rho c_2 + (1 - \rho)\tilde{A}_2\right)\varepsilon_{it+1} + (\rho c_1 + (1 - \rho)A_2)\beta_i \varepsilon_{t+1}
\end{aligned}
$$

and the (log) excess return is the (negative of the) conditional covariance with the SDF:

$$\log \mathbb{E}_t\left[R_{it+1}^e\right] = (\rho c_1 + (1 - \rho)A_2)\beta_i \gamma_t \sigma_\varepsilon^2$$

To solve for coefficients, use the Euler equation. First,

$$
\begin{aligned}
r_{it+1} + m_{it+1} &= (1-\rho)\left(\log\frac{P}{D} + A_{0i} - c_{0i}\right) + \rho c_3\phi_{0i} - \frac{1}{2}\gamma_0^2\sigma_\varepsilon^2 \\
&+ \left((\rho\rho_z - 1)c_2 + \rho c_3\phi_2 + (1-\rho)\tilde{A}_1\right)z_{it} \\
&+ \left(((\rho\rho_x - 1)c_1 + \rho c_3\phi_1 + (1-\rho)A_1)\beta_i - \gamma_0\gamma_1\sigma_\varepsilon^2\right)x_t \\
&+ \left((\rho\phi_3 - 1)c_3 + (1-\rho)A_3\right)k_{it} \\
&- \frac{1}{2}\gamma_1^2\sigma_\varepsilon^2 x_t^2 \\
&+ \left(\rho c_2 + (1-\rho)\tilde{A}_2\right)\varepsilon_{it+1} \\
&+ \left((\rho c_1 + (1-\rho)A_2)\beta_i - \gamma_0 - \gamma_1 x_t\right)\varepsilon_{t+1}
\end{aligned}
$$

The Euler equation implies

$$
\begin{aligned}
0 &= \mathbb{E}_t\left[r_{it+1} + m_{it+1}\right] + \frac{1}{2}\mathrm{var}\left(r_{it+1} + m_{it+1}\right) \\
&= (1-\rho)\left(\log\frac{P}{D} + A_{0i} - c_{0i}\right) + \rho c_3\phi_{0i} + \frac{1}{2}(\rho c_1 + (1-\rho)A_2)^2\beta_i^2\sigma_\varepsilon^2 - (\rho c_1 + (1-\rho)A_2)\beta_i\gamma_0\sigma_\varepsilon^2 \\
&+ \frac{1}{2}\left(\rho c_2 + (1-\rho)\tilde{A}_2\right)^2\sigma_{\tilde\varepsilon}^2 \\
&+ \left((\rho\rho_z - 1)c_2 + \rho c_3\phi_2 + (1-\rho)\tilde{A}_1\right)z_{it} \\
&+ \left((\rho\rho_x - 1)c_1 + \rho c_3\phi_1 + (1-\rho)A_1 - (\rho c_1 + (1-\rho)A_2)\gamma_1\sigma_\varepsilon^2\right)\beta_i x_t \\
&+ \left((\rho\phi_3 - 1)c_3 + (1-\rho)A_3\right)k_{it}
\end{aligned}
$$

and so by undetermined coefficients,

$$
\begin{aligned}
0 &= (1-\rho)\left(\log\frac{P}{D} + A_{0i} - c_{0i}\right) + \rho c_3\phi_{0i} + \frac{1}{2}(\rho c_1 + (1-\rho)A_2)^2\beta_i^2\sigma_\varepsilon^2 - (\rho c_1 + (1-\rho)A_2)\beta_i\gamma_0\sigma_\varepsilon^2 \\
&+ \frac{1}{2}\left(\rho c_2 + (1-\rho)\tilde{A}_2\right)^2\sigma_{\tilde\varepsilon}^2 \\
&= (\rho\rho_z - 1)c_2 + \rho c_3\phi_2 + (1-\rho)\tilde{A}_1 \\
&= (\rho\rho_x - 1)c_1 + \rho c_3\phi_1 + (1-\rho)A_1 - (\rho c_1 + (1-\rho)A_2)\gamma_1\sigma_\varepsilon^2 \\
&= (\rho\phi_3 - 1)c_3 + (1-\rho)A_3
\end{aligned}
$$

or

$$c_3 = \frac{(1-\rho)A_3}{1-\rho\phi_3}$$

$$c_2 = \frac{\rho c_3 \phi_2 + (1-\rho)\tilde{A}_1}{1-\rho\rho_z}$$

$$c_1 = \frac{\rho c_3 \phi_1 + (1-\rho)(A_1 - A_2\gamma_1\sigma_\varepsilon^2)}{1-\rho\rho_x + \rho\gamma_1\sigma_\varepsilon^2}$$

