The Liquidity-Augmented Model of Macroeconomic Aggregates

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ABSTRACT

We propose a new model of liquidity in the macroeconomy. It is simple and tractable, yet takes the foundations of liquidity seriously, and can thus be precise about the implementation, effects, and optimality of monetary policy. The model shines light on some important questions in macroeconomics: the tension between two channels through which the price of liquidity affects the economy (Friedman's real balance effect vs Mundell's and Tobin's asset substitution effect), the optimal rates of interest and inflation, the importance of distinguishing between the interest rates on liquid versus illiquid assets, and the liquidity trap.

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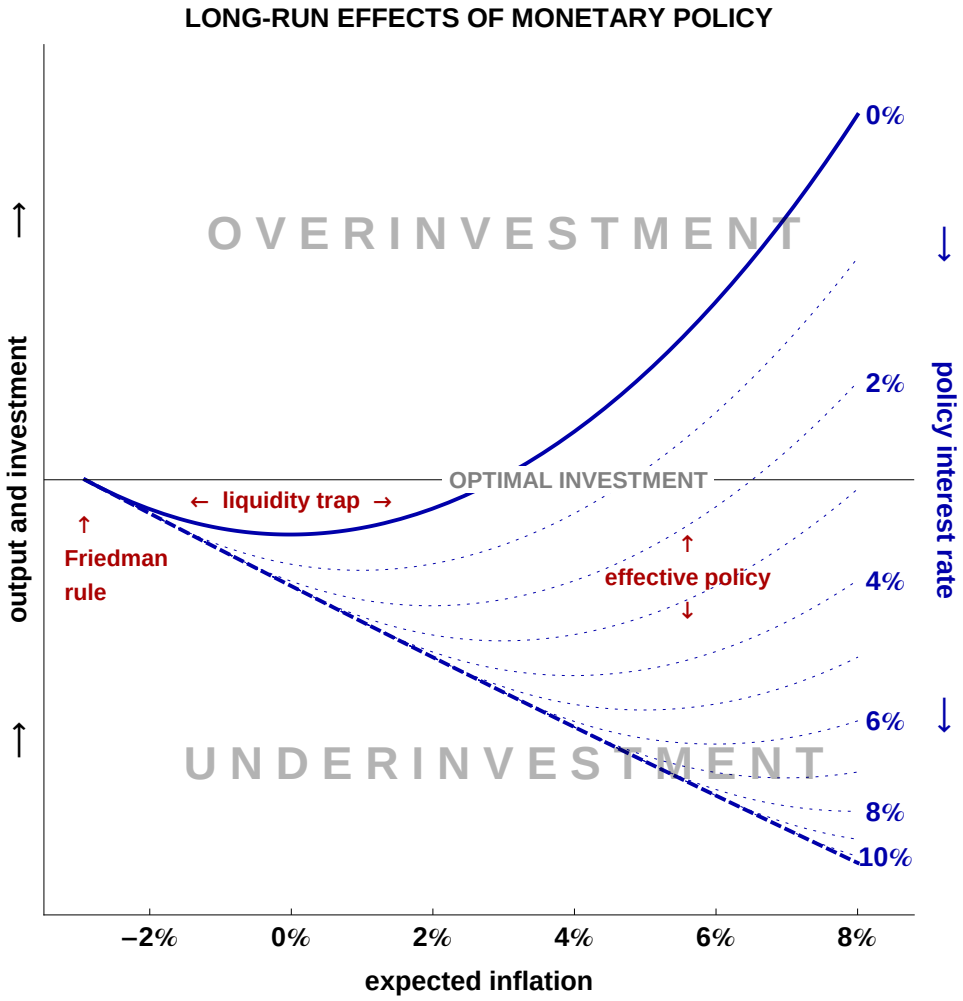
Some questions about the paper and model came up repeatedly. To do them justice, we answer them in an FAQ here: http://tiny.cc/LAMMA-FAQ

GRAPHICAL ABSTRACT



1 Introduction

Open any undergraduate macroeconomics textbook, and you will find agreement with the following facts: (1) Monetary policy is conducted through intervention in financial markets. (2) Financial assets tend to be imperfect substitutes and their demand curves slope down, which is what makes intervention effective in the first place. (3) The principal channel through which monetary policy affects the economy is the interest rate at which agents save and invest. (4) Generally, reductions in this interest rate increase investment and output. (5) Monetary policy is subject to boundary conditions: for instance, too much inflation is bad for the economy, and there seems to be something special about the case where the policy rate hits zero ("liquidity trap").

However, open any *graduate* textbook, and none of the workhorse models there are consistent with all five facts.[1] The model which has been most widely used as a guide to policy, the New Keynesian model, features a cashless economy where the driving friction is price stickiness. As a result, the model is not well suited to modeling monetary intervention in financial markets (it is cashless); moreover, its ability to explain a liquidity trap has been questioned.[2] These issues are easier to address in a New Monetarist model, where the driving frictions make liquidity emerge naturally (Lagos, Rocheteau, and Wright, 2017). However, this branch of the literature has mostly focused on inflation and the long-run real balance effect, at the expense of a realistic model of interest rates, their central role in monetary policy, and their effect on the economy.

Hence, we propose a model that can help: the Liquidity-Augmented Model of Macroeconomic Aggregates (LAMMA), which is parsimonious, tractable, and consistent with facts (1)-(5). The model nests the main workhorse model of macroeconomic aggregates – the neoclassical growth model – and augments it with a role for liquid assets. Due to frictions that we will describe precisely, a need for a medium of exchange arises in the economy, and in the model this role is played by fiat money. Government bonds and physical capital cannot be used as media of exchange, but they too are liquid, as agents with a need for money can sell their bonds and capital in a secondary asset market. A consolidated government controls the quantities of money and liquid bonds, and can therefore conduct open-market operations in that secondary market to target asset prices and interest rates, which is arguably the empirically relevant approach.

Monetary policy has real effects at all frequencies – certainly in the short run, but even in steady state. We can express the long-run effects in terms of two rates: the expected inflation

[1] For our claim, consider Ljungqvist and Sargent (2004), Woodford (2003), or Walsh (2003), and the workhorse models presented therein. Certainly, there are many good models for *subsets* of the five facts, and with enough complexity one can match anything. However, one of the merits of the model we propose here is that it is simple enough to be summarized in a graphical abstract.

[2] See Benhabib, Schmitt-Grohé, and Uribe (2001), Cochrane (2011, 2013), and Bullard (2015).

rate, and the interest rate on liquid bonds. Expected inflation makes people economize on money balances, which gives rise to two opposing forces. One is the inflation tax which falls on the productive economy, and this force tends to make money and capital complements in general equilibrium. The second force is the fact that as a somewhat liquid asset, capital can be an imperfect substitute to money (the Mundell-Tobin effect). The end result could be overinvestment or underinvestment, and which force prevails depends on the second instrument of monetary policy: the interest rate on bonds. Raising this interest rate by selling bonds in the asset market makes bonds a more desirable store of value, hence investment and output fall. Lowering this interest rate, by buying bonds in the asset market, does the opposite. With the right interest rate, investment and output are at their first-best levels.[3]

Hence, while the Friedman rule is *an* optimal long-run policy in this economy, it is not the *only* such policy. We examine the robustness of this conclusion by discussing several plausible model variations. When there are constraints on policy such as a lack of lump-sum taxes, or when there are additional distortions affecting investment, then every optimal policy involves inflation above the Friedman rule. On the other hand, for some changes in the structure of the economy (for example, if the economy consists of a monetary sector and a frictionless sector), the Friedman rule is the only first-best policy (if feasible).

When capital is hard to trade, the economy can be in a liquidity trap, which we define as a situation where (i) the policy interest rate is at a lower bound, (ii) output and investment are below their optimal levels, and (iii) raising interest rates would make things worse (roughly equivalent to saying that it would be desirable to lower interest rates further). This liquidity trap formalizes the long-held notion that saving is not automatically translated into investment, but requires a well-functioning financial system and an unconstrained interest rate. In such a trap, a variety of fiscal schemes may help, but there is also a simple monetary remedy: increase inflation permanently. In addition, there is nothing "short-run" about our mechanism; hence, there is no contradiction between a liquidity trap and stable, even positive, inflation. This fits with the experience of developed economies in the last decade (three decades in Japan), where near-zero interest rates have coexisted with stable inflation, but it can be considered a puzzle in some versions of the New Keynesian model.[4]

Finally, the model clarifies that the distinction between interest rates on liquid and illiquid assets is crucial for understanding the role of monetary policy, and for empirical analysis of its effects. In doing so, it also suggests a possible resolution to the recent "Neo-Fisherian" controversy about the causal link between nominal interest rates and inflation. Highly liquid assets are the closest substitutes for money, hence the real return on such assets is most easily

[3] There is also a set of parameters where the comparative statics described above are reversed, so that raising interest rates on liquid bonds stimulates investment and output. In that case, given positive inflation, the second-best monetary policy is to *raise* interest rates to the maximum. Due to its counterintuitive comparative statics, this case seems less relevant for modern economies. Of course, that may change in the future.

[4] But not in all. See Del Negro, Giannoni, and Schorfheide (2015) and García-Schmidt and Woodford (2015).

affected by monetary policy, and indeed monetary policy can be implemented by "setting" such rates. Highly illiquid assets, on the other hand, are poor substitutes for money, therefore their real return is insensitive to monetary policy, and their nominal return is simply the real one plus expected inflation.

Conceptually, our paper is related to Tobin (1969). Writing in the inaugural issue of the *Journal of Money, Credit and Banking*, he proposes a "general framework for monetary analysis":

> "Monetary policy can be introduced by allowing some government debt to take non-monetary form. Then, even though total government debt is fixed [...], its composition can be altered by open market operations. [...] It is assumed [that money, bonds and capital] are gross substitutes; the demand for each asset varies directly with its own rate and inversely with other rates."

Although today we can do better than "it is assumed", there is no doubt that Tobin's model contains the right ingredients: money, bonds, and capital. Our model contains the same ingredients, but also provides microfoundations of why these assets are liquid and how monetary policy can exploit their relationship and affect the macroeconomy. In order to give meaning to "monetary", we are explicit about the frictions that make monetary trade emerge.[5] In order to give meaning to "monetary policy", we add bonds that are imperfect substitutes to money, and a financial market where the monetary authority intervenes to "set interest rates". Finally, in order to capture the effects of monetary policy in a realistic way, our crucial addition is to recognize the *dual role of capital*: it is useful in production, as in the neoclassical model, and it can be traded (at least sometimes) in financial markets, making it liquid and making its yield integrated with the yields on monetary assets.

Our paper is part of a literature that studies how liquidity and monetary policy can shape asset prices, based on the New Monetarist paradigm (Lagos and Wright, 2005; Lagos et al., 2017). In papers like Geromichalos, Licari, and Suárez-Lledó (2007), Lester, Postlewaite, and Wright (2012), Nosal and Rocheteau (2012), Andolfatto and Martin (2013), and Hu and Rocheteau (2015), assets are 'liquid' in the sense that they serve directly as media of exchange (often alongside money).[6] An alternative approach highlights that assets may be priced at a liquidity premium not because they serve as media of exchange (an assumption often defied by real-world observation), but because agents can sell them for money when they need it (Geromichalos and Herrenbrueck, 2016a; Berentsen, Huber, and Marchesiani, 2014, 2016;

[5] Whether the medium of exchange is fiat money, or a broader aggregate that may include privately created money such as demand deposits, is not essential for the theory as long as the monetary authority controls the money supply at the margin.

[6] Some papers in this literature revisit well-known asset pricing puzzles and suggest that asset liquidity may be the key to rationalizing these puzzles. Examples include Lagos (2010), Geromichalos and Simonovska (2014), Lagos and Zhang (2015), and Geromichalos, Herrenbrueck, and Salyer (2016).

Mattesini and Nosal, 2015).[7] Herrenbrueck and Geromichalos (2017) dub this alternative approach *indirect* liquidity. In this paper, we make use of the indirect liquidity approach because it provides a natural way to mimic how central banks implement monetary policy in reality: they intervene in institutions where agents trade assets in response to short-term liquidity needs. That is exactly what the secondary asset market in our model represents.

One central question for us is the effect of monetary policy on capital, and we have argued that the dual role of capital, as a productive factor (affected by the inflation tax) and a liquid asset (competing with money as a store of value), is key to this (see also Herrenbrueck, 2014). Most of the literature has focused on one of these channels at a time. Aruoba, Waller, and Wright (2011) analyze how capital responds negatively to the inflation tax. In Rocheteau, Wright, and Zhang (2016), entrepreneurs can finance investment using money or credit, thus inflation also tends to depress investment. On the other hand, Lagos and Rocheteau (2008), Rocheteau and Rodriguez-Lopez (2014), and Venkateswaran and Wright (2014) explore the idea that capital could be valued for its potential liquidity properties as a substitute to money, which makes inflation cause *over*accumulation of capital unless offset by a negative externality or capital tax.[8]

The final important departure of our model from most of the New Monetarist literature is that we interpret the yield on liquid bonds as the main monetary policy instrument, rather than emphasizing money growth and the traditional real balance effect of expected inflation. In recent work, Rocheteau, Wright, and Xiao (2014) and Andolfatto and Williamson (2015) also emphasize the yield on liquid bonds, while Baughman and Carapella (2017) model the interest rate in interbank lending as the main policy instrument. These papers do not include physical capital or the effect of monetary policy on capital accumulation.

Our paper is also related to a large literature that studies the effect of monetary policy on macroeconomic aggregates in the presence of financial frictions. Notable examples include Bernanke and Gertler (1989), Kiyotaki and Moore (1997, 2012), and Cúrdia and Woodford (2011). Finally, our paper is related to the literature on policy in a liquidity trap, including Krugman, Dominquez, and Rogoff (1998), Eggertsson and Woodford (2003), Werning (2011), Williamson (2012), Guerrieri and Lorenzoni (2017), and Altermatt (2017).

The paper is organized as follows. Section 2 introduces and solves the dynamic model.

[7] In these papers, agents who receive an idiosyncratic consumption opportunity visit a market where they can sell financial assets and acquire money from agents who do not need it as badly. This idea is related to Berentsen, Camera, and Waller (2007), where the allocation of money into the hands of the agents who need it the most takes place through a (frictionless) banking system rather than through secondary asset markets.

[8] Empirical evidence does not resolve the question which one of the two effects dominates. First, the evidence that exists is ambiguous: in the long run, inflation seems to be positively related to investment at low levels, but negatively at higher levels (Bullard and Keating, 1995; Bullard, 1999); positively in the U.S. time series (Ahmed and Rogers, 2000), but negatively in the OECD cross-section (Madsen, 2003). Second, there is a strong theoretical reason why the evidence *should* be ambiguous; as we show, an optimal monetary policy *makes* investment unrelated to inflation in the long run. In other words, monetary policy works because it can exploit the Mundell-Tobin effect, but if this is done optimally, empirical evidence of the effect will be obscured.

Section 3 characterizes the steady-state solution, Section 4 applies the results to some open questions, and Section 5 concludes. Additional details and extensions of the model are provided in the Appendix.

2 The model

2.1 Environment

Time $t = 0, 1, \ldots$ is discrete and runs forever. The economy consists of a unit measure of households, an indeterminate measure of firms, and a consolidated government that controls fiscal and monetary policy.[9] Each household has two members: a worker and a shopper, who make decisions jointly to maximize the household's utility. The economy is subject to information and commitment frictions: all private agents are anonymous, therefore they cannot make long-term promises, and all trade must be quid-pro-quo.

