A Bayesian Semiparametric Competing Risk Model with Unobserved Heterogeneity[*]

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Abstract

This paper generalizes existing econometric models for censored competing risks by introducing a new flexible specification based on a piecewise linear baseline hazard, time-varying regressors, and unobserved individual heterogeneity distributed as an infinite mixture of Generalized Inverse Gaussian (GIG) densities, nesting the gamma kernel as a special case. A common correlated latent time effect induces dependence among risks. Our model is based on underlying latent exit decisions in continuous time while only a time interval containing the exit time is observed, as is common in economic data. We do not make the simplifying assumption of discretizing exit decisions – our competing risk model setup allows for latent exit times of different risk types to be realized within the same time period. In this setting, we derive a tractable likelihood based on scaled GIG Laplace transforms and their higher-order derivatives. We apply our approach to analyzing the determinants of unemployment duration with exits to jobs in the same industry or a different industry among unemployment insurance recipients on nationally representative individual-level survey data from the U.S. Department of Labor. Our approach allows us to conduct a counterfactual policy experiment by changing the replacement rate: we find that the impact of its change on the probability of exit from unemployment is inelastic.

JEL: C11, C13, C14, C41, J64
Keywords: competing risk model, Bayesian semiparametric model, unemployment insurance.

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1. Introduction

In an economic competing risk (CR) model with censoring, an individual is associated with a current state (e.g. of being unemployed) with the possibility to exit to one of several different states (e.g. finding a job in the same industry or a different industry). However, only one such exit is observed for some individuals, while other (censored) individuals are never observed to exit their current state. The state duration before exit to each potential new state or censoring time is modeled with a separate latent variable but only the shortest such duration is actually observed for each individual. A key ingredient of CR models is the survival function which captures the probability that the individual will remain the current state beyond any given time. CR analysis then typically seeks to determine the impact of observable characteristics of the individual and the various states on the survival function that can lead to policy recommendations.[5]

In this paper we introduce a new flexible model specification for the competing risk model, extending several strands of econometric and statistical literature on duration analysis. Our model encompasses three key features: (i) we estimate non-parametrically the density of unobserved individual heterogeneity; (ii) we model correlations between the different risk types; (iii) we allow for multiple latent exits within a time period with interval outcome data. Our model nests the single exit type (so-called duration model) as a special case. We apply our method to analyzing the determinants of unemployment duration among unemployment insurance recipients using data from the U.S. Department of Labor. We conduct a counterfactual experiment by changing the replacement rate. The counterfactual results show the impact of changing key policy variables such as the replacement rate on the survival function.

We will introduce each model feature in turn and discuss the advantages of using our model over the existing alternatives. First, our model provides a flexible approach to controlling for unobserved heterogeneity in competing risk data. A typical source of unobserved

[5] Applications of CR models in economics include analyzing unemployment duration (Flinn and Heckman, 1982; Katz and Meyer, 1990; Tysse and Vaage, 1999; Alba-Ramirez, Arranz, and Muñoz-Bullón, 2007), Ph.D. completion (Booth and Satchell, 1995), teacher turnover (Dolton and van der Klaauw, 1999), studies of age at marriage or cohabitation (Berrington and Diamond; 2000), mortgage termination (Deng, Quigley, and Van Order, 2000), school dropout decisions (Jakobsen and Rosholm, 2003), and manufacturing firms' exits from the market (Esteve-Perez, Sanchis-Llopis, and Sanchis-Llopis, 2010). A comprehensive overview is given in Van den Berg (2001).

heterogeneity is the omission of important but perhaps unobservable variables from the conditioning set. As an example, more motivated individuals may exit unemployment more quickly because they put more effort into the search for a new job.

It is well established that failure to account for unobserved heterogeneity biases the estimated hazard rate and the proportional effects of explanatory variables on the population hazard (Vaupel et al 1979; Lancaster 1979, 1990). A number of semi-parametric estimators for the single risk mixed proportional hazard (MPH) model have been proposed following Elbers and Ridder (1982) proof of MPH semi-parametric identification (Heckman and Singer, 1984; Honore 1990). Han and Hausman (1990) and Meyer (1990) propose an estimator for piecewise-constant baseline hazard and gamma distributed unobserved heterogeneity. Horowitz (1999) proposed a nonparametric estimator for both the baseline hazard and the distribution of the unobserved heterogeneity, under the assumption of constant time-invariant regressors. Hausman and Woutersen (2012) show that a nonparametric estimator of the baseline hazard with gamma heterogeneity yields inconsistent estimates for all parameters and functions if the true mixing distribution is not a gamma, stressing the importance of avoiding parametric assumptions on the unobserved heterogeneity.

As the second key feature of our model, our approach allows for correlations between the different risks in the CR model environment, even in the presence of the flexible individual heterogeneity infinite GIG mixture model component. This is important since the determinants of exit can differ depending on the risk type while being correlated across the risk types, and thus our approach provides additional information to the analyst compared to single-risk duration models.

Third, in our application, we deal with interval outcome data as is common in economics and other social sciences. Even though the underlying exit decision model is set in continuous time, only the broader period in which exit occurred is observable. Our data contain the week of exit from unemployment. Based on scaled GIG Laplace transforms and their higher-order derivatives, we provide a complete likelihood specification allowing for multiple latent exits within a single time period which is more realistic than simplifying the analysis by assuming that only one latent exit can occur in a given time period. Thus, we do not rule out by assumption an individual contemplating different job offers from firms in the same industry or in a different industry as their pre-unemployment industry within any given week.

We show that given the analytical forms newly provided in this paper our model can be implemented in a user-friendly way via a Bayesian nonparametric approach. One of the key benefits of Bayesian Markov chain Monte Carlo (MCMC) methods that we utilize is their ability to factorize a complicated joint likelihood model into a sequence of conditional tractable models, so-called Gibbs blocks, and by sampling each in turn deliver outcomes from the joint model. We detail this approach for our proposed model.

The bulk of the literature on CR model development is concentrated in the natural sciences. A recent overview of CR modeling in biostatistics is provided by Beyersmann, Schumacher, and Allignol (2012), and in medical research by Pintilie (2006). The associated estimation methods typically rely on continuous time data for the exact point of exit. In contrast, we observe only discrete time intervals within which latent exits occur.

Competing risk models suitable for economic interval outcome data have been proposed in various forms. Han and Hausman (1990), Fallick (1991), Sueyoshi (1992), and McCall (1996) provide model specifications either without or with parametric individual heterogeneity. Butler, Anderson, and Burkhauser (1989) propose a semiparametric CR model controlling for the correlation between unobserved heterogeneity components in each state, with quadratic time dependence. Lleras-Muney and Honoré (2006) analyze identification issues in a general class of competing risk models allowing for correlation among risk types. Their environment is free of many of the functional form and distributional assumptions that we impose here and hence a number of parameters are set-identified. Bierens and Carvalho (2007) consider Weibull baseline hazards and common flexible unobserved heterogeneity. Canals-Cerdá and Gurmu (2007) approximate unobserved heterogeneity distribution with Laguerre polynomials. They find that model selection rules (BIC, HQIC, and AIC) perform worse in determining the polynomial order than a naive approach of controlling for unobserved heterogeneity using simple models with a small number of points of support or a polynomial of small degree. Van den Berg, van Lomwel, and van Ours (2008) consider a model with nonparametric unobserved heterogeneity terms that is based on discrete time counts. Although the model can be derived as a time-aggregated version of an underlying continuous-time model, the latter is different from the continuous-time mixed proportional hazard model.

The literature on Bayesian nonparametric methods in the CR environment has been scant and, to our knowledge, has only been used in biostatistics for estimation of other objects of interest than individual heterogeneity.[6] Variants of Bayesian Dirichlet Process analysis have been used by Gasbarra and Karia (2000) for estimating nonparametically the overall hazard rate and in Salinas-Torres, Pereira, and Tiwari (2002) and Polpo and Sinha (2011) for the vector of risk-specific cumulative incidence functions. De Blasi and Hjort (2007) specify a beta-process prior for the baseline hazard, with asymptotic properties analyzed in De Blasi and Hjort (2009).

Identification results under various assumptions were established by Heckman and Honoré (1989), Sueyoshi (1992), Abbring and van den Berg (2003), and Lee and Lewbel (2013). In general, there have been three different approaches to identification (Honoré and Lleras-Muney, 2006): (a) to make no additional assumptions beyond the latent competing risk structure and estimate bounds on the objects of interest; (b), assume that the risks are independent conditional on a set of observed covariates and deal with a multiple duration models environment; and (c), to specify a parametric or semi-parametric model conditional on the covariates. Here we take the last approach. In particular, we do not assume that the risks are independent conditional on the observed covariates.

The remainder of the paper is organized as follows. Section 2 establishes the assumptions and building block results for a single risk duration model. Section 3 introduces assumptions and results for the competing risk model. Section 4 details our application and the counterfactual experiment and Section 5 concludes. Proofs or all theorems and additional empirical results are provided in the Appendix. The Online Appendix (Burda, Harding and Hausman, 2013) contains further results.

[6] In the single-risk, duration model case, Bayesian analysis with economics application was undertaken by Ruggiero (1994), Florens, Mouchart, and Rolin (1999), Campolieti (2001), Paserman (2004), and Li (2007).

2. Single Risk Duration Model

Denote by τ a continuous time variable with density $f(\tau)$ and distribution $F(\tau)$. Denote a latent failure (or exit) time of individual i by τ_i. Define the hazard rate $\lambda_i(\tau)$ as the failure rate at time τ conditional upon survival to time τ, $\lambda_i(\tau) = \lim_{\delta \to 0} Pr(\tau < \tau_i < \tau + \delta\ \tau_i \geq \tau)/\delta$ and denote the integrated hazard by:

(2.1)
$$\Lambda_i(\tau) = \int_0^{\tau} \lambda_i(u)du$$

with survivor function

(2.2)
$$S_i(\tau) = \exp\left(-\Lambda_i(\tau)\right)$$

Denote by t a generic time period $[\underline{\tau}, \overline{\tau})$ with end points $\underline{\tau}$ and $\overline{\tau}$, and by t_i the time period $[\underline{\tau}_i, \overline{\tau}_i) \ni \tau_i$ in which an individual i was observed to exit from a given state into another state.

ASSUMPTION (A1). *The data $\{t_i\}_{i=1}^{N}$ consists of single spells censored at time T and drawn from a single risk process.*

ASSUMPTION (A2). *The hazard rate is parameterized as*

(2.3)
$$\lambda_i(\tau) = \lambda_0(\tau)\exp(X_i(\tau)\beta + V_i)$$

where $\lambda_0(\tau)$ is the baseline hazard, $X_i(\tau)$ are observed covariates that are allowed to vary over time, β are model parameters, and V_i is an unobserved heterogeneity component.

Hence, using (2.1) and (2.3) the integrated hazard is given by

(2.4)
$$\Lambda_i(\tau) = \int_0^{\tau} \lambda_0(u)\exp\left(X_i(u)\beta + V_i\right)du$$

ASSUMPTION (A3). *The baseline hazard $\lambda_0(u)$ and the values of the covariates are constant for each time period t.*

Assumptions 1 and 2 are common in the literature. Assumption A3 is based on Han and Hausman (1990). Given Assumption A3, instead of $\lambda_i(\tau)$ we can consider the integrated baseline hazard in the form

(2.5)
$$\mu_{0t} = \int_{\underline{\tau}}^{\overline{\tau}} \lambda_0(u)du,$$

where we denote the vector $(\mu_{01}, \ldots, \mu_{0T})$ by μ_0.

For notational convenience, we will use subscripts for the time index and denote by Λ_{it} the quantity $\Lambda_i(\bar{\tau})$ at the end of the time period t, and similarly for other variables. Denote the probability of the exit event in time period t by $P(t_i = t)$. Conditional on V_i, for outcomes that are not censored ($t_i \leq T$),

$$P(t_i = t) = F_{it} - F_{i(t-1)} = (1 - S_{it}) - (1 - S_{i(t-1)}) = S_{i(t-1)} - S_{it} \tag{2.6}$$

When the duration observations are censored at the end of time period T,

$$P(t_i > T) = 1 - F_{iT} = S_{iT} \tag{2.7}$$

The following result is familiar in the literature and we include it here for the sake of completeness as a benchmark of comparison for the new competing risk model developed in the next Section.

RESULT 1. *Under Assumptions A1-A3, conditional on V_i, for uncensored observations*

$$P(t_i = t\; V_i) = \exp\left(-\sum_{j=1}^{t-1} \mu_{0j} \exp\left(X_{ij}\beta + V_i\right)\right) - \exp\left(-\sum_{j=1}^{t} \mu_{0j} \exp\left(X_{ij}\beta + V_i\right)\right) \tag{2.8}$$

and for the censored case

$$P(t_i > T\; V_i) = \exp\left(-\sum_{j=1}^{T} \mu_{0j} \exp\left(X_{ij}\beta + V_i\right)\right) \tag{2.9}$$

2.1. Parametric Heterogeneity

ASSUMPTION (A4). *Let $v_i \equiv \exp(V_i) \sim \mathcal{G}(v)$ where $\mathcal{G}(v)$ is a generic probability measure with density $g(v)$.*

From (2.4), (2.5), and Assumption A3, we have

$$\tilde{\Lambda}_{it} = \sum_{j=1}^{t} \mu_{0j} \exp\left(X_{ij}\beta\right) \tag{2.10}$$

$$\Lambda_{it} = v_i \tilde{\Lambda}_{it} \tag{2.11}$$

If v is a random variable with probability density function $g(v)$ then the Laplace transform of $g(v)$ evaluated at $s \in \mathbb{R}$ is defined as

$$\mathcal{L}(s) \equiv E_v[\exp(-vs)] \tag{2.12}$$

Using (2.2), (2.11), and (2.12), the expectation of the survival function can be linked to the Laplace transform of the integrated hazard function (Hougaard, 2000) as follows:

$$E_v\left[S_{it}\right] = \mathcal{L}(\widetilde{\Lambda}_{it}) \tag{2.13}$$

Using (2.10), (2.11), and (2.13) yields the unconditional exit probability of Result 1 as follows:

RESULT 2. *The expectation of (2.6) for the uncensored observations is*

$$E_{v_i}\left[P(t_i = t)\right] = \mathcal{L}(\widetilde{\Lambda}_{i(t-1)}) - \mathcal{L}(\widetilde{\Lambda}_{it}) \tag{2.14}$$

and the expectation of (2.7) for the censored observations takes the form

$$E_{v_i}\left[P(t_i > T)\right] = \mathcal{L}(\widetilde{\Lambda}_{iT}) \tag{2.15}$$

Since the individual heterogeneity term v_i defined in Assumption A4 is non-negative, a suitable family of distributions $\mathcal{G}(v)$ with support over $[0, \infty)$ and tractable closed-form Laplace transforms is Generalized Inverse Gaussian (GIG) class of distributions, whose special case is the gamma distribution popular in duration analysis.

ASSUMPTION (A5a). *The unobserved heterogeneity term v_i is distributed according to the Generalized Inverse Gaussian distribution, $\mathcal{G}(v) = \mathcal{G}^{GIG}(v; \kappa, \varphi, \theta)$.*

The GIG has the density

$$g^{GIG}(v; \kappa, \varphi, \theta) = \frac{2^{\kappa-1}}{K_\kappa(\varphi)}\frac{\theta}{\varphi^\kappa}(\theta v)^{\kappa-1}\exp\left\{-\theta v - \frac{\varphi^2}{4\theta v}\right\} \tag{2.16}$$

for $\varphi, \theta > 0$, $\kappa \in \mathbb{R}$, where $K_\kappa(\varphi)$ is the modified Bessel function of the second kind of order κ evaluated at φ (Hougaard, 2000). The GIG Laplace transform is given by

$$\mathcal{L}^{GIG}(s; \kappa, \varphi, \theta) = (1 + s/\theta)^{-\kappa/2}\frac{K_\kappa\left(\varphi\left(1 + s/\theta\right)^{1/2}\right)}{K_\kappa(\varphi)} \tag{2.17}$$

The GIG family includes as special cases the gamma distribution for $\varphi = 0$, the Inverse gamma distribution for $\theta = 0$, and the Inverse Gaussian distribution for $\kappa = -\frac{1}{2}$, among others.

Application of the Laplace transform of the GIG distribution (2.17) in Result 2 yields the following result that appears to not have been previously stated in the literature:

RESULT 3. *Under the Assumptions A1–A4, and A5a*

$$E_{v_i}^{GIG}\left[P(t_i = t)\right] = \left(1 + \frac{1}{\theta}\tilde{\Lambda}_{i(t-1)}\right)^{-\kappa/2} \frac{K_\kappa\left(\varphi\left(1+\frac{1}{\theta}\tilde{\Lambda}_{i(t-1)}\right)^{1/2}\right)}{K_\kappa\left(\varphi\right)}$$

$$- \left(1 + \frac{1}{\theta}\tilde{\Lambda}_{it}\right)^{-\kappa/2} \frac{K_\kappa\left(\varphi\left(1+\frac{1}{\theta}\tilde{\Lambda}_{it}\right)^{1/2}\right)}{K_\kappa\left(\varphi\right)} \tag{2.18}$$

and for the censored observations

$$E_{v_i}^{GIG}\left[P(t_i > T)\right] = \left(1 + \frac{1}{\theta}\tilde{\Lambda}_{iT}\right)^{-\kappa/2} \frac{K_\kappa\left(\varphi\left(1+\frac{1}{\theta}\tilde{\Lambda}_{iT}\right)^{1/2}\right)}{K_\kappa\left(\varphi\right)} \tag{2.19}$$

A special case of the GIG distribution is the gamma distribution, obtained from the GIG density function (2.16) when $\varphi = 0$. We use the gamma distribution for v_i as a benchmark model under the following alternative to Assumption 5a.

ASSUMPTION (A5b). *The unobserved heterogeneity term v_i is distributed according to the gamma distribution, $\mathcal{G}(v) = \mathcal{G}^G(v; \gamma, \theta)$.*

The gamma density is parameterized as

$$g^G(v; \gamma, \theta) = \frac{\theta}{\Gamma(\gamma)}\left(\theta v\right)^{\gamma-1}\exp(-\theta v) \tag{2.20}$$

and its Laplace transform is given by

$$\mathcal{L}^G(s; \gamma, \theta) = (1 + s/\theta)^{-\gamma} \tag{2.21}$$

In the gamma density (2.20) the parameter $\gamma > 0$ corresponds to the GIG parameter $\kappa \in \mathbb{R}$ in (2.16) restricted to the positive part of the real line. Using the gamma distribution in place of the GIG constitutes a special case of Result 3:

RESULT 4. *Under the Assumptions A1–A4, and A5b,*

$$E_{v_i}^{G}\left[P(t_i = t)\right] = \left(1 + \frac{1}{\theta}\tilde{\Lambda}_{i(t-1)}\right)^{-\gamma} - \left(1 + \frac{1}{\theta}\tilde{\Lambda}_{it}\right)^{-\gamma} \tag{2.22}$$

and

$$E_{v_i}^{G}\left[P(t_i > t)\right] = \left(1 + \frac{1}{\theta}\tilde{\Lambda}_{it}\right)^{-\gamma} \tag{2.23}$$

Result 4 was obtained in Han and Hausman (1990) and Meyer (1990).

In both gamma and GIG distributions, the scale parameter θ performs the same role. Specifically, for any $c \in \mathbb{R}_{+}$, if $v \sim \mathcal{G}^{G}(v;\gamma,\theta)$ then $cv \sim \mathcal{G}^{G}(v;\gamma,\theta/c)$, and if $v \sim \mathcal{G}^{GIG}(v;\kappa,\varphi,\theta)$ then $cv \sim \mathcal{G}^{GIG}(v;\kappa,\varphi,\theta/c)$. Due to this property, c and hence its inverse $s \equiv c^{-1}$ are not separately identified from θ in the Laplace transform expressions (2.17) and (2.21). Since all likelihood expressions are evaluated at $s = \widetilde{\Lambda}_{i(\cdot)}$ which is proportional to μ_{0j} for all j, as specified in (2.10), any change in θ only rescales the baseline hazard parameters μ_{0j}, leaving the likelihood unchanged. Hence, θ needs to be normalized to identify μ_{0j}. In the gamma case, typically this normalization takes the form $\theta = \gamma$ so that $E[v] = 1$. We use the equivalent normalization for the GIG case in order to nest the normalized gamma as a special case and to maintain the moment restriction $E[v] = 1$.

2.2. Flexible Heterogeneity

We now depart from the parametric form of the unobserved heterogeneity and consider a nonparametric infinite mixture for the distribution of v_i, as formulated in the following assumption.

ASSUMPTION (A6). *The prior for v_i $G \sim \mathcal{G}$ takes the form of the hierarchical model $G \sim DP(G_0, \alpha)$, $\alpha \sim g^{G}(a_0, b_0)$, $E[v_i] = 1$.*

In Assumption A6, G is a random probability measure distributed according to a Dirichlet Process (DP) prior (Hirano, 2002; Chib and Hamilton, 2002). The DP prior is indexed by two hyperparameters: a so-called baseline distribution G_0 that defines the "location" of the DP prior, and a positive scalar precision parameter α. The distribution G_0 may be viewed as the prior that would be used in a typical parametric analysis. The flexibility of the DP mixture model environment stems from allowing G to stochastically deviate from G_0. The precision parameter α determines the concentration of the prior for G around the DP prior location G_0 and thus measures the strength of belief in G_0. For large values of α, a sampled G is very likely to be close to G_0, and vice versa. Assumption A6 is then completed by specifying the baseline measure G_0. We consider two cases:

ASSUMPTION (A7a). *In Assumption A6, $G_0 = \mathcal{G}^{GIG}(\kappa,\varphi,\theta)$.*

Implementation of the GIG mixture model under Assumptions A1–A3, A6, and A7a uses the probabilities (2.8), (2.9), (2.18) and (2.19). Further implementation details are given in the Online Appendix (Burda, Harding and Hausman, 2013).

ASSUMPTION (A7b). *In Assumption A6,* $G_0 = \mathcal{G}^G(\gamma, \theta)$.

Under Assumptions A6 and A7b, as a special limit case, putting all the prior probability on the baseline distribution G_0 by setting $\alpha \to \infty$ would result in forcing $G = G_0 = \mathcal{G}^G(\gamma, \theta)$ which yields the parametric model of Han and Hausman (1990). Here we allow α and hence G to vary stochastically. Furthermore, the gamma baseline in Assumption A7b results as a special case of the GIG baseline in Assumption A7a for the hyperparameter value $\varphi = 0$. Hence, both the gamma flexible case with $G \sim DP(\mathcal{G}^G, \alpha)$ and the parametric benchmark Han and Hausman (1990) case with $G = \mathcal{G}^G$ are nested within our full GIG mixture model specification.

3. Competing Risk Model

We will now generalize the results from the single-risk case to the competing risk (CR) environment with several different potential types of exit. Let the risk type be indexed by $k = 1, \ldots, K$. Define the latent failure (or exit) times as $\tau_{1i}, \ldots, \tau_{Ki}$ corresponding to each risk type k, for each individual i. Define their minimum by $\tau_i \equiv \min(\tau_{1i}, \ldots, \tau_{Ki})$. In our CR model for interval outcome data, τ_i is not directly observed. Instead, the observed quantity is the time interval $[\underline{\tau}_i, \overline{\tau}_i)$ labeled as t_i which contains τ_i. This is in contrast to a large class of other types of CR models where the exact failure time τ_i is observed, as is typical in biostatistics. Intrinsically, the lifetimes of other risk types, τ_{ji} for $j \neq k$, and their corresponding time intervals, remain unobserved.

Denote by $f(u_1, u_2)$ the joint density of failure at time $u = (u_1, u_2)$. The functional form of $f(u_1, u_2)$ is provided in the Online Appendix, (OA.2.4). For two risk types with $K = 2$, this yields the probability of exit in time period t of the form

$$(3.1) \qquad P(t_{1i} = t, \ \tau_{2i} > \tau_{1i}) = \int_{\underline{\tau}_i}^{\overline{\tau}_i} \int_{u_1}^{\overline{\tau}_i} f(u_1, u_2) du_2 du_1 + \int_{\underline{\tau}_i}^{\overline{\tau}_i} \int_{\overline{\tau}_i}^{\infty} f(u_1, u_2) du_2 du_1$$

The first right-hand side term in (3.1) gives the probability that the second latent exit time occurred within the same time interval t as the first latent exit time. The second right-hand side term in (3.1) is then the probability that the second latent exit time occurred in a later time interval than t. A key difficulty with evaluating (3.1), precluding direct factorization, is the presence of the outer integrand u_1 in the lower bound in the inner integral of the first term. We deal with this issue and derive a closed-form solution for (3.1), under various assumptions on the latent model components. The joint density $f(u_1, u_2)$ is obtained as a function of covariates and unobserved heterogeneity from the parameterization of risk-specific hazard functions, in a direct analogy to the single-risk case. Previous work using CR interval outcome data has either bypassed this link (e.g. by assuming a multivariate Gaussian density for $f(u_1, u_2)$) or employed a discrete time approximation whereby only one exit type can occur per any one time period. Our model explicitly accounts for the continuous-time nature of the exit decisions. The statistical background for the stochastic environment of our CR model is given in the Online Appendix (Burda, Harding and Hausman, 2013).

For clarity of exposition, the numbering of the Assumptions and Theorems in this Section provides a direct counterpart to the Assumptions and Results of the single-risk case in the previous Section. We first treat the parametric case under the GIG and gamma distributions of unobserved heterogeneity, adding a common latent component for all risk types, and then proceed to infinite mixture modeling.

ASSUMPTION (B1). *The data consists of single spell data, drawn from a process with two risks $k = 1, 2$, and is censored at T_k.*

Assumption B1 readily generalizes to an arbitrary number of risks. Without loss of generality, suppose that the failure type is of type 1 so that $\tau_{1i} = \min(\tau_{1i}, \tau_{2i})$.

ASSUMPTION (B2). *The risk-specific hazard rate is parameterized as*

$$\lambda_{ki}(\tau) = \lambda_{0k}(\tau)\exp(X_i(\tau)\beta_k + V_{ki} + \zeta_k(\tau))$$

where $\lambda_{0k}(\tau) > 0$ is the baseline hazard whose logarithm is independent across k, $X_i(\tau)$ are covariates that are allowed to vary over time with full support on all of the real line for any given τ and one covariate common to all k, β_k are model parameters, V_{ki} is an unobserved heterogeneity component, and $\zeta_k(\tau)$ is a common time component correlated across k normalized to have mean zero in each k.

ASSUMPTION (B3). *For each k, the baseline hazard $\lambda_{0k}(\tau)$, the values of the covariates, and $\zeta_k(\tau)$ are constant within each time period t.*

Thus, the log-baseline hazard could also be stated as $\delta_{0kt} = \log(\lambda_{0kt}) + \zeta_{kt}$ where δ_{0kt} is correlated across k due to the presence of ζ_{kt}. Let $\zeta_k = (\zeta_{k1}, \ldots, \zeta_{kT})$; $\zeta = (\zeta_1, \zeta_2)$; $V_i = \{V_{ki}\}_{k=1}^{K}$; and $V = \{V_i\}_{i=1}^{N}$. The probability (3.1), conditional on (V_i, ζ) and a set of covariates, is

(3.2)

$$P(t_{1i} = t, \ \tau_{2i} > \tau_{1i} \ V_i; \zeta) = \int_{\underline{\tau}_i}^{\overline{\tau}_i} \int_{u_1}^{\overline{\tau}_i} f(u_1, u_2 \ V_i; \zeta) du_2 du_1 + \int_{\underline{\tau}_i}^{\overline{\tau}_i} \int_{\overline{\tau}_i}^{\infty} f(u_1, u_2 \ V_i; \zeta) du_2 du_1$$

Let $S_{kit} = \exp(-\Lambda_{kit})$ denote the risk-specific survivor function. We derive a closed-form solution to (3.2) in the following Theorem which extends Result 1 to our CR model environment.

THEOREM 1. *Under Assumptions B1–B3, conditional on the latent vector $(V_i; \zeta)$ and a set of covariates,*

(3.3)

$$P(t_{1i} = t, \ \tau_{2i} > \tau_{1i} \ V_i; \zeta) = S_{2i(t-1)} S_{1i(t-1)} \lambda_{1it} \left(\lambda_{2it} + \lambda_{1it}\right)^{-1} \left[1 - \exp\left(-\left(\lambda_{2it} + \lambda_{1it}\right)\right)\right]$$

for uncensored observations, and

$$P(t_{1i} > T, \ t_{2i} > T \ V_i; \zeta) = \left(1 - F_{1iT}\right)\left(1 - F_{2iT}\right) = S_{1iT} S_{2iT} \tag{3.4}$$

for censored observations.

The proof is provided in the Online Appendix.

3.1. Parametric Heterogeneity in the CR Model

ASSUMPTION (B4). *Let $v_{ki} \equiv \exp(V_{ki}) \sim \mathcal{G}_k(v_k)$ where $\mathcal{G}_k(v_k)$ is a generic probability measure with density $g_k(v_k)$. Let the correlation structure of ζ_{kt} be given by*

$$\begin{pmatrix} \zeta_{1t} \\ \zeta_{2t} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}; \left| \begin{matrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_2\sigma_1 & \sigma_2^2 \end{matrix} \right] \right)$$

with parameters ρ, σ_1, σ_2.

As in the single-risk case, we consider two alternative forms of the distribution of unobserved heterogeneity $\mathcal{G}(v)$ in Assumption B4. The first form is parametric given either by the GIG or gamma density. We provide new results regarding the model likelihood for the model B1–B4. These will be used in the nonparametric mixture model. This approach is different from Han and Hausman (1990) who considered the truncated multivariate Normal likelihood.

For the expected likelihood, we have two new expression for the expected probability of (3.3): one based on a quadrature, and another one with a series expansion without the need for a quadrature. The following Theorem extends Result 2 into the CR model environment.

THEOREM 2. *Under Assumptions B1–B4,*

$$(3.5)\quad E_v P(t_{1i} = t, \tau_{2i} > \tau_{1i}) = -\widetilde{\lambda}_{1it} \int_0^1 \mathcal{L}_2\left(\widetilde{\Lambda}_{2i(t-1)} + \widetilde{\lambda}_{2it} s_1\right) \mathcal{L}_1^{(1)}\left(\widetilde{\Lambda}_{1i(t-1)} + \widetilde{\lambda}_{1it} s_1\right) ds_1$$

or

$$\begin{aligned} E_v P(t_{1i} = t,\ \tau_{2i} > \tau_{1i}) \ &= \ \sum_{r_2=0}^{\infty} \sum_{r_1=0}^{\infty} \frac{(-1)^{2r_1+2r_2+1}}{r_2! r_1! \left(r_2 + r_1 + 1\right)} \widetilde{\lambda}_{1it}^{r_1+1} \widetilde{\lambda}_{2it}^{r_2} \\ (3.6) \qquad\qquad & \qquad \times \mathcal{L}_1^{(r_1+1)}\left(\widetilde{\Lambda}_{1i(t-1)}\right) \mathcal{L}_2^{(r_2)}\left(\widetilde{\Lambda}_{2i(t-1)}\right) \end{aligned}$$

for uncensored observations, and

$$(3.7)\qquad E_v P(t_{1i} > T;\ t_{2i} > T) = \mathcal{L}_1\left(\widetilde{\Lambda}_{1iT}\right) \mathcal{L}_2\left(\widetilde{\Lambda}_{2iT}\right)$$

for censored observations, where $\mathcal{L}_k^{(r)}(s)$ is the $r-th$ derivative of the Laplace transform.

The proof is given in the Online Appendix. Theorem 2 is derived for a generic distribution of the unobserved individual heterogeneity term v_i and provides a direct extension of (2.14) and (2.15) to the competing risk model environment. Specific alternative distributional assumptions with corresponding likelihood expressions are provided next.

ASSUMPTION (B5a). *The unobserved heterogeneity term v_i is distributed according to the Generalized Inverse Gaussian distribution, $\mathcal{G}(v) = \mathcal{G}^{GIG}(v; \kappa, \varphi, \theta)$.*

ASSUMPTION (B5b). *The unobserved heterogeneity term v_i is distributed according to the gamma distribution, $\mathcal{G}(v) = \mathcal{G}^{G}(v; \gamma, \theta)$.*

The derivatives of the Laplace transform in (3.5) and (3.6) Theorem 2 depend on the functional form of the density kernel of v_i, given in Assumptions B5a and B5b. The formulas for the derivatives of arbitrary order of the Laplace transform for the GIG or gamma densities do not appear to be available in the literature; we derive them in the Online Appendix. Using those expressions in Theorem 2 yields the following two Corollaries, extending Result 3 and 4, respectively, from the single-risk case to the competing risk model environment.