Substituting for c_1 we can solve for

$$\log \mathbb{E}_t\left[R_{it+1}^e\right] = \frac{\rho^2 c_3 \phi_1 + (1-\rho)(\rho A_1 + (1-\rho\rho_x)A_2)}{1-\rho\rho_x + \rho\sigma_\varepsilon^2\gamma_1}\beta_i\gamma_t\sigma_\varepsilon^2$$

Solving for

$$\rho A_1 + (1-\rho\rho_x)A_2 = \frac{\frac{1}{\rho} + \delta - 1 - \rho\theta\phi_1\phi_3}{\frac{1}{\rho} + \delta(1-\theta) - 1}$$

$$\rho^2 c_3 \phi_1 = \theta\frac{\rho^2(1-\rho)\phi_1\phi_3}{1-\rho\phi_3}\frac{\frac{1}{\rho}-\phi_3}{\frac{1}{\rho}+\delta(1-\theta)-1}$$

substituting into the return equation and simplifying, we obtain

$$\log \mathbb{E}_t\left[R_{it+1}^e\right] = \psi\beta_i\gamma_t\sigma_\varepsilon^2$$

where

$$\psi = \frac{\frac{1}{\rho}+\delta-1}{\frac{1}{\rho}+\delta(1-\theta)-1}\frac{1-\rho}{1-\rho\rho_x+\rho\gamma_1\sigma_\varepsilon^2}$$

which is equation (18) in the text.

The Sharpe ratio is the ratio of expected excess returns to the conditional standard deviation of the return:

$$SR_{it} = \frac{\psi\beta_i\gamma_t\sigma_\varepsilon^2}{\sqrt{\left(\rho c_2 + (1-\rho)\tilde{A}_2\right)^2\sigma_{\tilde{\varepsilon}}^2 + \psi^2\beta_i^2\sigma_\varepsilon^2}}$$

We can solve for

$$\rho c_2 + (1-\rho)\tilde{A}_2 = \frac{\frac{1}{\rho}+\delta-1}{\frac{1}{\rho}+\delta(1-\theta)-1}\frac{1-\rho}{1-\rho\rho_z}$$

and substituting and rearranging gives expression (21).

For a perfectly diversified portfolio (i.e., the integral over individual returns) idiosyncratic shocks cancel, i.e., $\sigma_{\tilde{\varepsilon}}^2 = 0$ and $SR_{mt} = \gamma_t\sigma_\varepsilon$.

A.5 Other Distortions

With other distortions, the derivations are similar to those in Appendix A.2.1. The Euler equation is given by

$$
\begin{aligned}
1 &= \mathbb{E}_t \left[M_{t+1} \left(\theta e^{\tau_{it+1}+z_{it+1}+\beta_i x_{t+1}} GK_{it+1}^{\theta-1} + 1 - \delta \right) \right] \\
&= (1-\delta)\,\mathbb{E}_t \left[M_{t+1} \right] + \theta GK_{it+1}^{\theta-1} \mathbb{E}_t \left[e^{m_{t+1}+\tau_{it+1}+z_{it+1}+\beta_i x_{t+1}} \right]
\end{aligned}
$$

Idiosyncratic distortions. Substituting for m_{t+1} and τ_{it+1} and rearranging,

$$
\begin{aligned}
\mathbb{E}_t \left[e^{m_{t+1}+\tau_{it+1}+z_{it+1}+\beta_i x_{t+1}} \right] &= \mathbb{E}_t \left[e^{\log \rho - \gamma_t \varepsilon_{t+1} - \frac{1}{2}\gamma_t^2 \sigma_\varepsilon^2 - \nu z_{it+1} - \eta_{it+1} + z_{it+1} + \beta_i x_{t+1}} \right] \\
&= \mathbb{E}_t \left[e^{\log \rho + (1-\nu)\rho_z z_{it} + (1-\nu)\varepsilon_{it+1} + \beta_i \rho_x x_t + (\beta_i - \gamma_t)\varepsilon_{t+1} - \frac{1}{2}\gamma_t^2 \sigma_\varepsilon^2 - \eta_{it+1}} \right] \\
&= e^{\log \rho + (1-\nu)\rho_z z_{it} + \beta_i \rho_x x_t + \frac{1}{2}(1-\nu)^2 \sigma_\varepsilon^2 + \frac{1}{2}\beta_i^2 \sigma_\varepsilon^2 - \beta_i \gamma_t \sigma_\varepsilon^2 - \eta_{it+1}}
\end{aligned}
$$

so that

$$
\theta GK_{it+1}^{\theta-1} = \frac{1 - (1-\delta)\rho}{e^{\log \rho + (1-\nu)\rho_z z_{it} + \beta_i \rho_x x_t + \frac{1}{2}(1-\nu)^2 \sigma_\varepsilon^2 + \frac{1}{2}\beta_i^2 \sigma_\varepsilon^2 - \beta_i \gamma_t \sigma_\varepsilon^2 - \eta_{it+1}}}
$$

and rearranging and taking logs,

$$
k_{it+1} = \frac{1}{1-\theta}\left(\tilde{\alpha} + \frac{1}{2}(1-\nu)^2 \sigma_\varepsilon^2 + \frac{1}{2}\beta_i^2 \sigma_\varepsilon^2 + (1-\nu)\rho_z z_{it} + \beta_i \rho_x x_t - \beta_i \gamma_t \sigma_\varepsilon^2 - \eta_{it+1} \right)
$$

where $\tilde{\alpha}$ and α are as defined in Appendix A.2.1.