Each period is divided into three sub-periods: an asset market (AM), a production market (PM), and a centralized market (CM). During the PM, shoppers buy goods from firms, and due to anonymity, they must pay for them with a suitable medium of exchange. The firms rent labor and capital from the households, and combine them to produce goods. In the CM, households divide the output goods between consumption and investment. Households also choose their asset portfolios for the next period – hence, the CM is the "primary" asset market. In the next morning, shoppers learn of a random opportunity trade with a firm during the PM. Since such trade requires a medium of exchange, shoppers may want to trade with other households to rebalance their portfolios; they can do so in the AM, which is therefore the "secondary" asset market. Households are active in all three periods; firms are active only during the PM, and the government is only active during the AM and CM subperiods. This timing is illustrated in Figure 1.

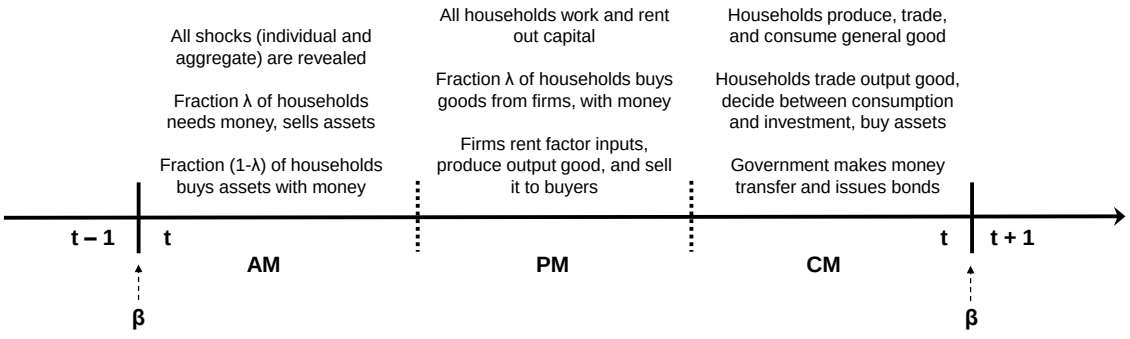


Figure 1: Timing of events.

[9] In Appendix A.4, we extend the model to distinguish explicitly between a fiscal and a monetary authority.

There are three assets in the economy: money (in aggregate supply M), nominal discount bonds (B), and physical capital (K). The government controls M and B, whereas capital is created by households through investment. Money is special in that it is the only asset that can be used during the PM, whereas bonds and capital cannot.[10] Capital is special in that it is both a tradable asset and a productive input. Bonds are special in that they are easier to trade than capital: within the AM, agents can sell all of their bonds, but only a fraction $\eta_t \in [0,1]$ of their capital.[11] Hence, money is the most liquid asset: it can be used to purchase anything. Bonds and capital cannot be used to purchase goods, but they can be sold for money when money is needed; thus, they have *indirect* liquidity properties.

During the PM, firms operate a technology that turns capital (k_t) and labor (h_t) into an output good y_t. Firms rent capital and labor from the worker-members of the households, on a competitive factor market. The production function is standard:

$$y_t = A_t k_t^{\alpha} h_t^{1-\alpha}$$

Due to anonymity and a lack of a double coincidence of wants, a medium of exchange is required to conduct trade in the PM, and, as already explained, money is the unique object that can serve this role. Additionally, there is a search friction: only a random fraction $\lambda_t \in (0,1)$ of shoppers will enter the PM. Once they are in the PM, trading with firms is competitive. (In Appendix A.3, we introduce search frictions and price posting by firms in the goods market, and show that competitive pricing arises as a limiting case when search frictions are small.)

All shocks to period-t variables are revealed at the beginning of that period, before the AM and PM open. Consequently, some shoppers learn that they will trade in the PM during the period, and others learn that they will not. As long as there is a positive cost of holding money, shoppers will never hold enough of it to satiate them in the goods market, but other shoppers will end up with money that they do not need in the same period. Hence, liquidity is misallocated. In order to correct this, shoppers visit the AM: those who need money seek

[10] This assumption is consistent with empirical observation (we rarely see bonds or capital serving directly as means of payment in transactions), but one may still ask why bonds and/or capital cannot be used as media of exchange. There are many potential reasons. For instance, physical capital is not a good candidate for a medium of exchange because it is usually made-for-purpose and non-portable. Furthermore, financial assets (including *claims* to capital) are not universally recognizable; thus, a seller may be reluctant to accept a bond as a medium of exchange, either because she does not know what a bond is supposed to look like or because it may not even be a tangible object (but just an entry in a computer). Finally, Rocheteau (2011) and Lester et al. (2012) show that if there is asymmetric information regarding the future returns of financial assets, then money will arise endogenously as a superior medium of exchange.

[11] This assumption allows us to capture the reasonable idea that capital is less tradable than bonds while maintaining the simplicity of our model. A bond delivers one dollar at the end of the current period; thus, any agent who does not have a current need for money will be happy to buy such bonds (at the right price). However, selling a piece of machinery or a building is less straightforward, as one first needs to find the *right* buyer(s) for these items. Hence, one can also think of η as the probability with which a suitable buyer is located.

to liquidate assets, while the others use their money to buy assets at a good price.[12] The government can also intervene in the secondary market by selling additional bonds, or by buying bonds with additional money. Pricing is competitive.

During the CM, households can buy or sell any asset, as well as the output good y, on a competitive market. They then choose how much of the output good is to be consumed (c_t), and they choose their asset holdings $(m_{t+1}, b_{t+1}, k_{t+1})$ for the next period. The government makes a nominal lump-sum transfer T_t to all households (a tax if negative), pays out the bond dividends to the households (one unit of money per bond), and issues new bonds. A fraction $\delta \in (0,1)$ of existing capital depreciates, and it can be replaced by investing some of the available output. Hence, the law of motion of the aggregate capital stock is:

$$k_{t+1} = y_t - c_t + (1-\delta)k_t$$

Also during the CM, households can produce, consume, and trade a "general" consumption good, $g \in \mathbb{R}$, where we interpret negative values as production and positive ones as consumption. As shown in Lagos (2010), this good is a convenient way to induce linear preferences and thereby collapse the portfolio problem into something tractable. The good has no other function in our paper; in particular, it cannot be used for investment.

Households discount the future at rate $\beta \equiv 1/(1+\rho)$, where $\beta < 1$ and/or $\rho > 0$. (In order to make equations more readable, we will use both β and ρ in the paper, but never both in the same equation or block of equations.) Households have the following utility function:

$$U_t(c_t, g_t) = u(c_t) + g_t,$$

where u is a twice continuously differentiable function that satisfies $u' > 0$ and $u'' < 0$. Labor generates no disutility, but a household's worker is only able to supply labor up to an endowment normalized to 1.

2.2 The social planner's solution

As a benchmark, consider a social planner who is not bound by commitment problems, and can freely transfer resources between agents. As all households and firms are the same, the planner will treat them symmetrically, and solve the following representative-agent problem, choosing a sequence of capital stocks and labor and consumption allocations:

$$\text{(SP)} \qquad V^{SP}(K_t) = \max_{\substack{K_{t+1}, \\ H_t, C_t, G_t}} \left\{ u(C_t) + G_t + \frac{1}{1+\rho}\, \mathbb{E}_t \left\{ V^{SP}(K_{t+1}) \right\} \right\}$$

[12] An alternative interpretation of our AM would be as a market where agents pledge their assets as collateral in order to obtain a secured (monetary) loan, as is the case in the repo market.

$$\text{subject to:} \quad C_t + K_{t+1} = A_t K_t^{\alpha} H_t^{1-\alpha} + (1-\delta)K_t, \quad H_t \leq 1, \quad \text{and} \quad G_t = 0$$

The initial capital stock K_0 is taken as given.

As the G-consumption good is in zero net supply, this is equivalent to the well-known neoclassical Ramsey problem. With perfectly inelastic labor supply, we must have $H_t = 1$, thus consumption and the capital stock satisfy:

$$\text{(EE)} \qquad u'(C_t) = \frac{1}{1+\rho}\,\mathbb{E}_t\left\{u'(C_{t+1})\left(\alpha A_{t+1} K_{t+1}^{\alpha-1} + 1 - \delta\right)\right\}$$

$$\text{(LOM)} \qquad C_t + K_{t+1} = A_t K_t^{\alpha} + (1-\delta)K_t$$

$$\text{(TVC)} \qquad 0 = \lim_{t\to\infty} \frac{u'(C_t) A_t K_t^{\alpha}}{(1+\rho)^t}$$

In steady state, we must have $Y = AK^{\alpha} = C + \delta K$, which we can use to solve:

$$\begin{aligned} Y^* &= A^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{\rho+\delta}\right)^{\frac{\alpha}{1-\alpha}} & K^* &= \frac{\alpha}{\rho+\delta} \cdot Y^* \\ C^* &= \frac{\rho + (1-\alpha)\delta}{\rho+\delta} \cdot Y^* & H^* &= 1,\ G^* = 0 \end{aligned} \tag{1}$$

2.3 Optimal behavior by private agents

We define the output good *in the CM* to be the numéraire. Because of the frictions in this economy, the price of that good *in the PM* will generally not be 1; denote it by q.

Begin the analysis with firms. Because of constant returns to scale in production, the firms' number is indeterminate, and the representative firm solves the static problem:

$$\text{(FP)} \qquad \max_{Y_t, H_t, K_t} \{q_t Y_t - w_t H_t - r_t K_t\} \quad \text{subject to: } Y_t = A_t K_t^{\alpha} H_t^{1-\alpha},$$

where w and r are the wage and rental rate on capital, denominated in terms of numéraire. (Hence, they represent the marginal *revenue* products of labor and capital, which differ from the marginal products by the output price q.) Solving this problem defines demand for labor and capital services:

$$\frac{w_t}{q_t} = A_t(1-\alpha)\left(\frac{K_t}{H_t}\right)^{\alpha} \qquad \frac{r_t}{q_t} = A_t \alpha \left(\frac{K_t}{H_t}\right)^{\alpha-1}$$

Since the supply of labor is capped but its marginal product is positive, we will have $H_t = 1$ in every equilibrium, which pins down the wage. The price of output thus satisfies:

$$q_t = \frac{r_t K_t^{1-\alpha}}{\alpha A_t} \tag{2}$$

This equation is central for the LAMMA. In steady state, the prices q and r will be determined by Euler equations. Thus, the long-run capital stock is governed by three sufficient statistics: productivity, the relative price of output between the PM and the CM, and the marginal revenue product of capital. Later, we will see that monetary policy affects the economy in the long run through q, r, or both.

Households make the dynamic decisions in this economy, thus they have a richer menu of choices which is easiest to describe in stages. Begin with the CM of period t, and consider a household coming in with portfolio (m_t, b_t, k_t, y_t) of money, bonds, capital, and output goods. The household chooses their consumption (c_t and g_t), as well as the asset portfolio $(m_{t+1}, b_{t+1}, k_{t+1})$ to be carried into the next period. The prices of general goods (p_t^G, in terms of numéraire), money (ϕ_t, in terms of numéraire), and bonds (p_t^B, in terms of money) are taken as given, and the transfer of money from the government (T_t) is also taken as given. Since new capital is created by not consuming output goods (the numéraire), the price of capital in the CM will simply be 1 – exactly as it is in the standard neoclassical model.

Let $\Lambda_{t+1} \in \{0, 1\}$ be the random variable indicating whether an individual shopper will be selected to shop in the next period. It is distributed i.i.d. with $\mathbb{P}\{\Lambda_{t+1} = 1\} = \lambda_{t+1}$, and we call it a "liquidity shock". Letting V^{CM} and V^{AM} denote the value functions in the CM and AM subperiods, respectively, we can describe the household's choice as follows:

$$V^{CM}(m_t, b_t, k_t, y_t) = \max_{\substack{c_t, g_t, m_{t+1}, \\ b_{t+1}, k_{t+1}}} \left\{ u(c_t) + g_t + \beta\, \mathbb{E}_t \left\{ V^{AM}(m_{t+1}, b_{t+1}, k_{t+1}, \Lambda_{t+1}) \right\} \right\} \tag{3}$$

$$\text{subject to:} \quad c_t + k_{t+1} + p_t^G g_t + \phi_t(m_{t+1} + p_t^B b_{t+1}) = y_t + (1-\delta)k_t + \phi_t(m_t + b_t + T_t)$$

At this point, one can confirm that the value function V^{CM} will be linear, and that a household's choice of consumption (c) is independent of its asset portfolio (details are provided in Appendix A.1). A household with few assets will work to produce general goods g, and sell them to be able to afford its desired level of c, and its desired future asset portfolio. Conversely, a household with many assets will be consuming general goods.

Working backwards through the period, consider the PM of period t. At this stage, the household decides how much labor (h) and capital services (x) to supply to firms, and how much of the output good y the shopper should buy (if applicable). The household takes factor prices and the price of goods as given. Letting V^{PM} denote the value function in the PM subperiod, we can describe the households' choices as follows:

$$V^{PM}(m_t, b_t, k_t, \Lambda_t) = \max_{y_t, x_t, h_t} \left\{ V^{CM}\left(m_t - \frac{q_t}{\phi_t} y_t + \frac{w_t}{\phi_t} h_t + \frac{r_t}{\phi_t} x_t, b_t, k_t, y_t \right) \right\}$$

$$\text{subject to:} \quad y_t \leq \Lambda_t \frac{\phi_t}{q_t} m_t, \quad x_t \leq k_t, \quad \text{and} \quad h_t \leq 1$$

Finally, consider the AM of period t. The liquidity shocks Λ_t have just been realized; money is the only asset that can be used to buy goods in the PM, therefore households with $\Lambda_t = 1$ will seek to sell other assets for money, and vice versa. Households can trade any amounts of money and bonds that they own, but they cannot sell them short; to short-sell is to create an asset, and bonds and money are special assets that can only be created by a trusted authority. With capital, households face an additional limited commitment problem: only a fraction $\eta_t \in [0,1]$ can be sold on the market. We denote the amounts of bonds and capital sold by $(\Lambda_t = 1)$-households by (χ_t, ξ_t), respectively. We denote the money spent to buy bonds and capital by the other households by (ζ_t^B, ζ_t^K), respectively. Households take the prices of bonds and capital as given; we denote them by s_t^B and s_t^K, in terms of money.[13] Hence, we can describe the households' choices as follows:

$$V^{AM}(m_t, b_t, k_t, 0) = \max_{\zeta_t^B, \zeta_t^K} \left\{ V^{PM}\left(m_t - \zeta_t^B - \zeta_t^K, b_t + \frac{\zeta_t^B}{s_t^B}, k_t + \frac{\zeta_t^K}{s_t^K}, 0 \right) \right\} \tag{4}$$
$$\text{subject to:} \quad \zeta_t^B + \zeta_t^K \leq m_t;$$

$$V^{AM}(m_t, b_t, k_t, 1) = \max_{\chi_t, \xi_t} \left\{ V^{PM}\Big(m_t + s_t^B \chi_t + s_t^K \xi_t, b_t - \chi_t, k_t - \xi_t, 1 \Big) \right\} \tag{5}$$
$$\text{subject to:} \quad \chi_t \leq b_t \quad \text{and} \quad \xi_t \leq \eta_t k_t$$

We relegate the detailed solution of the household's problem to Appendix A.1 and only review the highlights here. First, s^B and s^K are linked through a no-arbitrage equation, because the asset buyer (the household with $\Lambda_t = 0$) can choose to spend their money on either bonds or capital and must be indifferent in equilibrium:

$$\phi_t \, s_t^K = (r_t + 1 - \delta) \, s_t^B \tag{6}$$

That is, the real price of capital in the secondary market must equal the price of bonds, times the value of capital in subsequent markets. This value is the real marginal revenue product (r_t), plus the fraction remaining after depreciation ($1 - \delta$).