Corollary 1 (to Theorem 2). *Under Assumptions B1–B4 and B5a, the functional forms of Theorem 2 are given in (OA.2.43), (OA.2.44), and (OA.2.45) in the Online Appendix.*

Corollary 2 (to Theorem 2). *Under Assumptions B1–B4 and B5b, the functional forms of Theorem 2 are given in (OA.2.47), (OA.2.48), and (OA.2.49) in the Online Appendix*

3.2. Flexible Heterogeneity in the CR Model

We will now proceed to an infinite mixture model for the distribution of v_{ki}.

ASSUMPTION (B6). *The prior for v_{ki} $G_k \sim G_k$ is specified as the hierarchical model $G_k \sim DP(G_{0k}, \alpha_k)$, $\alpha_k \sim g^G(a_{0k}, b_{0k})$, $E[v_{ki}] = 1$.*

The roles of the individual model components are described in Assumption A6 and generalize to the CR framework. Similarly to the single risk model environment, we consider two cases for the functional form of the baseline measure G_0:

ASSUMPTION (B7a). *In Assumption B6, $G_{0k} = \mathcal{G}^{GIG}(\kappa_k, \varphi_k, \theta_k)$.*

ASSUMPTION (B7b). *In Assumption B6, $G_{0k} = \mathcal{G}^{G}(\gamma_k, \theta_k)$.*

Implementation of the mixture models under Assumptions B1–B3, B6, and B7a or B7b uses the probabilities derived in Theorem 1, Corollary 1, and Corollary 2. Further implementation details are given in the next Section and in the Online Appendix.

Heckman and Honoré (1989) show how the introduction of covariates allows identification of a large class of dependent competing risks models without invoking distributional assumptions. Nonetheless, normalization assumptions are necessary for parameter identification. The normalization constraints generalize directly from the single-risk case and we impose them for each risk type. Assumptions B2 and B4 impose explicit restrictions on the model behavior in continuous time within each time period. These restrictions allow us to invoke identification conditions of Heckman and Honorè (1989). A detailed discussion of identification is provided in the Online Appendix.

4. MCMC Posterior Sampling

4.1. Single Risk Model

All technical implementation details are provided in the Online Appendix. In this Section we summarize the main points. For the implementation of the Dirichlet Process Mixture model (Assumptions A6 and A7a,b) we used the Bayesian generalized Pòlya urn scheme (Neal 2000 Algorithm 2; West, Müller, and Escobar, 1994; Bush and MacEachern, 1996). Implementation of the GIG mixture model (Assumptions A1–A3, A6, and A7a) uses the functions (2.8), (2.9), (2.18), and (2.19). The gamma mixture model (Assumptions A1–A3, A6, and A7b) uses (2.8), (2.9), (2.22) and (2.23), respectively. The remaining model parameters were sampled in standard Gibbs blocks using Hybrid Monte Carlo (Neal 2011) with diffuse priors unless stated otherwise above. The results are discussed in our application below.

4.2. Competing Risk model

Similarly to the single-risk case, for the implementation of the Dirichlet Process Mixture model in the competing risk environment (Assumptions B6 and B7a,b) we also used the Bayesian generalized Pòlya urn scheme. Implementation of the GIG mixture model (Assumptions B1–B3, B6, and B7a) uses the functions (3.3) and (3.4) from Theorem 1, and Corollary 1 to Theorem 2. The gamma mixture model (Assumptions B1 B3, B6, and B7b) uses (OA.2.47) and (OA.2.49) from Corollary 2 to Theorem 2. The remaining model parameters were sampled in standard Gibbs blocks using Hybrid Monte Carlo (Neal 2011) with diffuse priors unless stated otherwise above, except the covariance matrix of Assumption B4 with parameters σ_1, σ_2, ρ for which we specify a proper Inverse Wishart prior with maximum possible dispersion, $IW(K+1, I_K)$ where I_K is the identity matrix of dimension $K = 2$. The results are discussed in our application below.

5. Application

Since its introduction in 1935 as part of Roosevelt's *Social Security Act*, unemployment insurance (UI) benefits provide partial insurance to workers who become unemployed. Most states offer unemployment insurance for up to 26 weeks. Neoclassical economic thought suggests that higher benefits also lead to reduced incentives to search for a job, thus prolonging the period of time an individual spends out of employment (Mulligan, 2012). As a result policy makers have placed increased emphasis on reforming the UI system by rewarding personal responsibility rather than bad luck. This has lead to a shift away from the unqualified provision of UI benefits towards a system that is search intensive, making benefits conditional on providing evidence that the potential recipient engaged in a certain minimum amount of job search. Additionally, schemes whereby individuals are provided one-off grants that attempt to alleviate temporary hardship rather than longer term UI benefits are advocated. At the same time the recent economic crisis has forced policy makers to extend the duration of UI benefits for up to 99 weeks.[7]

Applied economists require econometric tools to accurately estimate the impact of unemployment insurance on the duration of unemployment, while accounting for state unemployment rates, generosity of unemployment insurance benefits and workers' observed and unobserved heterogeneity. In this section we apply our approach to analyzing the determinants of unemployment duration among unemployment insurance recipients. We will stress the importance of relaxing the parametric assumptions of the econometric models and accounting for correlations in the competing exit choices faced by workers. One of the major advantages of our approach is that it is possible to simulate counterfactual policy changes which can inform policy makers on the relative merits of various changes that may be contemplated. We will illustrate this feature by evaluating the impact of a change in the replacement rate on the duration of unemployment.

5.1. Data

We use data from the Needels et. al. (2001) report submitted to the U.S. Department of Labor that is based on a nationally representative sample built from individual-level surveys of unemployment insurance (UI) recipients in 25 states between 1998 and 2001.

[7] The unemployment extension legislation was set to expire on January 1, 2013.

Candidates for the survey are selected on the basis of administrative records and are sampled from the pool of unemployed individuals that started collecting UI benefits at some point during the year 1998.

We are interested in analyzing the effect of unemployment insurance on the duration of unemployment. The duration of unemployment is measured in weeks. At the time of the survey and from the states that were included in the survey only two states provided UI benefits for a maximum of 30 weeks, the rest providing UI benefits for a maximum of 26 weeks. Theoretical models of the impact of UI benefits on unemployment duration, such as Mortensen (1977) and Moffitt and Nicholson (1982), predict an increasing hazard up to the point of benefit exhaustion and a flat one afterwards. We limit our study to the first 24 weeks of unemployment due to the recognized change in behavior in week 26 when UI benefits cease for a significant part of the sample (see, e.g., Han and Hausman, 1990), which would affect the econometric model.

The data contain individual-level information about labor market and other activities from the time the person entered the UI system through the time of the interview. The data include information about the individual's pre-UI job, other income or assistance received, and demographic information. We use two indicator variables, race (defined as an indicator for black) and age (defined as an indicator for over 50). We further use the replacement rate, which is the weekly benefit amount divided by the UI recipient's base period earnings. Lastly, we utilize the state unemployment rate of the state from which the individual received UI benefits during the period in which the individual filed for benefits. This variable changes over time. The Needels et. al. (2001) data shares certain similarities to the PSID dataset used in Han and Hausman (1990). The UI recipients are mostly white, young, poorly educated workers who find themselves below or very near the poverty line.[8]

Below we will estimate a single risk duration model for the duration to re-employment and also a competing risk duration model for the duration to re-employment using our proposed approach, which differentiates between workers who find a job in the same

[8] Note that the labor market conditions captured in this dataset are substantially different than the ones experienced today. According to the Bureau of Labor Statistics (BLS), the latest figures broken down by state for September 2012 show the mean state unemployment rate is 7.5% and varies between 3% and 11.8%. In contrast, in our dataset the state unemployment rate is approximately 4.5%.

industry and workers who find a job in a different industry. For both analyses we employ subsamples from the Needels et. al. (2001) survey data, after removing outliers and observations with missing values. For the single risk model we use a subsample consisting of 15,358 individuals. Summary statistics for this sample are given in Table 2. For a subsample of 1,243 of the Needels et. al. (2001) data we also know the SIC codes of the employer before and after the unemployment spell. We denote individuals who find a job in the same industry as individuals with risk type 1, while those who find a job in a different industry as individuals with risk type 2. Summary statistics for this subsample are given in Table 2. We note a marked difference in the unemployment durations of these two groups of individuals. Figure 1 provides plots of the number of individuals who exited in each time period, shown separately for workers who find a job in the same industry and those who do not (risk type 2). Individuals who find a job in the same industry exit unemployment much faster in the first few weeks after they lose their job but conditional on not having found employment by week 8 their exit pattern resembles that of the other individuals. This is empirically interesting as it points towards the importance of industry specific human capital. We would expect variation among the industry specific human capital to vary by industry but also by individual reflecting their level of experience and motivation. It is thus to be expected that some individuals have accumulated a significant degree of human capital which makes them attractive to other employers in the same industry. Switching industries usually entails learning new skills and the incentive to do so may be affected by the duration and generosity of the unemployment insurance benefits. We would thus expect substantial behavioral differences between workers facing these two competing risks.

5.2. Single Risk Duration Model with Flexible Heterogeneity

Estimation results of the semiparametric duration model with a flexible form of unobserved heterogeneity under GIG mixing (Assumptions A1–A3, A6, and A7a) are presented in Table 3. In the Online Appendix we also present estimation results for two benchmark parametric models. Estimation results of a model with parametric gamma heterogeneity (Han and Hausman, 1990; Meyer 1990), as specified in Assumptions A1 A4 and A5b, are given in Table OA.1. In Table OA.2 we present estimation results for another benchmark model with parametric GIG heterogeneity (Assumptions A1–A4 and A5a).

In addition to the above mentioned censoring at $T = 24$ weeks, we also include the benchmark cases where we censor at $T = 6$ and $T = 13$. In the GIG mixture model, all of our variables (state unemployment rate, race, age, and replacement rate) are estimated to have a negative significant impact on the hazard rate of exiting unemployment. Note however, that when comparing the estimates of the β coefficients, the scaling changes depending on the variance of the estimated heterogeneity distribution. Thus, the ratios of the coefficients should be compared, as opposed to their absolute values. This makes the interpretation of the coefficients less transparent. We note however that the coefficient estimates obtained from the flexible model are substantively different than those obtained from the parametric model. As discussed above, the parametric restriction on the heterogeneity distribution can lead to inconsistent estimates if the true mixing distribution does not exactly correspond to the parametric specification.

We would expect to obtain similar results irrespective of the truncation point and thus the coefficients obtained for the models truncated at 6, 13, and 24 weeks to be very similar. While the coefficient ratios are not constant they tend to be relatively similar. The one exception comes from the ratios involving the replacement rate, in particular for the model with censoring at 24 periods. This is similar to the results in Hausman and Woutersen (2012) and might be explained by behavioral changes as individuals approach the date of UI exhaustion.

The estimated distribution of the unobserved individual heterogeneity is presented in Figure 2. The estimated distributions can only be very roughly approximated by the gamma distribution. While in all three cases the mode is negative, as we increase the number of periods used in the estimation the distribution acquires a more pronounced left tail. This indicates that as we observe individuals over a longer period of time the model captures to a larger extent the part of unobserved heterogeneity which prevents workers from finding employment and thus becomes indicative of the propensity for long term unemployment.

The survival function estimates along with 95% confidence bands are presented in Figure 3, featuring the anticipated downward sloping shape. The smoothing parameter α of the Dirichlet Process (DP) Mixture model introduced in Assumption A6 controls the extent to which the DP draws mixture distributions that are more or less "similar" to the baseline parametric distribution G_0. In the limiting case of $\alpha \rightarrow \infty$ the mixture distribution becomes equivalent to G_0, while in the other extreme $\alpha \rightarrow 0$ the mixture distribution limits to a convolution of density kernels centered at each data point without any influence of the DP prior. The posterior distribution estimates of α are plotted in Figure OA.1 in the Online Appendix. The distributions are concentrated around a well-defined mode with a value of less than 1 indicating a strong influence of data relative to the baseline prior distribution thereby providing a high degree of support in favor of our nonparametric approach.

5.3. Competing Risk Model with Flexible Heterogeneity

We now present the results of our newly proposed competing risk model with a flexible form of unobserved heterogeneity using GIG mixing and correlated risks (Assumptions B1-B3, B6, and B7a). Note that in our example risk 1 corresponds to the event that a worker find a job in the same industry, while risk 2 corresponds to the event that she finds a new job in an industry that is different than the industry of her previous employer. We present the estimated coefficients in Table 4. For all three censoring times ($T = 6, 13, 24$), the partial effects of race and age are not statistically significant, with a few isolated exceptions. This could be due to smaller sample size available for the competing risk case as opposed to the single-risk case, with the former consisting of less than 10% of observations of the latter. For all three censoring times, the relative influence of the replacement rate is declining from $T = 13$ to $T = 24$ indicating the impact of benefit exhaustion.

The estimated density[9] of unobserved heterogeneity V_{ik} is shown in Figure 4 for the GIG mixture model for both risk types $k = 1, 2$, each centered at the time average of the risk-specific latent common time effect ζ_{kt} to reflect the overall influence of the unobserved heterogeneity component $\zeta_{kt} + V_{ik}$. The differences between the density of unobserved individual heterogeneity further highlight the importance of distinguishing between the different risk types in the competing risk model environment as compared to the single-risk duration case. In particular the two distributions of are distinct and well-separated

[9] In Table 4 we report the estimated GIG mixture model coefficients but as these enter all mixing kernel moments their interpretation is not immediate. Hence it appears more informative to examine the resulting mixture density estimate.

indicating that conditional on observed covariates there is a significant degree of sorting between workers finding jobs in the same or in a different industry. While the mode of both distributions is negative, workers who find a job in the same industry possess latent attributes that make them more desirable than workers who are not. This may indicate the presence of unobserved industry specific human capital for these workers which make them more attractive to employers in the same industry.

The survival function estimates along with 95% confidence bands are presented in Figure 5 for both types of risks. The differences in the shapes for the first few weeks are striking and also indicative of the differences in exit rates between workers finding a job in the same industry compared who workers that find a job in a different industry. The estimated survival function for workers who find a job in the same industry is convex while that for workers who do not is concave, indicating a much slower overall re-integration into the labor market. This appears to confer a long term advantage with the overall probability of being unemployed being substantially higher for workers with no apparent industry specific human capital and which have no particular advantage in terms of finding a job in the same industry as their pre-unemployment firm.

Figure 6 shows the estimated correlation structure of the latent time variables ζ_{kt} common to all individuals, defined in Assumption B4, for the GIG mixture model in terms of the estimated densities for the variances σ_1^2, σ_2^2, and the correlation coefficient ρ between ζ_{1t} and ζ_{2t} for the two different risk types. Interestingly, most probability mass for the density of ρ is negative for $T = 6$, around zero for $T = 13$ and positive for $T = 24$. This may correspond to a negative correlation of common shocks for jobs in the same and in different industries for the first several weeks of unemployment with a subsequent correlation reversal in later time periods, but may also be influenced by other factors, as estimates of other model elements also change. Nonetheless, such correlation pattern would explain the exit counts shown in Figure 1 for the two risks, with high ratios of jobs in the same industry to different industries for the first few weeks abetting to parity around week six.

The posterior distribution estimates of the smoothing parameter α of the Dirichlet Process Mixture model (Assumption B6) are plotted in Figure OA.2 in the Online Appendix.

The distributions are concentrated around a well-defined mode of value less than five, indicating a strong influence of data relative to the baseline prior distribution.

It is informative to contrast the estimates from our preferred model with those obtained under different modeling assumptions on the unobserved heterogeneity which are detailed in the Online Appendix. Table OA.3 presents the estimated coefficients from a model that ignores the presence of individual heterogeneity, Table OA.4 corresponds to a model which assumes parametric gamma distributed heterogeneity, Table OA.5 estimates a competing risk model with parametric GIG heterogeneity, and Table OA.6 presents the estimation results from a flexible model which estimates the unobserved semi-parametrically using an infinite mixture of GIG distributions but also further imposes the assumption of independence between the different risks. In Table OA.7 we present estimation results from our single risk GIG mixture model but applied to the subsample of observations which records the outcomes for the two competing risks.

Given the large number of parameters to be considered it is helpful to compare these different models in a graphical setting. In order to facilitate the comparison between the model which pools the two risks and the models which do not we can combine the two risks into a common survival function, as discussed in the Online Appendix. Thus, in Figure 7 we compare the estimated survival function of our CR GIG mixture model (Assumptions 1–3, B6 and B7a, labeled as "CR full") with its estimates in three restricted model versions: 1) the parametric GIG case (labeled as "CR param"), 2) the independent risks case where we estimate a single-risk model separately for each risk type of data and then merge their survival functions ex-post (labeled as "CR indep"); 3) the case without individual unobserved heterogeneity under the restriction $V_{ik} = 0$ (labeled as "CR no ihet"); and 4) the single-risk case where we do not distinguish between risk types in the competing risk data (labeled as "SR full").

Two features are particularly significant. First, we notice that if we enforce the assumption of independence of the two risk types, the resulting common survival function is severely downward biased. The magnitude of the bias dominates the other modeling choices which we make on the specification of unobserved heterogeneity. This could be due to the distributional effects of the risk correlations (Figure 7) that are absent in the independent risk model. Second, we plot the confidence bounds for our proposed model which allows for a flexible specification of the unobserved heterogeneity but also for correlated competing risks. We notice that all other more restrictive specifications are downward biased and the differences become statistically significant as the number of time periods increases.

5.4. Counterfactual Policy Evaluation

One of the advantages of our model consists in the explicit estimation of the unobserved heterogeneity components which enables us to evaluate the effectiveness of counterfactual policy experiments taking into account the distributional effects of individual heterogeneity. As discussed above, one of the main policy questions currently faced by economists is the extent to which the generosity of unemployment insurance benefits impacts the workers' incentives to find employment once their lose their job. On the one hand, more generous benefits are expensive to provide given the ongoing debt crisis and may actually prove detrimental in the long run as they may erode workers' incentives to find a job quickly. Thus they would ultimately contribute to increasing long-run unemployment. On the other hand, low levels of unemployment insurance benefits can make unemployment very difficult for many low income families. Poverty can also have a negative effect on their ability to find employment since job search is costly and in the absence of unemployment insurance benefits many workers may find themselves unable to support their families while also searching for an adequate job. As a result workers may end up underemployed or leave the labor market altogether. The relative magnitude of the impact of incentives over poverty is an empirical question and a counterfactual analysis using model estimates can provide some evidence in this debate.

In the context of our model we can consider changing the replacement rate in order to investigate its impact on the probability of exit from unemployment as captured by the survival function. Note that we assume that this policy change does not impact the distribution of heterogeneity. We can perform this policy counterfactual using both the single risk and the competing risk model. For clarity, we combine the two risk types in the CR model into a common survival function as described in the Online Appendix. The counterfactual experiment consists in increasing and decreasing the replacement rate by 10%. We present counterfactual results from our preferred specification which flexibly models the unobserved heterogeneity as an infinite GIG mixture. The estimated and counterfactual survival curves under the two scenarios are presented in Figure 8 (SR) and Figure 9 (CR).

Both Figures show that the survival function moves in the anticipated direction: for a replacement rate decrease the probability of staying unemployed is lower, and for replacement rate increase the probability of continued unemployment is higher. However, the changes are relatively small. For example, for $T = 24$ in the final period the survival function changes by -1.7% and 1.6% for the CR data, respectively. This suggests that while the estimated impact of a change in unemployment benefit generosity has the sign predicted by economic theory, the magnitude of the impact on the probability of unemployment exit is inelastic. Policy makers may thus wish to consider the extent to which cutting unemployment benefits may ultimately influence an unemployed worker's welfare.

In the Online Appendix we further report on the results of two extensions of the counterfactual experiment that we briefly summarize here. First, we split individuals into two different subsamples based on their unobserved heterogeneity component – below and above the median. The results indicate that individuals with higher unobserved heterogeneity react more to replacement rate changes and have better chances exiting unemployment faster than their counterparts with lower unobserved heterogeneity. Second, we specify a time-varying replacement rate counterfactual change under two scenarios: one with sharp initial change and one with sharp late change. On average the survival function changes more under the former scenario, albeit inelastically in absolute terms.

6. Conclusion

We introduced a new flexible model specification for the competing risk model with piecewise linear baseline hazard, time-varying regressors, risk-specific unobserved individual heterogeneity distributed as an infinite mixture of density kernels, and a common correlated latent effect. Unobserved individual heterogeneity is assumed to be distributed according a Bayesian Dirichlet Process mixture model with a data-driven stochastic number of mixture components estimated along with other model parameters. We derive a tractable likelihood for Generalized Inverse Gaussian (GIG) mixing based on scaled GIG Laplace transforms and their higher-order derivatives. We find that mixing under a special case of the GIG, the gamma kernel, leads to degenerate outcomes in nonparametric mixtures motivating the use of the more flexible GIG. We apply our approach to analyzing unemployment duration with exits to jobs in the same industry and to a different industry among unemployment insurance recipients on nationally representative individual-level survey data from the U.S. Department of Labor. We also conduct a counterfactual policy experiment that changes the replacement rate and find that the extent to which cuts in unemployment benefits incentivize unemployed workers is relatively very small.

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7. Appendix: Tables and Figures

Table 1. Overview of Assumptions

Heterogeneity type	*Single Risk*	*Competing Risks*
No heterogeneity	A1 A3	B1 B3
Parametric GIG	A1–A4, A5a	B1–B4, B5a
Parametric gamma	A1–A4, A5b	B1–B4, B5b
Flexible GIG mixture	A1–A3, A6, A7a	B1–B3, B6, B7a
Flexible gamma mixture	A1–A3, A6, A7b	B1–B3, B6, B7b

Table 2. Summary Statistics

	Duration Data				Competing Risk Data			
Variable	Mean	S.D.	Min	Max	Mean	S.D.	Min	Max
Duration	23.245	16.334	1	63	27.958	28.410	1	140
Race	0.117	0.321	0	1	0.107	0.310	0	1
Age	0.177	0.382	0	1	0.181	0.385	0	1
Replacement Rate	0.6558	0.3082	0.016	1.8434	0.660	0.324	0.015	2.173
State unemp rate:								
period 1	4.686	1.087	2	6.9	4.579	1.125	2	7.5
period 2	4.672	1.083	2	6.9	4.568	1.120	2	7.5
period 3	4.660	1.079	2	6.9	4.562	1.107	2	7.5
period 4	4.645	1.074	2	6.9	4.553	1.103	2	7.8
period 5	4.630	1.069	2	6.9	4.536	1.092	2	8.1
period 6	4.621	1.064	2	6.9	4.533	1.083	2	8.1
period 7	4.616	1.066	2	6.9	4.538	1.080	2	8.1
period 8	4.598	1.064	2	6.9	4.527	1.071	2	7.4
period 9	4.570	1.061	2	6.9	4.518	1.070	2	7.2
period 10	4.538	1.061	2	6.9	4.486	1.070	2	7.2
period 11	4.531	1.063	2	6.9	4.481	1.072	2	7.2
period 12	4.509	1.067	2	6.9	4.458	1.082	2	6.9
period 13	4.483	1.075	2	6.9	4.438	1.091	2	6.9
period 14	4.462	1.080	2	6.9	4.412	1.098	2	6.9
period 15	4.460	1.075	2	6.9	4.404	1.102	2	6.9
period 16	4.449	1.073	2	6.9	4.390	1.097	2	6.9
period 17	4.439	1.067	2	6.9	4.376	1.088	2	7.8
period 18	4.440	1.055	2	6.9	4.368	1.078	2	7.8
period 19	4.431	1.054	2	6.9	4.357	1.073	2	7.8
period 20	4.420	1.045	2	6.9	4.342	1.066	2	7.8
period 21	4.423	1.033	2	6.9	4.338	1.052	2	7.5
period 22	4.431	1.029	2	6.8	4.335	1.038	2	7.4
period 23	4.436	1.024	2	6.7	4.345	1.037	2	7.4
period 24	4.441	1.015	2	6.7	4.353	1.037	2	7.4
Observations		15,358				1,243		

Table 3. New Semiparametric Duration Model, GIG Mixture

		6 periods		13 periods		24 periods	
		Mean	s.e.	Mean	s.e.	Mean	s.e.
□		-0.963	0.039	-1.226	0.034	-1.659	0.101
□		2.284	0.110	3.022	0.097	4.240	0.285
Urate		-0.184	0.021	-0.214	0.014	-0.327	0.034
Race		-0.055	0.069	-0.126	0.042	-0.145	0.050
Age		-0.194	0.057	-0.178	0.035	-0.187	0.039
Rrate		-0.924	0.076	-0.473	0.051	-0.147	0.066
t	1	-2.142	0.116	-2.264	0.090	-1.971	0.181
	2	-1.716	0.109	-1.843	0.088	-1.573	0.175
	3	-2.026	0.112	-2.157	0.096	-1.878	0.182
	4	-1.733	0.112	-1.865	0.085	-1.578	0.182
	5	-2.070	0.112	-2.198	0.099	-1.928	0.185
	6	-1.833	0.102	-1.829	0.084	-1.547	0.184
	7			-2.336	0.100	-2.061	0.205
	8			-2.026	0.087	-1.743	0.189
	9			-2.347	0.097	-2.099	0.181
	10			-2.130	0.087	-1.871	0.189
	11			-2.347	0.087	-2.086	0.194
	12			-2.120	0.095	-1.856	0.196
	13			-2.277	0.074	-1.904	0.195
	14					-1.840	0.191
	15					-1.775	0.191
	16					-1.740	0.195
	17					-1.981	0.206
	18					-1.641	0.191
	19					-1.848	0.194
	20					-1.674	0.191
	21					-1.832	0.198
	22					-1.738	0.199
	23					-1.913	0.211
	24					-1.006	0.179

N $= 15,491$, *Urate* denotes the state unemployment rate, *Rrate* denotes the replacement rate.

Table 4. New Semiparametric Competing Risk Model. GIG Mixture

		6 periods				13 periods				24 periods			
		Risk 1		Risk 2		Risk 1		Risk 2		Risk 1		Risk 2	
		Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.
‖		-1.535	0.046	-1.516	0.046	-1.482	0.057	-1.554	0.062	-1.407	0.044	-1.492	0.047
□		3.890	0.129	3.838	0.304	3.741	0.162	3.944	0.176	3.532	0.126	3.771	0.134
Urate		-0.111	0.055	-0.103	0.083	-0.107	0.048	-0.117	0.063	-0.216	0.040	-0.095	0.056
Race		0.054	0.225	0.033	0.424	-0.044	0.183	-0.377	0.343	-0.046	0.157	0.312	0.256
Age		-0.008	0.176	-0.562	0.357	-0.016	0.154	-0.700	0.290	-0.036	0.130	-0.461	0.212
Rrate		-1.062	0.227	-0.465	0.390	-0.903	0.181	-0.411	0.288	-0.385	0.156	-0.331	0.226
t	1	-2.553	0.314	-4.864	0.516	-2.680	0.276	-4.741	0.503	-2.496	0.256	-4.947	0.511
	2	-1.993	0.296	-4.625	0.491	-2.113	0.255	-4.514	0.504	-1.933	0.234	-4.715	0.475
	3	-2.324	0.311	-4.937	0.565	-2.445	0.272	-4.830	0.558	-2.263	0.253	-5.027	0.554
	4	-2.022	0.302	-3.633	0.400	-2.138	0.263	-3.506	0.388	-1.962	0.243	-3.728	0.372
	5	-2.470	0.325	-3.638	0.413	-2.580	0..28	-3.516	0.391	-2.406	0.273	-3.729	0.376
	6	-3.854	0.290	-3.711	0.212	-3.044	0.322	-3.327	0.386	-2.872	0.308	-3.543	0.367
	7					-2.947	0.321	-4.361	0.518	-2.764	0.304	-4.599	0.508
	8					-2.493	0.295	-3.068	0.372	-2.318	0.277	-3.285	0.355
	9					-3.750	0.422	-3.984	0.479	-3.571	0.417	-4.203	0.460
	10					-2.620	0.305	-3.950	0.471	-2.449	0.293	-4.149	0.451
	11					-3.232	0.362	-3.925	0.480	-3.078	0.355	-4.129	0.449
	12					-2.791	0.326	-3.866	0.445	-2.625	0.305	-4.093	0.454
	13					-3.799	0.341	-3.829	0.409	-2.703	0.317	-4.057	0.453
	14									-2.738	0.329	-3.897	0.437
	15									-2.511	0.309	-3.634	0.406
	16									-3.074	0.376	-3.927	0.452
	17									-2.127	0.284	-3.872	0.453
	18									-2.564	0.335	-4.589	0.569
	19									-2.513	0.322	-3.285	0.388
	20									-2.702	0.347	-3.884	0.457
	21									-1.850	0.278	-3.569	0.424
	22									-3.795	0.540	-4.719	0.652
	23									-2.133	0.307	-3.780	0.464
	24									-3.640	0.322	-3.770	0.106

N = 1, 243. *Urate* denotes the state unemployment rate. *Rrate* denotes the replacement rate.