The realized mpk is given by (ignoring the variance terms)

$$
\begin{aligned}
mpk_{it+1} &= \log \theta + \pi_{it+1} - k_{it+1} \\
&= \log \theta + \log G + z_{it+1} + \beta_i x_{t+1} - (1-\theta)k_{it+1} \\
&= \log \theta + \log G + z_{it+1} + \beta_i x_{t+1} - \tilde{\alpha} - (1-\nu)\rho_z z_{it} - \beta_i \rho_x x_t + \beta_i \gamma_t \sigma_\varepsilon^2 + \eta_{it+1} \\
&= \alpha + \varepsilon_{it+1} + \beta_i \varepsilon_{t+1} + \nu \rho_z z_{it} + \beta_i \gamma_t \sigma_\varepsilon^2 + \eta_{it+1}
\end{aligned}
$$

which is equation (26). The conditional expected mpk is

$$
\mathbb{E}_t \left[mpk_{it+1} \right] = \alpha + \nu \rho_z z_{it} + \beta_i \gamma_t \sigma_\varepsilon^2 + \eta_{it+1}
$$

and the cross-sectional variance is

$$
\sigma_{\mathbb{E}_t[mpk_{it+1}]}^2 = (\nu\rho_z)^2 \sigma_z^2 + \sigma_\eta^2 + \left(\gamma_t \sigma_\varepsilon^2\right)^2 \sigma_\beta^2 \tag{29}
$$

Deriving stock returns follows closely the steps in Appendix A.4. Dividends are equal to

$$D_{it+1} = e^{\tau_{it+1}+z_{it+1}+\beta_i x_{t+1}} K_{it+1}^\theta - K_{it+2} + (1-\delta) K_{it+1} - \frac{\xi}{2} \left(\frac{K_{it+2}}{K_{it+1}} - 1 \right)^2 K_{it+1}$$

and log-linearizing,

$$d_{it+1} = \frac{\Pi}{D} \left(\tau_{it+1} + z_{it+1} + \beta_i x_{t+1} \right) + \left(\theta \frac{\Pi}{D} + (1-\delta) \frac{K}{D} \right) k_{it+1} - \frac{K}{D} k_{it+2} + \log D - \left(\theta \frac{\Pi}{D} - \delta \frac{K}{D} \right) k$$

where $k = \log K$.

Substituting for k_{it+1} and k_{it+2} from above,

$$d_{it+1} = A_0 + \tilde{A}_1 z_{it} + A_1 \beta_i x_t + \tilde{A}_2 \varepsilon_{it+1} + A_2 \beta_i \varepsilon_{t+1} + A_3 \eta_{it+1} + A_4 \eta_{it+2}$$

where

$$
\begin{aligned}
A_0 &= \log D - \left(\theta \frac{\Pi}{D} - \delta \frac{K}{D} \right) \left(k - \frac{\tilde{\alpha}}{1-\theta} \right) \\
A_1 &= \frac{1}{1-\theta} \left(\frac{\Pi}{D} + (1-\delta-\rho_x) \frac{K}{D} \right) \rho_x - \frac{1}{1-\theta} \left(\theta \frac{\Pi}{D} + (1-\delta-\rho_x) \frac{K}{D} \right) \gamma_1 \sigma_\varepsilon^2 \\
\tilde{A}_1 &= \frac{1-\nu}{1-\theta} \left(\frac{\Pi}{D} + (1-\delta-\rho_z) \frac{K}{D} \right) \rho_z \\
A_2 &= \frac{\Pi}{D} - \frac{1}{1-\theta} \frac{K}{D} \rho_x + \frac{1}{1-\theta} \frac{K}{D} \gamma_1 \sigma_\varepsilon^2 \\
\tilde{A}_2 &= \left(\frac{\Pi}{D} - \frac{1}{1-\theta} \frac{K}{D} \right) (1-\nu) \rho_z \\
A_3 &= -\frac{1}{1-\theta} \left(\theta \frac{\Pi}{D} + (1-\delta) \frac{K}{D} \right) \\
A_4 &= \frac{1}{1-\theta} \frac{K}{D}
\end{aligned}
$$