Finally, we arrive at the solution of the portfolio problem in the primary asset market (the CM). This solution must satisfy the following Euler equations for money, bonds, and capital:

$$u'(c_t)\phi_t = \beta \, \mathbb{E}_t \left\{ u'(c_{t+1})\phi_{t+1} \left(\lambda_{t+1} \frac{1}{q_{t+1}} + (1 - \lambda_{t+1}) \frac{1}{s_{t+1}^B} \right) \right\} \tag{7}$$

$$u'(c_t)\phi_t p_t^B = \beta \, \mathbb{E}_t \left\{ u'(c_{t+1})\phi_{t+1} \left(\lambda_{t+1} \frac{s_{t+1}^B}{q_{t+1}} + (1 - \lambda_{t+1}) \right) \right\} \tag{8}$$

[13] The letters p and s are intended to be mnemonics for "primary market price" and "secondary market price".

$$u'(c_t) = \beta\,\mathbb{E}_t\left\{u'(c_{t+1})(r_{t+1}+1-\delta)\left(\lambda_{t+1}\eta_{t+1}\frac{s^B_{t+1}}{q_{t+1}} + (1-\lambda_{t+1}\eta_{t+1})\right)\right\} \tag{9}$$

Naturally, the incentive to accumulate capital depends on conditions in the secondary market, including the liquidity of capital and its resale price s^K. The reason why s^K does not explicitly appear in (9) is that we have substituted it with the secondary market price of bonds, s^B, via the no-arbitrage equation. We write the Euler equation in this way because monetary policy is implemented via setting the secondary market price of bonds; thus, the equation makes transparent how monetary policy affects the value of capital in the primary market, where investment decisions happen, and how this effect is moderated by the expected tradability of capital in the secondary market (η_{t+1}).

Suppose, for a moment, that the future bond price (s^B_{t+1}) is known at time t (perhaps it is pegged by a policy, or we are in steady state). In that case, we can multiply both sides of the money demand equation (7) by s^B_{t+1}. The right-hand side that remains is identical to the right-hand side of the bond demand equation (8). Hence, in this case, we must have $p^B_t = s^B_{t+1}$: the primary market price of bonds (in the CM of period t) equals the secondary market price (in the AM of period $t+1$). Therefore, when we speak of a policy of "setting bond interest rates" henceforth, it does not always matter whether the primary or secondary market rate is being set. (It still matters sometimes; for example, if we allow state-contingent policies that respond to information revealed just before the AM opens.)

2.4 Market clearing and the government budget

The market clearing conditions of this economy are as follows, where integrals are to be taken over the measure of all households. In the AM, the demands for bonds and tradable capital must equal their respective supplies:

$$\lambda_t \int \chi_t \cdot s^B_t = (1-\lambda_t)\int \zeta^B_t \qquad \text{and} \qquad \lambda_t \int \xi_t \cdot s^K_t = (1-\lambda_t)\int \zeta^K_t$$

In the PM, where only a fraction λ of households is able to shop, but every household supplies factor services, individual choices must add up to the respective aggregate quantities that solve the firm's problem:

$$\lambda_t \int y_t = Y_t, \qquad h_t = H_t = 1, \qquad x_t = k_t, \qquad \text{and} \qquad \int k_t = K_t$$

And in the CM, demands for goods and assets must equal their respective supplies:

$$\int m_{t+1} = M_{t+1}, \qquad \int b_{t+1} = B_{t+1},$$

$$\int g_t = 0, \qquad \text{and} \qquad \int c_t + \int k_{t+1} = Y_t + (1-\delta)K_t$$

Excepting labor and capital services (h_t, x_t), only the total of individual-household variables has to equal the respective aggregate quantity. But since all households are ex-ante identical and have linear value functions, we may as well restrict attention to symmetric solutions in asset portfolios (allowing for temporary differentiation during the AM and PM, along with the differentiation in g this causes).

The consolidated government chooses the sequences $\{M_t, B_t\}_{t=0}^{\infty}$. Money is introduced (or withdrawn) via lump-sum transfers $\{T_t\}$, and bonds are sold to the public at the market price. (For now, we assume that both of these things happen in the CM – considering only steady states, it does not matter – but in Appendix A.4, we explicitly model a monetary authority that can buy and sell bonds for money in the AM, which is the obvious counterpart in our model to how monetary policy is implemented in the real world.) Hence, the government budget must satisfy an intertemporal budget constraint and a no-Ponzi constraint:

$$M_{t+1} + p_t^B B_{t+1} = M_t + B_t + T_t \qquad \text{for all } t \geq 0$$

$$\left\{ \frac{B_t}{M_t} \right\}_{t=0}^{\infty} \quad \text{is bounded}$$

Definition 1. An *equilibrium* of this economy consists of sequences of quantities $\{c_t, h_t, k_t, Y_t, m_t, b_t, \chi_t, \xi_t, \zeta_t^B, \zeta_t^K\}_{t=0}^{\infty}$ and prices $\{\phi_t, q_t, p_t^B, s_t^B, s_t^K\}_{t=0}^{\infty}$ that satisfy:

- The Euler equations (7)-(9)
- The market clearing equations and government budget constraints in this subsection
- The transversality conditions:

$$\lim_{t\to\infty} \beta^t \, u'(c_t)\phi_t = \lim_{t\to\infty} \beta^t \, u'(c_t)k_t = 0$$

An *equilibrium* is said to be *monetary* if $\phi_t > 0$ for all $t \geq 0$.

This completes our description of the economy. The following two sections analyze equilibrium and policy in steady state, and we will revisit stochastic equilibria in future work.

3 Equilibrium and policy in steady state

Clearly, a non-monetary equilibrium has $C = K = Y = 0$, hence it is not very interesting. For the rest of the paper, we focus on monetary equilibria.

The government has two (sequences of) choice variables: the supply of money and tradable bonds. In equilibrium, these sequences imply particular asset prices, or interest rates

(although the mapping is not one-to-one everywhere). It turns out that a certain pair of interest rates is a sufficient statistic for the effects of monetary policy on the macroeconomy.

Hence, we have a choice: we can define government policy in terms of particular quantities, or in terms of particular interest rates that the government is targeting. The next subsection describes the former case, and the one after that describes the latter case.

3.1 Policy is set in terms of quantities

In steady state, all variables except ϕ_t, s_t^K, M_t, and B_t must be constant. The latter three must grow at the same rate – call it μ in gross terms, or $(\mu-1)$ in net terms – and ϕ_t must decline at rate μ. The transversality condition requires that $\mu \geq \beta$.

The Euler equations thus take the following form in steady state:

$$\frac{\mu}{\beta} = \frac{\lambda}{q} + \frac{1-\lambda}{s^B} \tag{10}$$

$$p^B \frac{\mu}{\beta} = \lambda \frac{s^B}{q} + 1 - \lambda$$

$$\frac{1}{\beta} = (r + 1 - \delta) \cdot \left(1 + \eta \left[\frac{\mu}{\beta} s^B - 1\right]\right) \tag{11}$$

The first Euler equation represents the demand for money. The left-hand side is the cost of holding wealth in the form of money: inflation times impatience. The right-hand side is the benefit: the ability to buy goods at price q in the PM and then sell them at the higher price 1 in the CM (with probability λ), or the ability to buy bonds in the secondary market at price s^B and collect the full dividend (with probability $1-\lambda$).

The second Euler equation represents the demand for bonds. As said earlier, we can divide it by the money demand equation to confirm that $p^B = s^B$ in steady state.

The last Euler equation represents the demand for capital. The left-hand side is the cost of storing wealth in the form of capital: impatience. The first term on the right-hand side is the *fundamental* benefit: the ability to collect capital rents in the future. The second term is an additional value of capital: if $\eta > 0$, then capital also provides a *liquidity service*, and if $s^B > \beta/\mu$ – the price of bonds exceeds its own fundamental value – then both bonds and capital are priced for this service.

Suppose we have solved for prices q and r. Then the production side in the PM (Equation (2)) pins down the capital stock and the capital-output ratio:

$$\frac{K}{Y} = \frac{\alpha q}{r}$$

In steady state, aggregate consumption and capital depreciation must add up to output ($Y = C + \delta K$). Putting these together, we can solve for the rest of the real economy:

$$Y = A^{\frac{1}{1-\alpha}} \left(\frac{\alpha q}{r}\right)^{\frac{\alpha}{1-\alpha}}, \qquad K = \frac{\alpha q}{r} \cdot Y, \qquad \text{and} \qquad C = \left(1 - \frac{\alpha \delta q}{r}\right) \cdot Y \tag{12}$$

We see that the equilibrium quantities are fully pinned down by the prices q and r, and the Euler equations show that these prices are in turn determined by s^B, the secondary market price of bonds. Thus, everything hinges on conditions in the secondary market, and on the aggregate supplies of bonds and liquid capital relative to money.[14] It turns out that general equilibrium falls into one of three regions: (A) abundant bond supply, which is obtained when B/M is large; (B) an intermediate region; and (C) scarce bond supply, which is obtained when B/M is small. These regions are illustrated in Figure 2, and their boundaries are derived in detail in Appendix A.2.

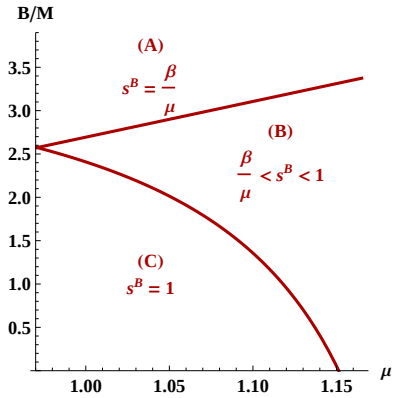


Figure 2: Regions of equilibrium, in terms of money growth μ and the bond-to-money ratio B/M. Parameters: $\beta = 1/1.03$, $\delta = 0.1$, $\alpha = 0.36$, $\lambda = 0.2$, $\eta = 0.5$.

Region (A): large bond supply

Consider the AM problem described in Equations (4)-(5) and solved in Appendix A.1. Because bonds are in large supply, the constraint on selling bonds will not bind. The associated first-order conditions (Equations (A.1)-(A.2)) show that if the constraint on selling bonds does not bind, then neither does the constraint on selling capital. Setting the corresponding Lagrange multipliers to zero and working through the first-order conditions, we learn that $s_t^B = q_t$ in any equilibrium. Hence, in steady state:

$$p^B = s^B = q = \frac{\beta}{\mu} \qquad \text{and} \qquad r = \frac{1}{\beta} + \delta - 1$$

And we can plug these prices into Equation (12), and substitute out the first-best level of output using Equation (1):

[14] One may also consider the possibility of private agents issuing liquid bonds. See Geromichalos and Herrenbrueck (2016b) and the references therein, as well as Lagos and Zhang (2018).

$$Y = \left(\frac{\beta}{\mu}\right)^{\frac{\alpha}{1-\alpha}} \cdot Y^*$$

This is the familiar form of the inflation tax. Output, consumption, and the capital stock are below their first-best levels unless money growth satisfies the Friedman rule: $\mu \to \beta$. The flow of expenditure in the PM must equal the value of output. In the AM all the money was channeled to the active shoppers, which gives us the following quantity equation for the PM:

$$\phi M = qY = \frac{\beta}{\mu} Y \tag{13}$$

This region satisfies "Wallace neutrality": changes in the supply of bonds, whether implemented by open-market operations or in any other way, are irrelevant. Money is neutral, too – it affects only the general price level ($1/\phi$) and nothing else – although of course not superneutral. One may think that the liquidity of bonds and/or capital is "irrelevant" here; however, that is not precisely true. The fact that bonds and capital allow agents to purchase money in the AM means that the demand for money is lower than it would otherwise be. This happens not to affect real variables in this region, but it does affect whether we can be in this region in the first place. Bonds and capital still provide liquidity services, it is just that they provide them inframarginally.

Region (B): intermediate bond supply

Now, suppose that B/M is smaller than in Region (A), but not much smaller. In that case, both buyers and sellers of assets in the AM will be constrained, and the market clearing equation in the AM becomes: $(s^B B + \eta\, s^K K) = (1-\lambda)\, M$. After using the no-arbitrage equation (6) to substitute s^K, we obtain:

$$\lambda\, s^B \left(B + \frac{r+1-\delta}{\phi}\eta K\right) = (1-\lambda)\, M \tag{14}$$

Because of CRS in production, capital owners receive a fraction α of total expenditure ϕM; that is, $rK = \alpha\phi M$. If we use this to substitute r, we obtain an equation that holds in any period – not just in steady state – and shows how an open-market operation that trades bonds for money in the AM affects the return on liquid bonds $1/s_t^B$ and the price level $1/\phi_t$, for given state variables (λ_t, η_t, K_t):

$$\frac{1}{s_t^B} = \frac{\lambda_t}{1-\lambda_t}\left(\frac{B_t}{M_t} + \alpha\eta_t + (1-\delta)\eta_t \frac{K_t}{\phi_t M_t}\right) \tag{15}$$

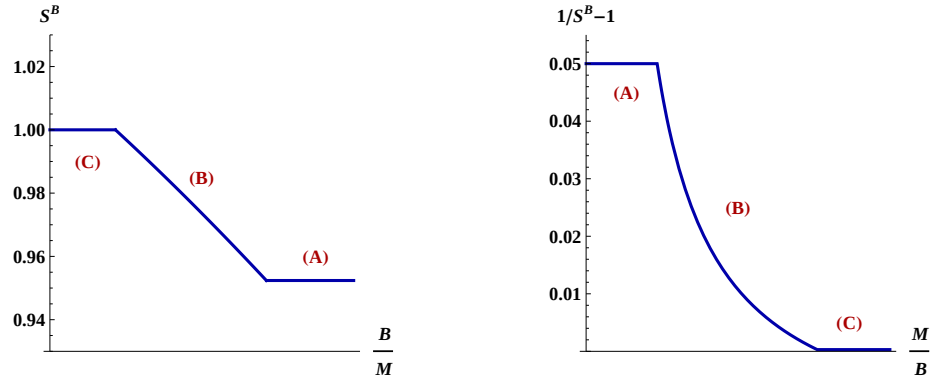
But since the focus of this section is on steady states, we instead use $rK = \alpha\phi M$ to substitute K in Equation (14), and the Euler equation to substitute r, and we define the auxiliary term

X to get the following expression relating the quantity of bonds with their price:

$$\frac{B}{M} = \frac{1-\lambda}{\lambda} \cdot \frac{1}{s^B} - \frac{\alpha\eta}{1-X}, \qquad \text{where:} \qquad X = \beta(1-\delta)\left[1+\eta\left(\frac{\mu}{\beta}s^B - 1\right)\right] \tag{16}$$

Since $dX/ds^B > 0$, we see that the quantity B/M must be negatively related to the price s^B, and that the equation has a unique implicit solution for s^B in terms of B/M.

Hence, Region (B) is the region of effective monetary policy. An open-market purchase which increases the quantity of money at the expense of bonds will increase the price of bonds and affect the real economy, in the short run through the level of real balances (Equation (15)), and in the long run through q and r (Equation (16), illustrated in Figure 3). Even a helicopter drop of money which left the quantity of bonds unchanged would work in the same direction and have, unless it was reversed, permanent effects.