Figure 1. Empirical Exit Count for Competing Risk Data

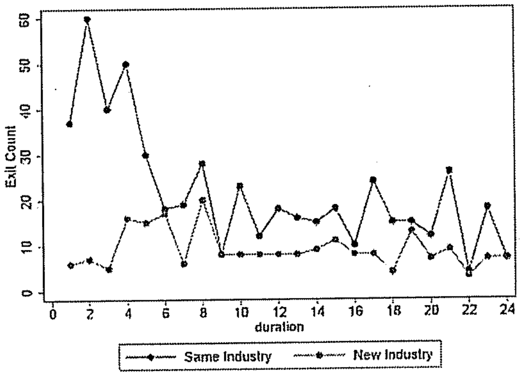


Figure 2. Density of individual heterogeneity component v_i, GIG mixture

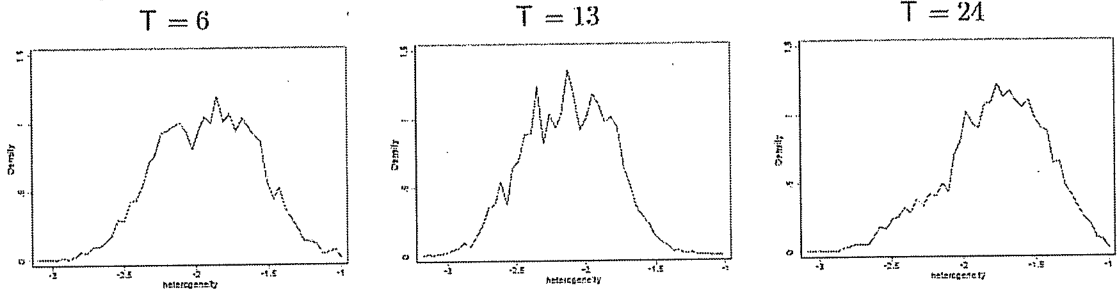


Figure 3. Survival function, GIG mixture

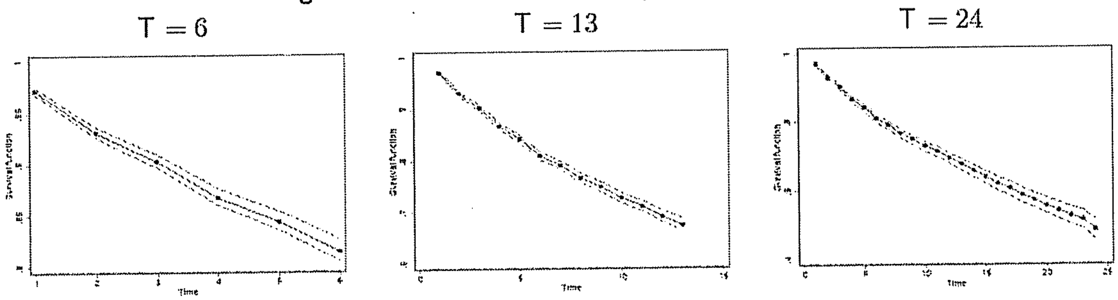


Figure 4. Heterogeneity density, GIG mixture

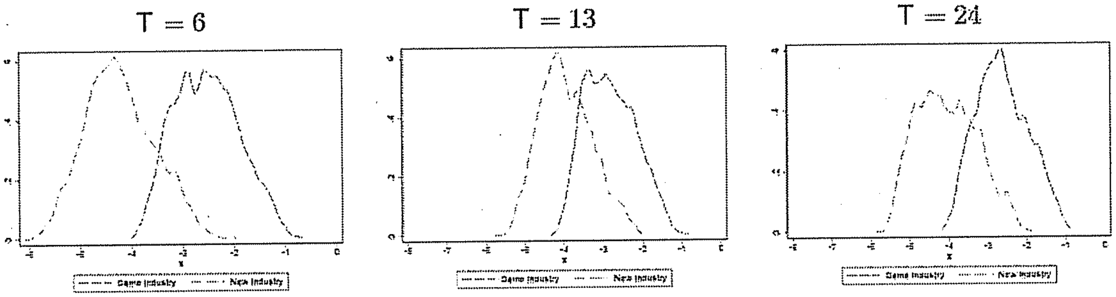


Figure 5. Survival function, GIG mixture

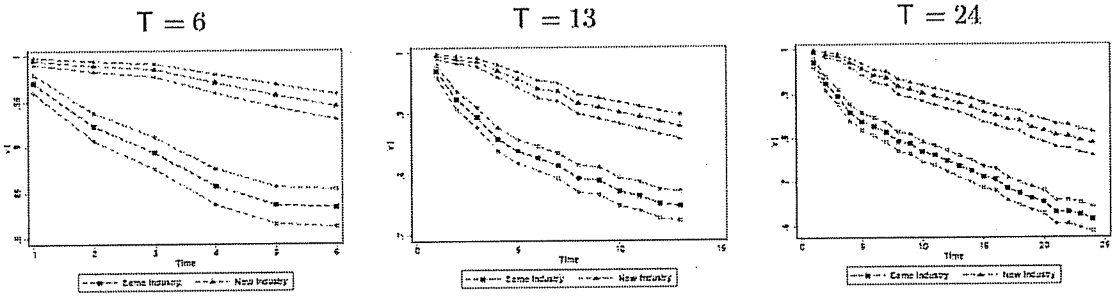


Figure 6. Correlation structure of ζ_t: density of σ_1^2, σ_1^2, and ρ, GIG mixture

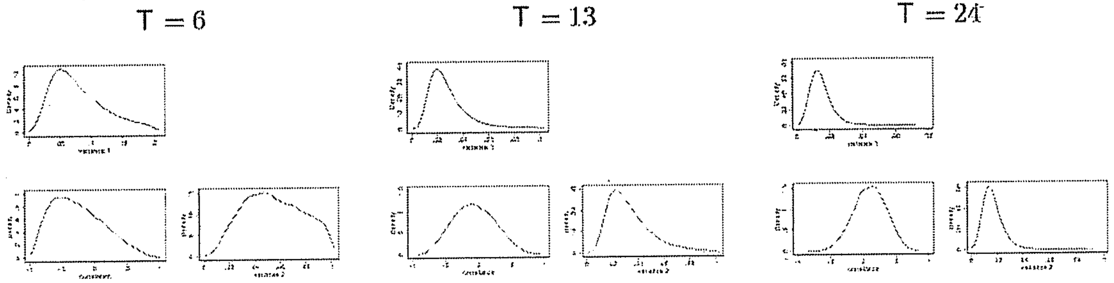


Figure 7. Model Comparison in Terms of Survival Functions

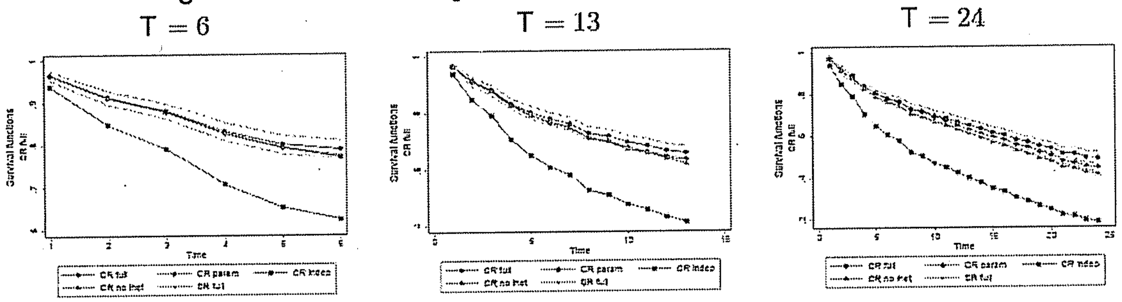


Figure 8. Counterfactual Experiment for the Single Risk GIG Mixture

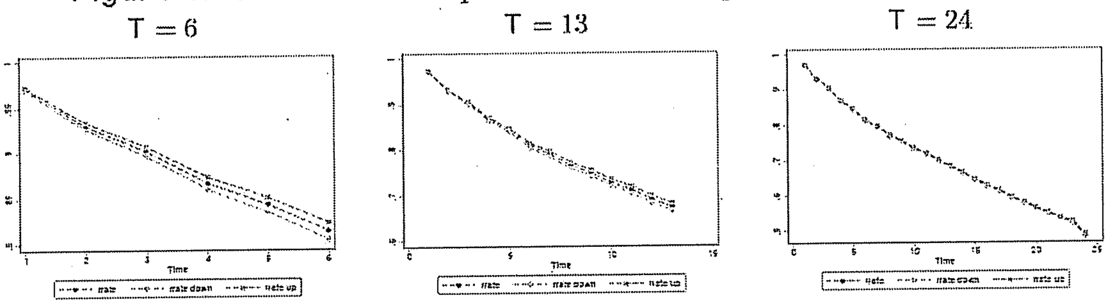
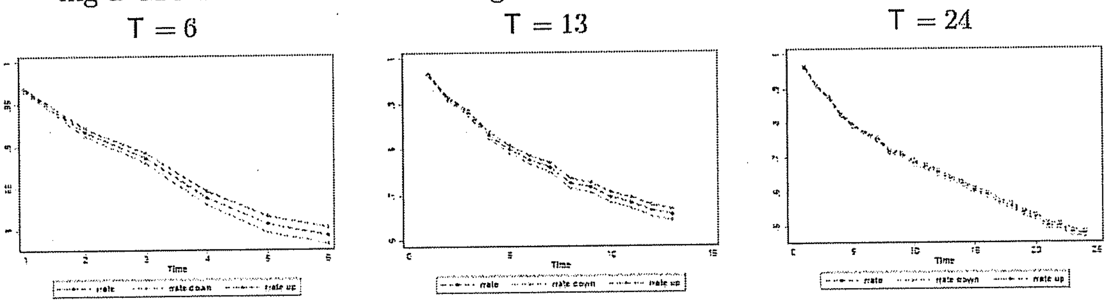


Figure 9. Counterfactual Experiment for the Competing Risks Model using a GIG Mixture and Combining the Risks.



Online Appendix

"A Bayesian Semiparametric Competing Risk Model with Unobserved Heterogeneity"
by Martin Burda, Matthew Harding, and Jerry Hausman
September 29, 2013

1. Details on MCMC Posterior Sampling

For our model implementation we utilize the Gibbs sampling scheme which belongs to the class of Markov Chain Monte Carlo (MCMC) simulation methods. An attractive feature of MCMC techniques is that samples of random draws can be generated from the joint posterior densities of parameters of interest indirectly, without the need to specify the exact analytical form of the joint densities. The Gibbs sampler uses an iterative procedure to create Markov chains by simulating from conditional densities instead which are analytically tractable. The sets of draws obtained in this way can be effectively considered as samples from the joint posterior densities.

1.1. Gibbs Blocks

For the single risk case, let $\square_{0t} = \log(\square_{0t})$, $\square_0 = (\square_{01}, \ldots, \square_{0T})$, and $V = \{V_i\}_{i=1}^{N}$. The model parameters consist of $\square$, $\square_0$, V, hyperparameters either of the GIG mixture $\square$ and $\square$ or the gamma mixture $\square$ (denote the hyperparameters generically by $\square$), and the DP concentration parameter $\square$. Under Assumptions A1–A7, the joint posterior density can be decomposed into the following Gibbs blocks:

(1) $\square, \square_0\, V, \square, \square$
(2) $V\ \square, \square, \square, \square_0$
(3) $\square\ \square, \square, \square_0, V$
(4) $\square\ \square, \square_0, V, \square$

In the competing risk case, all the above parameters are risk-specific. Moreover, due to Assumption B4, there are additional parameters $\square_{kt}$, $\square$, $\square_1$, and $\square_2$. Hence, let $\square_{0kt} = \log(\square_{0kt}) + \square_{kt}$ where $\square_{0kt} = \square_{0kt}$ since time intervals have unit length. Let further $\square = \{\square_k\}_{k=1}^{K}$, $\square_{0k} = (\square_{0k1}, \ldots, \square_{0kT})$, $\square_0 = (\square_{01}, \square_{02})$, $\square = \{\square_k\}_{k=1}^{K}$, $V_i = \{V_{ki}\}_{k=1}^{K}$, $V = \{V_i\}_{i=1}^{N}$, $\square = \{\square_k\}_{k=1}^{K}$, and $\square = \{\square_k\}_{k=1}^{K}$. In our application, $K = 2$. The Gibbs blocks are now as follows:

(1) $\square, \square_0\ \square, \square\, V, \square, \square$
(2) $\square, \square V, \square, \square, \square, \square_0$
(3) $V\ \square, \square, \square, \square_0, \square, \square$
(4) $\square\ \square, \square, \square_0, \square, \square\, V$
(5) $\square\ \square, \square_0, \square, \square\, V, \square$

This first Gibbs block is sampled using standard Hamiltonian Monte Carlo (HMC) with SR posterior (2.8) and (2.9), and CR posterior (3.3) and (3.4). For a detailed description of the HMC procedure, see e.g. Neal (2011), pp. 122–125. In the CR case, the covariance matrix of the second block is endowed with a proper Inverse Wishart prior with maximum dispersion, $IW(K+1, I_K)$ where I_K is the identity matrix of dimension K. Given the draws $\square_0$ and the Normal correlation structure in Assumption B4, the posterior can be found e.g. in Train (2003) on p. 301 and the sampling procedure on p. 302. Sampling individual heterogeneity V, hyperparameters □, and □ is detailed in the next section.

1.2. Individual Heterogeneity

The distribution of the unobserved heterogeneity component v_i is modeled as a mixture with countably infinite number of mixture components. In the Bayesian framework employing a prior distribution for mixing proportions, such as the Dirichlet Process that we adopt here, leads to a relatively few of the mixture components dominating in the posterior. Using a countably infinite mixture bypasses the need to determine the "correct" number of components in a finite mixture model.

DP mixture modeling is described in detail e.g. in Hjort et al (2010). In our implementation, we use Algorithm 2 of Neal (2000). Here we provide the essence of the procedure. The prior structure of the model for v_i is specified by our Assumptions A6 (SR) and B6 (CR). It is based on two levels of hierarchy, where the first one is formed by a random measure G that stochastically deviates from the baseline measure G_0 and the second level is given by the Dirichlet Process $DP(G_0, \square)$. The baseline measure G_0 is specified in our Assumptions A7a,b (SR) and B7a,b (CR).

The level formed by G can be integrated out to obtain a representation of the prior in terms of successive conditional distributions of a mixture form (Blackwell and MacQueen 1973):

$$(\text{OA.1.1}) \qquad v_i\ v_{-i} \sim \frac{1}{i-1+\square}\sum_{j=1}^{i-1}\square(v_j) + \frac{\square}{i-1+\square}G_0$$

where v_{-i} denotes the collection of v_j, $j \neq i$, and $\square(v_j)$ is the Dirac measure concentrated on the single point v_j. When combined with the likelihood, this yields the following conditional distribution for use in Gibbs sampling (Neal 2000):

$$(\text{OA.1.2}) \qquad \begin{aligned} v_i\ v_{-i}, t_i &\sim \sum_{j=1, j\neq i}^{N} q_{ij}\square(v_j) + q_0 H_i \\ q_{ij} &= bF(t_i, v_j) \\ q_0 &= b\square \int F(t_i, v)dG_0(v) \end{aligned}$$

where H_i is the posterior for v_i based on the prior $G_0(v)$ and the single observation t_i with likelihood denoted by $F(t_i, v_i)$, b is a normalizing constant such that $\sum_{j\neq i} q_{ij} + q_0 = 1$, and N is the sample size. In the SR case, implementation of the GIG mixture model (Assumptions A1–A3, A6, and A7a) uses (2.8) and (2.9) for $F(t_i, v)$, while $\int F(t_i, v)dG_0(v)$ is given by (2.18) and (2.19). The gamma mixture model (Assumptions A1 A3, A6, and A7b) uses (2.8), (2.9), (2.22) and (2.23), respectively. In the CR case, the GIG mixture model (Assumptions B1–B3, B6, and B7a) uses (3.3) and (3.4) for $F(t_i, v)$ from Theorem 1, and $\int F(t_i, v)dG_0(v)$, as derived in Corollary 1 to Theorem 2. The gamma mixture model (Assumptions B1–B3, B6, and B7b) uses (OA.2.47) and (OA.2.49) for the latter integral, from Corollary 2 to Theorem 2. The hyperparameters □ are then updated in a separate Gibbs block as given by Algorithm 2 (Neal

2000, p. 254). Gibbs updates of the concentration parameter □ are detailed in Escobar and West (1995).

1.3. Compilation and Runtime

All reported posterior means were obtained from Markov Chain Monte Carlo (MCMC) chains of total length of 30,000 steps with a 10,000 burn-in section. All models were implemented using the Intel Fortran 95 compiler on a 2.8GHz Unix machine under serial compilation. For a sample of 15,358 individuals, the single risk model implementation took approximately 3 hours for $\mathsf{T} = 6$, 4 hours for $\mathsf{T} = 13$, and 6 hours for $\mathsf{T} = 24$ to run. In contrast, with a sample of 1,317 individuals, the competing risk model implementation took approximately 2 hours for $\mathsf{T} = 6$, 6 hours for $\mathsf{T} = 13$, and 13 hours for $\mathsf{T} = 24$.

1.4. Gamma versus GIG

In the gamma mixture model we found that the probability mass of the individual heterogeneity component was accumulating at zero, with a thin right tail diverging to infinity, leading to a degenerate outcome. We believe this to be an artefact of the gamma density kernel shape with mode at zero for mean less than or equal to one. In contrast, for the GIG density under the Assumptions A6 and A7a (SR) or Assumptions B6 and B7a (CR), we obtained a well-defined stable nonparametric heterogeneity clustering without the degenerative tendencies of the gamma. We attribute this outcome to the more flexible functional form of the GIG with a well-defined mode at a strictly positive value for v for mean values smaller than one.

2. Details on Proofs and Derivations

2.1. CR Stochastic Environment

Consider the CR model setup with interval outcome data and latent exit times, as described in the main text. In this section we will initially omit the subscripts i and t and also covariates and heterogeneity variables to focus on the general model, without loss of generality. We will then include these elements into the model as needed. Denote the latent exit time variables by $\square = (\square, \ldots, \square_K)$ while the time integration variables by $\mathsf{u} = (\mathsf{u}_1, \ldots, \mathsf{u}_K)$, assumed conditionally independent.

The cause-specific hazard function for the k-th cause, which is the hazard from failing from a given cause in the presence of the competing risks, is defined as

$$\square_k(\mathsf{u}_k) = \lim_{h \to 0} \frac{\Pr(\mathsf{u}_k < \square_k \leq \mathsf{u}_k + \mathsf{h}; \mathsf{k}\ \square_k > \mathsf{u}_k)}{\mathsf{h}}$$

The joint hazard from all causes is

$$\begin{aligned} \square(\mathsf{u}) &= \lim_{h \to 0} \frac{\Pr(\mathsf{u} < \square \leq \mathsf{u} + \mathsf{h}\ \square > \mathsf{u})}{\mathsf{h}} \\ &= \sum_{k=1}^{K} \square_k(\mathsf{u}_k) \end{aligned}$$

where all inequalities are defined element-wise. The cause-specific integrated hazard is

(OA.2.1) $$\Lambda_k(\square_k) = \int_0^{T_k} \square_k(u_k)du_k$$

and the joint integrated hazard is

(OA.2.2) $$\Lambda(\square) = \int_0^T \square(u)du = \int_0^T \sum_{k=1}^K \square_k(u_k)du = \sum_{k=1}^K \int_0^T \square_k(u_k)du_k = \sum_{k=1}^K \Lambda_k(\square_k)$$

The joint survival function is

$$\begin{aligned} S(u) &= \Pr(\square > u) \\ &= \exp(-\Lambda(u)) \end{aligned} \quad \text{(OA.2.3)}$$

which is the complement of the probability of failure from any cause up to time $\square$ given by the overall cumulative distribution function

$$\begin{aligned} F(u) &= \Pr(\square \leq u) \\ &= 1 - S(u) \end{aligned}$$

For ease of exposition, we will focus on the case of two risk types with $K = 2$. The joint density of failure at time u is thus given by

$$\begin{aligned} f(u_1, u_2) &= \frac{\square^2 F(u_1, u_2)}{\square u_1 \square u_2} \\ &= -\frac{\square^2 S(u_1, u_2)}{\square u_1 \square u_2} \\ &= -\frac{\square^2 \exp(-\Lambda_1(u_1) - \Lambda_2(u_2))}{\square u_1 \square u_2} \\ &= \exp(-\Lambda_1(u_1) - \Lambda_2(u_2)) \square_1(u_1)\square_2(u_2) \end{aligned} \quad \text{(OA.2.4)}$$

Equation (OA.2.4) links $f(u_1, u_2)$ with the risk-specific hazard functions. Parametrization of the latter in terms of covariates and unobserved heterogeneity $(V_i, \square)$ is given by Assumption B2. We will now invoke this Assumption and reintroduce $(V_i, \square)$, while suppressing notational conditioning on the covariates X without loss of generality.

Note that conditional on X the failure times u_1 and u_2 are dependent since $\square_{1i}$ and $\square_{2i}$ are correlated. However, conditional on $X, V, \square$ the failure times u_1 and u_2 are independent. Hence $f(u_1, u_2 | V, \square)$ can be factorized into the product

$$f(u_1, u_2 | V, \square) = f(u_1 V, \square) f(u_2 V, \square)$$

From (OA.2.4) it follows that

(OA.2.5) $$f(u_k V, \square) = \exp(-\Lambda_k(u_k)) \square_k(u_k)$$

Define the function

(OA.2.6) $$S_k(u_k) \equiv \exp(-\Lambda_k(u_k))$$

for $k \in \{1, 2\}$. From (OA.2.5) and (OA.2.6) we have,

(OA.2.7) $$f(u_k V, \square) = S_k(u_k)\square_k(u_k)$$

From (OA.2.1), (OA.2.6), and (OA.2.7) it follows that

$$\int_{\underline{\mathcal{T}}_t}^{\overline{\mathcal{T}}_t} \mathsf{f}\,(\mathsf{u}_k\,\mathsf{V},\square)d\mathsf{u}_k = \mathsf{S}_{k(t-1)} - \mathsf{S}_{kt} \tag{OA.2.8}$$

The density (OA.2.7) should not be confused with the so-called subdensity function $\mathsf{f}_j(\mathsf{u}_j) = \mathsf{S}(\mathsf{u} = \mathsf{u}_j)\square_j(\mathsf{u}_j)$ that is sometimes used in CR analysis. Moreover, the function $\mathsf{S}_k(\mathsf{u}_k)$ defined in (OA.2.6) does not, in general, have the survival function interpretation for $\mathsf{K} > 1$. Nonetheless, examining (OA.2.2), (OA.2.3), and (OA.2.6) reveals that the product of $\mathsf{S}_k(\mathsf{u}_k)$ over k equals the joint survival function:

$$\mathsf{S}(\mathsf{u}) = \prod_{k=1}^{K} \mathsf{S}_k(\mathsf{u}_k) \tag{OA.2.9}$$

(for further details of interpretation of functions with survival-like properties see e.g. Porta, Gómez, and Calle 2008). In general, the unconditional product form of (OA.2.9) characterizes independent risks. However, dependence among risks can be introduced by conditioning each $\mathsf{S}_k(\mathsf{u}_k)$ on variables correlated across the risk types.

2.2. Competing Risk Model: Conditional Likelihood

From (3.2),

$$\mathsf{P}\,(\mathsf{t}_{1i} = \mathsf{t},\ \square_{2i} > \square_{1i}\,\mathsf{V}_i, \square) = \mathsf{A} + \mathsf{B} \tag{OA.2.10}$$

where

$$\begin{aligned}
\mathsf{A} &= \int_{\underline{\mathcal{T}}_i}^{\overline{\mathcal{T}}_t} \int_{u_1}^{\overline{\mathcal{T}}_t} \mathsf{f}\,(\mathsf{u}_1, \mathsf{u}_2 | \mathsf{V}_i, \square)d\mathsf{u}_2 d\mathsf{u}_1 \\
\mathsf{B} &= \int_{\underline{\mathcal{T}}_i}^{\overline{\mathcal{T}}_t} \int_{\overline{\mathcal{T}}_t}^{\infty} \mathsf{f}\,(\mathsf{u}_1, \mathsf{u}_2\,\mathsf{V}_i, \square)d\mathsf{u}_2 d\mathsf{u}_1
\end{aligned}$$

The expression A is more difficult to evaluate than B since in A the lower bound u_1 of the inner integral is an argument of the outer integral. In contrast, the two integrals in B are independent of each other and hence can be factorized.

Thus,

$$\begin{aligned}
\mathsf{A} &= \int_{\underline{\mathcal{T}}_i}^{\overline{\mathcal{T}}_i} \int_{u_1}^{\overline{\mathcal{T}}_i} \mathsf{f}_{it}(\mathsf{u}_1\,\mathsf{V}_{1i}, \square_1)\mathsf{f}_{it}(\mathsf{u}_2\,\mathsf{V}_{2i}, \square_2)d\mathsf{u}_2 d\mathsf{u}_1 \\
&= \int_{\underline{\mathcal{T}}_i}^{\overline{\mathcal{T}}_i} \left[\int_{u_1}^{\overline{\mathcal{T}}_i} \mathsf{f}_{it}(\mathsf{u}_2\,\mathsf{V}_{2i}, \square_2)d\mathsf{u}_2\right] \mathsf{f}_{it}(\mathsf{u}_1\,\mathsf{V}_{1i}, \square_1)d\mathsf{u}_1 \qquad \text{(OA.2.11)}
\end{aligned}$$

where $\mathsf{u}_k \in [\square_i, \square_i)$ for $\mathsf{k} \in \{1, 2\}$. For the inner integral in (OA.2.11), using (OA.2.8)

$$\int_{u_1}^{\overline{\mathcal{T}}_i} \mathsf{f}_{it}(\mathsf{u}_2\,\mathsf{V}_{2i}, \square_2)d\mathsf{u}_2 = \mathsf{S}_{2i}\,(\mathsf{u}_1) - \mathsf{S}_{2it} \tag{OA.2.12}$$

Let $\mathsf{s}_k = \mathsf{u}_k - \square_i$ so that $\mathsf{s}_k \in [0, 1)$. Then, from (OA.2.5), using piecewise constancy of the hazard function $\square_{ki}\,(\cdot)$ and hence piecewise linearity of the integrated hazard function $\Lambda_{ki}\,(\cdot)$

over time,

$$\begin{aligned} f_{it}(u_k\, V_{ki}, \square_k) &= \exp\left(-\Lambda_{ki}\left(u_k\right)\right) \square_{ki}(u_k) \\ &= \exp\left(-\Lambda_{ki(t-1)} - s_k \square_{kit}\right) \square_{kit} \\ &= \exp\left(-\Lambda_{ki(t-1)}\right) \exp\left(-s_k \square_{kit}\right) \square_{kit} \end{aligned} \tag{OA.2.13}$$

Similarly,

$$S_{ki}\left(u_j\right) = S_{ki(t-1)} \exp\left(-\left(s_j \square_{kit}\right)\right) \tag{OA.2.14}$$

for $k, j \in \{1, 2\}$. Using (OA.2.12), and integration by substitution with (OA.2.13) for $k = 1$ and with (OA.2.14) for $k = 2$, $j = 1$, in (OA.2.11) yields

$$\begin{aligned} A &= \int_{\mathcal{T}_t}^{\overline{\mathcal{T}}_i} \left[S_{2i}\left(u_1\right) - S_{2it}\right] f_{it}(u_1\, V_{1i}, \square) du_1 \\ &= S_{2i(t-1)} \int_0^1 \exp\left(-\left(s_1 \square_{2it}\right)\right) \exp\left(-\Lambda_{1i(t-1)}\right) \exp\left(-s_1 \square_{1it}\right) \square_{1it} ds_1 \\ &\quad -S_{2it} \int_0^1 \exp\left(-\Lambda_{1i(t-1)}\right) \exp\left(-s_1 \square_{1it}\right) \square_{1it} ds_1 \\ &= A_{11} + A_{12} \end{aligned} \tag{OA.2.15}$$

where

$$\begin{aligned} A_{11} &= S_{2i(t-1)} \int_0^1 \exp\left(-\left(s_1 \square_{2it}\right)\right) \exp\left(-\Lambda_{1i(t-1)}\right) \exp\left(-s_1 \square_{1it}\right) \square_{1it} ds_1 \\ &= S_{2i(t-1)} S_{1i(t-1)} \square_{1it} \left(\square_{2it} + \square_{1it}\right)^{-1} \int_0^1 \exp\left(-\left(s_1\left(\square_{2it} + \square_{1it}\right)\right)\left(\square_{2it} + \square_{1it}\right) ds_1 \\ &= -S_{2i(t-1)} S_{1i(t-1)} \square_{1it} \left(\square_{2it} + \square_{1it}\right)^{-1} \left[\exp\left(-\left(\square_{2it} + \square_{1it}\right)\right) - 1\right] \end{aligned} \tag{OA.2.16}$$

and

$$\begin{aligned} A_{12} &= -S_{2it} \int_0^1 \exp\left(-\Lambda_{1i(t-1)}\right) \exp\left(-s_1 \square_{1it}\right) \square_{1it} ds_1 \\ &= -S_{2it} S_{1i(t-1)} \int_0^1 \exp\left(-s_1 \square_{1it}\right) \square_{1it} ds_1 \\ &= S_{2it} S_{1i(t-1)} \left[\exp\left(-\square_{1it}\right) - 1\right] \end{aligned} \tag{OA.2.17}$$

Using (OA.2.16) and (OA.2.17) in (OA.2.15) yields

$$A = S_{2it} S_{1it} \left\{1 - \exp(\square_{1it}) - \square_{1it} \left(\square_{2it} + \square_{1it}\right)^{-1} \left[1 - \exp\left(\square_{2it1} + \square_{1it}\right)\right]\right\} \tag{OA.2.18}$$

The expression for B of (3.2) is given by

$$\begin{aligned} B &= \left[F_{1it} - F_{1i(t-1)}\right] \left[1 - F_{2it}\right] \\ &= \left[S_{1i(t-1)} - S_{1it}\right] S_{2it} \\ &= S_{1i(t-1)} S_{2it} - S_{1it} S_{2it} \\ &= S_{1it} S_{2it} \left[\exp(\square_{1it}) - 1\right] \end{aligned} \tag{OA.2.19}$$

Combining (OA.2.18) and (OA.2.19) in (OA.2.10) yields

$$\begin{aligned} \mathsf{P}(\mathsf{t}_{1i} &= \mathsf{t},\ \square_{2i} > \square_i\, \mathsf{V}_i, \square) = \mathsf{S}_{2i(t-1)}\mathsf{S}_{1i(t-1)}\square_{1it}\left(\square_{2it} + \square_{1it}\right)^{-1} \\ &\times \left[1 - \exp\left(-\left(\square_{2it} + \square_{1it}\right)\right)\right] \end{aligned}$$

with the resulting log-likelihood

$$\begin{aligned} \ln \mathsf{P}(\mathsf{t}_{1i} = \mathsf{t},\ \square_{2i} > \square_i\, \mathsf{V}_i, \square) &= -\Lambda_{2i(t-1)} - \Lambda_{1i(t-1)} + \log(\square_{1it}) - \log\left(\square_{2it} + \square_{1it}\right) \\ &\quad + \log\left(1 - \exp\left(-\left(\square_{2it} + \square_{1it}\right)\right)\right) \end{aligned}$$

2.3. Competing Risk Model: Integrated Likelihood

2.3.1. Quadrature Version

Here we derive an expression for the expectation of the exit probability (3.2) with respect to unobserved heterogeneity for each risk type, based on a simple quadrature. Taking the expectation of (3.2) yields

$$\begin{aligned} \mathsf{E}_v\mathsf{P}(\mathsf{t}_{1i} = \mathsf{t},\ \square_{2i} > \square_i) &= \mathsf{E}_v \int_{\underline{T}_i}^{\overline{T}_i} \int_{u_1}^{\overline{T}_i} \mathsf{f}\left(\mathsf{u}_1, \mathsf{u}_2\ \mathsf{V}_i, \square\right)\mathsf{du}_2\mathsf{du}_1 \\ &\quad + \mathsf{E}_v \int_{\underline{T}_i}^{\overline{T}_i} \int_{\overline{T}_i}^{\infty} \mathsf{f}\left(\mathsf{u}_1, \mathsf{u}_2\ \mathsf{V}_i, \square\right)\mathsf{du}_2\mathsf{du}_1 \\ &= \mathsf{E}_v \int_{\underline{T}_i}^{\overline{T}_i} \int_{u_1}^{\infty} \mathsf{f}\left(\mathsf{u}_1, \mathsf{u}_2\ \mathsf{V}_i, \square\right)\mathsf{du}_2\mathsf{du}_1 \\ &= \mathsf{E}_v \int_{\underline{T}_i}^{\overline{T}_i} \int_{u_1}^{\infty} \mathsf{f}_{it}(\mathsf{u}_1\ \mathsf{V}_{1i}, \square_1)\mathsf{f}_{it}(\mathsf{u}_2\ \mathsf{V}_{2i}, \square_2)\mathsf{du}_2\mathsf{du}_1 \\ &= \mathsf{E}_v \int_{\underline{T}_i}^{\overline{T}_i} \int_{u_1}^{\infty} \mathsf{f}_{it}(\mathsf{u}_2\ \mathsf{V}_{2i}, \square_2)\mathsf{du}_2\mathsf{f}_{it}(\mathsf{u}_1\ \mathsf{V}_{1i}, \square_1)\mathsf{du}_1 \end{aligned}$$

(OA.2.20) $$= \int_{\underline{T}_i}^{\overline{T}_i} \mathsf{E}_{v_{2i}}\left[\mathsf{S}_{2i}\left(\mathsf{u}_1\right)\right] \mathsf{E}_{v_{1i}}\left[\mathsf{f}_{it}(\mathsf{u}_1\ \mathsf{V}_{1i}, \square_1)\right] \mathsf{du}_1$$

From (OA.2.6),

(OA.2.21) $$\mathsf{E}_{v_{2i}}\left[\mathsf{S}_{2i}\left(\mathsf{u}_1\right)\right] = \mathcal{L}_2\left(\widetilde{\Lambda}_{2i}(\mathsf{u}_1)\right)$$

Using (OA.2.5),

$$\begin{aligned} \mathsf{E}_{v_{1i}}\left[\mathsf{f}_{it}(\mathsf{u}_1\ \mathsf{V}_{1i}, \square_1)\right] &= \mathsf{E}_{v_{1i}}\left[\exp\left(-\Lambda_{1i}\left(\mathsf{u}_1\right)\right) \square_{1i}(\mathsf{u}_1)\right] \\ &= \widetilde{\square}_{1i}(\mathsf{u}_1)\mathsf{E}_{v_{1i}}\left[\exp\left(-\mathsf{v}_{1i}\widetilde{\Lambda}_{1i}(\mathsf{u}_1)\right)\mathsf{v}_{1i}\right] \end{aligned}$$

(OA.2.22) $$= -\widetilde{\square}_{1i}(\mathsf{u}_1)\mathcal{L}_1^{(1)}\left(\widetilde{\Lambda}_{1i}(\mathsf{u}_1)\right)$$

where $\mathcal{L}^{(1)}\left(\mathsf{s}\right)$ is the first derivative of the Laplace transform $\mathcal{L}\left(\mathsf{s}\right)$ evaluated at s. Using (OA.2.21) and (OA.2.22) in (OA.2.20) yields