Using the log-linearized return equation,

$$r_{it+1} = \rho p_{it+1} + (1-\rho) d_{it+1} - p_{it} - \log \rho + (1-\rho) \log \frac{P}{D}$$

and conjecturing the stock price takes the form

$$p_{it} = c_{0i} + c_1 \beta_i x_t + c_2 z_{it} + c_3 \eta_{it+1}$$

gives

$$
\begin{aligned}
r_{it+1} = {}& -\log\rho + (1-\rho)\left(\log\frac{P}{D} + A_0 - c_0\right) \\
& + \left((\rho\rho_z - 1)\,c_2 + (1-\rho)\,\tilde{A}_1\right)z_{it} \\
& + \left((\rho\rho_x - 1)\,c_1 + (1-\rho)\,A_1\right)\beta_i x_t \\
& + \left(\rho c_2 + (1-\rho)\,\tilde{A}_2\right)\varepsilon_{it+1} + (\rho c_1 + (1-\rho)\,A_2)\,\beta_i\varepsilon_{t+1} \\
& + (\rho c_3 + (1-\rho)\,A_4)\,\eta_{it+2} + ((1-\rho)\,A_3 - c_3)\,\eta_{it+1}
\end{aligned}
$$

The (log) excess return is the (negative of the) conditional covariance with the SDF:

$$
\log\mathbb{E}_t\left[R_{it+1}^e\right] = (\rho c_1 + (1-\rho)\,A_2)\,\beta_i\gamma_t\sigma_\varepsilon^2
$$

A_2 is independent of ν and η. Following the same steps as in Appendix A.4, it is easily verified that c_1 is independent of these terms as well. Thus, expected returns are independent of distortions.

Aggregate distortions. Consider the first formulation, i.e.,

$$
\tau_{it+1} = -\nu_z z_{it+1} - \nu_x x_{t+1} - \eta_{it+1}
$$

Similar steps as above give expression (29). Expected stock market returns are similarly unaffected.

Next, consider the second formulation:

$$
\tau_{it+1} = -\nu_z z_{it+1} - \nu_x \beta_i x_{t+1} - \eta_{it+1}
$$

In this case, similar steps as above give the conditional expected mpk as

$$
\mathbb{E}_t\left[mpk_{it+1}\right] = \alpha + \nu_z\rho_z z_{it} + \nu_x\beta_i\rho_x x_t + (1-\nu_x)\,\beta_i\gamma_t\sigma_\varepsilon^2 + \eta_{it+1}
$$

and expected excess stock market returns as

$$
\log\mathbb{E}_t\left[R_{it+1}^e\right] = (1-\nu_x)\,\psi\beta_i\gamma_t\sigma_\varepsilon^2
$$

where ψ is as defined in expression (18). In other words, the risk-premium effect on expected mpk, as well as expected returns, are both scaled by a factor $1 - \nu_x$.

The mean level of expected *mpk* and return dispersion are, respectively,

$$
\mathbb{E}\left[\sigma^2_{\mathbb{E}_t[mpk_{it+1}]}\right] = \sigma^2_\eta + (\nu_z \rho_z)^2 \sigma^2_z + (\nu_x \rho_x)^2 \sigma^2_x \sigma^2_\beta
$$

$$
+ \left((1 - \nu_x) \sigma^2_\varepsilon\right)^2 \left(\gamma_0^2 + \gamma_1^2 \sigma^2_x\right) \sigma^2_\beta + 2\nu_x (1 - \nu_x) \rho_x \sigma^2_x \gamma_1 \sigma^2_\varepsilon \sigma^2_\beta
$$

$$
\mathbb{E}\left[\sigma^2_{\log \mathbb{E}_t[R^e_{it+1}]}\right] = \left((1 - \nu_x) \psi \sigma^2_\varepsilon\right)^2 \left(\gamma_0^2 + \gamma_1^2 \sigma^2_x\right) \sigma^2_\beta
$$

The last two terms of the first equation capture the *mpk* effects of risk premia. The last term there is new and does not have a counterpart in the second equation – in other words, using dispersion in expected returns would give the second to last term, as usual, but not the last. If $\nu_x < 0$, it is straightforward to verify that that term is positive (recall that γ_1 is negative). Then, we may be understating risk premium effects. If $\nu_x > 0$, the last terms is negative and we may be overstating them. At the estimated parameter values, the upper bound on ν_x discussed in Section 4.4 is about 0.065. Using this value, along with the other parameters, to calculate the last term in the equation gives the maximum upward bias to be 0.006, which is extremely small relative to our estimates of dispersion in *Empk* arising from the risk premium channel.