[a] Bond demand in terms of bond price [b] Money demand in terms of bond yield

Figure 3: Comparative statics of the bond-money ratio B/M in steady state, interpreted as long-run demand curves for these assets. Panel [b] is the inversion of Panel [a].
Parameters: $\beta = 1/1.03$, $\mu = 1.02$, $\delta = 0.1$, $\alpha = 0.36$, $\eta = 0.35$, $\lambda = 0.45$.

Having thus solved for s^B, we can use the Euler equations (10)-(11) to find q and r. Differentiating the Euler equations, we see that:

$$\frac{dq}{ds^B} < 0 \qquad \text{and} \qquad \frac{dr}{ds^B} < 0$$

This is intuitive: if bonds are more expensive in the secondary market, then asset buyers will not get such a good return on their money. Anticipating this (with probability $1-\lambda$), agents will carry less money in the first place. This goes on until the principal compensation for holding money – the mark-up earned by buying goods in the PM and selling them in the CM, $1/q$ – has increased enough. Furthermore, as bonds are more expensive in both markets, agents will prefer to hold capital as a store of value, leading to an increased accumulation of

capital; that is, until the return on capital has fallen enough to make them indifferent again.

Plug these results into Equation (12), and we see that the effect of s^B on steady-state output is generally ambiguous. Making bonds scarce (hence, increasing their price) takes away one way for agents to store their wealth and avoid the inflation tax. On the other hand, agents will respond by substituting into capital, which stimulates investment and, ultimately, output and consumption. It would then be good to know which effect dominates. We defer this analysis to the next subsection.

Region (C): low bond supply

In this region, the constraints on selling bonds and capital in the AM do bind, but the constraint on spending money does not. Setting the associated Lagrange multiplier to zero and working through the first-order conditions (see Appendix A.1), we learn that $s_t^B = 1$ in any equilibrium. This is intuitive: after the AM has closed, the only benefit the bonds have is to pay out one unit of money in the CM. Hence, in steady state:

$$\begin{aligned} p^B &= s^B = 1 \\ q &= \frac{\lambda\beta}{\mu - (1-\lambda)\beta} \\ r &= \frac{1}{\eta\mu + (1-\eta)\beta} + \delta - 1 \end{aligned}$$

In this region, bond prices are maximal and q and r are minimal. In the AM, agents with a shopping opportunity sell all their bonds and liquid capital ($\chi = B$ and $\xi_t = \eta_t K_t$; bonds are scarce if and only if capital is, too), but the asset buyers are not willing to spend all their money at such high prices. Therefore, the flow of spending in the PM no longer satisfies the standard quantity equation $\phi M = qY$. Instead, the general price level ($1/\phi$) is determined by a quantity equation that includes the bond supply:

$$\underbrace{\lambda\left[\phi\, M + \phi\, B + (r + 1 - \delta)\,\eta\, K\right]}_{\text{PM expenditure}} \quad = \quad \underbrace{q\, Y}_{\text{value of PM output}} \tag{17}$$

The left-hand side of the equation equals the real value of all money held by shoppers in the PM; this is less than ϕM, because bonds and tradable capital were so scarce in the AM that prospective shoppers were not able to buy up all the idle money in the economy.

What does this mean for monetary policy? As bonds are traded at a price of 1, open-market operations that swap money for bonds are neutral; they have no effect on any equilibrium variables, unless they happen to increase B/M sufficiently to exit Region (C). A helicopter drop of money is neutral for real variables, but it will still affect the general price level, $1/\phi$. Furthermore, money is not superneutral: an increase in steady-state money growth will

decrease both q and r, in the same way that an increase in bond prices did in Region (B).

3.2 Policy is set in terms of interest rates

Now that we understand the regions that a monetary equilibrium can be in, suppose that the government defines its monetary policy not in terms of sequences $\{M_t, B_t\}_{t=0}^{\infty}$, but in terms of the interest rates that these sequences imply, and lets the quantities adjust implicitly.

First, it will be convenient to consider the interest rate that would be paid on a bond that is nominal, one hundred percent default-free, but one hundred percent *illiquid*, in the sense that it must be held to maturity. Specifically, in our context, suppose that it is a one-period discount bond that is sold in the CM and pays of one unit of money in the subsequent CM. Call its interest rate i_t; in any monetary equilibrium, it must satisfy:

$$1 + i_t = \frac{u'(c_t)\phi_t}{\beta \, \mathbb{E}_t \left\{ u'(c_{t+1})\phi_{t+1} \right\}}$$

Therefore, in steady state:

$$1 + i = \frac{\mu}{\beta}$$

Such a bond – short-term, perfectly safe, yet perfectly illiquid – does not exist in the real world.[15] Hence its return is an abstract object that must be estimated, just like "the general price level" or "total factor productivity", and it should be referred to by a proper name. Since $i = 0$ is what defines the Friedman rule, we call i the **Friedman interest rate**.

Because this rate equals expected inflation divided by the discount factor (out of steady state approximately so), the real-world monetary policy instrument that is its closest counterpart is probably the inflation target rather than any particular interest rate.[16] Certainly, i is not the "policy rate" that is used to implement short-term monetary policy. We have a much better counterpart for that in our model: the price of liquid bonds which clears the secondary asset market. Consider:

> "The effective federal funds rate is the interest rate at which depository institutions ... borrow from and lend to each other overnight to meet short-term business needs."
>
> "Prior to ... 2007, the Federal Reserve bought or sold securities issued or backed by the U.S. government in the open market on most business days in order to keep ... the federal

[15] Safe assets tend to be more liquid (Lagos, 2010), and short term assets also tend to be more liquid (Geromichalos et al., 2016). To be really precise: when we say that an illiquid bond is one that has to be held to maturity and cannot be traded in between, we actually require that this maturity is so far off that the owner does not anticipate any particular liquidity need that the bond payout could be used for. For example, a 1-month bond cannot be terribly illiquid *by its very nature*; many unanticipated expenditures can be put off for a month or two, or paid for by dipping into a credit line, and then the bond payout can be used to pay off the loan.

[16] In developed economies with an inflation target, expected inflation has been stable for many years, and is thus not considered an important contributor to the business cycle (Hamilton, Harris, Hatzius, and West, 2016).

> funds rate at or near a target set by the Federal Open Market Committee."
>
> (Source: https://www.federalreserve.gov/aboutthefed/files/pf_3.pdf)

Or:

> "The Bank [of Canada] carries out monetary policy by ... raising and lowering the target for the overnight rate. The overnight rate is the interest rate at which major financial institutions borrow and lend one-day (or 'overnight') funds among themselves; the Bank sets a target level for that rate. This target for the overnight rate is often referred to as the Bank's policy interest rate."
>
> (Source: http://www.bankofcanada.ca/core-functions/monetary-policy/key-interest-rate/)

That is, monetary policy is *not* implemented by, say: "raising and lowering the target for the interest rate on safe yet illiquid bonds", or by: "manipulating money growth in order to set investors' inflation expectations". It is implemented by setting the target for *overnight loans to meet short-term business needs*, and backed by (explicit or implicit) *open-market operations*. Institutional details aside, this is exactly what is going on in our model: the "policy rate" is the price of bonds in the secondary market, where agents with "short-term business needs" meet to reallocate liquidity "overnight".

Therefore, exploiting the standard formula that links the price and interest rate of an asset, we define the **policy interest rate** to be:

$$1 + j_t \equiv \frac{1}{s_t^B}$$

As we have seen earlier, if j_{t+1} is known in period t, then it is also the interest rate on bonds in the primary market, no matter what other sources of uncertainty exist. In steady state, we simply have $j = 1/s^B - 1 = 1/p^B - 1$, and the policy rate has the bounds:

$$0 \leq j \leq i$$

Out of steady state, we must still have $j_t \geq 0$, but a temporary $j_t > i_t$ or $j_t > i_{t-1}$ is possible.

Using the results from the previous section, we can describe equilibrium in terms of two instruments: the policy rate j and the Friedman rate i (or, if one prefers, the policy rate j and the inflation target $i - \rho$). Setting $j \in (0, i)$ implies that equilibrium is in Region (B), and Regions (A) and (C) can only be reached at the boundaries $j = i$ and $j = 0$. Restating Equation (16) in terms of interest rates, we get:

$$\frac{B}{M} = \frac{1-\lambda}{\lambda} \cdot (1+j) \; - \; \frac{\alpha\eta}{1 - \beta(1-\delta)\left(1 - \eta + \eta\frac{1+i}{1+j}\right)}$$

As Panel [a] of Figure 4 illustrates, *holding the bond-to-money ratio fixed*, j is a monotonically increasing function of i. This fact, which holds true in all monetary models where bonds are not perfectly liquid, may tempt one to think that i and j must be positively linked in reality, too. Certainly, almost all of monetary theory treats i as the main instrument of monetary policy; comparative statics with respect to the bond supply may be considered, but are usually treated as secondary.

Here, we propose a very different approach, treating the interest rates i and j as *distinct policy instruments of equal standing*. The main instrument for the short run is j, the yield on liquid bonds, and j is implemented via open-market operations that alter the bond-to-money ratio. The Friedman rate i is determined by the implied path of inflation expectations; thus, depending on the prevailing shocks in the economy and the monetary policy regime, the empirical correlation between i and j may be positive, negative, or zero. We illustrate these possibilities in Panels [b]-[c] of Figure 4.[17]

Going back to the steady-state Euler equations (10)-(11), we can write them as follows. First, consider bond demand, which gives:

$$p^B = \frac{1}{1+j}$$

Since the fundamental price of a nominal discount bond is $1/(1+i)$, we can also write the bond price as the product of the fundamental price and a *liquidity premium*:

$$p^B = \frac{1}{1+i} \times (1+\ell),$$

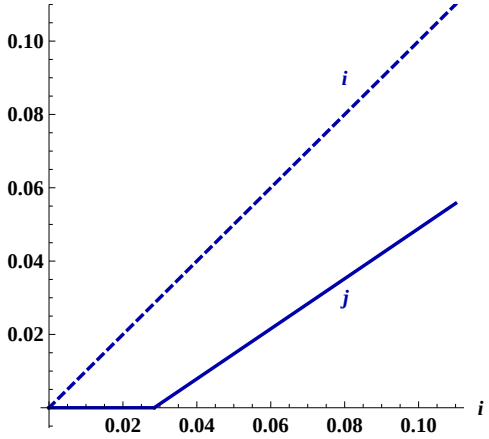
where the liquidity premium is thus defined as:

$$\ell \equiv \frac{i-j}{1+j},$$

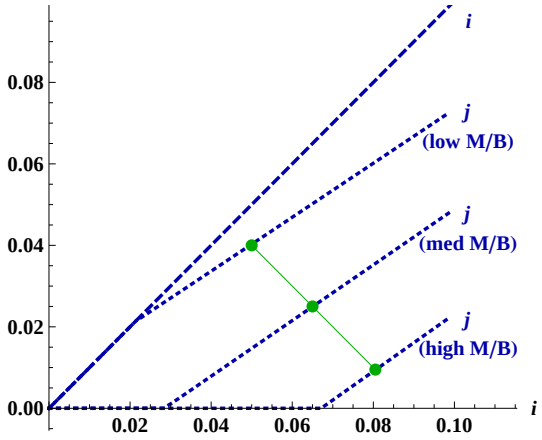
which must satisfy $0 \leq \ell \leq i$. In what follows, we will see that the most succinct way to write the equilibrium equations is in terms of (i, ℓ), but it should be kept in mind that this is a simple transformation of (i, j).

Second, money demand pins down the PM price of goods (q) in terms of monetary policy:

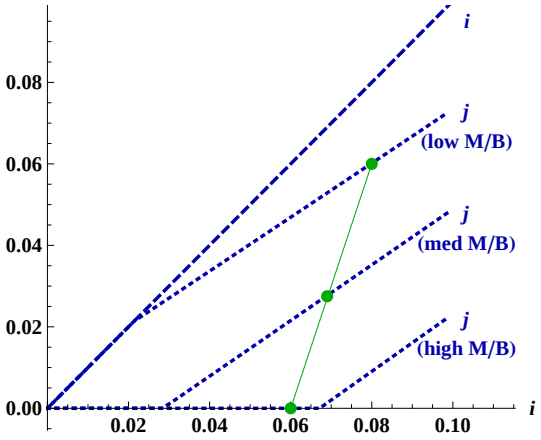
[17] To our knowledge, our model is the first to explicitly consider the secondary market interest rate on short-term, liquid bonds to be the main instrument of monetary policy. Andolfatto and Williamson (2015) and Rocheteau et al. (2014) are the closest to us on this count: they do not model a secondary market, but study the price of liquid bonds in the primary market. The vast majority of the New Monetarist literature uses the money growth rate as the only monetary policy instrument, and its principal influence on the economy comes through the inflation tax. Some New Monetarist papers also consider the quantity of liquid bonds as a tool of monetary policy (e.g. Williamson, 2012; Geromichalos and Herrenbrueck, 2016a; Herrenbrueck, 2014; Huber and Kim, 2017). The New Keynesian literature interprets i_t (which it inherits from the older money-in-the-utility-function and cash-in-advance literatures) as the monetary policy rate, and it is endowed with real effects on the economy via the assumption of sticky prices (Woodford, 2003).



[a] Standard view in monetary theory: j is a monotonic function of i. Comparative statics with respect to the bond supply may be considered, but are of secondary interest.



[b] If looser monetary policy increases the money stock and expected money growth at the same time, then i and j may be *negatively* correlated.



[c] If high expected inflation causes a tightening of monetary policy (for example via a Taylor rule), then i and j may be *positively* correlated.

Figure 4: Comparative statics of j, the yield on liquid bonds, with respect to the Friedman rate i and the bond-to-money ratio B/M, in steady state.
Parameters: $\rho = 0.03$, $\delta = 0.1$, $\alpha = 0.36$, $\eta = 0.35$, $\lambda = 0.45$ maintained.
Panel [a]: $B/M = 0.15$. Panels [b] and [c]: $B/M \in \{0.03, 0.15, 0.25\}$.

$$q = \frac{\lambda}{1 + i - (1 - \lambda)(1 + j)}$$

$$\Rightarrow \qquad q = \frac{1 + \ell}{(1 + i)\left(1 + \frac{\ell}{\lambda}\right)}$$

This is the place to recall Friedman's (1969) famous argument that money balances are optimized when the marginal cost of holding money is zero, which gave the policy $i = j = \ell = 0$ the name "Friedman rule". At the Friedman rule, $q = 1$, and away from it, $q < 1$; hence, q is a wedge that measures how far away the economy is from the Friedman rule, and we therefore call it the **Friedman wedge**. Notice that for a fixed i, the wedge is brought closest to 1 when $\ell = 0$ – that is, $j = i$, the policy rate being at the maximal level. The reason for this is that bonds represent a way for agents to avoid the inflation tax. When the rate of return on bonds is maximized, the impact of the inflation tax is minimized.[18]

Third, capital demand pins down the marginal revenue product of capital in terms of monetary policy – and it turns out that this is most conveniently shown by expressing time preference through the parameter $\rho = (1 - \beta)/\beta$:

$$1 + \rho = (r + 1 - \delta) \cdot \left(1 + \eta \frac{i - j}{1 + j}\right)$$

$$\Rightarrow \qquad r = \delta + \frac{\rho - \eta\ell}{1 + \eta\ell}$$

Thus, the liquidity premium ℓ is a sufficient statistic for the effect of monetary policy on r.