(OA.2.23) $$\mathsf{E}_v\mathsf{P}(\mathsf{t}_{1i} = \mathsf{t},\ \square_{2i} > \square_i) = -\int_{\underline{T}_i}^{\overline{T}_i} \widetilde{\square}_{1i}(\mathsf{u}_1)\mathcal{L}_2\left(\widetilde{\Lambda}_{2i}(\mathsf{u}_1)\right)\mathcal{L}_1^{(1)}\left(\widetilde{\Lambda}_{1i}(\mathsf{u}_1)\right)\mathsf{du}_1$$

Letting again $s_k = u_k - (t-1)$, $s_k \in [0,1)$, $k \in \{1,2\}$, and using piecewise constancy of $\sqcup_{ki}(\cdot)$ and piecewise linearity of $\Lambda_{ki}(\cdot)$, following a change of variables (OA.2.23) becomes (OA.2.24)

$$E_v P(t_{1i} = t,\ \square_{2i} > \square_{i}) = -\widetilde{\square}_{1it} \int_0^1 \mathcal{L}_2\left(\widetilde{\Lambda}_{2i(t-1)} + \widetilde{\square}_{2it} s_1\right) \mathcal{L}_1^{(1)}\left(\widetilde{\Lambda}_{1i(t-1)} + \widetilde{\square}_{1it} s_1\right) ds_1$$

2.3.2. Series Expansion

The series expansion expression for the expectation of (3.2) can be derived as follows. Using (OA.2.10) and taking expectations,

$$\begin{aligned} E_v P(t_{1i} = t,\ \square_{2i} > \square_{1i}) &= E_v \int_{\underline{\mathcal{T}}_i}^{\overline{\mathcal{T}}_i} \int_{u_1}^{\overline{\mathcal{T}}_i} f(u_1, u_2\ V_i, \square) du_2 du_1 \\ &\quad + E_v \int_{\underline{\mathcal{T}}_i}^{\overline{\mathcal{T}}_i} \int_{\overline{\mathcal{T}}_i}^{\infty} f(u_1, u_2\ V_i, \square) du_2 du_1 \\ \text{(OA.2.25)} \qquad &= E_v A + E_v B \end{aligned}$$

From (OA.2.11),

$$\text{(OA.2.26)} \qquad E_v A = \int_{\underline{\mathcal{T}}_i}^{\overline{\mathcal{T}}_i} E_{v2t} \left[\int_{u_1}^{\overline{\mathcal{T}}_i} f_{it}(u_2\ V_{2i}, \square_2) du_2 \right] E_{v1t} [f_{it}(u_1\ V_{1i}, \square_1)]\, du_1$$

For the expectation of the inner integral,

$$\begin{aligned} E_{v2t} \left[\int_{u_1}^{\overline{\mathcal{T}}_i} f_{it}(u_2\ V_{2i}, \square_2) du_2 \right] &= E_{v2t} [S_{2i}(u_1) - S_{2it}] \\ \text{(OA.2.27)} \qquad &= E_{v2i} [S_{2i}(u_1)] - \mathcal{L}_2\left(\widetilde{\Lambda}_{2it}\right) \end{aligned}$$

with the first right-hand side term intentionally not converted to the Laplace form in order to facilitate subsequent series expansion. Using (OA.2.27) in (OA.2.26),

$$\begin{aligned} E_v A &= \int_{\underline{\mathcal{T}}_i}^{\overline{\mathcal{T}}_i} \left[E_{v2t} [S_{2i}(u_1)] - \mathcal{L}_2\left(\widetilde{\Lambda}_{2it}\right) \right] E_{v1t} [f_{it}(u_1\ V_{1i}, \square)]\, du_1 \\ &= \int_{\underline{\mathcal{T}}_i}^{\overline{\mathcal{T}}_i} E_{v2i} [S_{2i}(u_1)]\, E_{v1i} [f_{it}(u_1\ V_{1i}, \square)]\, du_1 \\ &\quad - \mathcal{L}_2\left(\widetilde{\Lambda}_{2it}\right) E_{v1t} \int_{\underline{\mathcal{T}}_i}^{\overline{\mathcal{T}}_i} f_{it}(u_1\ V_{1i}, \square) du_1 \\ \text{(OA.2.28)} \qquad &= E_v A_1 + E_v A_2 \end{aligned}$$

Substituting with (OA.2.6) and (OA.2.7),

$$
\begin{aligned}
E_v A_1 &= \int_{\underline{T}_i}^{\overline{T}_i} E_{v_{2i}}\left[S_{2i}(u_1)\right] E_{v_{1i}}\left[f_{it}(u_1\ V_{1i}, \square)\right] du_1 \\
&= E_{v_{2i}} E_{v_{1i}} \int_{\underline{T}_i}^{\overline{T}_i} S_{2i}(u_1)\left[f_{it}(u_1\ V_{1i}, \square)\right] du_1 \\
&= E_{v_{2i}} E_{v_{1i}} \int_{\underline{T}_i}^{\overline{T}_i} \exp\left(-\Lambda_{2i}(u_1)\right)\exp\left(-\Lambda_{1i}(u_1)\right) \square_{1i}(u_1) du_1 \\
&= E_{v_{2i}} E_{v_{1i}} \int_{\underline{T}_i}^{\overline{T}_i} \exp\left(-v_{2i}\widetilde{\Lambda}_{2i}(u_1)\right)\exp\left(-v_{1i}\widetilde{\Lambda}_{1i}(u_1)\right) v_{1i}\widetilde{\square}_{1i}(u_1) du_1 \qquad \text{(OA.2.29)}
\end{aligned}
$$

Using integration by substitution with $s_k = u_k - \square_i$ in (OA.2.29) and piecewise constancy of $\widetilde{\square}_{ki}(s_k)$ for $s_k \in [0,1)$, $k \in \{1,2\}$,

$$
\begin{aligned}
E_v A_1 &= E_{v_{2i}} E_{v_{1i}} \exp\left(-v_{2i}\widetilde{\Lambda}_{2i(t-1)}\right)\exp\left(-v_{1i}\widetilde{\Lambda}_{1i(t-1)}\right) \\
&\quad \times \int_0^1 \exp\left(-v_{2i} s_1 \widetilde{\square}_{2it}\right)\exp\left(-v_{1i} s_1 \widetilde{\square}_{1it}\right) v_{1i}\widetilde{\square}_{1it} ds_1 \\
&= E_{v_{2i}} E_{v_{1i}} \exp\left(-v_{2i}\widetilde{\Lambda}_{2i(t-1)}\right)\exp\left(-v_{1i}\widetilde{\Lambda}_{1i(t-1)}\right) \\
&\quad \times \int_0^1 \sum_{r_2=0}^{\infty} \frac{(-1)^{r_2}}{r_2!}\left(v_{2i} s_1 \widetilde{\square}_{2it}\right)^{r_2} \sum_{r_1=0}^{\infty} \frac{(-1)^{r_1}}{r_1!}\left(v_{1i} s_1 \widetilde{\square}_{1it}\right)^{r_1} v_{1i}\widetilde{\square}_{1it} ds_1 \\
&= \sum_{r_2=0}^{\infty} \frac{(-1)^{r_2}}{r_2!} \sum_{r_1=0}^{\infty} \frac{(-1)^{r_1}}{r_1!} E_{v_{1i}}\left[\exp\left(-v_{1i}\widetilde{\Lambda}_{1i(t-1)}\right)\left(v_{1i}\widetilde{\square}_{1it}\right)^{r_1+1}\right] \\
&\quad \times E_{v_{2i}}\left[\exp\left(-v_{2i}\widetilde{\Lambda}_{2i(t-1)}\right)\left(v_{2i}\widetilde{\square}_{2it}\right)^{r_2}\right] \int_0^1 s_1^{r_2+r_1} ds_1 \\
&= \sum_{r_2=0}^{\infty} \frac{(-1)^{r_2}}{r_2!} \sum_{r_1=0}^{\infty} \frac{(-1)^{r_1}}{r_1!} E_{v_1}\left[A_{11}\right] E_{v_2}\left[A_{12}\right] A_{13} \qquad \text{(OA.2.30)}
\end{aligned}
$$

where

$$
\begin{aligned}
E_{v_1}\left[A_{11}\right] &= \widetilde{\square}_{1it}^{r_1+1} E_{v_{1i}}\left[\exp\left(-v_{1i}\widetilde{\Lambda}_{1i(t-1)}\right) v_{1i}^{r_1+1}\right] && \text{(OA.2.31)} \\
&= (-1)^{r_1+1}\widetilde{\square}_{1it}^{r_1+1}\mathcal{L}_1^{(r_1+1)}\left(\widetilde{\Lambda}_{1i(t-1)}\right) && \text{(OA.2.32)} \\
E_{v_2}\left[A_{12}\right] &= \widetilde{\square}_{2it}^{r_2} E_{v_{2i}}\left[\exp\left(-v_{2i}\widetilde{\Lambda}_{2i(t-1)}\right) v_{2i}^{r_2}\right] && \text{(OA.2.33)} \\
&= (-1)^{r_2}\widetilde{\square}_{2it}^{r_2}\mathcal{L}_2^{(r_2)}\left(\widetilde{\Lambda}_{2i(t-1)}\right) && \text{(OA.2.34)} \\
A_{13} &= \int_0^1 s_1^{r_2+r_1} ds_1 \\
&= \frac{1}{r_2+r_1+1} && \text{(OA.2.35)}
\end{aligned}
$$

whereby the time dimension of the previous quadrature has been parsed through following the series expansion linearization and integrated out in the remaining polynomial term in (OA.2.35).

Combining (OA.2.32) and (OA.2.34) and (OA.2.35) in (OA.2.30) results in

$$\begin{aligned}\mathsf{E}_v\mathsf{A}_1 &= \sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{\mathsf{r}_2!}\sum_{r_1=0}^{\infty}\frac{(-1)^{r_1}}{\mathsf{r}_1!}\frac{(-1)^{r_1+r_2+1}}{\mathsf{r}_2+\mathsf{r}_1+1}\widetilde{\square}_{1it}^{r_1+1}\widetilde{\square}_{2it}^{r_2}\\ &\quad\times\mathcal{L}_1^{(r_1+1)}\left(\widetilde{\Lambda}_{1i(t-1)}\right)\mathcal{L}_2^{(r_2)}\left(\widetilde{\Lambda}_{2i(t-1)}\right)\end{aligned}\tag{OA.2.36}$$

For the second part of (OA.2.28),

$$\begin{aligned}\mathsf{E}_v\mathsf{A}_2 &= -\mathcal{L}_2\left(\widetilde{\Lambda}_{2it}\right)\mathsf{E}_{v_{1i}}\int_{\underline{\tau}_i}^{\overline{\tau}_i}\mathsf{f}_{it}(\mathsf{u}_1\,\mathsf{V}_{1i},\square_1)\mathsf{du}_1\\ &= -\mathcal{L}_2\left(\widetilde{\Lambda}_{2it}\right)\mathsf{E}_{v_{1i}}\left|\mathsf{S}_{1i(t-1)}-\mathsf{S}_{1it}\right]\\ &= -\mathcal{L}_2\left(\widetilde{\Lambda}_{2it}\right)\left|\mathcal{L}_1\left(\widetilde{\Lambda}_{1i(t-1)}\right)-\mathcal{L}_1\left(\widetilde{\Lambda}_{1it}\right)\right]\end{aligned}\tag{OA.2.37}$$

Collecting (OA.2.36) and (OA.2.37) in (OA.2.28) yields

$$\begin{aligned}\mathsf{E}_v\mathsf{A} &= \sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{\mathsf{r}_2!}\sum_{r_1=0}^{\infty}\frac{(-1)^{r_1}}{\mathsf{r}_1!}\frac{(-1)^{r_1+r_2+1}}{\mathsf{r}_2+\mathsf{r}_1+1}\widetilde{\square}_{1it}^{r_1+1}\widetilde{\square}_{2it}^{r_2}\\ &\quad\times\mathcal{L}_1^{(r_1+1)}\left(\widetilde{\Lambda}_{1i(t-1)}\right)\mathcal{L}_2^{(r_2)}\left(\widetilde{\Lambda}_{2i(t-1)}\right)\\ &\quad-\mathcal{L}_2\left(\widetilde{\Lambda}_{2it}\right)\left|\mathcal{L}_1\left(\widetilde{\Lambda}_{1i(t-1)}\right)-\mathcal{L}_1\left(\widetilde{\Lambda}_{1it}\right)\right]\end{aligned}\tag{OA.2.38}$$

The expectation expression for B in (OA.2.25) is

$$\begin{aligned}\mathsf{E}_v\mathsf{B} &= \int_{\underline{\tau}_i}^{\overline{\tau}_i}\mathsf{E}_{v_{2i}}\left[\int_{\overline{\tau}_i}^{\infty}\mathsf{f}_{it}(\mathsf{u}_2\,\mathsf{V}_{2i},\square_2)\mathsf{du}_2\right]\mathsf{E}_{v_{1i}}\left[\mathsf{f}_{it}(\mathsf{u}_1\,\mathsf{V}_{1i},\square_1)\right]\mathsf{du}_1\\ &= \mathsf{E}_{v_{2i}}\left[\mathsf{S}_{2it}\right]\mathsf{E}_{v_{1i}}\left|\mathsf{S}_{1i(t-1)}-\mathsf{S}_{1it}\right]\\ &= \mathcal{L}_2\left(\widetilde{\Lambda}_{2it}\right)\left|\mathcal{L}_1\left(\widetilde{\Lambda}_{1i(t-1)}\right)-\mathcal{L}_1\left(\widetilde{\Lambda}_{1it}\right)\right]\end{aligned}\tag{OA.2.39}$$

Substituting (OA.2.38) and (OA.2.39) into (OA.2.25) yields

$$\begin{aligned}\mathsf{E}_v\mathsf{P}(\mathsf{t}_{1i}=\mathsf{t},\ \square_{2i}>\square_{1i}) &= \sum_{r_2=0}^{\infty}\sum_{r_1=0}^{\infty}\frac{(-1)^{2r_1+2r_2+1}}{\mathsf{r}_2!\mathsf{r}_1!(\mathsf{r}_2+\mathsf{r}_1+1)}\widetilde{\square}_{1it}^{r_1+1}\widetilde{\square}_{2it}^{r_2}\\ &\quad\times\mathcal{L}_1^{(r_1+1)}\left(\widetilde{\Lambda}_{1i(t-1)}\right)\mathcal{L}_2^{(r_2)}\left(\widetilde{\Lambda}_{2i(t-1)}\right)\end{aligned}\tag{OA.2.40}$$

2.3.3. Derivatives of the Laplace transform

In general,

$$\mathcal{L}^{(r)}(\mathsf{s}) = (-1)^r\int \mathsf{v}^r\exp(-\mathsf{sv})\,\mathsf{g}(\mathsf{v})\mathsf{dv}\tag{OA.2.41}$$

(see e.g. Hougaard 2000, p. 498) and $\mathcal{L}^{(r)}(\mathsf{s})$ exists for each $\mathsf{r} > \mathsf{c}$ such that $\mathsf{g}(\mathsf{v}) \leq \mathsf{K}\exp(\mathsf{cv})$ if $\mathsf{g}(\mathsf{v})$ is piecewise continuous over its domain.

In the GIG density function (2.16), replace □ with □/2, then let □ = □²/□ and then substitute the resulting expression into (OA.2.41) to obtain

$$\begin{aligned}
\mathcal{L}^{(r)GIG}(\mathsf{s}) &= (-1)^r \int \mathsf{v}^r \exp(-\mathsf{sv})\, \mathsf{g}^{GIG}(\mathsf{v})\mathsf{dv} \\
&= (-1)^r \int \mathsf{v}^r \exp(-\mathsf{sv}) \frac{(\square/\square)^{\kappa/2}}{2\mathsf{K}_{\kappa}\left((\square\square)^{1/2}\right)} \mathsf{v}^{\kappa-1} \exp\left\{-\frac{1}{2}\left(\square \mathsf{v} + \frac{\square}{\mathsf{v}}\right)\right\} \mathsf{dv} \\
&= (-1)^r \int \frac{(\square/\square)^{\kappa/2}}{2\mathsf{K}_{\kappa}\left((\square\square)^{1/2}\right)} \mathsf{v}^{\kappa+r-1} \exp\left\{-\frac{1}{2}\left((\square+2\mathsf{s})\,\mathsf{v} + \frac{\square}{\mathsf{v}}\right)\right\} \mathsf{dv} \\
&= (-1)^r \frac{2\mathsf{K}_{\kappa+r}\left(((\square+2\mathsf{s})\,\square)^{1/2}\right)((\square+2\mathsf{s})/\square)^{(\kappa+r)/2}}{2\mathsf{K}_{\kappa+r}\left(((\square+2\mathsf{s})\,\square)^{1/2}\right)((\square+2\mathsf{s})/\square)^{(\kappa+r)/2}} \\
&\quad \times \int \frac{(\square/\square)^{\kappa/2}}{2\mathsf{K}_{\kappa}\left((\square\square)^{1/2}\right)} \mathsf{v}^{\kappa+r-1} \exp\left\{-\frac{1}{2}\left((\square+2\mathsf{s})\,\mathsf{v} + \frac{\square}{\mathsf{v}}\right)\right\} \mathsf{dv} \\
&= (-1)^r \frac{\mathsf{K}_{\kappa+r}\left(((\square+2\mathsf{s})\,\square)^{1/2}\right)}{\mathsf{K}_{\kappa}\left((\square\square)^{1/2}\right)} \frac{(\square/\square)^{\kappa/2}}{((\square+2\mathsf{s})/\square)^{(\kappa+r)/2}} \\
&\quad \times \int \frac{((\square+2\mathsf{s})/\square)^{(\kappa+r)/2}}{2\mathsf{K}_{\kappa+r}\left(((\square+2\mathsf{s})\,\square)^{1/2}\right)} \mathsf{v}^{\kappa+r-1} \exp\left\{-\frac{1}{2}\left((\square+2\mathsf{s})\,\mathsf{v} + \frac{\square}{\mathsf{v}}\right)\right\} \mathsf{dv} \\
&= (-1)^r \frac{\mathsf{K}_{\kappa+r}\left(((\square+2\mathsf{s})\,\square)^{1/2}\right)}{\mathsf{K}_{\kappa}\left((\square\square)^{1/2}\right)} \frac{(\square/\square)^{\kappa/2}}{((\square+2\mathsf{s})/\square)^{(\kappa+r)/2}}
\end{aligned}$$

Reversing the substitution with □ = $\sqrt{\square\square}$ and then replacing □ with 2□ yields

(OA.2.42) $$\mathcal{L}^{(r)GIG}(\mathsf{s}) = (-1)^r \frac{\mathsf{K}_{\kappa+r}\left(\square(1+\mathsf{s}/\square)^{1/2}\right)}{\mathsf{K}_{\kappa}(\square)} \left(\frac{\square}{2\square}\right)^r (1+\mathsf{s}/\square)^{-(\kappa+r)/2}$$

The quadrature version for the GIG then follows from (2.17), (OA.2.24), and (OA.2.42).

$$
\begin{aligned}
\mathrm{E}_{v}^{GIG}\mathrm{P}\left(\mathrm{t}_{1i}=\mathrm{t},\ \square_{2i}>\square_{i}\right) &= \frac{\widetilde{\square}_{1it}\square_{1}}{2\square_{1}\mathrm{K}_{\kappa_1}\left(\square_1\right)\mathrm{K}_{\kappa_2}\left(\square_2\right)} \\
&\quad\times\int_0^1\left(1+\frac{1}{\square_1}\widetilde{\Lambda}_{1i(t-1)}+\frac{1}{\square_1}\widetilde{\square}_{1it}\mathrm{s}_1\right)^{-(\kappa_1+1)/2} \\
&\quad\times\left(1+\frac{1}{\square_2}\widetilde{\Lambda}_{2i(t-1)}+\frac{1}{\square_2}\widetilde{\square}_{2it}\mathrm{s}_1\right)^{-\kappa_2/2} \\
&\quad\times\mathrm{K}_{\kappa_1+1}\left(\square_1\left(1+\frac{1}{\square_1}\widetilde{\Lambda}_{1i(t-1)}+\frac{1}{\square_1}\widetilde{\square}_{1it}\mathrm{s}_1\right)^{1/2}\right) \\
&\quad\times\mathrm{K}_{\kappa_2}\left(\square_2\left(1+\frac{1}{\square_2}\widetilde{\Lambda}_{2i(t-1)}+\frac{1}{\square_2}\widetilde{\square}_{2it}\mathrm{s}_1\right)^{1/2}\right)\mathrm{ds}_1
\end{aligned}
\tag{OA.2.43}
$$

The series version follows from (OA.2.40) and (OA.2.42).

$$
\begin{aligned}
\mathrm{E}_{v}^{GIG}\mathrm{P}\left(\mathrm{t}_{1i}=\mathrm{t},\ \square_{2i}>\square_{1i}\right) &= \sum_{r_2=0}^{\infty}\sum_{r_1=0}^{\infty}\frac{(-1)^{3r_1+3r_2}}{\mathrm{r}_2!\mathrm{r}_1!\left(\mathrm{r}_2+\mathrm{r}_1+1\right)}\left(\frac{\widetilde{\square}_{1it}\square_1}{2\square_1}\right)^{r_1+1}\left(\frac{\widetilde{\square}_{2it}\square_2}{2\square_2}\right)^{r_2} \\
&\quad\times\left(1+\frac{1}{\square_1}\widetilde{\Lambda}_{1i(t-1)}\right)^{-(\kappa+r_1+1)/2}\left(1+\frac{1}{\square_2}\widetilde{\Lambda}_{2i(t-1)}\right)^{-(\kappa_2+r_2)/2} \\
&\quad\times\mathrm{K}_{\kappa_1+r_1+1}\left(\square_1\left(1+\frac{1}{\square_1}\widetilde{\Lambda}_{1i(t-1)}\right)^{1/2}\right)\left[\mathrm{K}_{\kappa_1}\left(\square_1\right)\right]^{-1} \\
&\quad\times\mathrm{K}_{\kappa_2+r_2}\left(\square_2\left(1+\frac{1}{\square_2}\widetilde{\Lambda}_{2i(t-1)}\right)^{1/2}\right)\left|\mathrm{K}_{\kappa_2}\left(\square_2\right)\right]^{-1}
\end{aligned}
\tag{OA.2.44}
$$

The censored case,

$$
\begin{aligned}
\mathrm{E}_{v}^{GIG}\mathrm{P}\left(\mathrm{t}_{1i}>\mathrm{T},\ \mathrm{t}_{2i}>\mathrm{T}\right) &= \left(1+\frac{1}{\square_1}\widetilde{\Lambda}_{1iT}\right)^{-\kappa_1/2}\left(1+\frac{1}{\square_2}\widetilde{\Lambda}_{2iT}\right)^{-\kappa_2/2} \\
&\quad\times\mathrm{K}_{\kappa_1}\left(\square_1\left(1+\frac{1}{\square_1}\widetilde{\Lambda}_{1iT}\right)^{1/2}\right)\left|\mathrm{K}_{\kappa_1}\left(\square_1\right)\right]^{-1} \\
&\quad\times\mathrm{K}_{\kappa_2}\left(\square_2\left(1+\frac{1}{\square_2}\widetilde{\Lambda}_{2iT}\right)^{1/2}\right)\left|\mathrm{K}_{\kappa_2}\left(\square_2\right)\right]^{-1}
\end{aligned}
\tag{OA.2.45}
$$

Expressions (OA.2.43), (OA.2.44), and (OA.2.45) are referenced in Corollary 1 to Theorem 2.

For the gamma density function (2.20),

$$\begin{aligned}
\mathcal{L}^{(r)G}(\mathsf{s}) &= (-1)^r \int \mathsf{v}^r \exp(-\mathsf{sv})\, \mathsf{g}^G(\mathsf{v})\mathsf{dv} \\
&= (-1)^r \int \mathsf{v}^r \exp(-\mathsf{sv})\, \Box^\gamma \frac{1}{\Gamma(\Box)} \mathsf{v}^{\gamma-1} \exp(-\Box \mathsf{v})\mathsf{dv} \\
&= (-1)^r \int \Box^\gamma \frac{1}{\Gamma(\Box)} \mathsf{v}^{\gamma+r-1} \exp(-(\Box+\mathsf{s})\,\mathsf{v})\mathsf{dv} \\
&= (-1)^r \frac{(\Box+\mathsf{s})^{\gamma+r}}{(\Box+\mathsf{s})^{\gamma+r}} \frac{\Gamma(\Box+\mathsf{r})}{\Gamma(\Box+\mathsf{r})} \\
&\quad \times \int \Box^\gamma \frac{1}{\Gamma(\Box)} \mathsf{v}^{\gamma+r-1} \exp(-(\Box+\mathsf{s})\,\mathsf{v})\mathsf{dv} \\
&= (-1)^r \frac{\Box^\gamma}{(\Box+\mathsf{s})^{\gamma+r}} \frac{\Gamma(\Box+\mathsf{r})}{\Gamma(\Box)} \\
&\quad \times \int (\Box+\mathsf{s})^{\gamma+r} \frac{1}{\Gamma(\Box+\mathsf{r})} \mathsf{v}^{\gamma+r-1} \exp(-(\Box+\mathsf{s})\,\mathsf{v})\mathsf{dv} \\
\text{(OA.2.46)} \qquad &= (-1)^r \frac{\Box^\gamma}{(\Box+\mathsf{s})^{\gamma+r}} \frac{\Gamma(\Box+\mathsf{r})}{\Gamma(\Box)}
\end{aligned}$$

The quadrature version for the gamma then follows from (2.21), (OA.2.24), and (OA.2.46).

$$\begin{aligned}
\mathsf{E}_v^G \mathsf{P}(\mathsf{t}_{1i}=\mathsf{t},\ \Box_{2i} > \Box_i) &= \Box_1 \frac{\widetilde{\Box}_{1it}}{\Box_1} \int_0^1 \left(1 + \frac{1}{\Box_2}\widetilde{\Lambda}_{2i(t-1)} + \frac{1}{\Box_2}\widetilde{\Box}_{2it}\mathsf{s}_1\right)^{-\gamma_2} \\
\text{(OA.2.47)} \qquad & \quad \times \left(1 + \frac{1}{\Box_1}\widetilde{\Lambda}_{1i(t-1)} + \frac{1}{\Box_1}\widetilde{\Box}_{1it}\mathsf{s}_1\right)^{-(\gamma_1+1)} \mathsf{ds}_1
\end{aligned}$$

The series version follows from (OA.2.40) and (OA.2.46).

$$\begin{aligned}
\mathsf{E}_v^G \mathsf{P}(\mathsf{t}_{1i}=\mathsf{t},\ \Box_{2i} > \Box_i) &= \sum_{r_2=0}^{\infty}\sum_{r_1=0}^{\infty} \frac{(-1)^{r_1+r_2}}{\mathsf{r}_1!\mathsf{r}_2!\,(\mathsf{r}_1+\mathsf{r}_2+1)} \left(\frac{\widetilde{\Box}_{1it}}{\Box_1}\right)^{r_1+1} \left(\frac{\widetilde{\Box}_{2it}}{\Box_2}\right)^{r_2} \\
& \quad \times \left(1 + \frac{1}{\Box_1}\widetilde{\Lambda}_{1i(t-1)}\right)^{-(\gamma_1+r_1+1)} \left(1 + \frac{1}{\Box_2}\widetilde{\Lambda}_{2i(t-1)}\right)^{-(\gamma_2+r_2)} \\
\text{(OA.2.48)} \qquad & \quad \times \Gamma(\Box_1+\mathsf{r}_1+1)\,[\Gamma(\Box_1)]^{-1}\,\Gamma(\Box_2+\mathsf{r}_2)\,[\Gamma(\Box_2)]^{-1}
\end{aligned}$$

For the censored case,

$$\text{(OA.2.49)} \qquad \mathsf{E}_v^G \mathsf{P}(\mathsf{t}_{1i} > \mathsf{T},\ \mathsf{t}_{2i} > \mathsf{T}) = \left(1 + \frac{1}{\Box_1}\widetilde{\Lambda}_{1iT}\right)^{-\gamma_1} \left(1 + \frac{1}{\Box_2}\widetilde{\Lambda}_{2iT}\right)^{-\gamma_2}$$

Expressions (OA.2.47), (OA.2.48), and (OA.2.45) are referenced in Corollary 2 to Theorem 2.

3. Model Identification

Cox (1962) and Tsiatis (1975) state that the simple competing risks model with no regressors is not identified. In particular, any competing risk model with correlated risks is observationally equivalent to some other competing risks model with independent risks. Heckman and Honore (1989), henceforth HH, establish an identification theorem for a general class of competing risks models with regressors. This class includes models with marginal distributions that follow proportional hazards, mixed proportional hazards, and accelerated hazards. The results are presented for two competing risks but generalize to any arbitrary finite number of risks. HH assume that the exact time of exit is observed.

Our competing risk (CR) model is based on continuous latent times of exit $\square_{1i}, \dots, \square_{Ki}$ with a minimum $\square_i = \min(\square_{1i}, \dots, \square_{Ki})$. We observe the time interval $[\square_i, \square_i)$, labeled as t_i, which contains $\square_i$. Nonetheless, our model assumptions impose more structure that would typically be implied by interval outcome data, which allows us to identify the structural model components. Assumption B2 explicitly parametrizes the time-varying model components as functions of the continuous time $\square$ In Assumptions B3 and B4, the values of these components are assumed constant within each time period t. Assumptions B2 and B4 thus allow us to adhere to a counterpart of the HH identification approach in our model setting.

As in the main text, we assume $K = 2$ risk types. HH identify the single-index structural parameters $\square_k$ up to scale from the ratio of the derivatives of the survival function with respect to a time increment of each risk type, evaluated at the time origin. Our counterpart is the ratio of the survival functions integrated over the first time period ($t = 1$). From Theorem 1,

$$\begin{aligned}\frac{P(t_{1i} = 1,\ \square_{2i} > \square_{1i}\, V, \square)}{P(t_{2i} = 1,\ \square_{1i} > \square_{2i}\, V, \square)} &= \frac{S_{2i0} S_{1i0} \square_{1i1} (\square_{2i1} + \square_{1i1})^{-1} [1 - \exp(-(\square_{2i1} + \square_{1i1}))]}{S_{1i0} S_{2i0} \square_{2i1} (\square_{1i1} + \square_{2i1})^{-1} [1 - \exp(-(\square_{1i1} + \square_{2i1}))]} \\ &= \frac{\square_{1i1}}{\square_{2i1}} \\ &= \frac{v_{i1} \exp(X_{i1} \square_1 + \square_{011})}{v_{i2} \exp(X_{i1} \square_2 + \square_{021})}\end{aligned}$$

Taking expectations with respect to $v_{ik} = \exp(V_{ik})$ and using the normalization restrictions $E[v_{ik}] = 1$ from Assumption B6, the absence of a constant term in X_{it}, and the support condition for X_{it} in Assumption B2 identifies the ratio of $\square_1$ and $\square_2$.

Conditional on $X = x$, HH assume the survival function structure

$$S(\square x) = \mathcal{K}[U_1(\square x), U_2(\square x)] \tag{OA.3.1}$$

where $U_k(\square x) = \exp[-Z_k(\square)\square_k(x)]$ and $\mathcal{K}$ is a joint distribution function on $[0, 1]^2$. In our case, the counterpart of (X) for period t outcomes, the joint expected survival function, can be expressed as

$$E_v S_t(x) = \mathcal{K}[U_{1t}(x), U_{2t}(x)] \tag{OA.3.2}$$

where $U_{kt}(x) = \exp\left(-\sum_{j=1}^{t} z_{kt} \square_{kt}(x)\right)$ with $\square_{kt}(x) = \exp(x_{kt} \square_k)$ and $z_{kt} = \exp(\square_{0kt})$, or equivalently $U_{kt}(x) = \widetilde{\Lambda}_{kt}(x)$. Our model assumptions uniquely determine the function $\mathcal{K}$ which is given in Theorem 2: for censored observations $\mathcal{K}$ is a product of the Laplace transforms of

$\widetilde{\Lambda}_{1t}(x)$ and $\widetilde{\Lambda}_{2t}(x)$, and for non-censored observations an expression involving the derivatives of the respective Laplace transforms.