A.6   Autocorrelation of Investment

To derive the autocorrelation of investment, define net investment as $\Delta k_{it+1} = k_{it+1} - k_{it}$. We use the following:

$$
\begin{aligned}
\mathrm{cov}\left(\Delta z_{it}, z_{it}\right) &= \mathrm{cov}\left((\rho_z - 1) z_{it-1} + \varepsilon_{it}.\rho_z z_{it-1} + \varepsilon_{it}\right) \\
&= \rho_z (\rho_z - 1) \sigma^2_z + \sigma^2_{\tilde{\varepsilon}} \\
&= \frac{1}{1 + \rho_z} \sigma^2_{\tilde{\varepsilon}} \\
\mathrm{cov}\left(\Delta k_{it}, z_{it}\right) &= \mathrm{cov}\left(\Delta k_{it}, \rho_z z_{it-1} + \varepsilon_{it}\right) \\
&= \rho_z \mathrm{cov}\left(\Delta k_{it}, z_{it-1}\right) \\
&= \rho_z \mathrm{cov}\left(\phi_1 \beta_i \Delta x_{t-1} + \phi_2 \Delta z_{it-1} + \phi_3 \Delta k_{it-1}, z_{it-1}\right) \\
&= \rho_z \left(\mathrm{cov}\left(\phi_2 \Delta z_{it-1}, z_{it-1}\right) + \phi_3 \mathrm{cov}\left(\Delta k_{it-1}, z_{it-1}\right)\right) \\
&= \rho_z \phi_2 \frac{1}{1 + \rho_z} \sigma^2_{\tilde{\varepsilon}} + \rho_z \phi_3 \mathrm{cov}\left(\Delta k_{it-1}, z_{it-1}\right)
\end{aligned}
$$

so that

$$
\mathbb{E}\left[\mathrm{cov}\left(\Delta k_{it}, z_{it}\right)\right] = \frac{\rho_z}{1 + \rho_z} \frac{\phi_2 \sigma^2_{\tilde{\varepsilon}}}{1 - \phi_3 \rho_z}
$$

Next,

$$
\begin{aligned}
\operatorname{cov}\left(\Delta k_{it+1}, \Delta z_{it+1}\right) &= \operatorname{cov}\left(\phi_1 \beta_i \Delta x_t + \phi_2 \Delta z_{it} + \phi_3 \Delta k_{it}. \left(\rho_z - 1\right) z_{it} + \varepsilon_{it+1}\right) \\
&= \phi_2 \left(\rho_z - 1\right) \operatorname{cov}\left(\Delta z_{it}, z_{it}\right) + \phi_3 \left(\rho_z - 1\right) \operatorname{cov}\left(\Delta k_{it}, z_{it}\right) \\
&= \phi_2 \left(\rho_z - 1\right) \frac{1}{1+\rho_z} \sigma_{\tilde{\varepsilon}}^2 + \frac{\rho_z}{1+\rho_z} \frac{\phi_3 \left(\rho_z - 1\right) \phi_2 \sigma_{\tilde{\varepsilon}}^2}{1 - \phi_3 \rho_z} \\
&= \frac{\rho_z - 1}{1+\rho_z} \frac{\phi_2 \sigma_{\tilde{\varepsilon}}^2}{1 - \phi_3 \rho_z}
\end{aligned}
$$

Similar steps give

$$
\operatorname{cov}\left(\Delta k_{it+1}, \Delta x_{t+1}\right) = \frac{\rho_x - 1}{1+\rho_x} \frac{\phi_1 \beta_i \sigma_{\varepsilon}^2}{1 - \phi_3 \rho_x}
$$

Combining these gives the variance of investment:

$$
\begin{aligned}
\sigma_{\Delta k}^2 &= \phi_1^2 \beta_i^2 \operatorname{var}\left(\Delta x_t\right) + \phi_2^2 \operatorname{var}\left(\Delta z_{it}\right) + \phi_3^2 \sigma_{\Delta k}^2 \\
&\quad + 2\phi_1 \phi_3 \beta_i \operatorname{cov}\left(\Delta x_t, \Delta k_{it}\right) + 2\phi_2 \phi_3 \operatorname{cov}\left(\Delta z_{it}, \Delta k_{it}\right) \\
&= \phi_1^2 \beta_i^2 \frac{2}{1+\rho_x} \sigma_{\varepsilon}^2 + \phi_2^2 \frac{2}{1+\rho_z} \sigma_{\tilde{\varepsilon}}^2 + \phi_3^2 \sigma_{\Delta k}^2 \\
&\quad + \frac{2\phi_1^2 \phi_3 \beta_i^2 \sigma_{\varepsilon}^2}{1 - \phi_3 \rho_x} \frac{\rho_x - 1}{1+\rho_x} + \frac{2\phi_2^2 \phi_3 \sigma_{\tilde{\varepsilon}}^2}{1 - \phi_3 \rho_z} \frac{\rho_z - 1}{1+\rho_z} \\
&= \phi_3^2 \sigma_{\Delta k}^2 + 2\phi_1^2 \beta_i^2 \frac{\sigma_{\varepsilon}^2}{1+\rho_x}\left(1 + \frac{\phi_3 \left(\rho_x - 1\right)}{1 - \phi_3 \rho_x}\right) + 2\phi_2^2 \frac{\sigma_{\tilde{\varepsilon}}^2}{1+\rho_z}\left(1 + \frac{\phi_3 \left(\rho_z - 1\right)}{1 - \phi_3 \rho_z}\right) \\
&= \frac{2}{1+\phi_3}\left(\phi_1^2 \beta_i^2 \sigma_{\varepsilon}^2 \frac{1}{1+\rho_x} \frac{1}{1 - \phi_3 \rho_x} + \phi_2^2 \sigma_{\tilde{\varepsilon}}^2 \frac{1}{1+\rho_z} \frac{1}{1 - \phi_3 \rho_z}\right)
\end{aligned}
$$