This is the place to recall Mundell's (1963) and Tobin's (1965) famous argument that inflation should stimulate capital accumulation, since it makes holding money more costly and money and capital are substitutes as stores of value. Since the first-best level of r is $\rho + \delta$, we can define a wedge that measures how far away the return on capital is from its benchmark:

$$\frac{\rho + \delta}{r} = \frac{1 + \eta\ell}{1 - \frac{1-\delta}{\rho+\delta}\eta\ell}$$

This wedge describes how a positive liquidity premium on bonds ($\ell > 0$) stimulates the accumulation of capital, therefore we call it the **Mundell-Tobin wedge**. When viewed in terms of the monetary policy instruments (i, j), we see that a high illiquid interest rate i (achieved, for example, through higher inflation expectations) stimulates capital accumulation, but it is a *low* level of the policy rate j that does the same.

In order to complete the characterization, we return to Equation (12) and divide by the first-best level (Equation (1)) in order to eliminate the constant. We get:

[18] This is essentially a translation of the main result of Berentsen et al. (2007) from an economy with banking to an economy with secondary asset markets.

$$Y = \left(\frac{\rho+\delta}{r}\, q\right)^{\frac{\alpha}{1-\alpha}} \cdot Y^*$$

It is now clear that output will equal its first-best level if and only if the Friedman wedge and the Mundell-Tobin wedge *exactly offset* one another. Substituting the two wedge terms for q and r, we get:

$$\left(\frac{Y}{Y^*}\right)^{\frac{1-\alpha}{\alpha}} \;=\; \Omega(i,\ell) \;\equiv\; \frac{1+\ell}{(1+i)\left(1+\frac{\ell}{\lambda}\right)} \times \frac{1+\eta\ell}{1-\frac{1-\delta}{\rho+\delta}\,\eta\ell}$$

We call Ω the **monetary wedge**. Its direction is defined such that a higher value of the wedge causes higher investment and output.[19] The effect of monetary policy on macroeconomic aggregates, in steady state, is fully described by the monetary wedge:

$$\begin{aligned} Y &= A^{\frac{1}{1-\alpha}} \left(\frac{\Omega\alpha}{\rho+\delta}\right)^{\frac{\alpha}{1-\alpha}} \\ K &= \frac{\Omega\alpha}{\rho+\delta} \cdot Y \\ C &= \frac{\rho+(1-\Omega\alpha)\delta}{\rho+\delta} \cdot Y \end{aligned}$$

3.3 Optimal policy in steady state

Comparing these equations with the social planner's solution from Section 2.2, it is clear that the economy is at its optimum if and only if $\Omega = 1$. When $\Omega > 1$, then output is inefficiently large. When $\Omega < 1$, then output is inefficiently small.

But what values does Ω take? Consider first the extreme case corresponding to Region (A), where $j = i$ and therefore $\ell = 0$ (the liquidity premium is zero, indicating that neither bonds nor capital are priced for their liquidity services). In that case:

$$\Omega_A(i) = \frac{1}{1+i}$$

The only policy that achieves $\Omega = 1$ with a zero liquidity premium is the Friedman rule, $i = 0$. Note, for later, the derivative of $\log(\Omega_A)$ at $i = 0$:

$$\left.\frac{d\log(\Omega_A)}{di}\right|_{i=0} = -1$$

Consider next the other extreme, corresponding to the "zero lower bound" region (C) where $j = 0$ and therefore $\ell = i$ (bonds are so scarce that the liquidity premium is maximal):

[19] Consumption is increasing in Ω iff $\Omega < (\rho+\delta)/\delta$. Beyond this, higher Ω would push capital accumulation beyond its Golden Rule level, and steady-state consumption would decrease.

$$\Omega_C(i) = \frac{1}{1+\frac{i}{\lambda}} \frac{1+\eta i}{1-\frac{1-\delta}{\rho+\delta}\eta i} \tag{18}$$

This term may be greater or smaller than 1, although it also satisfies $\Omega_C(0) = 1$ (there is no distortion at the Friedman rule). The term blows up when $i \to (\rho+\delta)/[(1-\delta)\eta]$. Hence, if inflation is high enough, the zero lower bound on the bond interest rate cannot be attained; the demand for bonds will hit zero at a positive interest rate, and equilibrium in the asset market must be in the interior.

For low inflation, we can classify equilibria into three cases – illustrated in Figure 5 – depending on the derivative of $\log(\Omega_C)$ at $i = 0$:[20]

$$\left.\frac{d\log(\Omega_C)}{di}\right|_{i=0} = -\frac{1}{\lambda} + \frac{\rho+1}{\rho+\delta}\eta \tag{19}$$

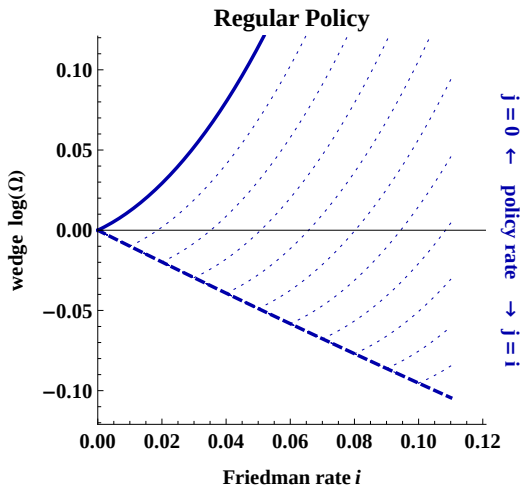
High η: "regular policy". Suppose the term (19) is zero or positive. Then, $\Omega_C > 1$ for all $i > 0$, which means that there exists an interior policy interest rate $j \in (0, i)$ that achieves the first-best $\Omega = 1$. Hence, while $j \geq 0$ is a lower bound on the policy rate, it is not a "trap"; the policy maker should never want $j = 0$ in the first place.

Intermediate η: "liquidity trap". Suppose the term (19) is within $[-1, 0)$. Then, for low inflation rates we have $\Omega_C \in (\Omega_A, 1)$. This means that the lower bound $j \geq 0$ is a binding constraint on policy: the policy maker would like to achieve $\Omega = 1$, but cannot do so by setting j alone. Instead, there are *two* ways to escape the trap: reduce inflation to the Friedman rule, or increase it sufficiently so that $\Omega_C \geq 1$ again, which is always possible if $(1-\delta)\eta > 0$.

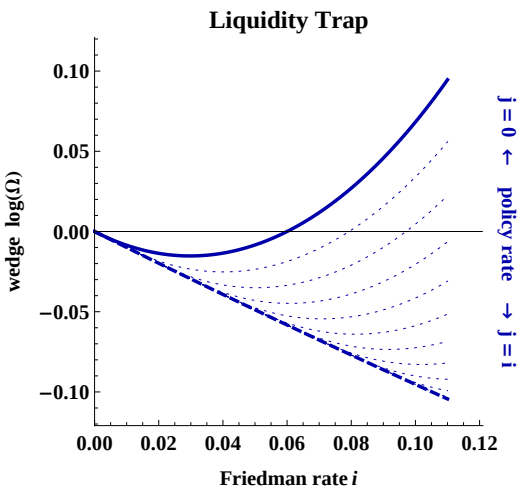
Low η: "reversal". Suppose the term (19) is less than (-1). In that case, for low enough inflation rates we have $\Omega_C < \Omega_A$, a reversal of the previous ranking. The prescription for achieving the first-best is the same as in the previous case: reduce i to zero or increase it until $\Omega_C \geq 1$. However, conditional on a fixed $i > 0$, the second-best policy is to ramp up the policy rate j to its *maximum*, $j \to i$. The Friedman effect is so strong that it completely dominates the Mundell-Tobin effect, and if the policy maker cannot enact the Friedman rule for some reason, then the second-best policy is to maximize bond interest rates in order to give a boost to money demand.

The "reversal" case is similar to several New Monetarist models where a higher bond sup-

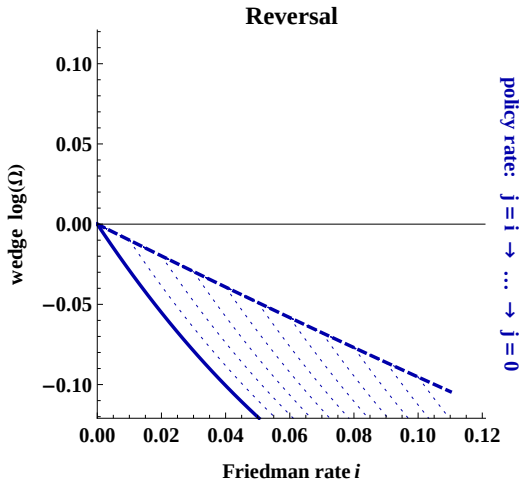
[20] *Proof.* First, solve for $\Omega_C = 1$. This gives a quadratic equation with two generic solutions: $i = 0$ and one more, call it i_1. Recall that Ω_C blows up as i increases (unless $(1-\delta)\eta = 0$, in which case there is only the trivial solution $i = 0$). Therefore, either $i_1 < 0$, in which case $\Omega_C > 1 > \Omega_A$ for all $i > 0$, or $i_1 > 0$, in which case $\Omega_C < 1$ for low enough i. Second, solve for $\Omega_C = \Omega_A$. Again, there exist two generic solutions: $i = 0$ and one more, call it i_2. Because Ω_C blows up but Ω_A does not, $i_2 < 0$ implies $\Omega_C > \Omega_A$ for all $i > 0$. Conversely, $i_2 > 0$ implies that $\Omega_C < \Omega_A < 1$ for low enough i. Third, since $i = 0$ is always a solution of $\Omega_C = \Omega_A = 1$, we can classify the ranking of (Ω_C, Ω_A) for low inflation by comparing their log derivatives at $i = 0$.



[a] High η



[b] Intermediate η



[c] Low η

Figure 5: The cone of policy options, for the three cases described in the text.
The continuous line is $\log(\Omega_C)$ (policy rate is zero), and the dashed line is $\log(\Omega_A)$ (policy rate is maximal). Dotted lines indicate policy rates of 1% to 10%.
Positive values of $\log(\Omega)$ represent overinvestment and overproduction, negative values vice versa, and any point on the horizontal axis represents first-best.
Maintained parameters: $\rho = 0.03$, $\delta = 0.1$, $\lambda = 0.2$. Varying parameter: $\eta = \{0.25, 0.5, 0.75\}$.

ply (or bond interest rate) always increases output, and often increases welfare (Williamson (2012) and Williamson (2017) are representative). The LAMMA replicates this result at the limit $\eta \to 0$, but the conclusions – and policy prescriptions – of the models radically diverge when capital is liquid enough.

4 Applications to open questions in macroeconomics

This section includes three subsections: the liquidity trap, the question of optimal inflation and the Friedman rule, and the Neo-Fisherian controversy about interest rates. They can be read independently.

4.1 The liquidity trap

The term "liquidity trap" is widely thrown about but rarely defined, and as a result, there are many models of such a trap in the literature, and they are not all that close. The term itself was adapted from Robertson (1940) – "liquidity [...] is a trap for savings" – but the concept was introduced by Keynes (1936): "almost everyone prefers cash to holding a debt which yields so low a rate of interest" (which, in context, is something bad). During the late 20th century, the memory of low nominal interest rates had faded and the concept fell into disregard until it was revived by Krugman et al. (1998) using the New-Keynesian model. In that model, zero interest rates can be a trap because prices are sticky; the economy 'wants' either lower prices today, or higher prices tomorrow, but when sticky prices constrain the former and an inflation target constrains the latter, equilibrium cannot be reached. Later, Williamson (2012) was the first to talk about the liquidity trap in a model which was explicit about the frictions that made the economy monetary, or defined exactly how the central bank could "set" interest rates. However, in that model it was no longer clear what was so bad about zero interest rates, or what made them a "trap" for policy: the zero interest rate is an indication that liquid bonds are scarce, so the right thing to do for a fiscal authority is to create more, and the right thing to do for a monetary authority is to not buy them up.

Because there are so many competing uses of the term and no clear definition, for our purposes we define a **liquidity trap** as follows:

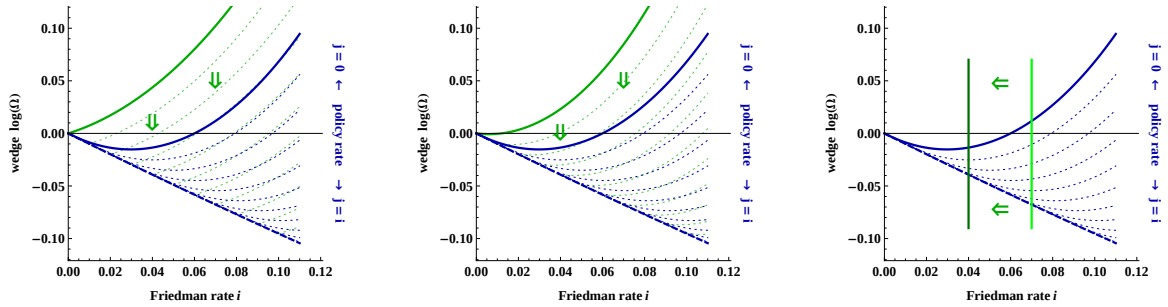
(i) The policy interest rate is at a lower bound (which could be zero or something else), but interest rates on less liquid assets are not (the economy is not at the Friedman rule)

(ii) Output and investment are below their optimal levels

(iii) Raising interest rates would make things worse (hence, "trap")

When is the LAMMA economy in a liquidity trap? The first criterion is $j = 0 < i$ (or, equivalently, $\ell = i > 0$; the liquidity premium is maximal as the policy rate is minimal).

As shown in Section 3.3, the rest depends on $\Omega(i,i)$, the value of the monetary wedge when the policy interest rate is zero. If $\Omega(i,i) < 1$, then the second criterion is satisfied, and if furthermore $\Omega(i,0) < \Omega(i,i) < 1$, then the third criterion is satisfied, too.



[a] Fall in λ (from 0.3 to 0.2)

[b] Fall in η (from 0.6 to 0.5)

[c] Fall in i (from 7% to 4%)

Figure 6: How an economy can "fall" into a liquidity trap: three plausible candidates. Parameters: $\rho = 0.03$, $\delta = 0.1$, $\eta = 0.5$, $\lambda = 0.2$ (except where indicated otherwise).

But what could push the economy into the liquidity trap? The mathematical requirement of $\Omega(i,0) < \Omega(i,i) < 1$ can be translated as saying that the term (19) is negative but not too negative, and that i is not too large. This suggests three possible culprits, as illustrated in Figure 6:

(a) A fall in the frequency of liquidity needs ($\lambda \downarrow$). Relatively more households want to buy assets in the secondary asset market, and fewer want to sell them. The equilibrium prices of bonds and capital rise, until their returns hit the respective lower bounds.

(b) A fall in the tradability of capital ($\eta \downarrow$). Capital becomes harder to sell in the secondary asset market; hence, both bonds and the remaining saleable part of capital become more valuable, until their returns hit the respective lower bounds.

(c) A fall in the Friedman interest rate ($i \downarrow$), which could be due to increasing patience or a fall in expected inflation. Either way, keeping $j = 0$ constant, the liquidity premium $\ell = i$ falls. As this premium compensates capital investors for the inflation tax – which falls, too – the combined effect may be to increase or reduce welfare, as Panel [b] of Figure 5 shows.

It is worth noting that there is no mechanism in the model whereby being in the liquidity trap would cause deflation on its own. On the contrary, a liquidity trap is consistent with any inflation rate that preserves inequality (A.3). Moreover, the price level in the liquidity trap is still governed by a quantity equation (Equation (17)); it is just not the usual one.