Let $\square_{2t}(x) \to 0$ while holding $\square_{1t}(x)$ fixed, which is feasible by the full support condition for the covariates. Then

$$E_v S_t(x) = \mathcal{K}\left|\exp\left(-\sum_{j=1}^{t} z_{1t} \exp\left(x_{1t}\square_1\right)\right), 1\right]$$

Since $\mathcal{K}$ and $\square_{1t}(x)$ are known and $\mathcal{K}$ is increasing in both arguments, z_{1t} can be identified for any t, and similarly for z_{2t}. Identification of $\square$ $\square_1^2$, and $\square_2^2$ follow directly from identification of z_{kt} and Assumptions B2 and B4.

Using (OA.2.9) the joint expected survivor function can be expressed explicitly in terms of v_1 and v_2 as

$$E_v S_t(x) = \int_{\Omega} \exp\left[-v_1 U_{1t}(x)\right] \exp\left[-v_2 U_{2t}(x)\right] dG(v_1, v_2) \tag{OA.3.3}$$

HH show nonparametric identification of G for a special case with $v_1 = v_2 = \exp(c_2\square)$. Honore (1993) provides the proof in full generality, albeit in that paper (OA.3.3) was obtained from a multi-spell background. The argument is that if the marginal distributions of G along with other model components are identified, then G is nonparametrically identified by the uniqueness of the multivariate Laplace transform. The same argument can be used when (OA.3.3) is obtained from a multiple risk background as we consider here.

The marginal distributions of G are in turn identified under the following Elbers and Ridder (1982) assumptions:

ER1: v_k is non-negative, with $E\left|v_k\right] = 1$.

ER2:. The function $z_k(\square)$ defined on $|0, \infty)$ can be written as the integral of a non-negative function $\square$.

ER3': There are two points in the support of X, x_0 and x_1, such that $\square(x_0) \neq \square(x_1)$. Furthermore, $\square(x_0) = 1$.

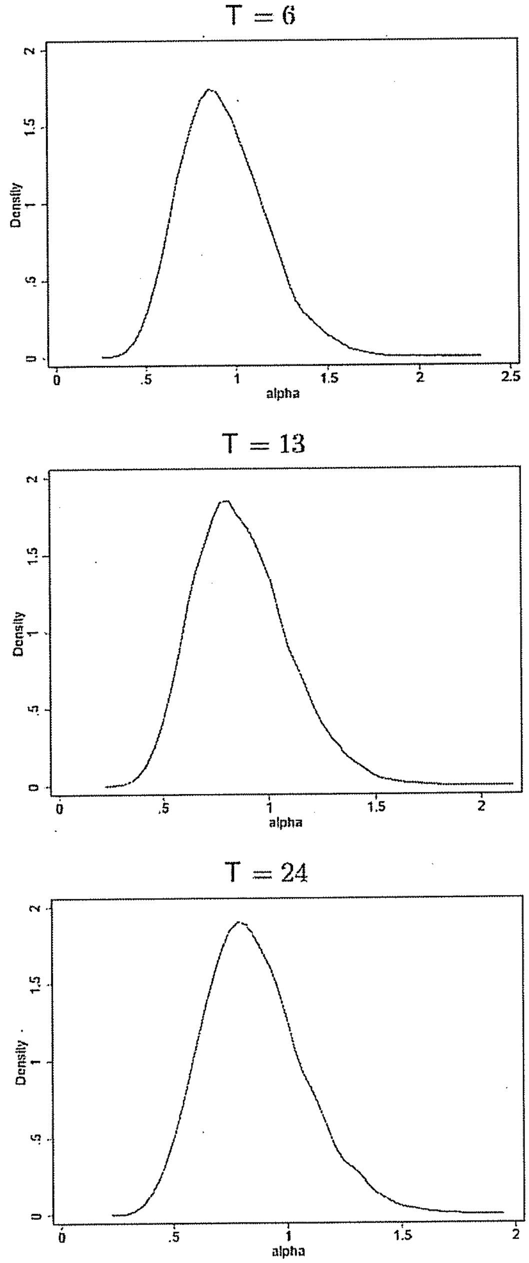
Assumption ER1 is satisfied by our Assumption B6, and Assumptions ER2 and ER3' are satisfied by our Assumptions B2, B6 and the definition of integrated baseline hazard.

Identification of $\square_k$ in HH relies on a limit result with the time variable approaching zero. An alternative proof of nonparametric identification of a general class of CR models that does not rely on a time limit at zero is provided in a recent paper by Lee and Lewbel (2013), henceforth LL. Their approach also does not depend on exclusion restrictions and allows for discrete regressors as long as some are continuously distributed.

LL define mappings $B_k(s\, x)$ and $C(s\, x)$ that are identified directly from data and whose unique solution is the accelerated failure time nonparametric regression function $g(x)$. Both $B_k(s|x)$ and $C(s|x)$ are expressed as integrals over the continuous time domain which we can evaluate as well under our assumptions using the formula for the density of the continuous latent time of exit.

Other than regularity conditions that are satisfied in our model, LL rely on a key rank assumption stating that the columns of the Frèchet derivative of $C^*(s,h) = C(s\,x)$ with respect to its functional argument h are linearly independent and that C^* is a proper mapping preserving compactness under inverse image. These conditions generally require that X contain at least K continuously distributed elements and also that no one element of $g(x)$ can be expressed as a function of the other elements of $g(x)$. LL show that the conditions can be met under parametric assumptions preventing non-degeneracy of the correlation structure between the competing risks, and hence we conclude that these will hold in our model.

Figure OA.1. Posterior Density of the Dirichlet Process Concentration Parameter α, GIG mixture



T = 6

T = 13

T = 24

Table OA.1. Duration model with parametric gamma heterogeneity (Han and Hausman, 1990)

		6 periods		13 periods		24 periods	
		Mean	s.e.	Mean	s.e.	Mean	s.e.
□		0.210	0.034	0.242	0.017	0.316	0.020
Urate		-0.480	0.036	-0.457	0.021	-0.402	0.012
Race		-0.184	0.070	-0.186	0.060	-0.195	0.055
Age		-0.403	0.077	-0.325	0.072	-0.284	0.048
Rrate		-1.449	0.104	-0.941	0.074	-0.383	0.052
t	1	-0.346	0.215	-0.774	0.111	-1.374	0.058
	2	0.305	0.237	-0.168	0.123	-0.819	0.058
	3	0.194	0.263	-0.303	0.137	-1.012	0.059
	4	0.688	0.289	0.153	0.139	-0.588	0.062
	5	0.525	0.322	-0.046	0.154	-0.826	0.077
	6	1.095	0.337	0.499	0.144	-0.324	0.072
	7			0.131	0.162	-0.721	0.078
	8			0.571	0.161	-0.322	0.080
	9			0.351	0.195	-0.566	0.100
	10			0.700	0.179	-0.262	0.088
	11			0.590	0.169	-0.389	0.100
	12			0.945	0.193	-0.063	0.102
	13			1.007	0.212	-0.015	0.101
	14					0.163	0.120
	15					0.305	0.096
	16					0.463	0.125
	17					0.307	0.132
	18					0.712	0.142
	19					0.658	0.153
	20					0.916	0.152
	21					0.853	0.165
	22					1.063	0.166
	23					0.995	0.202
	24					1.283	0.176

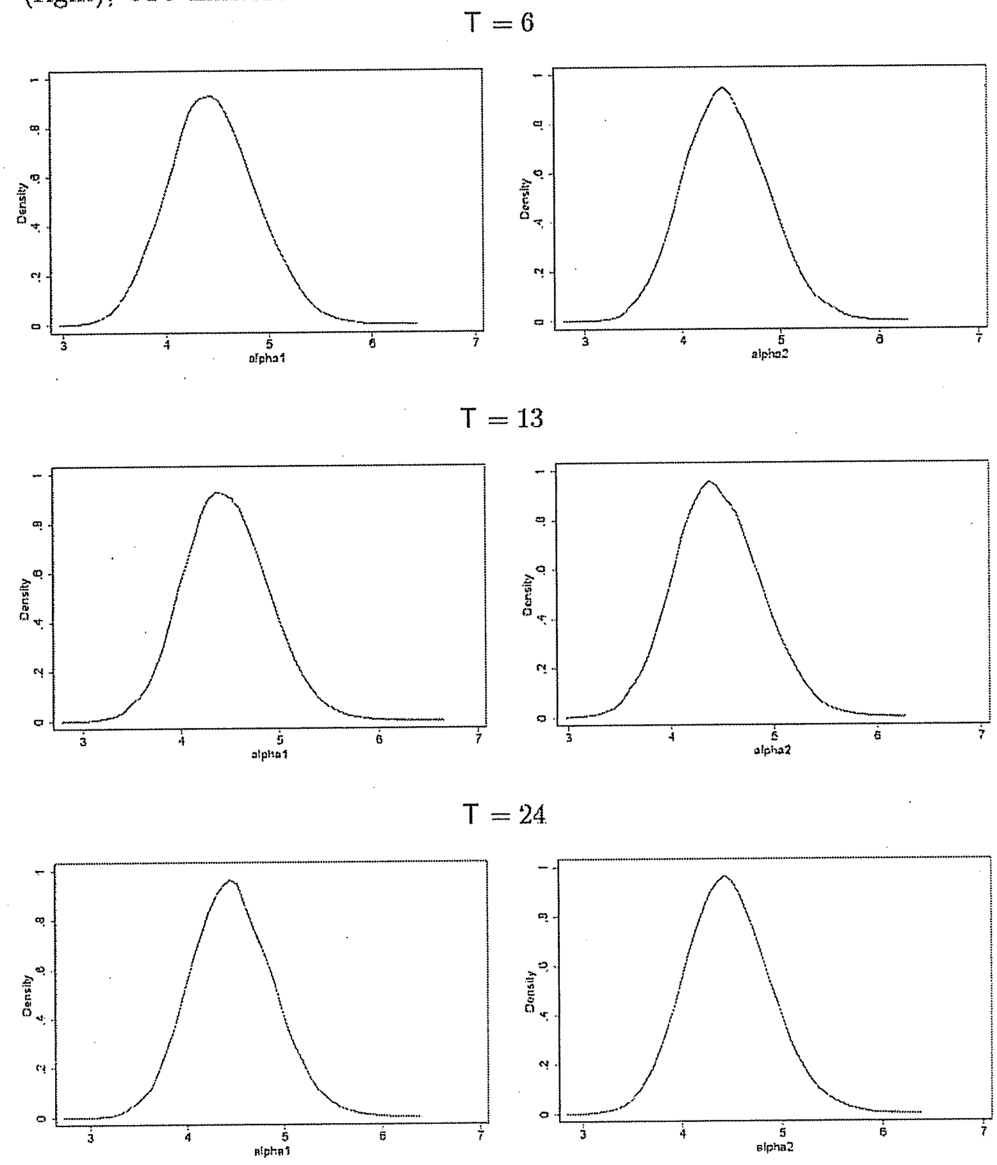
N = 15, 491, *Urate* denotes the state unemployment rate, *Rrate* denotes the replacement rate.

Table OA.2. Duration Model with Parametric GIG Heterogeneity

		6 periods		13 periods		24 periods	
		Mean	s.e.	Mean	s.e.	Mean	s.e.
□		-0.825	0.054	-1.176	0.048	-1.501	0.072
□		1.900	0.151	2.882	0.136	3.792	0.203
Urate		-0.307	0.018	-0.285	0.014	-0.259	0.017
Race		-0.108	0.056	-0.152	0.043	-0.111	0.044
Age		-0.273	0.058	-0.223	0.041	-0.178	0.038
Rrate		-1.190	0.069	-0.708	0.043	-0.039	0.059
t	1	0.115	0.150	0.938	0.102	1.345	0.068
	2	0.584	0.139	1.397	0.092	1.734	0.095
	3	0.309	0.145	1.124	0.099	1.510	0.069
	4	0.640	0.138	1.442	0.098	1.800	0.090
	5	0.334	0.139	1.131	0.095	1.510	0.073
	6	0.734	0.104	1.538	0.103	1.891	0.117
	7			1.059	0.117	1.429	0.071
	8			1.400	0.098	1.718	0.090
	9			1.066	0.101	1.438	0.068
	10			1.311	0.100	1.623	0.082
	11			1.096	0.103	1.462	0.054
	12			1.356	0.098	1.665	0.084
	13			1.334	0.074	1.676	0.072
	14					1.669	0.093
	15					1.788	0.097
	16					1.819	0.100
	17					1.677	0.066
	18					1.931	0.125
	19					1.775	0.097
	20					1.939	0.142
	21					1.798	0.089
	22					1.922	0.113
	23					1.752	0.074
	24					1.790	0.096

N $= 15,491$, *Urate* denotes the state unemployment rate, *Rrate* denotes the replacement rate.

Figure OA.2. Competing Risk Model; Posterior Density of the Dirichlet Process Concentration Parameter α, Type 1 Risk (left) and Type 2 Risk (right), GIG mixture



T = 6

T = 13

T = 24

Table OA.3. Competing Risk Model without Individual Heterogeneity

		6 periods				13 periods				24 periods			
		Risk 1		Risk 2		Risk 1		Risk 2		Risk 1		Risk 2	
		Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.
Urate		-0.295	0.061	-0.305	0.081	-0.199	0.045	-0.116	0.067	-0.199	0.037	-0.095	0.055
Race		-0.003	0.220	-0.044	0.422	-0.083	0.181	-0.329	0.312	-0.035	0.136	-0.289	0.215
Age		-0.155	0.175	-0.807	0.429	-0.097	0.135	-0.596	0.271	-0.011	0.119	-0.437	0.200
Rrate		-1.461	0.265	-0.865	0.506	-1.092	0.222	-0.407	0.302	-0.330	0.138	-0.374	0.226
t	1	-1.312	0.343	-3.434	0.644	-1.928	0.302	-4.495	0.578	-2.402	0.255	-4.643	0.494
	2	-0.759	0.322	-3.210	0.609	-1.389	0.300	-4.288	0.540	-1.865	0.230	-4.426	0.462
	3	-1.118	0.339	-3.538	0.669	-1.739	0.323	-4.583	0.601	-2.210	0.250	-4.736	0.524
	4	-0.829	0.335	-2.236	0.549	-1.448	0.303	-3.320	0.459	-1.946	0.243	-3.442	0.354
	5	-1.285	0.363	-2.226	0.547	-1.898	0.329	-3.335	0.482	-2.387	0.263	-3.444	0.360
	6	-1.763	0.384	-2.058	0.527	-2.407	0.369	-3.146	0.445	-2.891	0.299	-3.294	0.352
	7					-2.297	0.358	-4.205	0.571	-2.779	0.303	-4.342	0.498
	8					-1.854	0.326	-2.922	0.452	-2.349	0.278	-3.031	0.336
	9					-3.146	0.443	-3.813	0.529	-3.587	0.405	-3.947	0.468
	10					-2.004	0.362	-3.792	0.538	-2.503	0.289	-3.951	0.445
	11					-2.671	0.384	-3.798	0.533	-3.147	0.332	-3.940	0.448
	12					-2.187	0.360	-3.741	0.536	-2.715	0.308	-3.875	0.421
	13					-2.287	0.357	-3.721	0.521	-2.753	0.318	-3.832	0.426
	14									-2.833	0.329	-3.683	0.438
	15									-2.592	0.329	-3.439	0.405
	16									-3.169	0.381	-3.764	0.434
	17									-2.241	0.277	-3.713	0.414
	18									-2.695	0.316	-4.388	0.571
	19									-2.681	0.318	-3.145	0.393
	20									-2.831	0.341	-3.763	0.460
	21									-2.020	0.275	-3.467	0.400
	22									-4.002	0.576	-4.664	0.660
	23									-2.307	0.308	-3.651	0.455
	24									-3.277	0.433	-3.600	0.449

$N = 1,243$. *Urate* denotes the state unemployment rate. *Rrate* denotes the replacement rate.

Tabl e OA.4. Competing Risk Model with Parametric Gamma Heterogeneity

		6 periods				13 periods				24 periods			
		Risk 1		Risk 2		Risk 1		Risk 2		Risk 1		Risk 2	
		Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.
||		0.480	0.112	0.240	0.091	0.420	0.061	0.128	0.028	0.350	0.051	0.207	0.045
Urate		-0.361	0.060	-0.327	0.106	-0.298	0.058	-0.199	0.080	-0.342	0.054	-0.167	0.071
Race		0.024	0.272	-0.160	0.440	-0.130	0.236	-0.541	0.416	-0.067	0.217	-0.330	0.296
Age		-0.052	0.212	-0.766	0.386	0.002	0.197	-1.072	0.398	0.089	0.192	-0.723	0.291
Rrate		-1.763	0.257	-1.141	0.536	-1.434	0.235	-0.737	0.401	-0.957	0.223	-0.507	0.333
t	1	-0.844	0.321	-3.151	0.647	-1.278	0.306	-3.846	0.539	-1.381	0.296	-4.172	0.466
	2	-0.171	0.287	-2.907	0.604	-0.605	0.295	-3.578	0.490	-0.678	0.269	-3.973	0.474
	3	-0.401	0.285	-3.171	0.577	-0.825	0.304	-3.782	0.537	-0.876	0.319	-4.190	0.682
	4	-0.009	0.265	-1.792	0.487	-0.431	0.282	-2.427	0.426	-0.442	0.297	-2.891	0.373
	5	-0.390	0.280	-1.719	0.463	-0.798	0.321	-2.309	0.435	-0.791	0.315	-2.799	0.399
	6	-0.759	0.227	-1.500	0.403	-1.205	0.340	-1.967	0.417	-1.192	0.371	-2.562	0.357
	7					-1.061	0.338	-2.909	0.531	-1.056	0.368	-3.507	0.420
	8					-0.566	0.292	-1.490	0.376	-0.526	0.321	-2.141	0.357
	9					-1.762	0.406	-2.309	0.443	-1.756	0.419	-3.000	0.418
	10					-0.607	0.318	-2.242	0.435	-0.585	0.362	-2.946	0.397
	11					-1.186	0.359	-2.065	0.465	-1.142	0.385	-2.746	0.390
	12					-0.693	0.328	-1.962	0.472	-0.677	0.346	-2.759	0.434
	13					-0.876	0.345	-2.073	0.325	-0.709	0.393	-2.719	0.415
	14									-0.667	0.397	-2.433	0.400
	15									-0.333	0.379	-2.105	0.393
	16									-0.913	0.445	-2.385	0.440
	17									0.109	0.361	-2.345	0.442
	18									-0.263	0.402	-2.869	0.508
	19									-0.204	0.415	-1.525	0.358
	20									-0.230	0.401	-2.216	0.456
	21									0.679	0.384	-1.709	0.389
	22									-1.313	0.748	-2.859	0.576
	23									0.469	0.408	-1.835	0.421
	24									-1.060	0.346	-1.594	0.318

N $= 1,243$. *Urate* denotes the state unemployment rate, *Rrate* denotes the replacement rate.

Tabl e OA.5. Competing Risk Model with Parametric GIG Heterogeneity

		6 periods				13 periods				24 periods			
		Risk 1		Risk 2		Risk 1		Risk 2		Risk 1		Risk 2	
		Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.
||		-1.117	0.043	-1.145	0.081	-1.121	0.036	-1.157	0.056	-1.134	0.029	-1.153	0.047
□		2.715	0.122	2.794	0.227	2.726	0.101	2.827	0.159	2.762	0.083	2.817	0.132
Urate		-0.215	0.049	-0.259	0.077	-0.182	0.043	-0.265	0.057	-0.295	0.044	-0.198	0.052
Race		0.059	0.259	-0.104	0.401	-0.110	0.227	-0.472	0.342	-0.073	0.190	-0.346	0.226
Age		-0.026	0.192	-0.733	0.418	-0.056	0.162	-0.760	0.269	-0.039	0.142	-0.553	0.224
Rrate		-1.323	0.241	-0.797	0.423	-1.150	0.236	-0.631	0.318	-0.568	0.180	-0.473	0.202
t	1	-1.737	0.272	-3.399	0.351	-1.949	0.255	-3.695	0.504	-1.798	0.276	-4.022	0.452
	2	-1.101	0.255	-3.096	0.332	-1.316	0.249	-3.485	0.539	-1.158	0.261	-3.834	0.450
	3	-1.365	0.272	-3.271	0.356	-1.588	0.264	-3.657	0.491	-1.440	0.276	-4.107	0.622
	4	-1.030	0.264	-2.257	0.265	-1.243	0.259	-2.395	0.373	-1.093	0.265	-2.844	0.372
	5	-1.453	0.281	-2.228	0.281	-1.639	0.267	-2.397	0.359	-1.495	0.294	-2.849	0.346
	6	-2.208	0.286	-2.195	0.215	-2.104	0.302	-2.185	0.353	-1.931	0.303	-2.623	0.356
	7					-1.982	0.317	-3.306	0.546	-1.821	0.341	-3.631	0.479
	8					-1.527	0.280	-1.868	0.343	-1.354	0.284	-2.325	0.334
	9					-2.751	0.445	-2.821	0.418	-2.651	0.448	-3.230	0.414
	10					-1.627	0.297	-2.772	0.446	-1.464	0.312	-3.188	0.396
	11					-2.232	0.383	-2.711	0.505	-2.105	0.373	-3.102	0.473
	12					-1.768	0.320	-2.597	0.443	-1.632	0.335	-3.016	0.435
	13					-2.215	0.271	-2.285	0.309	-1.696	0.331	-3.053	0.396
	14									--1.694	0.360	-2.848	0.389
	15									-1.474	0.325	-2.640	0.410
	16									-1.998	0.372	-2.924	0.396
	17									-1.103	0.304	-2.870	0.482
	18									-1.508	0.369	-3.572	0.513
	19									-1.461	0.351	-2.223	0.367
	20									-1.638	0.352	-2.919	0.469
	21									-0.772	0.316	-2.481	0.394
	22									-2.598	0.526	-3.746	0.687
	23									-1.068	0.333	-2.695	0.404
	24									-2.241	0.358	-2.279	0.391

N $= 1,243$, *Urate* denotes the state unemployment rate, *Rrate* denotes the replacement rate.

Table OA.6. Competing Risk Model with Independent Risks, GIG Mixture

		6 periods				13 periods				24 periods			
		Risk 1		Risk 2		Risk 1		Risk 2		Risk 1		Risk 2	
		Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.
||		-1.321	0.046	-1.015	0.058	-1.444	0.045	-1.314	0.056	-1.550	0.041	-1.466	0.056
□		3.288	0.129	2.430	0.163	3.634	0.126	3.270	0.159	3.933	0.115	3.698	0.160
Urate		-0.153	0.048	-0.210	0.072	-0.120	0.042	-0.014	0.059	-0.170	0.037	-0.160	0.052
Race		0.011	0.220	-0.031	0.406	-0.085	0.177	-0.203	0.329	-0.066	0.145	-0.305	0.243
Age		-0.187	0.174	-0.487	0.398	-0.208	0.134	-0.255	0.277	-0.246	0.113	-0.174	0.204
Rrate		-1.252	0.233	-0.517	0.371	-1.067	0.189	-0.226	0.261	-0.494	0.145	-0.151	0.197
t	1	-1.526	0.369	-4.443	0.383	-1.759	0.257	-3.392	0.484	-1.873	0.246	-3.656	0.487
	2	-0.951	0.355	-4.176	0.356	-1.185	0.236	-3.212	0.458	-1.302	0.223	-3.478	0.461
	3	-1.263	0.370	-4.408	0.387	-1.499	0.253	-3.556	0.520	-1.621	0.242	-3.829	0.528
	4	-0.947	0.368	-3.453	0.302	-1.187	0.245	-2.309	0.357	-1.313	0.233	-2.572	0.354
	5	-1.384	0.392	-3.486	0.307	-1.624	0.273	-2.339	0.365	-1.751	0.259	-2.603	0.357
	6	-2.524	0.352	-3.846	0.319	-2.085	0.309	-2.168	0.350	-2.220	0.305	-2.435	0.346
	7					-1.988	0.310	-3.229	0.489	-2.121	0.297	-3.509	0.493
	8					-1.543	0.276	-1.938	0.335	-1.673	0.266	-2.207	0.338
	9					-2.795	0.412	-2.857	0.435	-2.935	0.404	-3.128	0.437
	10					-1.665	0.289	-2.861	0.449	-1.806	0.278	-3.114	0.435
	11					-2.297	0.355	-2.825	0.440	-2.435	0.349	-3.086	0.439
	12					-1.835	0.306	-2.801	0.436	-1.982	0.301	-3.067	0.438
	13					-2.661	0.349	-2.522	0.366	-2.060	0.314	-3.055	0.441
	14									-2.088	0.322	-2.904	0.420
	15									-1.855	0.300	-2.666	0.385
	16									-2.424	0.370	-2.976	0.437
	17									-1.469	0.275	-2.954	0.434
	18									-1.893	0.317	-3.703	0.586
	19									-1.842	0.320	-2.400	0.371
	20									-2.031	0.346	-3.018	0.455
	21									-1.170	0.267	-2.725	0.417
	22									-3.087	0.566	-3.929	0.661
	23									-1.434	0.299	-2.952	0.456
	24									-2.777	0.242	-2.690	0.461

N = 1,243, *Urate* denotes the state unemployment rate, *Rrate* denotes the replacement rate.

Table OA.7. Single Risk Model with Competing Risk Data, GIG mixture

		6 periods		13 periods		24 periods	
		Mean	s.e.	Mean	s.e.	Mean	s.e.
□		-1.230	0.035	-1.417	0.035	-1.534	0.037
□		3.032	0.101	3.560	0.099	3.887	0.105
Urate		-0.150	0.038	-0.136	0.034	-0.167	0.031
Race		0.035	0.192	-0.139	0.157	-0.117	0.122
Age		-0.153	0.157	-0.183	0.125	-0.136	0.099
Rrate		-0.998	0.204	-0.765	0.155	-0.303	0.116
t	1	-2.075	0.225	-2.242	0.220	-2.395	0.211
	2	-1.567	0.206	-1.740	0.203	-1.894	0.190
	3	-1.907	0.224	-2.084	0.221	-2.240	0.213
	4	-1.463	0.208	-1.639	0.203	-1.798	0.194
	5	-1.789	0.224	-1.966	0.222	-2.127	0.219
	6	-2.416	0.239	-2.176	0.236	-2.335	0.226
	7			-2.477	0.258	-2.640	0.251
	8			-1.775	0.218	-1.937	0.209
	9			-2.862	0.299	-3.024	0.293
	10			-2.157	0.243	-2.324	0.234
	11			-2.571	0.276	-2.734	0.269
	12			-2.274	0.255	-2.444	0.249
	13			-2.636	0.237	-2.493	0.254
	14					-2.464	0.252
	15					-2.238	0.239
	16					-2.693	0.279
	17					-2.068	0.230
	18					-2.561	0.274
	19					-2.128	0.239
	20					-2.487	0.272
	21					-1.818	0.224
	22					-3.448	0.408
	23					-2.086	0.244
	24					-2.761	0.238

N = 1,243, *Urate* denotes the state unemployment rate, *Rrate* denotes the replacement rate.

4. Extended Counterfactual Policy Experiment

In the main text we have reported the results of a counterfactual policy experiment whereby we simulated a change in the replacement rate and estimated its impact on the probability of exit from unemployment as captured by the survival function. Here we further provide the details on two extensions of the counterfactual experiment: first estimating potential differences between the policy impact on individuals with different unobserved heterogeneities, and second estimating the impact of varying the changes of the replacement rate over time. A summary of the findings is presented in the main text.

4.1. Counterfactuals for Split Samples Based on Unobserved Heterogeneity

The unobserved heterogeneity term v_i can be interpreted as a factor which also contributes to the variation in the hazard rates but is not included among the observed explanatory variables and instead inferred indirectly from the model. One of the key advantages of estimating v_i is that it enables us to differentiate among various groups of individuals based on their unobserved qualities. In our model specification, increasing v_i increases the cumulative hazard function and hence decreases the survival function of unemployment. Thus, individuals with higher v_i have better chances exiting unemployment faster, while individuals with lower v_i are more likely to be long term unemployed. It is difficult to interpret the exact meaning of the unobserved individual component. Nonetheless, given the way it influences the hazard function, v_i can perhaps be thought of as individual ability or quality of labor market characteristics.

As the MCMC output we obtained a Markov chain of draws for each v_i. Denote its mean by $\bar{v}_i$ and the median of the individual means by v_{med}. For both single risk and competing risk model, we split the sample into two parts: one for individuals with $\bar{v}_i \leq v_{med}$ (label these as "low type") and individuals with $\bar{v}_i > v_{med}$ (label these as "high type"). We then ran the counterfactual experiment changing the replacement rate by 10% for each subsample separately, for the case $T = 24$. The resulting % change of the survival function are reported in Table OA.8 (single risk) and Table OA.9 (competing risks) below. In each model, high type individuals react more to the replacement rate changes than low type individuals, for either direction of the change. In the single risk model, the relative ratio of the survival function changes of high type to low type individuals is just over 20%, while in the competing risk case the corresponding figure is approximately 15%. This finding is consistent with the literature estimating the policy effect of training and job placement effects.[10]

[10] We would like to thank an anonymous referee for pointing this out.

Tabl e OA.8. % Change in Survival Function, Single Risk, GIG mixture, T=24

	Pooled		High Type		Low Type	
t	down	up	down	up	down	up
1	-0.052	0.045	-0.057	0.050	-0.047	0.041
2	-0.128	0.111	-0.140	0.121	-0.117	0.101
3	-0.182	0.158	-0.200	0.174	-0.167	0.145
4	-0.255	0.221	-0.282	0.245	-0.234	0.203
5	-0.305	0.265	-0.340	0.295	-0.281	0.244
6	-0.377	0.328	-0.421	0.366	-0.348	0.302
7	-0.421	0.366	-0.477	0.415	-0.388	0.338
8	-0.480	0.418	-0.548	0.477	-0.443	0.386
9	-0.521	0.453	-0.601	0.523	-0.481	0.419
10	-0.570	0.497	-0.662	0.576	-0.528	0.460
11	-0.609	0.531	-0.713	0.621	-0.566	0.493
12	-0.657	0.573	-0.771	0.671	-0.613	0.534
13	-0.701	0.612	-0.828	0.722	-0.657	0.574
14	-0.746	0.651	-0.879	0.766	-0.705	0.615
15	-0.794	0.693	-0.940	0.820	-0.754	0.658
16	-0.838	0.732	-0.987	0.861	-0.802	0.701
17	-0.869	0.760	-1.018	0.889	-0.840	0.734
18	-0.919	0.803	-1.076	0.940	-0.892	0.780
19	-0.953	0.834	-1.107	0.967	-0.932	0.816
20	-0.998	0.874	-1.154	1.009	-0.981	0.859
21	-1.033	0.905	-1.203	1.052	-1.019	0.893
22	-1.071	0.938	-1.225	1.072	-1.062	0.930
23	-1.102	0.966	-1.292	1.132	-1.096	0.960
24	-1.187	1.041	-1.387	1.215	-1.183	1.038

"down" denotes counterfactual decrease of the replacement rate by 10%, and "up" denotes increase by 10%.

Table OA.9. % Change in Survival Function, Competing Risks, GIG mixture, T=24

	Pooled		High Type		Low Type	
t	down	up	down	up	down	up
1	-0.141	0.120	-0.154	0.131	-0.109	0.093
2	-0.370	0.316	-0.406	0.347	-0.286	0.244
3	-0.522	0.446	-0.567	0.485	-0.411	0.351
4	-0.750	0.642	-0.817	0.701	-0.601	0.514
5	-0.896	0.768	-0.988	0.848	-0.735	0.630
6	-1.015	0.870	-1.126	0.967	-0.842	0.721
7	-1.103	0.946	-1.228	1.055	-0.924	0.792
8	-1.274	1.095	-1.417	1.220	-1.081	0.929
9	-1.327	1.141	-1.481	1.275	-1.135	0.975
10	-1.450	1.248	-1.633	1.408	-1.246	1.071
11	-1.526	1.314-	-1.722	1.485	-1.317	1.134
12	-1.630	1.404	-1.835	1.584	-1.414	1.218
13	-1.721	1.484	-1.936	1.672	-1.506	1.298
14	-1.816	1.568	-2.084	1.802	-1.600	1.380
15	-1.936	1.673	-2.245	1.943	-1.717	1.483
16	-1.997	1.726	-2.318	2.007	-1.792	1.548
17	-2.130	1.844	-2.472	2.143	-1.933	1.672
18	-2.195	1.901	-2.506	2.173	-2.020	1.749
19	-2.299	1.994	-2.614	2.269	-2.148	1.862
20	-2.371	2.058	-2.735	2.378	-2.239	1.943
21	-2.538	2.206	-2.990	2.604	-2.417	2.101
22	-2.526	2.196	-2.874	2.499	-2.453	2.132
23	-2.664	2.318	-3.076	2.678	-2.588	2.253
24	-2.686	2.338	-3.101	2.717	-2.641	2.300

”down” denotes counterfactual decrease of the replacement rate by 10%, and ”up” denotes increase by 10%.