Next,

$$
\begin{aligned}
\operatorname{cov}\left(\Delta k_{it+1}, \Delta k_{it}\right) &= \operatorname{cov}\left(\phi_1 \beta_i \Delta x_t + \phi_2 \Delta z_{it} + \phi_3 \Delta k_{it}, \Delta k_{it}\right) \\
&= \phi_1 \beta_i \operatorname{cov}\left(\Delta x_t, \Delta k_{it}\right) + \phi_2 \operatorname{cov}\left(\Delta z_{it}, \Delta k_{it}\right) + \phi_3 \sigma_{\Delta k}^2 \\
&= \phi_1^2 \beta_i^2 \sigma_{\varepsilon}^2 \frac{\rho_x - 1}{1+\rho_x} \frac{1}{1 - \phi_3 \rho_x} + \phi_2^2 \sigma_{\tilde{\varepsilon}}^2 \frac{\rho_z - 1}{1+\rho_z} \frac{1}{1 - \phi_3 \rho_z} + \phi_3 \sigma_{\Delta k}^2
\end{aligned}
$$

and the autocorrelation is:

$$
\operatorname{corr}\left(\Delta k_{it+1}, \Delta k_{it}\right) = \phi_3 + \frac{1+\phi_3}{2} \frac{\phi_1^2 \beta_i^2 \sigma_{\varepsilon}^2 \frac{\rho_x - 1}{1+\rho_x} \frac{1}{1 - \phi_3 \rho_x} + \phi_2^2 \sigma_{\tilde{\varepsilon}}^2 \frac{\rho_z - 1}{1+\rho_z} \frac{1}{1 - \phi_3 \rho_z}}{\phi_1^2 \beta_i^2 \sigma_{\varepsilon}^2 \frac{1}{1+\rho_x} \frac{1}{1 - \phi_3 \rho_x} + \phi_2^2 \sigma_{\tilde{\varepsilon}}^2 \frac{1}{1+\rho_z} \frac{1}{1 - \phi_3 \rho_z}} \tag{30}
$$

Notice that this approaches

$$
\operatorname{corr}\left(\Delta k_{it+1}, \Delta k_{it}\right) = \phi_3 + \left(1 - \phi_3\right) \frac{\rho_x - 1}{2}
$$

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as ρ_z and ρ_x become close. Further, in the case both shocks follow a random walk, the auto-correlation is simply equal to ϕ_3.

A.7 Multifactor Model

There are J aggregate risk factors in the economy. Firm have heterogeneous loadings on these factors, so that the profit function (in logs) takes the form

$$\pi_{it} = \beta_i x_t + z_{it} + \theta k_{it} \tag{31}$$

where β_i is a vector of factor loadings of firm i and x_t the vector of factor realizations at time t, i.e.,

$$\beta_i = \begin{bmatrix} \beta_{1i} \\ \beta_{2i} \\ \vdots \\ \beta_{Ji} \end{bmatrix}' \qquad x_t = \begin{bmatrix} x_{1t} \\ x_{2t} \\ \vdots \\ x_{Jt} \end{bmatrix}'$$

Each factor, indexed by j, follows an AR(1) process

$$x_{jt+1} = \rho_j x_{jt} + \varepsilon_{jt+1}, \quad \varepsilon_{jt+1} \sim \mathcal{N}\left(0, \sigma_{\varepsilon_j}^2\right) \tag{32}$$

where the innovations are potentially correlated across factors. Denote by Σ_f the covariance matrix of factor innovations, i.e.,

$$\Sigma_f = \begin{bmatrix} \sigma_{\varepsilon 1}^2 & \sigma_{\varepsilon 1, \varepsilon 2} & \cdots & \sigma_{\varepsilon 1, \varepsilon J} \\ \sigma_{\varepsilon 2, \varepsilon 1} & \sigma_{\varepsilon 2}^2 & \cdots & \sigma_{\varepsilon 2, \varepsilon J} \\ \vdots & \vdots & & \vdots \\ \sigma_{\varepsilon J, \varepsilon 1} & \sigma_{\varepsilon J, \varepsilon 2} & \cdots & \sigma_{\varepsilon J}^2 \end{bmatrix}$$