What kind of policy works for an economy in the liquidity trap? Since the trap manifests itself as depressed investment, we can speculate that a variety of fiscal policies targeting investment could be useful (such as a tax credit, or direct spending by the fiscal authority), but to investigate them properly is beyond the scope of this paper. What we know is that there are two *monetary* policy options: run the Friedman rule, or increase expected inflation until the lower bound on the policy rate no longer binds. In the strict context of our model, both options are equally good, but applied to reality each may have pros and cons, as we discuss in Section 4.2 below.

One thing the model does make clear is that short-term interventions will have short-term effects, whereas the liquidity trap is in principle a long-term phenomenon. Thus – unless the trap is caused by some temporary shock, such as a fall in η during a financial crisis that can be expected to abate over time – escaping a liquidity trap is not a matter of "priming the pump". If the conditions that pushed the economy into the trap (λ, η, etc.) are expected to last, then medium-run forward guidance about interest rates is likely to be less successful than a permanently higher inflation target.

Paradoxes

Since its inception, the liquidity trap has been associated with a **paradox of thrift**. The argument is that a higher desire by agents to save would normally lead to more investment and output; however, this increase in saving requires a well-functioning financial system and an unconstrained interest rate in order to be *translated* into investment. The LAMMA can capture this mechanism: in the liquidity trap, the policy interest rate is zero and therefore the liquidity premium on capital is maximal, but the premium is still too low to stimulate investment to its optimal level.[21]

What would an increase in the "desire to save" do in this case? The term can be given two possible meanings: first, patience increases ($\rho \downarrow$), and second, agents perceive a lack of spending opportunities ($\lambda \downarrow$). A quick look at Equation (18) and Panel [b] of Figure 5 clarifies that a fall in λ always reduces investment and output – not just in the liquidity trap, but whenever policy interest rates are held fixed. A rise in patience, on the other hand, is generally ambiguous, because it is the inverse of asking what happens if expected inflation falls. We already know that in the liquidity trap, reducing inflation to the Friedman rule and increasing inflation to a certain higher level are *both* welfare improving policies; hence, the analysis of ρ is inherited from the analysis of i.

Much younger than the paradox of thrift are two other paradoxes, proposed in the New Keynesian literature: the **paradox of toil** and the **paradox of flexibility** (Eggertsson, 2011;

[21] However, the LAMMA also clarifies that this reasoning does not always apply. If capital tradability is altogether too low, then we are in the "reversal" region and investment can be increased by *raising* interest rates.

Eggertsson and Krugman, 2012). The former states that higher potential output (e.g., TFP) reduces output at the zero lower bound, and the latter states that higher price flexibility (less stickiness) does the same. These two paradoxes do not exist in the LAMMA, even in the liquidity trap. First, lower TFP or a lower capital stock always reduce output and welfare; this is consistent with evidence that contractionary supply shocks are contractionary, even at the zero lower bound (Wieland, 2014). Second, the model shows that a liquidity trap can be understood as a phenomenon of monetary and financial frictions which is altogether independent of price stickiness.

4.2 Should we run the Friedman rule?

One of the classic questions in monetary economics is the optimal rate of inflation. In the vast majority of models where agents have any reason to hold money (from money-in-the-utility-function all the way to the most modern treatments of frictions), higher inflation induces agents to economize on holding money balances, and thereby receive less of whatever it is money gives them in the particular model (direct utility, more efficient transactions, etc.). This is the basis for Friedman's rule: to set equal the private marginal cost of holding money to its social cost of creation, which in most models is zero.[22]

Papers in which the best policy (in a first-best or second-best sense) is not the Friedman rule tend to fall into three classes. First, the Friedman rule may not be feasible for a constrained government, because deflation requires retiring money balances via taxes. If taxes are distortionary or hard to collect, then zero or positive inflation may be second-best. Second, there may be distortions in the real economy that make the equilibrium suboptimal even at the Friedman rule. Third, there may be nominal distortions due to information or other frictions that push the optimal inflation rate towards zero. Without explicitly modeling these considerations, the LAMMA still contains lessons for all three classes of papers.

No lump-sum taxes

First, properly introducing distortionary taxes is beyond the scope of the present paper, but we can take a (rough) first pass at it by assuming that lump-sum taxes are not possible but lump-sum transfers are.[23] Setting $T \geq 0$ in the government's budget constraint does two things. One, it makes the Friedman rule inaccessible. Two, any interest payments on government bonds need to be financed with seigniorage revenue, so zero inflation implies $B = 0$.

[22] A few papers delve more deeply into Friedman's wording. If agents differ in patience, there is no single "private marginal return" that could be set to zero for everyone (e.g., Boel and Camera, 2006; Boel and Waller, 2015). And a literal reading of Friedman's rule actually prescribes positive inflation if inside money can do some things that outside money cannot do, and the social cost of creating inside money is not zero (such as in some models of banking; e.g., Dong, Huangfu, Sun, and Zhou, 2016).

[23] The fiscal benefit of the inflation tax is the focus of Phelps (1973), and the difficulty to implement lump-sum transfers is emphasized by Andolfatto (2013) and Gomis-Porqueras and Waller (2017).

Depending on all the other parameters, $B = 0$ may imply that bonds are scarce in the secondary market, hence $j = 0$. In order to achieve the first-best, bonds must become abundant enough that policy interest rates drop ("tight monetary policy"), which requires further inflation. The counterintuitive implication is that positive inflation may be required in order to make "tight" monetary policy possible in the first place. This result is illustrated in Panel [a] of Figure 7.

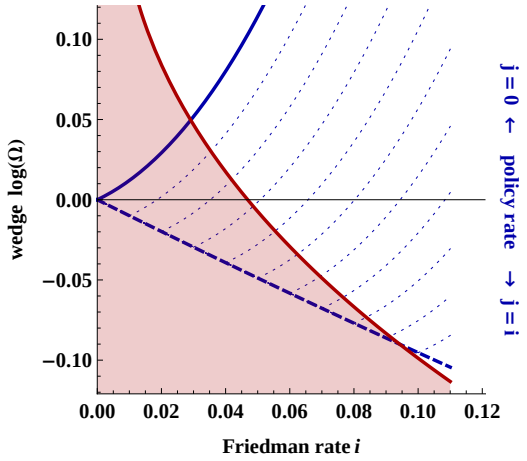
Additional distortions

Second, suppose that there are additional distortions or externalities that push the economy towards overinvestment or underinvestment (without affecting any other margins). To keep the focus on the monetary mechanism, we are deliberately agnostic as to what these distortions may actually be, but for the sake of concreteness, call them "animal spirits".[24] Suppose that there was no secondary asset market, so that both bonds and capital were completely illiquid, as is assumed in most of macroeconomics. There would be two cases: one, where the animal spirits distortion is negative, and two, where it is positive. In the former case, the Friedman rule is second-best, and because the social welfare function is not maximal at the Friedman rule, any deviation $i > 0$ implies a first-order welfare loss. In the latter case, some positive $i > 0$ is the unique first-best policy. This case is illustrated in Panel [b] of Figure 7.

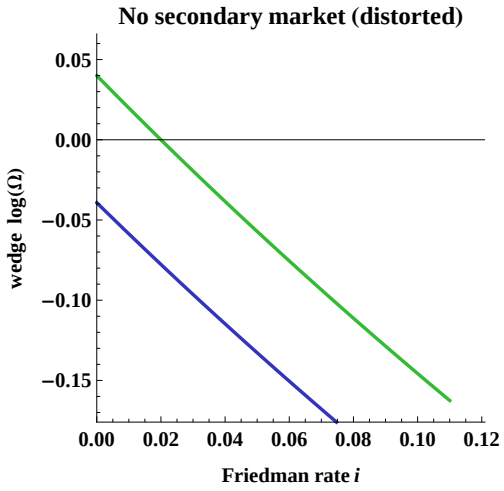
Now, suppose we treat the "animal spirits" distortion as a random variable that is close to zero on average, but may be positive or negative; it could be a time-varying shock, or simply reflect our imperfect understanding of the world. Without any information, we could adopt the neutral prior that it could be positive or negative with equal probability (or half of the time). If anything, we might suppose that it is more likely to be negative, because of the "Anna Karenina principle" (there are more ways to be unhappy than to be happy). With a healthy dose of skepticism about our understanding of the world, this should make us err on the side of caution: keep inflation as low as possible.

However, the full LAMMA – with the secondary asset market that makes bonds and capital indirectly liquid – reveals that this conclusion is premature: as long as capital is liquid enough, and liquid bonds are scarce enough that policy interest rates are close to zero, further inflation stimulates investment rather than discourage it. As Panel [c] of Figure 7 reveals, the picture has become symmetric: a positive distortion shifts the cone up and a negative one shifts it down, but *in both cases* the first-best outcome can be achieved with a positive inflation rate. For example, suppose the distortion is negative, which makes the inflation tax worse; surprisingly, the optimal policy response requires inflation to be high and interest rates to be low. (An "accommodative stance" of policy.) Hence, there is no more reason to say that low

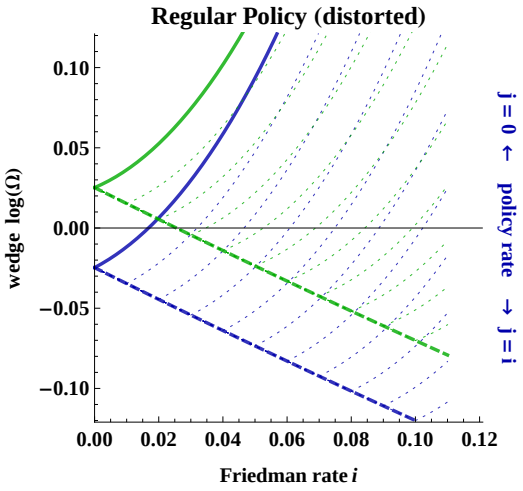
[24] Papers where the inflation tax is magnified by additional distortions (leading to underproduction and/or underinvestment) include Lagos and Wright (2005) and Aruoba et al. (2011). Papers where the reverse is true include Head and Kumar (2005) and Herrenbrueck (2017), and both can happen in Rocheteau and Wright (2005).



[a] Shaded area is not feasible without lump-sum taxes



[b] With distortion (positive and negative), assuming the AM is shut down



[c] With distortion (positive and negative), assuming the "regular policy" regime

Figure 7: The cone of policy options, considering feasibility and distortions. Parameters: $\rho = 0.03$ and $\delta = 0.1$ maintained. Panels [a] and [c]: $\lambda = 0.2$, $\eta = 0.75$. Panel [b]: $\lambda = 0.5$, $\eta = N/A$.

inflation errs on the side of caution. On the contrary: the less sure we are about the possible direction and size of all the distortions that may exist in reality, the more sure we should be that positive inflation is part of an optimal policy.

Nominal frictions

Third, how does the LAMMA interact with models of nominal frictions?[25] First and foremost, of course, the LAMMA clarifies how monetary policy can have real effects even in the absence of nominal frictions. This is not to say that such frictions do not exist – but to the extent that in the literature, large nominal frictions were *inferred* in order to explain observed real effects of money, the true impact of such frictions may be smaller than previously thought. Either way, there is no contradiction in taking the forces in the LAMMA seriously in addition to others. For example, consider the observed asymmetry that workers may be less willing to accept nominal wage cuts than real ones delivered through inflation, which is one commonly cited rationale for a positive inflation target. Following most of monetary theory, there is a tension between such considerations and the need to minimize the inflation tax (e.g., Kim and Ruge-Murcia, 2011). According to the LAMMA, by contrast, positive inflation may be part of a first-best monetary policy, as long as the monetary authority is able to offset its effect with appropriate interest rates.

Advantages of the Friedman rule

On the other hand, even from the point of view of the LAMMA the Friedman rule has advantages. Two in particular are prominent: the Friedman rule requires less knowledge on part of the monetary authority, and it is more robust to some reasonable model extensions than positive inflation is. For the first point, go back to the argument that the existence of an unknown (and possibly time varying) externality implies that the Friedman rule is certainly not first-best, but a positive inflation rate may be part of a first-best policy. One may question whether a monetary authority which is not able to figure out the size or direction of the externality is instead able to figure out the optimal interest rate required to offset the inflation tax. Apart from the externality itself, this would also require knowing the parameters (ρ, λ, η). And policy goals such as "stabilize output and inflation" are not precise enough: as we have shown, stable inflation is consistent with a range of output levels and interest rates.

The second argument in favor of the Friedman rule is that the LAMMA is a little bit special in how the right interest rate policy can *exactly* offset the inflation tax. For example, suppose that there are multiple sectors in the economy that differ by "cash intensity": per-

[25] The literature on nominal frictions is too vast to survey here, but it is worth noting that the mere observation of delayed or incomplete nominal price adjustments is not enough to conclude that there must be frictions preventing complete adjustments (Head, Liu, Menzio, and Wright, 2012; Burdett, Trejos, and Wright, 2015).

haps credit is easier to extend in markets for durable goods because the goods themselves can be used as collateral (and there could be many other reasons). In that case, the inflation tax does not fall equally on all sectors, but the offsetting channel – the liquidity premium on capital that responds to monetary policy – affects all capital goods equally. Without additional distortions, the Friedman rule would be the unique optimal policy again. *With* additional distortions, there may be a unique first-best or second-best combination of i and j, but its level would be sensitive to the specifics.

A similar consideration would apply when the model is extended to include an elastic labor supply margin, depending on how exactly this is done. The real balance effect on labor supply can still be offset through the right interest rate policy, but the interest rate that optimizes the labor supply margin may not be the same one that optimizes the investment margin. Again, without additional distortions, the Friedman rule would be the unique optimal policy. *With* additional distortions – including those that push output *below* the first-best at the Friedman rule – the second-best policy tends to involve positive inflation.

4.3 The Neo-Fisherian Controversy

In recent years, there has been a controversy regarding the effect of interest rates on the macroeconomy, and the associated question of the optimal interest rate policy. At the heart of the disagreement is the Fisher equation, approximately stated as:

$$\text{nominal interest rate} = \text{real interest rate} + \text{expected inflation}$$

The traditional understanding of this equation, espoused in most textbooks, goes as follows: (1) There is a fundamental, "natural", level of the real interest rate that is independent of monetary factors (such as inflation). (2) Monetary policy sets nominal interest rates. (3) Higher interest rates cause lower investment and output, which put downward pressure on prices and inflation. Hence, if the Fisher equation is understood as a statement about the "natural" real interest rate, it cannot hold in the short run, and in fact it only holds in the long run if monetary policy follows the "Taylor principle", whereby the monetary authority raises interest rates aggressively in response to higher inflation.

The contrary view, named "neo-Fisherian" (see Bullard, 2015, for a summary), is that the Fisher equation holds more or less always, and causality runs the other way: higher interest rates cause higher inflation. There are various ways to make this true in a New Keynesian model (Cochrane, 2014), and three ways to make this true in a New Monetarist model. First, suppose that the monetary authority was committed to manipulating inflation expectations in order to target a particular level of the Friedman interest rate (i in the LAMMA), taking the path of all other variables as given. If so, then a higher-order target for i could be said to "cause" a lower-order inflation target. Second, suppose that the monetary authority raises

nominal interest rates forever, which increases the cost to the fiscal authority of servicing its debt; further suppose that the fiscal authority responds to this by issuing even more debt in order to make its payments, and for some reason agents expect the monetary authority to accommodate this with higher inflation (Andolfatto, 2014). Third, suppose the monetary authority conducted an open market sale of liquid bonds, thereby raising the interest rate on such bonds. Such an action can decrease the steady-state level of real balances in some monetary models (e.g., Herrenbrueck, 2014, or Andolfatto and Williamson, 2015). In other words, the price level rises. If fiscal policy (taxes, transfers, debt) is denominated in *real* terms, rather than the nominal terms that most monetary models assume, then the resulting outcome is an increase in inflation, both along the transition path and in steady state.