4.2. Counterfactuals for Time-varying Changes in the Replacement Rate

In this section we further explore two additional scenarios for the counterfactual policy change: first, a sharply declining replacement rate at the beginning of the spell, and second a scenario where the rate declines sharply only at the end of the spell. For this purpose we construct a decreasing function $r : [0,1] \to [0,1]$ defining the factor $r(t/T)$ by which we multiply the original replacement rate in each time period t with T being the final period. Let $u \in [0,1]$. In the first scenario,

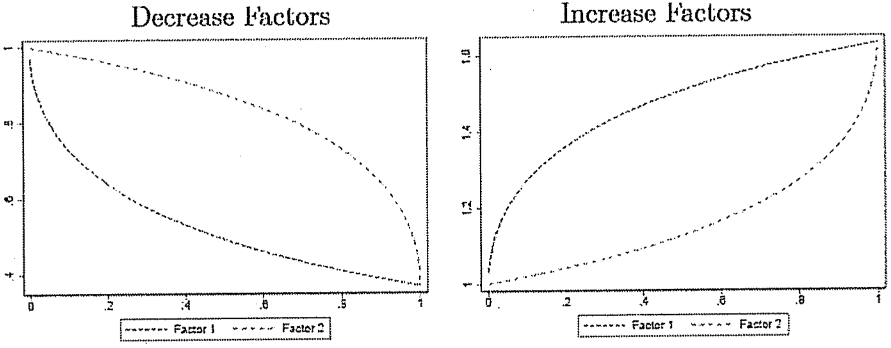
$$r_1 = \exp\left(-u^{\delta}\right)$$

and in the second scenario,

$$r_2 = 1 + \exp(-1) - \exp(-(1-u)^{\delta})$$

A mirror image of r_1 and r_2 increasing from 1 is also used for a counterfactual increase of the replacement rate. It is important to note that at the end of the observation time window both factors become equal, $r_1(1) = r_2(1)$. The constant □$\in (0,1)$ controls the degree of curvature within the exponential change of r_1 and r_2, which smaller □yielding shaper curvature. We set □$= 1/2$, resulting in r_1 and r_2 as shown in Figure OA.3.

Figur e OA.3. Replacement Rate Counterfactual Decrease Factors



The results are presented in Table OA.10. In both SR and CR models, factor 1 (sharp initial change of the replacement rate) leads to an overall larger change in the survival function than factor 2 (sharp change towards the end of the observation time period). In the SR model the change for factor 1 relative to factor 2 at $T = 24$ is more than twofold, and in the CR model it is close to threefold. This indicates that on average individuals respond more to incentives provided early in their unemployment spells relative to ones provided later, even if the response is still overall inelastic.

Table OA.10. % Change in Survival Function, Time-varying Replacement Rate, GIG mixture, T=24

	Single Risk				Competing Risk			
	Factor 1		Factor 2		Factor 1		Factor 2	
t	down	up	down	up	down	up	down	up
1	-0.055	0.100	-0.006	0.029	-0.264	0.219	-0.010	0.009
2	-0.253	0.220	-0.011	0.014	-0.855	0.702	-0.047	0.041
3	-0.428	0.319	-0.052	0.022	-1.330	1.085	-0.085	0.075
4	-0.648	0.505	-0.074	0.018	-2.133	1.732	-0.164	0.143
5	-0.816	0.645	-0.094	0.034	-2.708	2.194	-0.231	0.201
6	-1.117	0.823	-0.173	0.025	-3.210	2.594	-0.297	0.259
7	-1.256	0.987	-0.171	0.070	-3.608	2.913	-0.358	0.311
8	-1.454	1.215	-0.175	0.136	-4.419	3.565	-0.495	0.430
9	-1.603	1.375	-0.187	0.181	-4.685	3.777	-0.546	0.474
10	-1.855	1.527	-0.268	0.189	-5.318	4.288	-0.678	0.586
11	-2.025	1.686	-0.301	0.232	-5.725	4.612	-0.771	0.666
12	-2.239	1.885	-0.348	0.291	-6.297	5.073	-0.916	0.790
13	-2.442	2.082	-0.395	0.356	-6.823	5.500	-1.065	0.916
14	-2.659	2.279	-0.459	0.422	-7.385	5.957	-1.238	1.063
15	-2.853	2.534	-0.491	0.538	-8.104	6.546	-1.479	1.266
16	-3.067	2.759	-0.558	0.638	-8.502	6.869	-1.636	1.398
17	-3.263	2.898	-0.653	0.687	-9.342	7.574	-1.986	1.691
18	-3.476	3.184	-0.717	0.844	-9.795	7.953	-2.209	1.877
19	-3.616	3.411	-0.762	0.985	-10.499	8.551	-2.578	2.184
20	-3.783	3.714	-0.815	1.187	-10.989	8.973	-2.870	2.425
21	-3.959	3.926	-0.916	1.330	-12.090	9.921	-3.557	2.989
22	-4.175	4.136	-1.070	1.481	-12.099	9.930	-3.645	3.059
23	-4.423	4.253	-1.290	1.568	-12.999	10.713	-4.368	3.637
24	-5.311	4.369	-2.260	1.725	-13.205	10.880	-4.720	3.895

"down" denotes counterfactual time-varying decrease of the replacement rate, and "up" denotes time-varying increase.

Environmental Justice:
Evidence from Superfund Cleanup Durations

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Abstract

This paper investigates the extent to which cleanup durations at Superfund sites reflect demographic biases incongruent with the principles of Environmental Justice. We argue that the duration of cleanup, conditional on a large number of site characteristics, should be independent of the race and income profile of the neighborhood in which the site is located. Since the demographic composition of a neighborhood changes during the cleanup process, we explore whether cleanup durations are related to neighborhood demographics recorded at the time when the cleanup is initiated. We estimate a semiparametric Bayesian proportional hazard model, which also allows for unobserved site specific heterogeneity, and find that sites located in black, urban and lower educated neighborhoods were discriminated against at the beginning of the program but that the degree of bias diminished over time. Executive Order 12898 of 1994 appears to have re-prioritized resources for the faster cleanup of sites located in less wealthy neighborhoods. We do not find that the litigation process is an impediment in the cleanup process, and support the notion that community involvement plays an important role.

JEL: Q53, Q58, C41
Keywords: Environmental Justice, Superfund, semiparametric Bayesian duration analysis

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1. Introduction

This paper investigates the extent to which the cleanup process of toxic waste sites, known as Superfund sites, over the last 30 years was implemented in a fair way without inherent demographic biases. The Environmental Protection Agency (EPA) defines *Environmental Justice* as "the fair treatment and meaningful involvement of all people regardless of race, color, national origin, or income with respect to the development, implementation, and enforcement of environmental laws, regulations, and policies". Environmental Justice considerations were formally established in 1994 when President Bill Clinton signed Executive Order 12898 which aimed to prevent discrimination in the implementation of environmental protection policies.

Evaluating Environmental Justice presents substantial challenges due to the inherent selection of the location of productive activity and residential sorting decisions taken over a long period of time. These may lead to the spurious correlation between neighborhood demographics and the presence of a hazardous waste site. This papers takes a novel identification approach to evaluating Environmental Justice claims. We analyze separate milestones in the cleanup process conditional on a large set of site characteristics (both observable and unobservable) and investigate whether the resulting *duration* of cleanup was in any way influenced by the demographic characteristics of the affected population. Since cleanups take many years to complete, we expect neighborhood demographics to also change as a result of the cleanup process itself. We avoid this potential source of endogeneity by relating the duration of the cleanups to neighborhood demographics at the very beginning of the cleanup process. This allows us to treat the factors driving the cleanup process as pre-determined with respect to the cleanup duration.

Our identification strategy requires us to model the cleanup duration conditional on a large set of observed and unobserved site characteristics. In spite of the richness of our data, which describes the nature of the contamination at a given site in detail, it is not possible to account for all site specific features which may influence the duration of the cleanup process. We therefore rely on a state of the art econometric model that accounts for the presence of unobserved nonparametrically distributed site specific effects. This added flexibility helps diminish potential biases due to model misspecification.

We further evaluate the extent to which demographic biases may have changed over time and in particular the degree to which the 1994 legislative change, which emphasized Environmental Justice considerations, altered the way Superfund cleanups are conducted. We find that sites located in black, urban, lower educated communities were discriminated against at the beginning of the Superfund program in the early 1980s. The degree of bias does diminish over time though and the emphasis placed on Environmental Justice after 1994 lead to faster cleanup times for Superfund sites located in poor neighborhoods. After the cleanup is completed, the time to return a site to general use depends almost exclusively on the economic health of the neighborhood.

We also investigate whether the observed demographic or economic biases may in fact reflect different aspects of the bargaining process between the government, the responsible parties and the local community. We do not find evidence that the Superfund litigation process is delaying Superfund cleanups. We do however find that community involvement plays an important role in the cleanup duration.

Various aspects of Superfund sites have been under scrutiny in the previous academic literature. Environmental Justice concerns were initially introduced by a number of correlation based studies which documented the presence of a relationship between the location of hazardous waste sites and the demographic composition of the adjacent neighborhoods (United Church of Christ (UCC) 1987). While considerable disagreement exists regarding how best to define a neighborhood, a number studies have documented the presence of racial and income inequalities in the geographic location of Superfund sites (Stretsky and Hogan 1998, Smith 2009, Sigman and Stafford 2010). A related strand of the literature investigates the process through which hazardous waste sites are designated as Superfund sites and finds that sites located in communities with a higher percentage of minorities are less likely to be listed on the National Priorities List (NPL) thereby delaying the cleanup process (Anderton, Oakes, and Egan 1997). It is not clear however to what extent the resulting biases documented in both strands of the literature reflect actual biases or the influence of unobserved factors that initially determined the nonrandom distribution of production activity and hazardous waste location in the country. Wolverton (2009) shows that when plant locations are associated with current demographic characteristics, both race and income predict plant locations. However, when plant locations are associated with demographic characteristics at the time of the siting race is no longer a significant predictor.

Limited attention has been given to the duration of cleanup at Superfund sites. Beider (1994) uses a survey of EPA site managers to investigate the main reasons for the long cleanup durations and concludes that the primary reasons are the inherent difficulty of cleanup (i.e. the extent and nature of the contamination process) and the associated legal process which may involve many parties. Sigman (2001) is the only study we are aware of which employs a formal econometric model for Superfund cleanup durations. The paper finds that the extent of contamination and the nature of the liable parties explain the durations. However, higher income communities were found to have longer cleanup durations.

The benefits of cleanup are substantial. Currie, Greenstone, and Moretti (2011) report that Superfund cleanups reduce the incidence of congenital anomalies in newly born babies by up to 25%. In general though, it is difficult to quantify the cleanup influence on human health precisely and incorporate it in a traditional cost benefit analysis (Hamilton and Viscusi 1999). For example, measuring human health benefits in terms of the number of cancer cases avoided requires assumptions on any number of behavioral and environmental confounders over a life time. One of the difficulties also comes from the fact that we often have to rely on indirect approaches, e.g. by looking at the impact of Superfunds on the housing market, which may conflate the true benefit of Superfund cleanups with informational or reputational considerations (Gayer, Hamilton, and Viscusi 2000, Greenstone and Gallagher 2008).

This paper proceeds as follows. In Section 2 we discuss the cleanup process and distinguish between the various milestones in the cleanup of a Superfund site. Section 3 introduces the available data. We elaborate on our approach to identifying the presence of demographic biases that may be incongruent with Environmental Justice considerations. Since our identification strategy requires the estimation of a complex econometric model, we also discuss our estimation strategy in detail. Section 4 presents the main empirical results, while Section 5 explores the robustness of these results to alternative explanations based on the degree of bargaining power between the different parties involved in the cleanup process. In particular we investigate the role of litigation and community involvement activities. Section 6 concludes.

2. The Superfund Cleanup Process

Over the years policy makers have become increasingly aware of both the need to regulate dangerous substances and also to address the existing stock of hazardous waste sites. The most well-known effort to clean up hazardous waste sites, commonly known as Superfund, provides broad federal authority to the Environmental Protection Agency (EPA) to clean up or compel the responsible parties to clean up the most hazardous of these sites.

Waste is an inevitable part of the production process. The 2010 census counted more than 5.7 million firms and over 7.3 million establishments. It has been estimated that over 600,000 establishments are currently generating waste which can be classified as hazardous to human health (Sigman and Stafford 2010). This includes many types of substances which are known to be toxic, ignitable, radioactive, or in some other fashion present a real danger to the nearby population. In addition there are many hazardous waste sites resulting from production activity or inappropriate storage in past decades which resulted in soil and water contamination, such as abandoned factories and warehouses, landfills and military installations.

In this paper we explicitly focus on and model the durations of two main stages in the Superfund cleanup process, which we will briefly review.[3] To become a Superfund site, a hazardous waste site must go through an evaluation process. This process consists of discovery, evaluation, and nomination of contamination sites to the Superfund National Priorities List (NPL) as defined in the Comprehensive Environmental Remediation, Compensation, and Liabilities Act (CERCLA).

The Superfund process begins with the discovery of a Superfund site or notification to EPA of the possible release of hazardous substances. Site discovery can be initiated by a number of different parties, including citizens, businesses, State or local government and EPA regional

[3] More detailed information can be found on the EPA website: `http //www epa gov/superfund/`

offices. Once a site has been discovered, it is entered into the Comprehensive Environmental Response, Compensation, and Liability Information System (CERCLIS). The site is then evaluated to determine whether it meets the qualifications for listing on the NPL.

The first step in this evaluation is a Preliminary Assessment (PA) to determine if the site has the potential to qualify for the NPL. This is a limited screening investigation to distinguish sites that pose little to no potential threat. During this stage, readily available information about the site is collected. If it is determined that the site indeed poses little to no threat, then the process stops here. If instead the evaluation determines that the site may pose a threat to human health or the environment and therefore may qualify for the NPL, Site Inspection (SI) will commence. At this point environmental and waste related data is collected and analyzed. This data is then used to determine if the site qualifies for the NPL. The data will also be used to score the site based on the Hazard Ranking System (HRS). The HRS is a quantitative based tool to assess the relative degree of risk to the environment and human health by a potential or actual release of hazardous substances.

The proposal to list the site on the NPL and the HRS package is placed on the Federal Register. After a preliminary investigation, if the site is still found to qualify for NPL, then it will be placed on the NPL and the remedial process will begin. For our purpose we consider the NPL listing date as the initial starting point of the cleanup process.

Once a site is listed on the NPL the first stage of the cleanup process, the "remedial program" begins. First, a detailed examination of the site ensues which determines the precise nature of the contamination and the technical requirements for cleaning up the site. At this stage the EPA is required to solicit public opinion in the evaluation of the various cleanup options. Once this evaluation is completed a Record of Decision (ROD) is issued which describes the precise nature of the cleanup process to be implemented and the nature of the eventual cleanup target. After this, the various actions listed in the ROD commence and it will normally take years for the actions to be implemented. This is not unexpected given the technical challenges encountered in the process of removing the hazardous substances involved and containing or cleaning the contamination of surrounding soil and water. The *first milestone* in the cleanup process consists of the date when a site is labeled as "construction complete". This indicates that all physical or engineering tasks have been completed and both immediate and long term threats have been addressed. Note that construction complete does not mean that all threats have been neutralized and the cleanup goals have been achieved. For example, it is possible for the source of the contamination to have been completely removed but the surrounding media to remain toxic and thus not ready for being returned to general use.

The post construction complete phase may involve a number of different activities necessary for achieving the ultimate clean up goals. For example, ground water restoration may require prolonged ongoing treatment. Other hazardous sites may require ongoing monitoring and restricted access for many years after the engineering effort has ceased. This process is subject to regular reviews until it is determined that all cleanup goals have been met and no further action is required. At that point, the site reaches the *second milestone* in the cleanup process, when it is "deleted" from the NPL. Depending on the nature of the site it may then be reused or redeveloped for a new purpose.

In this paper we use two different measures of the cleanup durations in the analysis. Since the processes involved for reaching the two different milestones are different we expect each measure to be informative in its own right. Therefore, we do not restrict the model parameters across the duration types and estimate separate models for each duration. The durations are as follows: (1) the duration between a site being listed on NPL and the construction being completed at the site; (2) the duration between the construction being completed and the site being deleted from NPL.

3. Data and Empirical Strategy

In this paper we use data obtained from the EPA on all sites listed on NPL between 1983 and the end of 2010. In Figure 1 we plot the histograms for the two durations. Many of the observations are censored and this feature will need to be accounted for in the estimation. The mean values for the two durations are 13.8 years, and 9.0 years respectively. The first milestone is reached by most sites within 20 years. Sites for which the construction complete process has not been reached within 20 years are substantially more likely to be censored by the end of 2010. In contrast if the second milestone is reached, then it is reached for most sites within 5 years, indicating that the cleanup goals are achieved relatively soon after construction is completed. Nevertheless, for a substantial number of the sites this milestone is not reached indicating that only a fraction of the sites have been returned to general use so far.

For each site we observe its location and also a very comprehensive description of the form of contamination at that site. In particular we see the nature of the contaminated media (debris, groundwater, sediment, surface water, or waste) and the type of contaminants from acids to radioactive substances and volatile organic compounds (VOC). We believe this to be both an accurate and comprehensive description of the challenges encountered at the site and the degree of difficulty to clean it up. In particular note that many sites have both varied contaminated media and numerous contaminants that need to be addressed. The presence of a type of contaminated media or contaminant at a site is recorded in the form of an indicator variable. In Table 1 we report the means and standard deviations of the contaminants and contaminated media of all sites listed for each of the decades 1980s, 1990s, and 2000s. We notice a substantial degree of heterogeneity both across contaminants and decades. In particular the presence and extent of contamination appears to be decreasing over time. This is consistent with the notion that the most hazardous and challenging sites were detected in the early years of the Superfund program and that advances in regulation have reduced, although not eliminated, the occurrence of new hazardous waste. No new sites were listed during the 2000s that were contaminated with radioactive materials or where the contaminated media consisted of debris (often in the form of building remains contaminated with asbestos).

During the preliminary assessment and site inspection, each site is allocated an HRS score on the EPA Hazard Ranking System. The score is computed by aggregating along a number of different dimensions such as the characteristics of the waste (toxicity and quantity), the extent of hazardous waste released or expected to be released into the environment, the intensity with which people may be affected, and the degree to which ground water, surface water, soil and air have been exposed. The HRS score is designed to capture the nature of the site's hazard used to decide whether the site should be placed on the NPL. Note however, that according to the EPA[4] the HRS is *not* sufficient to prioritize the cleanup process at a Superfund site. In particular, given resource constraints, a high HRS score does not imply the reallocation of funds from existing cleanups already in process. Thus, while HRS is correlated with the degree to which a site is hazardous and it plays an important role in the placement of a site on the NPL, we expect it to be only weakly related to the cleanup duration itself. Table 1 reveals a small increase in the HRS scores at listing for sites over the three decades.[5]

A crucial component for determining the cleanup strategy consists in compiling the Record of Decision (ROD). The ROD presents details on the planned cleanup implementation. The costs recorded in the RODs are projected for the alternative selected from many possible options. These include capital costs, transaction costs, and operation and management costs. We have individually reviewed the RODs for all NPL sites and extracted from them a measure of the estimated present value of the cleanup costs at an assumed interest rate of 7%. Table 1 does not indicate any consistent trend in the costs associated with the cleanup process over time.

Note however that about 5% of the sites on NPL have a recorded costs of 0. In this case the selected alternative was "no further action". This could happen due to two possible reasons: (1) upon further consideration it was determined that there was no threat to human life or the environment, and (2) an immediate threat required removal action and by the time the rest of the procedures (everything up to the ROD) were completed, no further action was needed. We consider these sites to be different from other sites and assign them a separate indicator variable.

For each site we use the site location to obtain the population demographics in the zip code in which the Superfund site is located at the time of listing. We use the 1980 census to capture the demographics for a site listed between 1981 and 1989, and similarly for other decades. We record the median household income, and the fractions of the population which are college educated, black, and urban. Furthermore we record the fractions of the population by age. Table 1 shows that the demographic composition of the neighborhoods in which the hazardous sites were located varied with the time when the site was listed on the NPL. Sites listed earlier were more likely to be located in affluent, white neighborhoods, while sites listed later were more likely to be located in urban neighborhoods with a higher percentage of college educated

[4] `http://www.epa.gov/superfund/programs/npl_hrs/hrsint.htm`

[5] Note that this does not mean that sites listed later are more contaminated. A less contaminated site can have a larger score if the contamination presents a risk to a larger population.

residents.[6] Sites listed earlier were also more likely to be located in neighborhoods with younger residents. We condition on the event of the sites' NPL listing and control for their demographic characteristics.

The Superfund discovery is a distinct process beyond the scope of our analysis. Recall that earlier studies have found that sites located in neighborhoods with a higher percentage of minority residents seem less likely to be placed on the NPL (Anderton, Oakes, and Egan 1997). It would be difficult to convincingly model the discovery process itself, as the location and nature of the contamination were determined in most cases decades before the discovery process was initiated.

3.1. Identification

This paper focuses on evaluating the extent to which biases based on demographic characteristics such as income and race may be affecting the cleanup durations at Superfund sites in violation of the principles of Environmental Justice. The main assumption, which drives the identification of our model, is that the duration of cleanup is based purely on a rational cost-benefit analysis which depends on a wide range of site specific factors (both observed and unobserved by the econometrician) and a common baseline hazard which reflects macroeconomic trends and potentially the variation in the Superfund budget. Departures from the cost-benefit framework, e.g. in the form of faster cleanups observed in wealthier neighborhoods, indicate the presence of demographic biases. Our approach to identification is similar to that chosen by Viscusi and Hamilton (1999), who interpret departures from the cost-benefit analysis in the decisions taken by regulators regarding the chemical cleanup targets at Superfund sites as evidence of departures from rationality, behavioral biases, and risk misperceptions.

Our framework assumes that the following set of factors provides a comprehensive model explaining the durations between the different cleanup stages:

(1) the set of contaminants recorded at each site;
(2) the set of contaminated media at each site;
(3) the HRS score for each site;
(4) the engineering estimate of the cost of cleanup based on the original ROD;
(5) the time when a site was listed;
(6) information on the parties involved in litigation;
(7) information on the degree to which the community was involved in the cleanup decision process;
(8) an aggregate trend capturing the impact of the macroeconomy or the Superfund budget common to all sites;
(9) a site specific time invariant random effect.

[6] Note that in the tables for this paper we use the name Bachelor+ to refer to the percentage of the population which has obtained at least a BA degree.

In order to test for the presence of biases we augment this model with demographic variables *pre-determined* for each site at the time of listing. This avoids the potential endogenous feedback between the duration of the cleanup and subsequent demographic and environmental changes. In Table 2 we explore the extent to which neighborhood demographics change after a site is designated a Superfund site. Income, in particular, declines sharply during the cleanup period. This effect is not limited to the early years after a cleanup begins, which may be driven by residents leaving an area once they become aware of the presence of a Superfund site in their neighborhood. Median income continues to decline even after 20 years of cleanup activities. If we were to correlate the duration of cleanup with the change in the demographic composition of the neighborhood we would find a large negative correlation between the duration of cleanup and the change in median income. It would however be misleading to interpret this correlation as implying that wealthy neighborhoods are cleaned up faster, since it is likely that the composition of the a neighborhood changes as wealthier households leave a neighborhood with a Superfund site that is being cleaned up. Therefore, we only use the demographic composition of a site at the time of its listing to explain the subsequent cleanup duration. In order to test for possible violations of Environmental Justice we then test for the significance of the demographic variables at the time when a site was listed on NPL. This approach is similar to that used by Wolverton (2009) who investigates the relationship between firm locations and neighborhood demographics by focusing on the demographic composition of the neighborhood at the time when the location decision was made.

Our econometric model allows for the the cleanup duration to also depend on a site specific effect, which our Bayesian hierarchical model allows to be correlated with the observed site attributes. Our estimation procedure will estimate the distribution of these effects in the sample. The rationale behind including a site specific effect is that in spite of the richness of our data, which captures many of the observed site characteristics, it is nevertheless possible that not all features of the site which are relevant for the cleanup process have been recorded and which may lead to an omitted variables bias. Consider for example the period of time a site was contaminated before the cleanup process was initiated. We do not observe this in the data and it may be correlated with the severity of the contamination. Furthermore, sites that have been contaminated for a longer period of time may be inherently more difficult to clean up or may require more intensive and time consuming engineering processes. This variable may also be correlated with neighborhood characteristics, since the timing of the location could have been driven by the latter. Below we introduce the econometric model and its technical assumptions.

3.2. Econometric Model

In order to quantify the degree to which the duration of the cleanup process is biased by the demographic characteristics of the neighborhoods in which the Superfund sites are located, we develop a state of the art econometric model of the duration between the different milestones in the Superfund cleanup process. The model builds on the recent work of Burda, Harding, and

Hausman (2012) (BHH) who introduce a flexible semiparametric Bayesian proportional hazard duration model. The model allows for the presence of time variant or invariant observables but also models the baseline hazard and the site specific unobserved heterogeneity nonparametrically. While BHH devised their model for interval outcome data whereby only a general time period of the duration outcome was observed, here we alter their model to make use of the exact timing of the duration with point-in-time outcomes.

Denote by t_i the point in time elapsed when a site i was observed to exit from a given state into another state. Define the hazard rate $\square_{it}$ as the failure rate at time t conditional upon survival to time t, $\square_{it} = \lim_{\square\square 0} Pr(t < t_i < t + \square)/\square$ and denote the integrated hazard by:

$$\Lambda_{it} = \square_0^t \square_\square d\square \tag{3.1}$$

The survivor function S_{it} and the distribution function F_{it} of t are defined as

$$S_{it} = \exp(\square\Lambda_{it}) \tag{3.2}$$

$$F_{it} = 1 \square S_{it} \tag{3.3}$$

Hence, the conditional density function of exit at t is given by

$$\begin{aligned} f_{it} &= F_{it}^{\square} \\ &= \square S_{it}^{\square} \\ &= \exp(\square\Lambda_{it})\square_{it} \\ &= S_{it}\square_{it} \end{aligned} \tag{3.4}$$

which forms the contribution to the conditional likelihood function for non-censored data. For observations censored at time T, all we know under non-informative censoring is that the lifetime exceeds T. The probability of this event, and therefore its contribution to the likelihood is

$$\begin{aligned} P(t_i > T) &= 1 \square F_{iT} \\ &= S_{iT} \end{aligned} \tag{3.5}$$

The likelihood terms (3.4) and (3.5) can be written as the single expression

$$L_i(t_i) = S_{it}\square_{it}^{d_i} \tag{3.6}$$

where d_i is a censoring indicator variable taking the value of 1 if $t_i \square T$, or the value of 0 if $t_i > T$, in which case t_i is set to equal T in (3.6).

ASSUMPTION (1). *The data $\{t_i\}_{i=1}^N$ consists of single spells censored at time T and drawn from a single risk process.*

ASSUMPTION (2). *The hazard rate is parameterized as*

$$\square_{it} = \square_{0t}\exp(X_{it}\square + V_i) \tag{3.7}$$

where $\square_{0t}$ is the baseline hazard, X_{it} are observed covariates that are allowed to vary over time, $\square$ are model parameters, and V_i is an unobserved heterogeneity component.

ASSUMPTION (3). *The baseline hazard* $\sqcup_{0t}$ *and the values of the covariates* X_{it} *are constant over time intervals* $[t_{j\sqcap 1}, t_j)$ *for* $j = 1, \ldots, J$.

Assumptions 1 and 2 are common in the literature. Assumption 3 is based on Han and Hausman (1990). Given Assumption 3, we can consider the integrated baseline hazard in the form

(3.8) $$\sqcap_{0j} = \square_{t_{j-1}}^{t_j} \sqcap_{0\square} d\sqcap$$

where we denote the vector $(\square_{01}, \ldots, \square_{0J})$ by $\square_0$.

Denote by $[\underline{t_i}, \overline{t_i}) \sqcap t_i$ the time interval during which i's exit occurred, with endpoints $\underline{t_i} \sqcap \{t_0, \ldots, t_{J\sqcap 1}\}$ and $\overline{t_i} \sqcap \{t_1, \ldots, t_J\}$ with $\{t_j\}_{j=0}^{J}$ as defined in Assumption 3. Define the variable

(3.9) $$\sqcap_{ij} = \begin{cases} 1 \text{ if } t_i \not\sqcup [t_{j\sqcup 1}, t_j) \\ t_i \sqcup \underline{t_i} / (\overline{t_i} \sqcup \underline{t_i}) \text{ if } t_i \sqcup [t_{j\sqcap 1}, t_j) = [\underline{t_i}, \overline{t_i}) \end{cases}$$

Using Assumptions 1–3 and the notation in (3.9), the conditional likelihood (3.6) can now be rewritten as

(3.10) $$L_i(t_i; V_i) = \exp \sqcup \sqcap \sum_{j=1}^{\square_{\overline{i}}} \sqcap_{ij} \sqcap_{0j} \exp(X_{ij}\sqcap + V_i) \sqcup \square \sqcup\sqcup \sqcap_{0\overline{t_i}} / (\overline{t_i} \sqcap \underline{t_i}) \sqcup \exp(X_{i\overline{t_i}}\sqcap + V_i) \sqcup^{d_i}$$

3.3. Parametric Heterogeneity

ASSUMPTION (4). *Let*

$$v_i \square \exp(V_i) \square G(v)$$

where $G(v)$ *is a probability distribution function with density* $g(v)$.

Using Assumption 4, denote by tilde the part of the hazard without the heterogeneity term:

(3.11) $$\sqcup_{ij} = v_i \tilde{\square}_{ij}$$

where, from Assumption 3 and (3.7),

$$\tilde{\square}_{ij} = (\sqcap_{0j} / (t_j \sqcap t_{j\sqcap 1})) \exp(X_{ij}\sqcap)$$

Hence, at the time of exit t_i,

(3.12) $$\tilde{\square}_{it_i} = \square_{0\overline{t_i}} / (\overline{t_i} \square \underline{t_i}) \square \exp(X_{i\overline{t_i}}\square)$$

Similarly, using Assumption 4, let

(3.13) $$\Lambda_{it} = v_i \tilde{\Lambda}_{it}$$

where, from (3.1),

$$\tilde{\Lambda}_{it} = \sqcap_0^t \square_{0\sqcap} \exp(X_{i\sqcap}\square) d\square$$

Due to Assumption 3, (3.8), (3.9), and (3.13), at the time of exit t_i,

$$\bar{\Lambda}_{it_i} = \sum_{j=1}^{\square t_i} \square_j \, (\square_{0j} / (t_j \square t_{j \square 1})) \exp (X_{ij} \square) \tag{3.14}$$

If v is a random variable with probability density function $g(v)$ then the Laplace transform of $g(v)$ evaluated at $s \sqcup R$ is defined as

$$\begin{aligned} L(s) \ \sqcap \ \overset{\square}{} \ \exp(\sqcap vs)g(v)dv \\ = \ E_v|\exp(\square vs)] \end{aligned} \tag{3.15}$$

and its r-th derivative is

$$L^{(r)}(s) \square (\square 1)^r \ \overset{\sqcap}{} \ v^r \exp(\square vs)g(v)dv \tag{3.16}$$

Using (3.11), (3.13), and (3.15), the expectation of the survival function can be linked to the Laplace transform of the integrated hazard function (Hougaard, 2000) as

$$E_v\,|S_{it}] = L(\bar{\Lambda}_{it}) \tag{3.17}$$

which forms the expected likelihood for censored observations.