The idiosyncratic component of firm productivity follows

$$z_{it+1} = \rho_z z_{it} + \varepsilon_{it+1}, \quad \varepsilon_{it+1} \sim \mathcal{N}\left(0, \sigma_{\tilde{\varepsilon}}^2\right) \tag{33}$$

The stochastic discount factor takes the form

$$m_{t+1} = \log \rho - \gamma \varepsilon_{t+1} - \frac{1}{2}\gamma \Sigma_f \gamma' \tag{34}$$

where γ is a vector of factor exposures and ε_{t+1} the vector of innovations in each factor, i.e.,

$$\gamma = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_J \end{bmatrix}' \qquad \varepsilon_{t+1} = \begin{bmatrix} \varepsilon_{1t+1} \\ \varepsilon_{2t+1} \\ \vdots \\ \varepsilon_{Jt+1} \end{bmatrix}$$

For simplicity, we have assumed that the factor exposures are constant, although the setup can be extended to include time-varying exposures as well. Expressions (31), (32), (33) and (34) are simple extensions of (8), (6) and (7).

Following a similar derivation as A.2.1, we can derive the realized mpk:

$$mpk_{it+1} = \alpha + \varepsilon_{it+1} + \beta_i \varepsilon_{t+1} + \beta_i \Sigma_f \gamma'$$

where β_i and ε_{t+1} denote vectors of factor loadings and shocks. The expected mpk and its cross-sectional dispersion are given by

$$\mathbb{E}_t \left[mpk_{it+1} \right] = \alpha + \beta_i \Sigma_f \gamma', \quad \sigma^2_{\mathbb{E}_t[mpk]} = \gamma \Sigma'_f \Sigma_\beta \Sigma_f \gamma'$$

where Σ_β is the covariance matrix of factor loadings across firms, i.e.,

$$\Sigma_\beta = \begin{bmatrix} \sigma^2_{\beta_1} & \sigma_{\beta_1,\beta_2} & \cdots & \sigma_{\beta_1,\beta_J} \\ \sigma_{\beta_2,\beta_1} & \sigma^2_{\beta_2} & \cdots & \sigma_{\beta_2,\beta_J} \\ \vdots & \vdots & & \vdots \\ \sigma_{\beta_J,\beta_1} & \sigma_{\beta_J,\beta_2} & \cdots & \sigma^2_{\beta_J} \end{bmatrix}$$

Similar steps as Appendix A.4 gives

$$\mathbb{E}_t \left[R^e_{it+1} \right] = \beta_i \psi \Sigma_f \gamma', \quad \sigma^2_{\mathbb{E}_t[R]} = \gamma \Sigma'_f \psi' \Sigma_\beta \psi \Sigma_f \gamma'$$

where ψ is a diagonal matrix with

$$\psi_{jj} = \left(\frac{1-\rho}{1-\rho\rho_j} \right) \frac{\frac{1}{\rho} + \delta - 1}{\frac{1}{\rho} + (1-\theta)\delta - 1}$$

B   Data

B.1   Compustat/CRSP

B.2   Aggregate Productivity Series

Solow residuals.   To build a series of Solow residuals, we obtain data on real GDP and aggregate labor and capital from the Bureau of Economic Analysis. Data on real GDP are from BEA Table 1.1.3 ("Real Gross Domestic Product"), data on labor are from BEA Table 6.4 ("Full-Time and Part-Time Employees") and data on the capital stock are from BEA Table 1.2 ("Net Stock of Fixed Assets"). The data are available annually from 1929-2016. With these data we compute $x_t = y_t - \theta_1 k_t - \theta_2 n_t$. We extract a linear time-trend and then estimate the autoregression in equation (6).

Firm-level series.   To construct the alternative series for aggregate productivity from the firm-level data, we use the following procedure. First, we compute firm-level productivity as $z_{it} + \beta_i x_t = va_{it} - \theta k_{it}$. We then average these values across all firms in each year. Because z_{it} is mean-zero and independent across firms, this yields a scaled measure of aggregate productivity, $\bar{\beta} x_t$, where $\bar{\beta}$ is the mean beta across firms, which under our assumptions, is approximately two. We extract a linear time-trend from this series and then estimate the autoregression. The coefficient from this regression gives ρ_x. The standard deviations of the residuals gives $\bar{\beta}\sigma_\varepsilon$ and after dividing by $\bar{\beta}$ gives the true volatility of shocks. Applying this procedure to the set of Compustat firms over the period 1962-2016 yields values of $\rho_x = 0.92$ and $\sigma_\varepsilon = .0245$.

C   Numerical Procedure

Our numerical approach to parameterize the model is as follows. For a given set of the parameters γ_0, γ_1 and σ_β^2, we compute the autocorrelation using equation (30). We solve the model for a wide grid of beta-types centered around the mean beta. We use an 11 point grid ranging from -3 to 7 (the results are not overly sensitive to the width of the grid). We simulate a time series of excess returns for a large number of firms of each type. We then average the returns across firms in each time period, which yields a series for the market excess return, and compute the mean and standard deviation of this series.