However, does either of these three arguments provide a plausible explanation for a positive causal link from interest rates to inflation? One may have doubts. The first explanation is literally accurate but stretches the meaning of "cause". At any rate, most of the time central banks do not try to manipulate inflation expectations directly, aside of course from setting a long-run inflation target. The second explanation depends on a particular fiscal policy rule, and one may question whether inflation-targeting central banks would be willing to accommodate a fiscal push for higher inflation, or whether private agents would believe that the central bank would do this. The third explanation is plausible qualitatively (governments often do specify spending in real terms: "10 fighter planes" rather than "as many planes as $100 million will buy") but not quantitatively (we can measure expected inflation, and it varies much less than interest rates over the business cycle; see Hamilton et al., 2016).

So, viewing the controversy through the lens of the LAMMA, we propose the following resolution. The Fisher equation holds, for any asset, once we acknowledge that of course different assets can have different equilibirium rates of return, and that comparing "the" nominal interest rate with "the" real interest rate is meaningless if they do not refer to *the same asset*. Regarding causality, the key distinction is between liquid versus illiquid assets. The older interpretation of the Fisher equation – associated with Fisher, Friedman, and Monetarism – that the real interest rate is governed by "fundamentals" and the nominal rate is the real rate plus expected inflation, is correct for a very illiquid asset. The alternative interpretation, that the nominal interest rate is governed by monetary policy and the real rate is the nominal rate minus expected inflation, applies to a very liquid asset. Assets with intermediate (or time-varying) liquidity fall somewhere in between.[26] Consider capital in the LAMMA, whose nominal rate of return, net of depreciation, is approximately:

$$\tilde{r} \equiv r - \delta + \pi \approx (1 - \eta)i + \eta j$$

[26] The fact that the real interest rate on a not-perfectly-illiquid asset is related to expected inflation has been noted many times in the New Monetarist literature. E.g., Geromichalos et al. (2007) for equity, Geromichalos and Herrenbrueck (2016a) for bonds, and Venkateswaran and Wright (2014) for bonds, housing, and capital. The latter paper also reviews the empirical support for this proposition, which is considerable.

The distinction between long-term and short-term rates which one finds in many textbooks is irrelevant in theory, but not in practice, because long-term assets do tend to be less liquid than short-term ones (Geromichalos, Herrenbrueck, and Salyer, 2016). And it is important to keep in mind that a perfectly default-free, short-term, yet perfectly illiquid asset does not exist. Hence, its return i must be estimated as an extrapolation, or perhaps as the upper envelope of the yields of safe assets along the observed liquidity spectrum.

5 Summary

The LAMMA is a formalization of the following intuitive concepts:

(i) Due to certain frictions (made explicit), we live in a monetary economy, where many assets are valued – and priced – for their liquidity.
(ii) Financial assets are (generally) imperfect substitutes, and their demand curves (generally) slope down; thus, a central bank that controls the supply of certain assets can "set" interest rates.
(iii) The principal way this is done is via intervention in secondary asset markets where agents rebalance their portfolios in response to short-term liquidity needs.
(iv) The principal channel through which monetary policy affects the economy is the interest rate at which agents save and invest.

As a result, we obtain the following conclusions and lessons for monetary policy:

1. Monetary policy can have real and realistic effects in a tractable model without sticky prices. Such effects are not limited to the short run, but can persist even in steady state.
2. There exists both a real balance effect (inflation causes underproduction) and a Mundell-Tobin effect (inflation causes overproduction). With the right interest rate policy, these effects offset, thus monetary policy may be able to achieve the first-best outcome at positive inflation rates.
3. Reductions in the policy rate generally increase investment and output, but the effect can reverse in certain cases. (Specifically: if liquidity needs arrive rarely, and if capital assets are hard to trade.) In such a case, the Friedman rule is the only first-best policy.
4. There are two meaningful interpretations of the Fisher equation. One applies to perfectly illiquid assets: their real return is governed by "fundamentals", and their nominal return is the real return plus expected inflation. Another applies to liquid assets: since they are close substitutes to money, their nominal return is a monetary object, responding to monetary policy. Their real return is the nominal return *minus* expected inflation. Most real-world assets are somewhere in between.
5. There can exist a liquidity trap where the second-best policy is to lower the policy rate to zero, and a first-best policy involves increasing inflation expectations. An economy

is likely to be in the trap after a fall in the frequency of liquidity needs ("desire to save rather than spend"), a fall in tradability of capital assets ("shortage of liquid assets"), and a fall in expected inflation ("binding lower bound on real interest rates").
Conceptually, this liquidity trap is similar to that of Keynes (1936) and Hicks (1937) and only a distant cousin of the New Keynesian liquidity trap. For example, there is a paradox of thrift, but improvements in technology still increase output. For another, falling inflation can cause a liquidity trap, but being in a liquidity trap does not (by itself) cause falling inflation.

These results are representative of the topics that the LAMMA can address, but by no means exhaustive. A coequal contribution of the paper is to offer a *framework* facilitating the communication between monetary theory and business cycle macroeconomics. Monetary policy is represented in an empirically relevant way – implemented in secondary asset markets, and treating the policy rate on short-term liquid bonds and the inflation rate as distinct instruments – and is shown to have plausible effects both in the short run and in the long run. Monetary, financial, and real frictions can be extended as needed in future work. The potential that such a framework has for quantitative policy analysis is obvious.

Appendix

A.1 Derivation: from value functions to Euler equations

The representative household's value function in the CM subperiod, V^{CM}, is given by Equation (3). It is easy to confirm that it will be linear; the first-order conditions with respect to the two consumption goods yield $u'(c_t) = 1/p_t^G$, and the envelope conditions are:

$$\partial_m V^{CM} = \partial_b V^{CM} = \frac{\phi_t}{1-\delta} \cdot \partial_k V^{CM} = \phi_t \cdot \partial_y V^{CM} = \phi_t \cdot u'(c_t)$$

Hence, a household's consumption (c) is independent of its asset portfolio.

Next, we turn to the household's problem in the AM, where the value functions of asset buyers and sellers are given by Equations (4)-(5). There are three constraints; asset buyers cannot spend more money than they have, asset sellers cannot sell more bonds than they have, and asset sellers can only sell a fraction η_t of their capital. Denote the Lagrange multipliers on the three constraints by $(\theta_t^M, \theta_t^B, \theta_t^K)$, respectively, and shorten notation by writing V_Λ^{PM} for $V^{PM}(\ldots, \Lambda)$. Hence, the first-order conditions with respect to supply and demand of assets can be written as follows:

$$\partial_m V_0^{PM} + \theta_t^M = \frac{1}{s_t^B}\, \partial_b V_0^{PM} \qquad\qquad \partial_b V_1^{PM} + \theta_t^B = s_t^B\, \partial_m V_1^{PM} \tag{A.1}$$

$$\partial_m V_0^{PM} + \theta_t^M = \frac{1}{s_t^K}\,\partial_k V_0^{PM} \qquad\qquad \partial_k V_1^{PM} + \theta_t^K = s_t^K\,\partial_m V_1^{PM} \tag{A.2}$$

Combining the two equations on the left, and substituting the envelope conditions for V^{PM} and V^{CM}, we obtain the asset market no-arbitrage equation (6):

$$\begin{aligned}
\frac{\partial_b V_0^{PM}}{s_t^B} = \frac{\partial_k V_0^{PM}}{s_t^K} \quad &\Leftrightarrow \quad \frac{\partial_b V^{CM}}{s_t^B} = \frac{\frac{r_t}{\phi_t}\,\partial_m V^{CM} + \partial_k V^{CM}}{s_t^K} \\
&\Leftrightarrow \quad \frac{\partial_m V^{CM}}{s_t^B} = \frac{(r_t + 1 - \delta)\,\partial_m V^{CM}}{\phi_t s_t^K} \\
&\Rightarrow \quad s_t^K = \frac{r_t + 1 - \delta}{\phi_t}\,s_t^B
\end{aligned}$$

To solve the portfolio problem in the primary asset market (the CM), we take the first-order conditions of problem (3) with respect to $(m_{t+1}, b_{t+1}, k_{t+1})$. As we have already established that the marginal value of a unit of numéraire in the CM equals $u'(c_t)$, we can write:

$$\begin{aligned}
u'(c_t)\phi_t &= \beta\,\mathbb{E}_t\Big\{\lambda_{t+1}\partial_m V_{1,t+1}^{AM} + (1-\lambda_{t+1})\partial_m V_{0,t+1}^{AM}\Big\} \\
u'(c_t)\phi_t p_t^B &= \beta\,\mathbb{E}_t\Big\{\lambda_{t+1}\partial_b V_{1,t+1}^{AM} + (1-\lambda_{t+1})\partial_b V_{0,t+1}^{AM}\Big\} \\
u'(c_t) &= \beta\,\mathbb{E}_t\Big\{\lambda_{t+1}\partial_k V_{1,t+1}^{AM} + (1-\lambda_{t+1})\partial_k V_{0,t+1}^{AM}\Big\}
\end{aligned}$$

Taking envelope conditions in the AM, and substituting the AM optimality conditions shown above, we get:

$$\begin{aligned}
\partial_m V_{0,t+1}^{AM} &= \partial_m V_{0,t+1}^{PM} + \theta_{t+1}^M = \frac{1}{s_{t+1}^B}\,\partial_b V_{0,t+1}^{PM} \\
\partial_m V_{1,t+1}^{AM} &= \partial_m V_{1,t+1}^{PM} \\
\\
\partial_b V_{0,t+1}^{AM} &= \partial_b V_{0,t+1}^{PM} \\
\partial_b V_{1,t+1}^{AM} &= \partial_b V_{1,t+1}^{PM} + \theta_{t+1}^B = s_{t+1}^B \partial_m V_{1,t+1}^{PM} \\
\\
\partial_k V_{0,t+1}^{AM} &= \partial_k V_{0,t+1}^{PM} \\
\partial_k V_{1,t+1}^{AM} &= \partial_k V_{1,t+1}^{PM} + \eta_{t+1}\theta_{t+1}^K = \eta_{t+1} s_{t+1}^K \partial_m V_{1,t+1}^{PM} + (1-\eta_{t+1})\partial_k V_{0,t+1}^{PM} \\
&= \eta_{t+1}\frac{r_{t+1} + 1 - \delta}{\phi_{t+1}}\,s_{t+1}^B \partial_m V_{1,t+1}^{PM} + (1-\eta_{t+1})\partial_k V_{0,t+1}^{PM}
\end{aligned}$$

Finally, we substitute the AM, PM and CM envelope conditions into the first-order conditions to obtain the Euler equations for money (7), bonds (8), and capital (9).

A.2 Derivation: region boundaries

Regions (A)-(B): The boundary can be found by computing the trade volume in the AM conditional on being in Region (A). In this Region, we have $\zeta^B + \zeta^K = M$, thus the combined trade volume of bonds and capital is $(1-\lambda)M$. If bonds and capital are to be plentiful in AM trade, then their nominal value ($\lambda(s^B B + s^K \eta K)$, to be evaluated at Region-(A) prices) cannot be any smaller. Using the no-arbitrage equation (6), the quantity equation (13), and the earlier results for this region, we see that equilibrium is in Region (A) if:

$$\lambda(s^B B + s^K \eta K) \geq (1-\lambda)M \quad \Rightarrow \quad s^B \left(\frac{B}{M} + (r+1-\delta)\,\eta\,\frac{K}{\phi M} \right) \geq \frac{1-\lambda}{\lambda}$$
$$\Rightarrow \quad \frac{B}{M} \geq \frac{1-\lambda}{\lambda} \cdot \frac{\mu}{\beta} - \frac{\alpha\eta}{1-\beta+\beta\delta},$$

a threshold which is increasing in the rate of money growth, μ. (It is increasing because a higher inflation rate decreases the bond price, thus making it less likely that a given quantity of bonds will be enough to purchase the available money.) The threshold can also be negative; in that case, the economy will be in Region (A) for any positive quantity of bonds.

Regions (B)-(C): This boundary can be found by plugging the bond price upper bound $s^B = 1$ into Equation (16). We see that equilibrium is in Region (C) if:

$$\frac{B}{M} \leq \frac{1-\lambda}{\lambda} - \frac{\alpha\eta}{1-(1-\delta)\left[(1-\eta)\beta+\eta\mu\right]} \tag{A.3}$$

This threshold is decreasing in the parameters α (capital intensity of production), λ (frequency of liquidity needs), η (tradability of capital), and μ (money growth). It can be negative; in that case, the economy will be in Region (A) or (B) for any positive quantity of bonds. In particular, as μ increases, the second fraction will eventually blow up; this means that high money growth in steady state is incompatible with a zero interest rate on bonds.

A.3 Extension: trading frictions in the PM

There are two reasons for introducing search frictions explictly into the goods market. One is the fact that we have already assumed that shoppers are anonymous and unable to commit to promises. This fits more naturally with the idea that shoppers meet with only a small number of firms, and trade bilaterally. The second reason is that search frictions give rise to market power (firms receive some of the gains from trade), and to mismatch (some shoppers do not trade). The result of these two things is to make the velocity of money in the goods market endogenous (or, at any rate, more flexible than it was in the main text; see Equation (13), which reflected the fact that at least outside of the liquidity trap, every dollar in the economy got spent in the goods market). For future empirical applications, this additional flexibility is

likely to be important. The reader may in any case be interested in a version of the LAMMA where firms have market power.

Suppose that there are N firms (where N is large), who are price takers in the factor market; they rent labor and capital at market prices w and r exactly as in the main text. However, they have the ability to post output prices, and shoppers face search frictions: they are subject to a lottery whereby they observe the price of n firms with probability ψ_n (Burdett and Judd, 1983). Draws are independent across shoppers, firms, and time. After observing their set of prices (or none, if $n = 0$), shoppers will choose to spend all their money at the firm with the lowest price. There is no recall of prices seen in previous periods.

Because there is a continuum of consumers and a finite number of firms, the law of large numbers applies and each firm can perfectly forecast demand for its product, conditional on the price it has set. (Hence, this set-up abstracts away from inventory or unemployment concerns.) Now, what is that demand? Each shopper has a certain amount of money to spend, and a constant marginal rate of substitution between money and goods (ϕ_t, as derived in Section 2.3). Since ϕ_t is the same for all shoppers, and independent of their money holdings, all shoppers follow the same optimal strategy: spend all of their money on the firm with the cheapest price available, unless that price exceeds $1/\phi_t$. In the latter case, spend nothing. Thus, for any firm charging a price below this reservation price, the intensive margin of demand is unit elastic. Based on this intensive margin alone, the best thing for a firm to do would be to set their price equal to the reservation price.