For uncensored observations, collecting (3.11), (3.12), (3.13), and (3.14) in (3.10), yields

$$L_i(t_i; V_i, d_i = 1) = \exp \ \overset{\square}{} \sqcup v_i \bar{\Lambda}_{it_i} \ \overset{\square}{} v_i \square_{t_i}$$

Taking expectations and using (3.16) we-obtain

$$\begin{aligned} E_{V_i}\,|L_i(t_i; V_i)] \ &= \ E_{V_i} \ \overset{\square}{} \exp \ \overset{\square}{} \sqcup v_i \bar{\Lambda}_{it_i} \ \overset{\square}{} v_i \square_{t_i} \overset{\square}{} \\ &= \ \square_{t_i} E_{V_i} \ \overset{\square}{} \exp \ \overset{\square}{} \sqcup v_i \bar{\Lambda}_{it_i} \ \overset{\square}{} v_i \overset{\square}{} \\ &= \ \square \square_{t_i} L^{(1)}(\bar{\Lambda}_{it_i}) \end{aligned} \tag{3.18}$$

The expected likelihood terms (3.17) and (3.18) are summarized in the following Result:

RESULT 1. *The expectation of the likelihood (3.6) with respect to unobserved heterogeneity, distributed according to a generic probability measure as given by Assumption 4, is for uncensored observations*

$$E_{V_i}\,[L_i(t_i; V_i, d_i = 1)] = \square \square_{t_i} L^{(1)}(\bar{\Lambda}_{it_i}) \tag{3.19}$$

and for censored observations

$$E_{V_i}\,[L_i(t_i; V_i, d_i = 0)] = L(\bar{\Lambda}_{iT}) \tag{3.20}$$

Since the site heterogeneity term v_i defined in Assumption 4 is non-negative, a suitable family of distributions $G(v)$ with support over $[0, \square)$ and tractable closed-form Laplace transforms is Generalized Inverse Gaussian (GIG) class of distributions, whose special case, among others, is the gamma distribution popular in duration analysis.

ASSUMPTION (5). *The unobserved heterogeneity term* v_i *is distributed according to the Generalized Inverse Gaussian distribution,*

$$G(v) = G^{GIG}(v; \square, \square, \square)$$

The GIG has the density

$$g^{GIG}(v; \square, \square, \square) = \frac{2^{\square\square 1}}{K_{\square}(\square)} \frac{\square}{\square^{\square}} (\square v)^{\square\square 1} \exp \overset{\square}{\square \square v \square} \frac{\square^2}{4\square v} \overset{\square}{} \quad (3.21)$$

for $\square, \square > 0$, $\square \square R$, where $K_{\square}(\square)$ is the modified Bessel function of the second kind of order $\square$ evaluated at $\square$ (Hougaard, 2000). The GIG Laplace transform is given by

$$L^{GIG}(s; \square, \square, \square) = (1 + s/\square)^{\square\square/2} \frac{K_{\square} \overset{\square}{\square (1 + s/\square)^{1/2}} \overset{\square}{}}{K_{\square}(\square)} \quad (3.22)$$

and its derivatives by

$$L^{(r)GIG}(s) = (\square 1)^r \frac{K_{\square + r} \overset{\square}{\square (1 + s/\square)^{1/2}} \overset{\square}{}}{K_{\square}(\square)} \overset{\square}{} \frac{\square}{2\square} \overset{\square}{} (1 + s/\square)^{\square(\square + r)/2} \quad (3.23)$$

The GIG family includes as special cases the gamma distribution for $\square = 0$, the Inverse gamma distribution for $\square = 0$, and the Inverse Gaussian distribution for $\square = \square\frac{1}{2}$, among others.

Application of the Laplace transform of the GIG distribution (3.22) and its derivatives (3.23) in Result 1 yields the following result:

RESULT 2. *Under the Assumptions 1 5,*

$$E_{v_i}[L_i(t_i; V_i, d_i = 1)] = \frac{\square \square_{t_i}}{2 \; \square} \overset{\square}{} 1 + \frac{\square_{t_i}}{\square} \overset{\square_{\square(\square+1)/2}}{} [K_{\square}(\square)]^{\square 1} K_{\square+1} \overset{\square}{\square} \square \; 1 + \frac{\square_{t_i}}{\square} \overset{\square}{} \overset{\square_{1/2}}{} \square \quad (3.24)$$

and for the censored observations

$$E_{v_i}[L_i(t_i; V_i, d_i = 0)] = \overset{\square}{} 1 + \frac{\square_{iT}}{\square} \overset{\square_{\square\square/2}}{} [K_{\square}(\square)]^{\square 1} K_{\square} \overset{\square}{\square} \square \; 1 + \frac{\square_{iT}}{\square} \overset{\square}{} \overset{\square_{1/2}}{} \square \quad (3.25)$$

A special case of the GIG distribution is the gamma distribution, obtained from the GIG density function (3.21) when $\square = 0$ and $\square$ is restricted to the positive part of the real line.

The scale parameter $\square$ has the feature that for any $c \square R_+$, if $v \square G^{GIG}(v; \square, \square, \square)$ then $cv \square G^{GIG}(v; \square, \square, \square/c)$. Due to this property, c and hence its inverse $s \square c^{\square 1}$ are not separately identified from $\square$ in the Laplace transform (3.22). Since all likelihood expressions are evaluated at $s = \bar{\Lambda}_{it}$ which is proportional to $\square_{0j}$ for all j, as specified in (3.8), any change in □ only rescales the baseline hazard parameters $\square_{0j}$, leaving the likelihood unchanged. Hence, □ needs to be normalized to identify $\square_{0j}$ by the moment restriction $E[v] = 1$.

3.4. Flexible Heterogeneity

We now depart from the parametric form of the unobserved heterogeneity and instead consider a nonparametric infinite mixture for the distribution of v_i, as formulated in the following assumption.

ASSUMPTION (6). *The prior for* v_i *takes the form of the hierarchical model*

$$\begin{aligned} t_i &\;\square\; F(v_i) \\ v_i|G &\;\square\; G \\ G &\;\square\; DP(G_0, \square) \\ \square &\;\square\; \Gamma(a_0, b_0) \\ E[v_i] &= 1 \end{aligned}$$

In Assumption 6, G is a random probability measure distributed according to a Dirichlet Process (DP) prior (Hirano, 2002; Chib and Hamilton, 2002). The DP prior is indexed by two hyperparameters: a so-called baseline distribution G_0 that defines the "location" of the DP prior, and a positive scalar precision parameter □. The distribution G_0 may be viewed as the prior that would be used in a typical parametric analysis. The flexibility of the DP mixture model environment stems from allowing G to stochastically deviate from G_0. The precision parameter □ determines the concentration of the prior for G around the DP prior location G_0 and thus measures the strength of belief in G_0. For large values of □, a sampled G is very likely to be close to G_0, and vice versa. Assumption 6 is then completed by specifying the baseline measure G_0 as follows:

ASSUMPTION (7). *In Assumption 6,*

$$G_0 = G^{GIG}(\square, \square, \square) \tag{3.26}$$

Implementation of the GIG mixture model under Assumptions 1 3, 6, and 7 uses the probabilities (3.6), (3.24) and (3.25).

Under Assumptions 6 and 7, as a special limit case, putting all the prior probability on the baseline distribution G_0 by setting □ □ □ would result in forcing $G = G_0 = G^{GIG}(v; \square, \square, \square)$ which yields a parametric model. Here we allow □ and hence G to vary stochastically and the parametric benchmark specification is nested as a special case in our model.

3.5. Marginal Effects

One of the challenges of interpreting the economic significance of the empirical results lies in the inherent difficulty of computing marginal effects in this highly non-linear setting. Thus, while the estimated coefficients correctly capture the sign of the effect of interest it is non-trivial to translate the magnitude into an easily interpretable quantity. While we follow the established statistical practice of reporting the estimated coefficients, we also go a step further and use a simulation based approach to computing the economic significance of the statistically significant coefficients that are likely to be of particular interest to the reader.

There is no unique way of computing marginal effects in this type of non-linear model. We choose a simulation based approach which computes the average marginal effects for a discrete change in the variable of interest over the sample and using a large number of repeated draws from the distribution of unobserved heterogeneity. The economic significance of a coefficient is most easily interpretable in terms of time and we thus report the impact of a discrete change in the variable of interest as a fraction or multiple of 1 year of additional cleanup.

The expectation of a non-negative random variable t truncated at T is given by

$$E\,[t|t \square T] = \frac{1}{F(T)} \square_{0}^{T} tf(t)dt \tag{3.27}$$

where $f(t)$ and $F(t)$ are pdf and cdf of t, respectively.

In our model where t denotes duration to cleanup,

$$\begin{aligned} f_i(t) &= \exp(\square\Lambda_i(t))\,\square_i(t) \\ F_i(t) &= 1 \square \exp(\square\Lambda_i(t)) \end{aligned}$$

Under the assumption of piece-wise constant baseline and covariates over time

$$E\,[t|t \square T] = \frac{1}{1 \square \exp(\square\Lambda_{iT})} \square_{j=1}^{T} j \exp(\square\Lambda_{ij})\,\square_j \tag{3.28}$$

where

$$\begin{aligned} \square_j &= \square_{0j} \exp(X_{ij}\square + V_i) \\ \Lambda_{ij} &= \square_{s=1}^{j} \square_s \end{aligned}$$

is the hazard and cumulative hazard, respectively.

For the effect of a □change of X_{ijk} on $E\,[t|t \square T]$, we evaluate

$$\Delta E\,[t|t \square T] = E\,[t|t \square T, X_{ijk} + \square] \square E\,[t|t \square T, X_{ijk}]. \tag{3.29}$$

Since, the choice of □is arbitrary for continuous variables, we follow Sigman (2001) and simulate the economic significance of a change of one standard deviation in the relevant covariate. In accordance with the censoring time of our observations, we truncate the simulated distribution at the latest date available in the sample. In the text below we report the economic significance of the main variables when describing the empirical results. More detailed tables are available from the authors.

4. Empirical Findings

Our current empirical framework allows us to investigate a number of important hypotheses regarding the main factors driving the cleanup durations between the two milestones in the cleanup process. It is important to note that throughout our identification strategy rules out the impact of sorting on the cleanup process. We then proceed to measure the extent to which the cleanup process was driven by cost-benefit factors associated with the engineering decisions regarding the technological aspects of the cleanup.

To the extent that demographic variables remain significant drivers of the cleanup durations we then proceed to investigate whether this reflects some form of direct discrimination or is perhaps a more indirect form of discrimination resulting from the differential bargaining ability of the different agents involved in the cleanup. In particular we differentiate between:

(1) the role of the legal system and the bargaining power of the responsible parties,
(2) the impact of general collective action as proxied by home ownership in the community,
(3) a direct measure of Superfund related involvement as measured by EPA reported community activities.

4.1. Cleanup Durations

We consider a series of model specifications designed to estimate the factors determining the two cleanup durations of interest: the duration between listing and construction completion (LC), and the duration between construction completion and deletion (CD) from the NPL list. Recall that listing refers to the time when a site is listed on NPL, completion refers to the time when the remedial process has been completed, and deletion refers to the time when the site is removed from NPL and returned to general use. These models capture our baseline identification approach and are developed to test a number of hypotheses of interest.

Our aim is to control for site characteristics (both observable and unobservable) and also for the demographic characteristics of the households potentially impacted by the site. Under our identification assumption we expect the presence of statistically significant coefficients on the demographic characteristics to be indicative of biases potentially incompatible with Environmental Justice considerations. In all specifications we model the conditional hazard rate for each site, yielding the probability that a site reaches the next milestone in the cleanup process. An estimated *negative* coefficient implies a lower probability of reaching the next milestone and a *slower* cleanup (longer cleanup duration).

In Table 3 we first estimate a simple duration model without unobserved heterogeneity which relates the two durations of interest, LC and CD, to neighborhood demographics only. These models are misspecified as a result of omitting a number of potentially important explanatory variables. Here, neighborhood demographics are strong predictors of the cleanup durations. Higher income, unemployment and the fraction of the population which is black are all associated with slower cleanup times. We then add observable site characteristics to the specification. These include engineering cost estimates and the description of the contaminants and contaminated media. If we now re-evaluate the relevance of the neighborhood demographics we find that their impact has been greatly diminished and most coefficients on the demographic variables become statistically insignificant.

In Table 4 we estimate the specification with both neighborhood demographics and observed site characteristics (columns 5-8 in Table 3) while also allowing for the presence of unobserved site specific effects. For each duration we estimate the corresponding model allowing for either a parametric specification or a nonparametric specification of the unobserved heterogeneity. While there are some noticeable differences, the models are comparable. From an econometric perspective, we consider the nonparametric model to be superior to the parametric one, in that the former nests the latter as a special case which may or may not be supported by the data evidence. This implies that in the nonparametric model the coefficient estimates of the demographic characteristics are likely to be more accurate and less confounded by the presence of unobserved site specific factors. Therefore, in all other tables, we will only report estimates derived from the nonparametric model.

First, consider the baseline model for the duration between listing and completion in Table 4. The impact of the HRS score is small, negative, and statistically significant. This is consistent with the EPA strategy of using the HRS scores to determine whether a site should be listed on NPL but not using the HRS scores directly to prioritize the cleanup activities, even though it reflects the extent to which a site is hazardous. The engineering cost estimates for the clean-up constitute a large and significant LC duration predictor. These costs are determined by the choice of remedy adopted and proxy for the complexity of the engineering process involved. We also include in our model an indicator for sites who have zero cost recorded in the documents available from the EPA. These are sites that were considered a priority and the cleanup was initiated immediately before an ROD was compiled because of the imminent danger to the population and the environment. The coefficient on this variable is an order of magnitude larger the one on the cost variable, reflecting the severity of the contamination at these sites.

The nature of contamination and the inherent technical difficulties involved in the cleanup process are major determinants of the cleanup durations. As we would expect sites containing metals, radioactive or PCB waste take longer to clean up. The contaminated media also represent an important factor. Sites where the waste takes the form of debris or waste which can be easily removed are much faster to clean up than sites where the sediment or soil is contaminated.

When considering the demographic variables we do not find statistical evidence that sites in minority neighborhood or low income neighborhoods are cleaned up slower. In fact we find that sites in wealthier neighborhoods are cleaned up slower but that sites located in neighborhoods with a large fraction of the population over 65 are cleaned up faster. In general we expect both wealthier and retired people to be more actively engaged in the construction decision process. Their incentives will vary however. Wealthy households are likely to prefer a comprehensive remedial process which will safeguard house prices by implementing more detailed and costly engineering approaches. On the other hand, older retired households may prefer a fast remedial process.

Let us now consider the corresponding models for the duration between completion and deletion. Sites with higher cleanup costs have longer durations. Contamination with metals, pesticides, and VOC impose additional challenges and extend the period it takes for the EPA to release a site for general use. Sites with contaminated groundwater are particularly challenging to clean and return back to the community, and increase the duration to be deleted from the NPL.

We do not find biases associated with either income, race or education, or the fraction of children. However, we find that the fraction of residents in the neighborhood which is unemployed is a large negative predictor for the duration to deletion, as is the fraction of college educated individuals. Contaminated sites in areas suffering from high unemployment are thus less likely to be returned to general use and may linger on contaminated for quite some time. This reflects the possibility that in already economically depressed areas re-purposing a past Superfund site is not easily accomplished.

The baseline hazard is estimated as a flexible partial linear function in all models but is not reported in the tables due to space limitations. In all models for LC durations we have found the baseline hazard to be monotonically increasing which is consistent with the cleanup following a well-defined process driven by engineering milestones. The baseline hazard for the CD durations however is estimated to be non-monotonic reflecting the fact that after the construction is completed the site undergoes regular but not continuous reviews to determine progress and whether it can be returned to general use. In Section 5.3 we discuss how the estimated unobserved site specific heterogeneity can be interpreted and what insights we can gain from it.

4.2. Time of Listing

One important consideration is the fact that the timing of the discovery of Superfund sites is not random. It is thus possible that Superfund sites may spuriously correlate to neighborhood characteristics in virtue of the time when they were listed unless we also control for the year of listing. In Table 5 we present estimation results from models for the two durations of interest that also control for the year of listing.

We find that this virtually does not change the impact of the engineering characteristics of the site such as the cost, contamination type, and contaminated media. We do see, however, some changes in the estimated effect of the demographic features of the neighborhood. When considering the LC duration, we continue to find that neighborhoods with a larger proportion of the population over 65 are cleaned up faster but the relationship to income becomes statistically insignificant. For the CD duration, we continue to find that sites located in areas with high unemployment take longer to be released for general use. We now also find a small negative impact of income.

It is rather surprising that in the above specifications the relationship between income and the two durations of interest is sensitive to the inclusion of the controls for the time of listing. This may indicate that the relationship itself is time varying and requires additional model specifications.

In the history of Superfund there are two distinct periods in the development of the program itself that need to be considered. They are separated by a very important milestone in the development of the Superfund program, Executive Order 12898, "Federal Actions to Address Environmental Justice in Minority Populations and Low-Income Populations", signed by President Bill Clinton in February 1994, which directed the attention of federal agencies to issues of environmental equity. In particular it explicitly focuses on the problems faced by low income and minority populations living near a Superfund site.

We explore the effect of the 1994 policy change by interacting an indicator variable capturing the period of listing 1994-2010 with all the demographic variables used in the model (in addition to controlling for the time of listing). If the Executive Order did not change the prioritization of cleanup procedures, we would not expect the interaction terms to be statistically significant. We present the results for the two durations of interest LC and CD in Table 5. For the LC duration, neighborhoods with a high proportion of residents over 65 continue to be cleaned up faster overall, but now it is also the case that sites located in low income areas and areas with high unemployment are cleaned up faster after 1994 than before that year. These large negative coefficients for median income and unemployment indicate that after 1994 the prioritization of resources was effectively directed towards speeding up the cleanups in economically depressed neighborhoods. It is interesting to note that areas with highly educated residents also experience a faster cleanup after 1994. The 1994 policy change also included provisions for greater transparency and community involvement, which seems to be reflected in the faster cleanup durations.

In contrast, the results for the CD duration do not change much with the inclusion of the interaction between the demographics and the post-1994 period, indicating that the policy change had a much smaller impact on the process that leads to a site being deleted from the NPL list. We continue to find that the primary demographic driver is whether a site is located in an economically depressed neighborhood.

Another important feature of the Superfund NPL listing timeline is the distinction between the first listing wave in 1983 and sites that were listed after that year. The initial Superfund site discovery process started already in 1980 but the discovered sites were only listed upon the official launch of the cleanup program in 1983. Beider (1994) interviews site managers who argue that the sites that were initially listed on the NPL were quintessentially different than sites listed in later years and presented a number of technical challenges that had to be overcome which affected the cleanup duration. We therefore split the sample into sites that were listed in 1983 and sites that were listed after that year. We estimate separate models for each split sample for both the LC and CD durations. The estimated coefficients are presented in Tables 6 and 7.

First, consider the results for the LC duration for sites listed in the first wave in 1983. It is particularly notable that the nature of contamination does not appear to drive the durations at all. The only exception consists of sites with contaminated sediment which take longer to clean up. At the same time, the impact of the demographic variables is large and significant. Sites with a large share of urban and black population take much longer to be cleaned up while sites with a highly educated population are cleaned up faster. In contrast, when we consider the sites listed after 1983, it appears that their cleanup duration is driven largely by costs and the nature of the contamination and not by the demographic characteristics. Sites located in neighborhoods with a larger share of the population over 65 are cleaned up faster (although the coefficient is not significant in these specifications). If we now consider the CD duration, we find that for both sites listed before and after 1983, the single largest determinant of the duration is the economic health of the neighborhood as measured by the fraction of the population which is unemployed. For sites listed more recently, the fraction of the population under 18 also seems to be a significant driver for speeding up the release of the site for general use. In both cases contaminated groundwater is a major delay factor.

4.3. Economic Significance

The models estimated above reveal that the factors identified to be statistically significant in driving the durations between the different milestones in the cleanup process are also economically very significant. We use the methodology described in Section 3.5 to quantify the economic significance of a discrete change in a variable of interest and determine what the implied counterfactual change in the expected cleanup duration is. A one standard deviation increase in the expected cost of a cleanup increases the LC cleanup duration by 4.8 years. Similarly the effects of the contaminants and contaminated media are also very significant. The presence of metal increases the LC duration by 1.4 years while the presence of radioactive substances increases the duration by 5.1 years. The CD duration is somewhat less determined by cost and contaminants. A one standard deviation increase in the expected cost increases the CD duration by 1.8 years. The contaminated media is however much more important. Contaminated groundwater increases the CD duration by an average of 5.3 years.

The impact of the demographics is also substantial. An increase in one standard deviation in the fraction of the population over 65 reduces the LC duration by approximately 8 months. In contrast a one standard deviation in the fraction of the population that is unemployed delays deletion from the NPL list by an average of 3.6 years.

After 1994 we see that sites located in neighborhoods which are one standard deviation poorer reach the construction complete milestone 1.2 years faster. Similarly, sites in neighborhoods with higher unemployment also have an LC duration which is 2-3 months shorter. In contrast we measure that sites listed in 1983 reached the first cleanup milestone 5.7 years sooner in more educated neighborhoods but 2.5 years later in areas with a higher black population. This confirms that the economic significance of the observed demographic discrimination during the initial phase of the Superfund program was quite substantial.

5. The Role of Bargaining Power

To the extent that we found that cleanup durations are a function of community characteristics, it is important to assess whether the estimated effects are a result of policy bias in terms of the implementation of cleanup activities or whether they result from the differential use of bargaining power by the parties involved in the cleanup (including the community). The first possibility would be an indicator of direct discrimination based on neighborhood demographics, while the second might reflect the extent to which different parties are involved in the process itself while the degree of involvement may correlate with the demographic characteristics. From an econometric perspective, if the demographic variables are really capturing the degree to which the parties influence the cleanup process, we would expect that once we control for proxies describing the involvement of the different parties, the effect of the demographics will diminish.

Below we consider two measures of involvement. One characterizes the litigation process associated with the cleanup, and the other measures the extent to which the communities were actively involved in deciding the course of the cleanup.

5.1. Litigation

The EPA searches for the Principal Responsible Parties (PRP) associated with a Superfund site as a part of the litigation process. Following a letter of determination these parties are asked to contribute financially to the cleanup. It is important to note that in many cases no such responsible parties can be found. This is generally because the associated entities no longer exist, such as companies that dumped hazardous waste but have since been dissolved. If the parties refuse to pay, legal action will be initiated.

While the EPA list of PRPs is available, it is not possible to find out detailed information about these companies in a comprehensive fashion. Most of the parties are quite small and no longer exist. Thus, they are not tracked by databases such as Bloomberg or Compustat. With these limitations in mind we create indicator variables for the case where no PRP exists for a site (PRP 0), where the number of parties is between 2-10 (PRP 2-10), and the case where the number of parties is greater than 10 (PRP 10+). These provide a rough approximation of the liability share of each party which will then impact the subsequent litigation and potentially cleanup duration.

In Table 8 we show the coefficient estimates for both the baseline model and the model with year of listing indicators for both the LC and CD durations where we add the PRP indicators. We find that sites with more than 10 PRPs experience faster construction completion times but that the number of PRPs does not influence the time it takes to return the site to general use. Since litigation happens at the beginning of the cleanup process, it makes sense for the litigation process to only affect the LC but not the CD durations.

At first glance it may seem counterintuitive, that a larger number PRPs is associated with shorter cleanup durations. This is consistent with the existing literature on Superfund litigation though, which suggests that the existence of multiple parties does improve the odds of settlement thereby reducing the length of the litigation process and reducing the LC duration (Rausser and Simon 1998, Sigman 1998, Chang and Sigman 2000). The intuition is that it is easier to obtain settlements from litigation with many small parties than one large corporation which can sustain a prolonged court battle. When sites have a small number of PRPs, it usually indicates that the site is owned by a large corporation. In such a case, as earlier studies have shown, the large corporation has an incentive to minimize its liability and require lengthy reviews, thereby delaying the cleanup process. Furthermore, the presence of many PRPs can also be associated with mostly local entities who may have a more direct concern or benefit from the the cleanup. The impact of the joint liability framework is also economically significant. Sites where the number of PRPs is larger than 10 complete the LC duration an average of 2 years earlier.

Concerning our main hypothesis, we seek to assess whether the observed demographic biases reflect policy biases or are driven by the extent to which neighborhoods with different demographic characteristics are also host to different types of businesses. Since the litigation process involves the PRPs operating in that community, delays due to the litigation process may be falsely attributed to neighborhood characteristics. Table 8 however reveals that this is not the case. The coefficients on the demographic variables do not change much with the addition of the PRP variables.

5.2. Community Involvement

While we do not have a direct measure of the extent to which a community is concerned about the timing and nature of the cleanup of a local Superfund site, we do attempt to proxy for community involvement in two different ways. First, we investigate whether the fraction of home ownership in the community impacts the cleanup durations. We report the estimated coefficients in Table 9. While home ownership does not have a significant effect on the LC duration it does increase the probability that a site is deleted from the NPL list substantially. A one standard deviation increase in the proportion of homeowners reduces the CD duration by almost 4 years. We note moreover that adding home ownership to the model leads to a small decline of the effect of the percent of the population over 65 on the CD duration, but since it is highly correlated with the percent of the population which is unemployed it removes the statistical significance of the latter variable in the CD specification. This makes it difficult to interpret home ownership as a proxy for community involvement. While the results appear to suggest that homeowners are somewhat more likely to be involved in the cleanup process, it is also likely that the variable captures an aspect of the economic vibrancy of a community.

Second, we evaluate the extent to which the community was involved in the cleanup decision process as recorded by the EPA. This involvement can happen at any point in the process but does require coordination with the EPA site manager. Community involvement can take many forms of dialogue between the EPA and the public such as public meetings. The data does not record precise details on the process of community involvement, but it does report whether community relations activities were conducted to address concerns raised by the local community.

Using the available data, we construct a site specific indicator which records whether the community was involved in the cleanup process. Since Executive Order 12898 placed a much heavier emphasis on community involvement as part of its requirement to promote Environmental Justice, we also create an indicator variable which captures whether community relations activities were performed for sites listed after 1994.

In Table 10, we report results for both the baseline model and the model with year of listing indicators for both the LC and CD durations. We add the above indicators for community involvement and find that community involvement is a significant predictor of shorter LC durations, but not for CD durations in the models which account for the year of listing. Moreover, the magnitude of this effect is several times larger after 1994. This reflects the extent to which community involvement was made a policy priority after 1994. At the same time, for the LC duration model which controls for time of listing, we see that adding controls for community involvement removes the statistical significance of the demographic variables. The coefficient on the fraction of the population over 65 is reduced from 3.234 to 1.424 and becomes statistically insignificant. We do not find a corresponding effect for community involvement on the CD duration. The impact of community involvement is economically significant. Before 1994 we estimate that sites with active community involvement completed the LC duration on average one year earlier than sites without community involvement. After 1994 sites with community involvement activities reached the first cleanup milestone on average of 5.4 years sooner.

This indicates that community involvement plays an important role in explaining the heterogeneity between cleanup durations, even after accounting for technical factors related to the nature and extent of the contamination. It is difficult to interpret this finding causally, however, since community activities are often initiated by the EPA site manager. Thus, while it is certainly probable that communities with a population over 65 are more likely to be engaged in the cleanup process and participate in community events, we cannot exclude the possibility that at least some neighborhoods were discriminated against by not engaging the local community in the cleanup process. The analysis seems to confirm this view by finding a much larger impact of community involvement after 1994, when Environmental Justice considerations prioritized community involvement in the cleanup process.

5.3. Unobserved Site Heterogeneity

Figure 2 shows the nonparametric estimate of the unobserved site heterogeneity estimated from each of the baseline models corresponding to the two durations of interest. The density estimate indicates that the distribution of heterogeneity can be characterized by two modes and a thick right tail. Thus, a small number of sites corresponding to heterogeneity estimates close to zero suffer from conditions which slow down the clean up process. At the other extreme, there is a substantial number of sites that benefit from additional unobserved factors that speed up the cleanup process.

The estimated unobserved individual heterogeneity of Superfund sites can be interpreted as a factor which also contributes to the variation in the cleanup or deletion duration but is not included among the observable explanatory variables. The heterogeneity term thus acts as another explanatory variable in itself, albeit not directly measured but rather inferred indirectly from the model. The distribution of heterogeneity across all sites is normalized to have mean one, reflecting the multiplicative way in which it enters the hazard model parameterization. Its influence is exhibited as deviations beyond the mean effects captured by the measured observables and the baseline hazard parameters. Heterogeneity is thus essentially estimated as explaining the deviations of durations from the mean model prediction once the effect of the observables has been accounted for. We do not constrain the distribution of heterogeneity to any specific parametric shape, but rather endow it with a flexible nonparametric model in order to mitigate any potential model misspecification biases. At the same time, under the Bayesian hierarchical model framework, the distribution of unobserved heterogeneity is allowed to be correlated with the observed explanatory variables. An analysis of this correlation pattern may indicate the source of heterogeneity.

In a post-estimation analysis, we investigate the extent to which the estimated heterogeneity at the individual site level correlates with the site and neighborhood characteristics by regressing individual heterogeneity on the full set of covariates for each type of duration. Statistically significant partial correlation of heterogeneity was detected for some demographic characteristics of the neighborhoods with Superfund sites for the completion to deletion duration, namely income (negative), higher education (positive), and fraction of urban population (positive). This suggests that the influence of the unobserved individual component on faster deletion duration decreases with higher income but increases with education and urbanization.

It is difficult to interpret the exact meaning of the unobserved individual component. Nonetheless, since virtually no heterogeneity correlation was detected for the site physical characteristics we can conclude that the influence of any unobservables beyond the mean effect captured in the main model rests either with the neighborhood characteristics (as opposed to the site attributes) or other factors orthogonal to the variables included in the model.

States are also involved to some degree in the cleanup process and thus one possibility is that the unobserved heterogeneity captures funding or political economy differences across States. However, we could not detect any statistically significant differences between the State level heterogeneity averages across States. Any State mean differences in terms of the observables (such as income or fraction of urban population) are controlled for at the individual site level and it appears that there is no residual spatial pattern of unobserved differences on the aggregate level.

6. Conclusion

This paper introduces a more nuanced analysis of Environmental Justice in Superfund cleanups than has previously been available. Given the inherent demographic bias resulting from the geographic location decisions made by firms producing hazardous waste, we focus on the duration of Superfund cleanups which is subject to decisions made by the various parties involved in the cleanup process.

Our identification assumption relies on the observation that conditional on a large number of observable site characteristics, a rational cleanup process subject to cost-benefit analysis will depend only on the site characteristics and not on the demographic composition of the neighborhood. We use a state of the art econometric model to further account for the presence of unobserved site heterogeneity.

The empirical results strongly suggest that the nature of demographic biases changed over time. In particular we find that the cleanup of Superfund sites listed in the initial phase of the program in the early 1980s suffered from a number of biases against sites located in black, urban neighborhoods but in favor of sites located in areas with a highly educated population. These biases appear to diminish over time however, largely following the 1994 Executive Order which formally establishes Environmental Justice as a policy concern. After 1994 we see in fact a prioritization of cleanups in economically disadvantaged neighborhoods. Furthermore, some of these biases may have manifested themselves through the extent to which the community was involved with the cleanup process. We do not find the associated litigation process to be an impediment to Superfund cleanups. The return of a site to general use remains slow and driven by the overall economic health of the community. This suggests that additional resources ought to be made available to assist with the process of deleting Superfund sites from the NPL list in underprivileged areas.

While we believe that, in general faster, cleanups are beneficial to the communities where Superfund sites it is important to note that based on the analysis in this paper we do not have the ability to make concrete social welfare statements. Although we don't have any data or evidence to this effect, we cannot exclude the possibility that longer durations may in fact be associated with higher quality cleanups, or reflect unobserved underlying preferences or sensitivity to environmental damage of the communities involved.