Next we compute the expected return for each beta-type as the mean of the conditional expectation of returns, i.e., $\mathbb{E}_t [R_{it+1}] = \mathbb{E}_t \left[\frac{D_{it+1}+P_{it+1}}{P_{it}} \right]$. We then use these values to calculate the dispersion in expected returns, σ_{Er}^2, interpolating for values of β that are not on the grid. Finally, we find the set of the four parameters, γ_0, γ_1, σ_β^2 and ξ that make the simulated moments

consistent with the empirical ones, i.e., expected excess market returns, market Sharpe ratio, dispersion in expected returns and autocorrelation of investment. As noted in the text, we implement this procedure for returns using a fourth-order approximation in Dynare++.

D   Empirical Predictions

Computation of Betas and Expected Returns   We compute stock market betas for equity returns by running time series regressions of excess equity returns on factor portfolio returns for each firm. We compute stock market betas using monthly returns and a two-year rolling window horizon. We also compute "MPK Betas" as an alternative measure of firm exposure to the aggregate shock, regressing firm $log(Y/K)$ on factor portfolio returns. As firm $log(Y/K)$ is only observed at the quarterly frequency, we use quarterly returns and a five-year rolling window horizon.

To compute expected returns, we first run Fama-Macbeth regressions to estimate market prices of risk for each factor. We then compute two measures of expected returns. We then compute $E_t[r_{i,t}()] = \sum_j \beta_{i,j}\lambda_j + \bar{\epsilon}_i$, where j denotes the factor, i the firm, λ_j is the market price of risk for factor j, and $\bar{\epsilon}_i$ is the average residual for firm i in the Fama-Macbeth regressions. We then compute $E_t[r_{i,t}(\beta)] = \sum_j \beta_{i,j}\lambda_j$, a measure of expected returns coming just from the factors and the market prices of risk (and not mis-pricing). The results reported in the body of the text consider the Fama-French model, but we have replicated qualitatively similar results using a number of other factor models.

Supplemental Tables   Table 9 displays a cross-sectional regression of the standard deviation of mpk (within each industry) on the standard deviation of betas and expected returns (within each industry).

Table 9: Cross-sectional Industry Regression of mpk Dispersion

	(1) $\sigma(LMPK)$	(2) $\sigma(LMPK)$	(3) $\sigma(LMPK)$	(4) $\sigma(LMPK)$
$\sigma(E_{FF}[ret])$	2.994***			
	(11.38)			
$\sigma(E_{FF,\beta}[ret])$		16.64***		
		(11.26)		
$\sigma(\beta_{MKT})$			0.263***	
			(2.68)	
$\sigma(\beta_{HML})$			0.170***	
			(3.11)	
$\sigma(\beta_{SMB})$			0.217***	
			(3.41)	
$\sigma(\beta_{MKT,MPK})$				0.252***
				(4.41)
$\sigma(\beta_{HML,MPK})$				-0.0157
				(-0.39)
$\sigma(\beta_{SMB,MPK})$				0.145***
				(5.84)
Constant	0.377***	0.198***	0.0500	0.251***
	(9.26)	(3.55)	(0.79)	(4.62)
Observations	206	206	206	113
R^2	0.388	0.383	0.450	0.521

Notes: $\mathbb{E}_t[r_t]$ is the expected return computed from a Fama-Macbeth regression. $\mathbb{E}_t[r_t(\beta)]$ is the expected return predicted from the β's of that regression alone. β' denotes the stock return β on the FF factors and β_{MPK} the mpk β on the same factors. t-statistics are in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 10: Panel Industry Regression of mpk Dispersion, Year FE

	(1)	(2)	(3)	(4)
$\sigma(E[ret])$	2.244***			
	(27.35)			
$\sigma(E_\beta[ret])$		12.66***		
		(27.69)		
$\sigma(\beta_{MKT})$			0.267***	
			(10.36)	
$\sigma(\beta_{HML})$			0.107***	
			(7.33)	
$\sigma(\beta_{SMB})$			0.129***	
			(7.85)	
$\sigma(\beta_{CAPM,MPK})$				0.138***
				(6.32)
$\sigma(\beta_{HML,MPK})$				0.0961***
				(6.16)
$\sigma(\beta_{SMB,MPK})$				0.0703***
				(6.33)
Observations	2721	2746	2734	1427
R^2	0.219	0.221	0.275	0.291

Notes: $\mathbb{E}_t[r_t]$ is the expected return computed from a Fama-Macbeth regression. $\mathbb{E}_t[r_t(\beta)]$ is the expected return predicted from the β's of that regression alone. β' denotes the stock return β on the FF factors and β_{MPK} the mpk β on the same factors. t-statistics are in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$