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Table 1. Summary Statistics

	1980		1990		2000	
	Mean	S.D.	Mean	S.D.	Mean	S.D.
hrs	41.044	9.357	44.306	9.744	47.915	7.931
Cost ($m)	15.840	8.772	8.624	9.896	11.909	15.560
Acids	0.490	0.500	0.365	0.482	0.214	0.415
Dioxins Dibenzofurans	0.133	0.339	0.150	0.358	0.119	0.327
Inorganics	0.338	0.473	0.296	0.457	0.071	0.260
Metals	0.775	0.417	0.772	0.420	0.738	0.445
PAH	0.555	0.497	0.520	0.500	0.333	0.477
PCBs	0.320	0.466	0.247	0.432	0.095	0.297
Pesticides	0.299	0.458	0.308	0.462	0.214	0.415
Radioactive	0.040	0.196	0.056	0.232		
VOC	0.798	0.401	0.796	0.403	0.571	0.500
Other Contaminants	0.170	0.376	0.154	0.362	0.142	0.354
Debris	0.188	0.391	0.093	0.291		
Groundwater	0.863	0.344	0.873	0.332	0.714	0.457
Sediment	0.320	0.466	0.329	0.470	0.214	0.415
Surface Water	0.249	0.432	0.247	0.432	0.166	0.377
Soil	0.797	0.402	0.768	0.422	0.809	0.397
Waste	0.232	0.422	0.105	0.308	0.095	0.297
Other Contaminated Media	0.130	0.337	0.109	0.313	0.190	0.397
N	774		246		42	
Household Median Income	36,767	10,134	24,217	8,837	19,306	7,020
Fraction of Unemployed	0.041	0.019	0.039	0.019	0.035	0.023
Fraction of Bachelor plus	0.051	0.030	0.112	0.074	0.139	0.089
Fraction of Black	0.079	0.149	0.087	0.153	0.094	0.158
Fraction of Urban	0.456	0.457	0.656	0.367	0.722	0.334
Fraction Age 0-17	0.292	0.051	0.260	0.047	0.250	0.046
Fraction Age 65 plus	0.104	0.045	0.122	0.047	0.144	0.052
N	1062		1062		1062	

Table 2. Demographics during the cleanup period.

	Change over time				Correlation	
	All	LC □ 10	10 < LC □ 20	LC > 20	All	Listed after 1983
ln(income)	-0.484	-0.414	-0.463	-0.595	-0.346	-0.324
Fraction of Unemployed	-0.003	-0.001	-0.005	-0.003	-0.029	-0.012
Fraction of Bachelor plus	0.065	0.053	0.059	0.088	0.231	0.082
Fraction of Black	0.007	0.009	0.005	0.011	0.033	0.022
Fraction of Urban	0.188	0.159	0.187	0.220	0.084	0.091
Fraction Age 0-17	-0.031	-0.030	-0.030	-0.033	-0.066	-0.025
Fraction Age 65 plus	0.027	0.016	0.028	0.036	0.254	0.216

LC denotes list to construction completion duration (N = 1,062, uncensored = 787, censored = 275).

Table 3. Models without Site Heterogeneity

Model Type	LC		CD		LC		CD	
Variable	Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.
hrs					-0.013**	0.004	-0.002	0.009
ln(cost)					-0.198**	0.021	-0.213**	0.039
Cost zero indicator					-2.515**	0.369	-1.487**	0.596
Acids					0.146*	0.084	-0.126	0.172
Dioxins Dibenzofurans					-0.119	0.110	0.247	0.246
Inorganics					0.110	0.083	0.171	0.161
Metals					-0.234	0.099	0.427**	0.197
PAH					0.086	0.092	0.088	0.166
PCBs					-0.216**	0.093	0.098	0.176
Pesticides					-0.154	0.094	-0.328	0.208
Radioactive					-0.663**	0.204		
VOC					-0.171	0.109	-0.451**	0.191
Other Contaminants					-0.399**	0.113	0.251	0.204
Debris					0.253**	0.096	-0.406*	0.213
Groundwater					-0.098	0.121	-1.185**	0.194
Sediment					-0.274**	0.089	-0.035	0.174
Surface Water					-0.104	0.094	0.183	0.192
Soil					-0.245**	0.087	0.070	0.189
Waste					0.171*	0.087	0.137	0.197
Other contaminated media					-0.129	0.117	-0.378*	0.238
ln(income)	-0.652**	0.126	-0.250*	0.143	-0.517**	0.105	0.403**	0.186
Fraction of Unemployed	-3.996*	2.094	-6.137	3.854	-1.597	2.076	-6.026	3.885
Fraction of Bachelor+	-0.924	1.029	-3.639*	2.044	0.468	0.995	-3.633*	2.051
Fraction of Black	-0.477*	0.277	0.656	0.463	-0.174	0.288	0.978*	0.480
Fraction of Urban	-0.285**	0.085	-0.060	0.175	-0.156	0.098	0.181	0 .17
Fraction Age 0-17	-1.457	1.004	2.439	1.529	-0.569	1.000	2.347	1.666
Fraction Age 65 plus	-0.727	0.939	-0.728	1.421	0.412	0.929	1.930	1.567

LC denotes list to construction completion duration (N = 1,062, uncensored = 787, censored = 275).
CD denotes construction completion to deletion duration (N = 787, uncensored = 205, censored = 582).

Table 4. Base Model with Site Heterogeneity

Model Type	Parametric LC		Non-parametric LC		Parametric CD		Non-parametric CD	
Variable	Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.
hrs	-0.012**	0.005	-0.011**	0.004	-0.007	0.011	-0.007	0.009
ln(cost)	-0.191**	0.028	-0.180**	0.025	-0.368**	0.057	-0.293**	0.049
Cost zero indicator	-2.121**	0.470	-2.068**	0.431	-2.929**	0.863	-2.434**	0.746
Acids	0.131	0.110	0.106	0.100	-0.040	0.279	-0.117	0.184
Dioxins Dibenzofurans	-0.117	0.151	-0.092	0.125	0.316	0.364	0.298	0.294
Inorganics	0.094	0.106	0.086	0.094	0.256	0.214	0.193	0.177
Metals	-0.226*	0.122	-0.212**	0.107	0.489**	0.251	0.424**	0.202
PAH	0.016	0.119	0.035	0.109	0.119	0.247	0.111	0.196
PCBs	-0.289**	0.117	-0.237**	0.101	0.191	0.265	0.163	0.192
Pesticides	-0.169	0.120	-0.154	0.102	-0.485	0.316	-0.381*	0.230
Radioactive	-0.914**	0.255	-0.710**	0.216				
VOC	-0.182	0.139	-0.164	0.123	-0.652**	0.276	-0.515**	0.208
Other Contaminants	-0.394**	0.135	-0.371**	0.112	0.413	0.348	0.262	0.246
Debris	0.278*	0.131	0.252**	0.114	-0.631**	0.302	-0.445**	0.214
Groundwater	-0.057	0.150	-0.068	0.134	-1.918**	0.311	-1.371**	0.221
Sediment	-0.363**	0.117	-0.319**	0.102	-0.030	0.294	-0.054	0.208
Surface Water	-0.072	0.123	-0.070	0.110	0.306	0.277	0.183	0.213
Soil	-0.234*	0.122	-0.215**	0.101	-0.058	0.287	0.042	0.207
Waste	0.198	0.120	0.189*	0.107	0.309	0.276	0.197	0.200
Other contaminated media	-0.181	0.139	-0.150	0.122	-0.460	0.360	-0.362	0.298
ln(income)	-0.136	0.144	-0.212**	0.102	0.551**	0.269	0.140	0.215
Fraction of Unemployed	0.770	2.454	0.762	2.274	-7.405	5.554	-7.624*	4.462
Fraction of Bachelor+	2.018*	0.886	1.581	1.001	-4.636	2.956	-4.476*	2.422
Fraction of Black	0.105	0.351	0.023	0.310	1.256*	0.740	0.917	0.548
Fraction of Urban	-0.051	0.118	-0.080	0.101	0.242	0.288	0.129	0.200
Fraction Age 0-17	2.406*	1.312	1.502	0.995	4.132*	2.573	1.253	1.931
Fraction Age 65 plus	4.253**	1.390	2.930**	0.916	2.801	2.532	-0.031	1.869

LC denotes list to construction completion duration (N = 1, 062, uncensored = 787, censored = 275).

CD denotes construction completion to deletion duration (N = 787, uncensored = 205, censored = 582).

Table 5. Base Model with List Year Dummies

Model Type	LC		CD		LC		CD	
Variable	Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.
hrs	-0.005	0.005	-0.001	0.009	-0.005	0.005	-0.003	0.009
ln(cost)	-0.218**	0.030	-0.265**	0.053	-0.222**	0.030	-0.288**	0.051
Cost zero indicator	-2.438**	0.497	-1.916**	0.796	-2.294**	0.494	-2.175**	0.805
Acids	0.093	0.104	-0.165	0.190	0.100	0.107	-0.152	0.202
Dioxins Dibenzofurans	-0.158	0.139	0.335	0.253	-0.139	0.141	0.347	0.272
Inorganics	0.054	0.103	0.202	0.179	0.057	0.103	0.212	0.187
Metals	-0.239**	0.118	0.451**	0.209	-0.229**	0.115	0.470**	0.214
PAH	0.011	0.119	0.112	0.196	-0.005	0.113	0.112	0.208
PCBs	-0.220**	0.111	0.068	0.204	-0.184*	0.109	0.089	0.213
Pesticides	-0.155	0.110	-0.301	0.218	-0.161	0.112	-0.320	0.232
Radioactive	-0.915**	0.241			-0.891**	0.241		
VOC	-0.235	0.140	-0.444**	0.213	-0.201	0.134	-0.461**	0.229
Other Contaminants	-0.397**	0.135	0.222	0.232	-0.396**	0.134	0.241	0.245
Debris	0.249**	0.117	-0.450**	0.223	0.274*	0.119	-0.479**	0.237
Groundwater	-0.143	0.153	-1.349**	0.211	-0.169	0.153	-1.415**	0.223
Sediment	-0.342**	0.112	0.017	0.206	-0.355**	0.113	-0.007	0.213
Surface Water	-0.077	0.116	0.210	0.209	-0.099	0.116	0.224	0.218
Soil	-0.211*	0.112	-0.005	0.204	-0.241**	0.114	-0.012	0.214
Waste	0.213*	0.114	0.147	0.193	0.210*	0.114	0.160	0.198
Other contaminated media	-0.202	0.132	-0.413	0.266	-0.199	0.134	-0.411	0.287
ln(income)	0.018	0.160	-0.495*	0.273	0.144	0.154	-0.615**	0.279
Fraction of Unemployed	2.995	2.403	-7.468*	4.360	2.280	2.407	-9.013**	4.334
Fraction of Bachelor+	0.808	1.437	1.121	2.845	-0.065	1.575	0.473	2.807
Fraction of Black	-0.040	0.348	0.729	0.479	0.122	0.360	0.684	0.570
Fraction of Urban	-0.053	0.115	0.295	0.202	-0.083	0.115	0.272	0.208
Fraction Age 0-17	1.507	1.219	3.472	2.230	0.290	1.184	2.979	2.172
Fraction Age 65 plus	3.234**	1.062	0.830	1.962	2.355**	1.066	0.044	1.983
L1984-86	0.253**	0.122	0.011	0.214	0.243**	0.123	0.026	0.203
L1987-89	0.588**	0.141	-0.236	0.223	0.565**	0.134	-0.250	0.217
L1990-92	0.547**	0.200	-1.010**	0.387	0.615**	0.190	-0.994**	0.352
L1993-95	-0.159	0.299	-3.984**	1.122	1.976	4.100	-0.260	0.852
L1996+	0.710**	0.283	-1.409**	0.636	2.373	4.063	0.103	0.785
L94-10□ln(income)					-1.824**	0.890	-0.566	0.364
L94-10□Fraction of Unemployed					3.709**	1.098	-0.407	0.912
L94-10□Fraction of Bachelor+					7.243**	3.723	-0.049	1.029
L94-10□Fraction of Black					-0.421	1.159	0.177	0.943
L94-10□Fraction of Urban					-0.362	0.455	-0.284	0.769
L94-10□Fraction Age 0-17					7.651	5.305	-0.099	1.014
L94-10□Fraction Age 65 plus					7.672	5.906	0.009	0.813

LC denotes list to construction completion duration (N = 1,062, uncensored = 787, censored = 275).
CD denotes construction completion to deletion duration (N = 787, uncensored = 205, censored = 582).

Table 6. Split Samples, List to Construction Completion Duration

Model Type	List Year 1983		List Years 1984–2010	
Variable	Mean	s.e.	Mean	s.e.
hrs	0.001	0.008	-0.006	0.006
ln(cost)	-0.274□□□	0.055	-0.191□□□	0.034
Cost zero indicator	-2.650□□□	0.983	-2.142□□□	0.578
Acids	0.082	0.174	0.100	0.125
Dioxins Dibenzofurans	0.025	0.232	-0.190	0.161
Inorganics	-0.111	0.183	0.166	0.114
Metals	-0.307	0.204	-0.217	0.137
PAH	-0.007	0.203	-0.007	0.144
PCBs	-0.086	0.172	-0.290□□	0.138
Pesticides	0.145	0.181	-0.258□	0.140
Radioactive	-0.554	0.548	-0.968□□	0.279
VOC	0.276	0.231	-0.314□□	0.152
Other Contaminants	0.167	0.189	-0.632□□	0.172
Debris	-0.120	0.204	0.340	0.144
Groundwater	-0.350	0.230	0.034	0.192
Sediment	-0.632□□□	0.180	-0.136	0.136
Surface Water	-0.203	0.204	-0.008	0.135
Soil	-0.146	0.194	-0.211	0.143
Waste	-0.141	0.190	0.286□□□	0.130
Other contaminated media	-0.151	0.228	-0.276□	0.163
ln(income)	0.797	0.296	-0.247	0.187
Fraction of Unemployed	-2.889	4.207	2.624	3.122
Fraction of Bachelor+	13.130□□	3.487	1.687	1.353
Fraction of Black	-2.244□□	0.657	-0.468	0.388
Fraction of Urban	-0.575□□	0.193	0.046	0.137
Fraction Age 0-17	-1.218	1.933	0.424	1.394
Fraction Age 65 plus	2.152	1.804	1.757	1.176
L1987-89			0.329□□	0.141
L1990-92			0.129	0.209
L1993-95			-0.533	0.330
L1996+			0.280	0.281

For list year 1983, N = 294, uncensored = 233, censored = 61.

For list years 1984–2010, N = 768, uncensored = 554, censored = 214.

Tabl e 7. Split Samples, Construction Completion to Deletion Duration

Model Type	List Year 1983		List Years 1984–2010	
Variable	Mean	s.e.	Mean	s.e.
hrs	-0.015	0.015	0.006	0.012
ln(cost)	-0.167	0.104	-0.317$^{\square\square}$	0.037
Cost zero indicator	-1.385	1.662	-2.365$^{\square\square}$	0.514
Acids	-0.121	0.350	-0.167	0.253
Dioxins Dibenzofurans	0.603	0.474	-0.002	0.369
Inorganics	-0.094	0.338	0.367	0.221
Metals	0.718$^{\square\square}$	0.358	0.269	0.253
PAH	-0.043	0.369	0.189	0.245
PCBs	-0.321	0.352	0.382	0.256
Pesticides	-0.886$^{\square\square}$	0.409	0.015	0.275
VOC	-0.021	0.467	-0.593$^{\square\square}$	0.260
Other Contaminants	0.482	0.393	-0.088	0.300
Debris	-0.715$^{\square}$	0.424	-0.332	0.249
Groundwater	-1.371$^{\square\square}$	0 .38	-1.557$^{\square\square}$	0.270
Sediment	0.331	0.334	-0.247	0.273
Surface Water	0.134	0.387	0.275	0.269
Soil	0.597	0.383	-0.171	0.242
Waste	0.405	0.316	-0.017	0.235
Other contaminated media	0.359	0.381	-0.886$^{\square\square}$	0.406
ln(income)	-0.175	0.528	-0.386	0.349
Fraction of Unemployed	-16.486$^{\square\square}$	7.660	-7.334$^{\square\square}$	3.043
Fraction of Bachelor+	-8.363	7.832	2.446	3.017
Fraction of Black	1.595	1.133	0.696	0.653
Fraction of Urban	0.459	0.403	0.172	0.258
Fraction Age 0-17	-0.996	4.113	5.386$^{\square\square}$	2.620
Fraction Age 65 plus	-2.007	3.362	2.948	2.225
L1987-89			-0.285	0.221
L1990-92			-1.106$^{\square\square}$	0.434
L1993-95			-2.907$^{\square\square}$	1.406

For list year 1983, N = 233, uncensored = 68, censored = 165.

For list years 1984–2010, N = 556, uncensored = 137, censored = 417.

Tabl e 8. Potentially Responsible Parties (PRP) Variables

Model Type	LC		CD		LC		CD	
Variable	Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.
hrs	-0.011**	0.004	-0.009	0.009	-0.007	0.005	-0.002	0.009
ln(cost)	-0.190**	0.024	-0.294**	0.047	-0.220**	0.032	-0.248**	0.046
Cost zero indicator	-2.211**	0.391	-2.327**	0.714	-2.425**	0.570	-1.634**	0.706
Acids	0.137	0.096	-0.088	0.195	0.117	0.107	-0.140	0.196
Dioxins Dibenzofurans	-0.087	0.125	0.233	0.285	-0.131	0.137	0.288	0.260
Inorganics	0.100	0.091	0.193	0.187	0.070	0.102	0.197	0.176
Metals	-0.236**	0.104	0.434**	0.218	-0.251**	0.119	0.461**	0.202
PAH	0.047	0.103	0.116	0.200	0.017	0.116	0.091	0.195
PCBs	-0.250**	0.100	0.175	0.215	-0.240**	0.105	0.062	0.204
Pesticides	-0.130	0.102	-0.433*	0.232	-0.134	0.110	-0.322	0.221
Radioactive	-0.664**	0.230			-0.823**	0.244		
VOC	-0.189	0.120	-0.553**	0.210	-0.231	0.133	-0.494**	0.214
Other Contaminants	-0.382**	0.117	0.284	0.233	-0.391**	0.133	0.269	0.229
Debris	0.238**	0.111	-0.451**	0.236	0.228*	0.120	-0.480**	0.232
Groundwater	-0.079	0.129	-1.463**	0.217	-0.146	0.147	-1.364**	0.216
Sediment	-0.322**	0.102	-0.028	0.213	-0.342**	0.109	0.032	0.201
Surface Water	-0.074	0.106	0.206	0.216	-0.080	0.114	0.212	0.208
Soil	-0.229**	0.102	0.022	0.212	-0.246**	0.116	0.032	0.204
Waste	0.173	0.105	0.238	0.204	0.208*	0.110	0.171	0.194
Other contaminated media	-0.172	0.123	-0.389	0.278	-0.227	0.139	-0.404	0.279
ln(income)	-0.347**	0.108	0.158	0.228	-0.158	0.161	-0.161	0.263
Fraction of Unemployed	-0.119	2.192	-7.754*	4.665	2.288	2.319	-6.895	4.284
Fraction of Bachelor+	1.434	0.984	-5.013**	2.556	0.945	1.318	-0.176	2.611
Fraction of Black	-0.078	0.302	0.942*	0.553	-0.164	0.345	0.834	0.528
Fraction of Urban	-0.104	0.098	0.129	0.204	-0.064	0.115	0.297	0.205
Fraction Age 0-17	0.939	0.985	0.935	2.049	1.035	1.148	3.643*	1.969
Fraction Age 65 plus	2.191**	0.916	-0.155	2.096	2.449**	1.057	1.940	1.838
L1984-86					0.253**	0.121	0.014	0.207
L1987-89					0.553**	0.133	-0.227	0.217
L1990-92					0.466**	0.197	-0.793**	0.351
L1993-95					-0.207	0.319	-2.676**	1.362
L1996+					0.661**	0.245	-1.263**	0.648
PRP 0	0.102	0.141	0.271	0.288	0.093	0.158	0.349	0.270
PRP 2 10	0.248	0.152	-0.049	0.302	0.246	0.163	0.000	0.286
PRP 10+	0.328**	0.153	0.233	0.323	0.340**	0.169	0.278	0.300

LC denotes list to construction completion duration (N = 1, 062, uncensored = 787, censored = 275).

CD denotes construction completion to deletion duration (N = 787, uncensored = 205, censored = 582).

Table 9. Home Ownership Variables

Model Type	LC		CD	
Variable	Mean	s.e.	Mean	s.e.
hrs	-0.009	1.005	-0.004	0.009
ln(cost)	-0.213**	0.029	-0.251**	0.040
Cost zero indicator	-2.420**	0.497	-1.826**	0.611
Acids	0.118	0.106	-0.117	0.194
Dioxins Dibenzofurans	-0.143	0.136	0.270	0.267
Inorganics	0.051	0.102	0.157	0.172
Metals	-0.186	0.116	0.469**	0.213
PAH	0.017	0.111	0.097	0.193
PCBs	-0.269**	1.110	0.163	0.218
Pesticides	-0.129	0.113	-0.269	0.220
Radioactive	-0.887**	0.255		
VOC	-0.230*	0.130	-0.514**	0.211
Other Contaminants	-0.386**	0.131	0.365	0.235
Debris	0.277**	0.116	-0.446**	0.212
Groundwater	-0.112	0.147	-1.388**	0.213
Sediment	-0.376**	0.110	-0.075	0.198
Surface Water	-0.013	1.116	0.271	0.207
Soil	-0.194*	0.112	-0.012	0.209
Waste	0.278**	0.115	0.212	0.193
Other contaminated media	-0.174	0.132	-0.235	0.273
ln(income)	0.084	0.160	-0.638**	0.288
Fraction of Unemployed	2.049	2.584	-6.365	4.356
Fraction of Bachelor+	1.441	1.285	-0.110	2.601
Fraction of Black	-0.098	0.356	0.776	0.511
Fraction of Urban	-0.070	0.118	0.303	0.205
Fraction Age 0-17	0.743	1.169	0.856	2.048
Fraction Age 65 plus	2.851**	1.093	1.226	1.838
L1984-86	0.107	0.125	0.104	0.209
L1987-89	0.262*	0.132	-0.049	0.219
L1990-92	0.531**	0.195	-0.403	0.333
L1993-95	0.168	0.503	-3.334**	1.464
L1996+	0.129	0.572	-3.263**	1.377
Fraction of homeowners	-0.219	0.371	1.998**	0.695
L(94-10)□Fràc homeown	-0.004	0.713	2.460	1.703

LC denotes list to construction completion duration (N = 1,056, uncensored = 786, censored = 270).
CD denotes construction completion to deletion duration (N = 786, uncensored = 205, censored = 581).

Table 10. Community Involvement Variables

Model Type	LC		CD		LC		CD	
Variable	Mean	s.e.	Mean	s.e.	Mean	s.e.	Mean	s.e.
hrs	-0.011**	0.004	-0.009	0.010	-0.007	0.005	-0.002	0.009
ln(cost)	-0.185**	0.025	-0.346**	0.052	-0.205**	0.026	-0.280**	0.051
Cost zero indicator	-2.134**	0.405	-3.094**	0.786	-2.212**	0.462	-2.093**	0.804
Acids	0.121	0.097	-0.118	0.218	0.110	0.102	-0.161	0.196
Dioxins Dibenzofurans	-0.105	0.127	0.310	0.305	-0.166	0.137	0.335	0.272
Inorganics	0.086	0.094	0.203	0.197	0.051	0.102	0.215	0.187
Metals	-0.215**	0.106	0.464**	0.229	-0.220**	0.114	0.470**	0.212
PAH	0.041	0.106	0.097	0.224	0.005	0.113	0.095	0.204
PCBs	-0.237**	0.101	0.165	0.235	-0.232**	0.106	0.082	0.209
Pesticides	-0.146	0.104	-0.435*	0.250	-0.130	0.111	-0.322	0.228
Radioactive	-0.723**	0.226			-0.910**	0.249		
VOC	-0.193	0.122	-0.514**	0.236	-0.214	0.134	-0.434*	0.223
Other Contaminants	-0.380**	0.118	0.292	0.258	-0.393**	0.134	0.244	0.238
Debris	0.254**	0.111	-0.502**	0.258	0.265**	0.119	-0.472**	0.228
Groundwater	-0.060	0.131	-1.514**	0.242	-0.140	0.154	-1.410**	0.223
Sediment	-0.330**	0.104	-0.063	0.236	-0.337**	0.114	0.020	0.209
Surface Water	-0.071	0.109	0.234	0.239	-0.069	0.117	0.242	0.219
Soil	-0.228**	0.105	0.073	0.235	-0.241**	0.112	0.021	0.210
Waste	0.181*	0.106	0.210	0.208	0.209*	0.119	0.156	0.197
Other contaminated media	-0.170	0.126	-0.382	0.311	-0.238*	0.141	-0.365	0.287
ln(income)	-0.246**	0.109	0.224	0.242	-0.318	0.334	-0.926	1.104
Fraction of Unemployed	0.458	2.242	-8.519*	4.782	-0.030	0.160	-0.539*	0.281
Fraction of Bachelor+	1.548	1.012	-5.472	2.680	2.123	1.846	-8.304**	4.340
Fraction of Black	0.020	0.309	1.163*	0.612	0.618	1.407	1.355	2.455
Fraction of Urban	-0.078	0.104	0.128	0.228	-0.062	0.337	0.694	0.564
Fraction Age 0-17	1.456	1.022	0.679	2.172	-0.038	0.112	0.277	0.214
Fraction Age 65 plus	2.711**	0.946	0.039	1.875	1.424	1.235	3.601*	2.089
L1984-86					2.791**	1.086	1.002	1.759
L1987-89					0.209*	0.123	0.016	0.215
L1990-92					0.591**	0.135	-.2634	0.224
L1993-95					0.520**	0.196	-1.074**	0.369
L1996+					-0.000	0.378	-2.650**	1.437
Community	0.111	0.092	-0.222	0.199	0.185*	0.110	-0.203	0.185
L(94-10)□Community					0.878**	0.300	-1.193	0.837

LC denotes list to construction completion duration (N = 1,062, uncensored = 787, censored = 275).
CD denotes construction completion to deletion duration (N = 787, uncensored = 205, censored = 582).

Figure 1. Distributions of Durations (in Years) , for the different cleanup milestones.

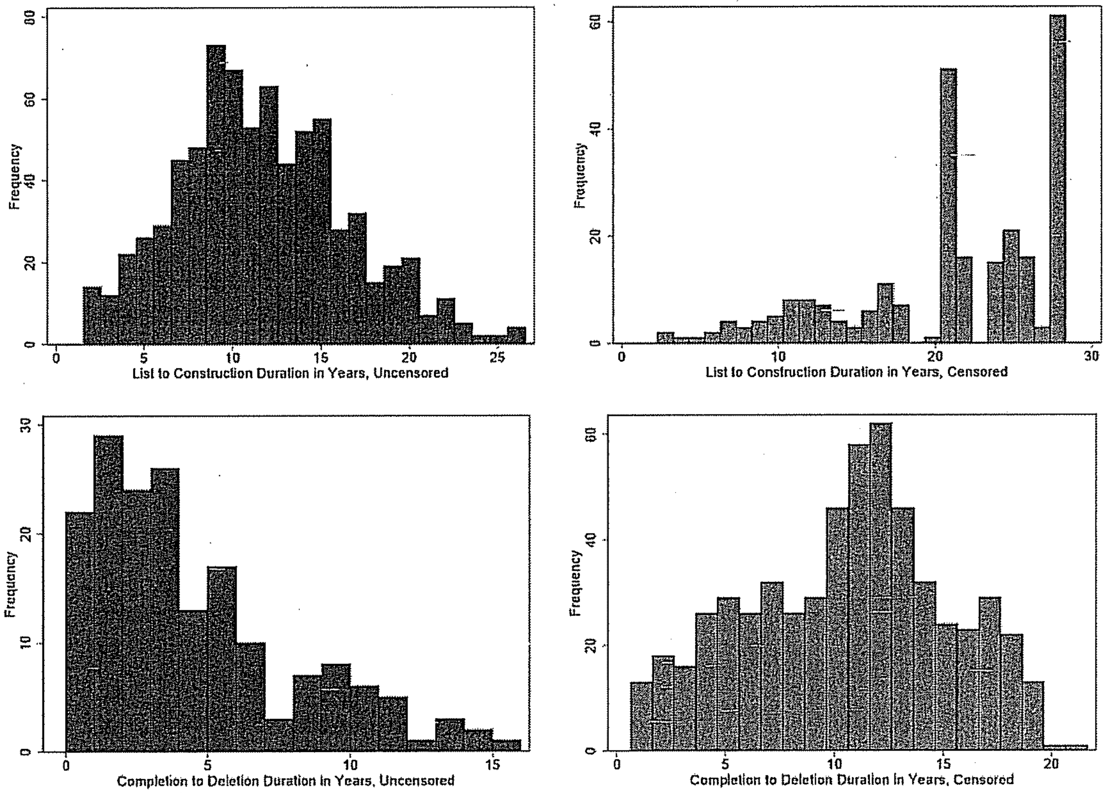
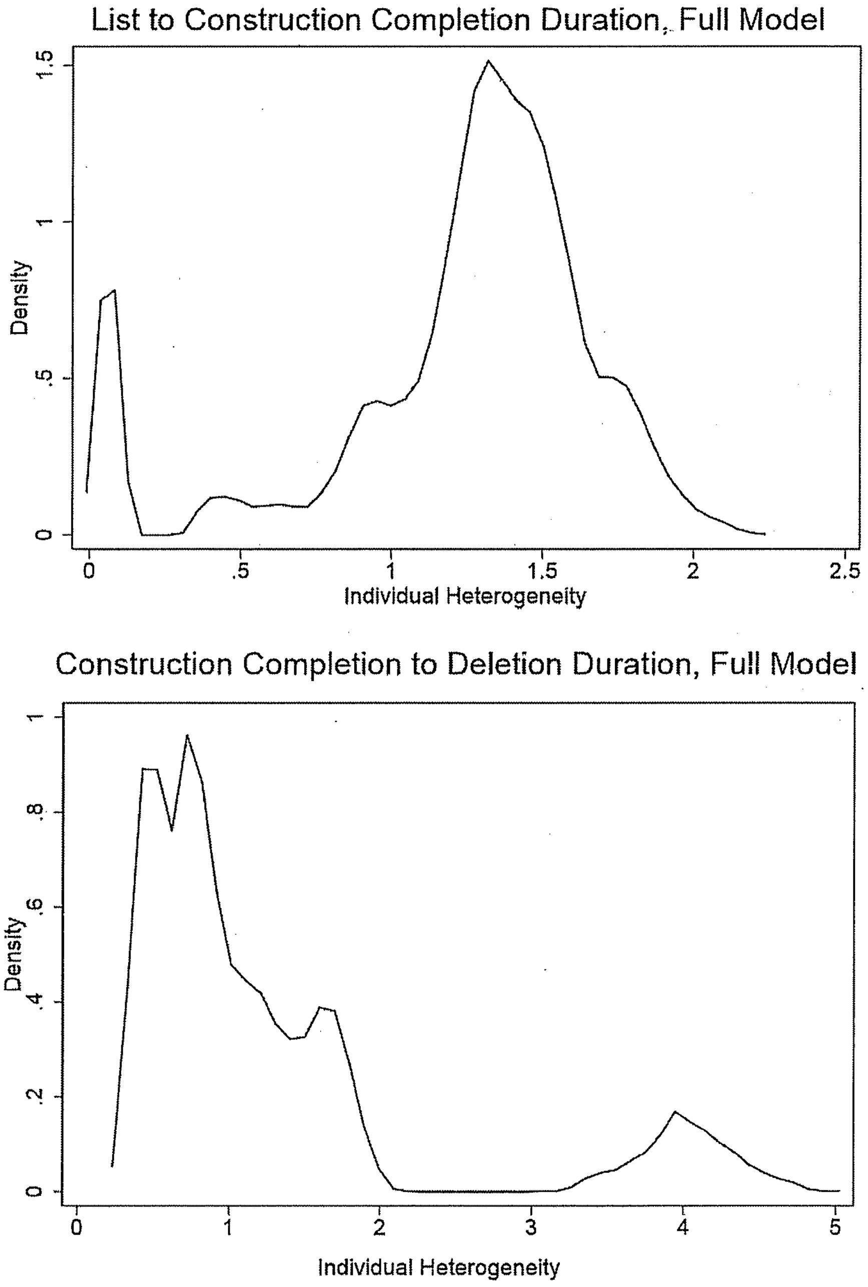


Figure 2. Estimated Density of Individual Heterogeneity



List to Construction Completion Duration, Full Model

Construction Completion to Deletion Duration, Full Model