However, there is also the extensive margin to be considered. Suppose that the c.d.f. of posted prices is $F(p)$, and that it has no mass points; then, a firm charging price p' will almost surely sell to $a(F(p'))$ shoppers, where:

$$a(F) = \sum_{n=0}^{\infty} \psi_n n (1-F)^{n-1}$$

Therefore, writing $q_t \equiv A_t^{-1}\alpha^{-\alpha}(1-\alpha)^{\alpha-1}r_t^{\alpha}w_t^{1-\alpha}$ for the real unit cost of producing one unit of output, and writing M_t' for the amount of money held by all shoppers in the PM, nominal profits of a firm with price $p' \leq 1/\phi_t$ are equal to:

$$M_t' \left(1 - \frac{q_t}{\phi_t p'}\right) a(F(p'))$$

Burdett and Judd (1983) proved that – as long as both $\psi_1 > 0$ and $\psi_n > 0$ for some $n \geq 2$ – the only equilibrium of this price-setting game is endogenous price dispersion, where all firms post the price distribution $F(p)$ as a mixed strategy and make equal profits in expectation. The equilibrium $F(p)$ indeed has no mass points, and some firms do charge the reservation price ($F(p) < 1$ for $p < 1/\phi_t$). Furthermore, Herrenbrueck (2017) showed that the total

amount of output purchased equals:

$$Y_t = \left(\psi_0 \cdot 0 + \psi_1 \cdot 1 + (1 - \psi_0 - \psi_1)\frac{1}{q_t}\right) \cdot \phi_t M_t'$$

In words: a fraction ψ_0 of shoppers is mismatched and does not purchase anything (although they still get to hold on to their money). The rest of the solution is surprisingly simple: even though almost all shoppers spend a price in between the efficient price (q_t/ϕ_t) and their reservation price ($1/\phi_t$), the equilibrium is *as if* a fraction ψ_1 of them spent the reservation price and everyone remaining (who was matched with $n \geq 2$ firms) spent the efficient price, equal to marginal cost.

Since the marginal disutility of a dollar of spending is ϕ_t (the output good in the CM is the numéraire), and allowing for shocks to the matching parameters ψ_n, the Euler equations representing asset demands change as follows from Equations (7)-(9). First, define the ex-post liquidity premium ℓ_{t+1}:

$$\ell_{t+1} \equiv \lambda_{t+1} s_{t+1}^B \left(\psi_{0,t+1} + \psi_{1,t+1} + \frac{1 - \psi_{0,t+1} - \psi_{1,t+1}}{q_{t+1}}\right) - \lambda_{t+1}$$

Then:

$$u'(c_t)\phi_t = \beta\, \mathbb{E}_t \left\{ u'(c_{t+1})\phi_{t+1}\, \frac{1 + \ell_{t+1}}{s_{t+1}^B} \right\}$$

$$u'(c_t)\phi_t p_t^B = \beta\, \mathbb{E}_t \left\{ u'(c_{t+1})\phi_{t+1}\, (1 + \ell_{t+1}) \right\}$$

$$u'(c_t) = \beta\, \mathbb{E}_t \left\{ u'(c_{t+1})(r_{t+1} + 1 - \delta)\, (1 + \eta_{t+1}\ell_{t+1}) \right\}$$

Note that if $\psi_0 = \psi_1 = 0$, i.e., all shoppers see at least two prices, then PM trade is *effectively* competitive and the Euler equations are the same as before. And, more precisely, these Euler equations hold under the assumption that the number of prices a shopper observes (n) is only revealed *after* the AM subperiod has concluded. If this was revealed at the beginning of a period, then shoppers with low n or high observed prices would choose to use their money to buy assets rather than goods in that period (Chen, 2015).

In steady state, and using the two interest rates $1 + j \equiv 1/s^B$ and $1 + i \equiv (1 + j)(1 + \ell)$ again, we obtain the following expression for the Friedman wedge:

$$q = \frac{1 - \psi_0 - \psi_1}{\frac{1+i}{1+\ell}\left(1 + \frac{\ell}{\lambda}\right) - \psi_0 - \psi_1}$$

The equations for the Mundell-Tobin wedge and for PM clearing (Equation 2) stay the same. Hence, the resulting monetary wedge follows the same formula as before:

$$\Omega(i,\ell) = q \cdot \frac{\rho+\delta}{r},$$

the only difference being that q now incorporates the matching friction terms ψ_0 and ψ_1. Their effect will be to push the wedge Ω down compared to the main text. First, if $\psi_1 > 0$, then firms have market power and shoppers will give up some surplus. Second, if $\psi_0 > 0$, then there is mismatch, and some shoppers will not be able to make a purchase. However, at the Friedman rule, $i = \ell = 0$ implies $q = 1$, as before. Thus, the effect of matching frictions in the goods market is to *rotate* the cone of policy options downwards around the origin – see Figure A.1 for an illustration. The result of this is that for any given policy interest rate, the inflation tax bites more keenly, and output is lower; equivalently, for any given inflation rate the optimal policy interest rate has to be lower.[27]

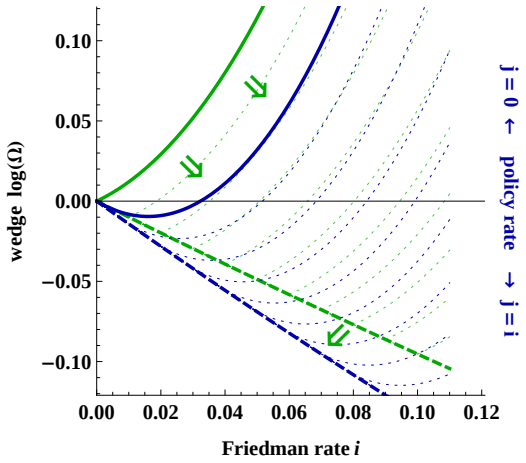


Figure A.1: Menu of policy options without (blue) and with (green) goods market frictions. Maintained parameters: $\rho = 0.03$, $\delta = 0.1$, $\lambda = 0.2$, $\eta = 0.75$. Varying: $(\psi_0 + \psi_1) \in \{0, 0.3\}$.

On the income side, how does the money held by shoppers get distributed after the PM? First, a fraction ψ_0 is unspent by the shoppers, hence they keep it. It can be shown (Herrenbrueck, 2017) that a fraction $(1-q)\psi_1/(1-\psi_0)$ of the remainder – or, equivalently, $(1-q)\psi_1$ of the total – goes to the owners of the firms as profits. The rest gets paid to the owners of factor inputs. Hence, a fraction $\alpha[1-\psi_0-(1-q)\psi_1]$ of the shoppers' money holdings goes to capital owners, and a fraction $(1-\alpha)[1-\psi_0-(1-q)\psi_1]$ goes to workers.

It is now straightforward to take the limit $(\psi_0, \psi_1) \to 0$, meaning that every shopper sees the prices of *at least two* firms. Then, the equations of the frictional model become identical to those of the competitive model in the main text.

Once firms make profits, it may be interesting to model firm equity explicitly. In par-

[27] Unless we are in the "reversal" region of the parameter space, in which case the second-best level of the policy rate, for a given $i > 0$, is maximal – as before.

ticular, equity might be considered an indirectly liquid asset that can be sold in the AM, in the same way that capital is (see also Rocheteau and Rodriguez-Lopez, 2014). Other details can be added. For example, there may be firm entry subject to a cost, and entry by more firms may have the effect to improve the matching probabilities by shoppers in the sense of a FOSD shift in the distribution $\{\psi\}$ (see also Herrenbrueck, 2017).

A.4 Extension: interaction between the fiscal and monetary authorities

In this section, we split the consolidated government into a fiscal authority (in charge of bond issuance, B) and a monetary authority (in charge of the money supply, M), and analyze these authorities' policy options separately. We will not take a deeper look into "fiscal policy", which could also include cyclical policy, government spending on a public good, or distortionary taxation. All of these issues are also important, of course.

Suppose that the fiscal authority controls the sequence of bond issues, $\{B_t^F\}_{t=1}^{\infty}$, and it seeks to finance a sequence of nominal lump-sum transfers $\{T_t\}_{t=0}^{\infty}$ (taxes if negative). The fiscal authority is only active during the CM.

The monetary authority is active during the AM and the CM, and it controls the sequence of money supplies, $\{M_t\}_{t=1}^{\infty}$. It can intervene in the AM by buying up bonds with money, or selling bonds for money; denote the monetary authority's bond holdings at the beginning of period t by B_t^M, and assume that the monetary authority is not able to sell more bonds than it has: $B_t^M \geq 0$. Since the bond here is a one-period discount bond, this means that if the monetary authority wishes to be able to conduct an open-market sale in period $t+1$, it must buy some newly issued bonds in the CM of period t.

With this choice of notation, B_t^F indicates bonds *issued* by the fiscal authority, whereas B_t^M indicates bonds *held* by the monetary authority. The stock of bonds held by the public, at the beginning of period t, will then be $B_t \equiv B_t^F - B_t^M$.

At the end of a period, the monetary authority makes a seigniorage transfer to the fiscal authority, S_t. Here, we do not take a stand on whether this transfer can be negative as well as positive, or whether the monetary authority has authority over choosing its level. For example, it may be realistic to assume that the monetary authority has full authority over choosing *positive* levels of S_t, but requires the cooperation of the fiscal authority if it wants to collect a tax. Alternatively, a monetary authority may have limited power to increase inflation if a more hawkish fiscal authority refuses to increase its spending (Andolfatto, 2015); arguably, this has been the case in the Eurozone in recent years (Bützer, 2017).

Since the fiscal authority issues bonds in the primary market (the CM), where the bond price is p^B, it must obey the following budget constraint, for all $t \geq 0$:

$$p_t^B B_{t+1}^F + S_t = B_t^F + T_t,$$

along with the no-Ponzi condition that B_t^F/M_t remains bounded. The monetary authority must obey the following budget constraint, for all $t \geq 0$:

$$M_{t+1} - M_t + B_t^M = S_t + s_t^B(B_{t+1}^M - B_t^M) + p_t^B B_{t+1}^M$$

We can interpret this constraint as follows. On the left hand side is the 'revenue' of the monetary authority in period t: newly created money ($M_{t+1} - M_t$) and payments from redemption of the bonds in its portfolio (B_t^M). On the right hand side are the things this revenue can be spent on: the seigniorage transfer to the fiscal authority (S_t), open-market purchases of bonds from the public ($s_t^B(B_{t+1}^M - B_t^M)$), and purchases of newly issued bonds from the fiscal authority ($p_t^B B_{t+1}^M$).

Two things can be noted from these budget constraints. First: M_t is the money held by the public at the *beginning* of period t; hence, it is the amount of money available to be spent on bonds and capital in the AM. However, the amount of money held by the public during the PM, i.e., the amount of money available to be spent on goods, is $M_t + s_t^B(B_{t+1}^M - B_t^M)$. And the amount of money held by the public at the *end* of period t, i.e., at the end of the CM, equals M_{t+1}. Second, we can add up the budget constraints of the two authorities. If we also assume that $B_t^M = 0$, i.e., the monetary authority does not carry a balance sheet but simply makes seigniorage transfers to the fiscal authority in the CM, then the budget constraint of the consolidated government is exactly the one from Section 2.4.

As in the main body of the paper, let us proceed by looking at steady states. Assume that the fiscal authority is committed to increasing the supply of nominal bonds at (gross) rate μ^B, for a long period of time.[28] Does that mean that the long-run inflation rate will be μ^B? Not necessarily, because it is still the monetary authority that controls the money stock. But it is now impossible for monetary policy to achieve every point on the 'cone of policy options' derived in Section 3.3 and illustrated in Figure 5. Instead, the monetary authority is left with three options:

(i) Grow the money stock at (gross) rate $\mu^M < \mu^B$ (or shrink it if $\mu^M < 1$). In that case, B/M grows large, and eventually equilibrium must be in Region (A), where the policy rate is maximal, and governed by the rate of money growth: $j = i = \mu^M/\beta - 1$.

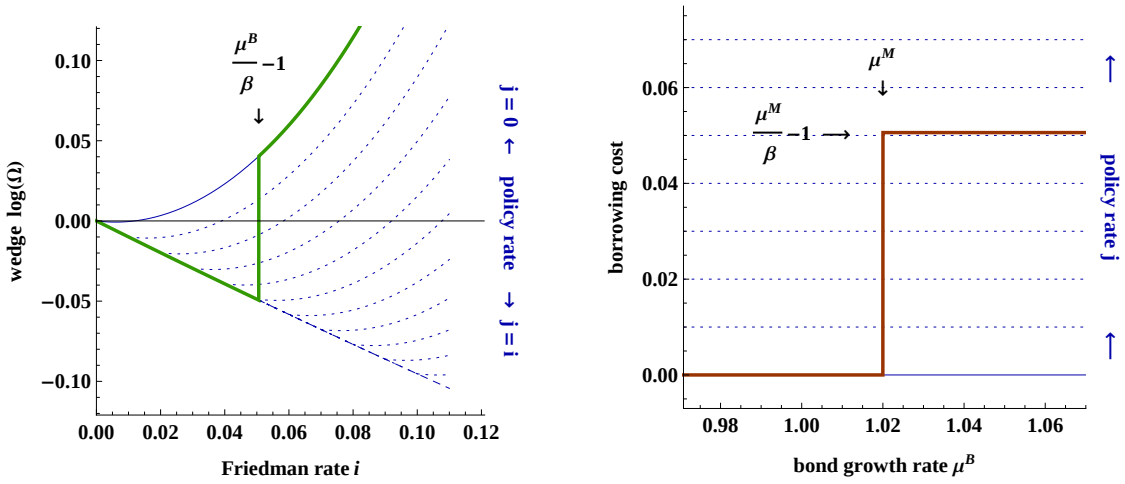
(ii) Grow the money stock at exactly $\mu^M = \mu^B$. In that case, any policy interest rate $j \in [0, i]$ is achievable for the monetary authority.

(iii) Grow the money stock at a faster rate than the supply of bonds: $\mu^M > \mu^B$. In that case, $B/M \to 0$, and eventually equilibrium must be in Region (C), where the policy interest rate is at the zero lower bound: $j = 0$.

This menu of monetary policy options is illustrated in Panel [a] of Figure A.2. In every case

[28] Not, strictly speaking, forever, as the fiscal no-Ponzi condition would be violated if $B/M \to \infty$.

$i = \mu^M/\beta - 1$; that is, the monetary authority controls inflation. However, a benevolent monetary authority seeking to maximize social welfare will have strong incentives to match the money growth rate to the bond supply growth rate, because that is the only way the policy rate can be set to the first-best level. Unless, of course, the monetary authority can implement the Friedman rule; but since this requires $S < 0$, negative seigniorage, they may not be able to do this without cooperation from the fiscal authority.



[a] Monetary policy options (green), constrained by bond growth rate μ^B

[b] Fiscal policy options (red), constrained by money growth rate μ^M

Figure A.2: Menu of long-run options for the monetary and fiscal authorities.

On the other hand, the fiscal authority may have incentives, too. Even if it is not fully benevolent, it probably still prefers low borrowing costs (policy interest rate j) to high ones. Panel [b] of Figure A.2 illustrates this. Taking the money growth rate μ^M as given, the fiscal authority is left with three options:

(i) Grow the bond supply at (gross) rate $\mu^B > \mu^M$, at least for a while (it cannot be forever due to the no-Ponzi condition). In that case, B/M grows large, and eventually equilibrium must be in Region (A), where the borrowing cost is maximal: $j = i = \mu^M/\beta - 1$.

(ii) Grow the bond supply at exactly $\mu^B = \mu^M$. In that case, the monetary authority chooses both i and j, and it is likely to choose $j < i$.

(iii) Grow the bond supply at a slower rate than the money stock: $\mu^B < \mu^M$. In that case, $B/M \to 0$, and eventually equilibrium must be in Region (C), where the borrowing cost is minimal: $j = 0$.

It is beyond the scope of this paper to take a stand on the particular incentives that the two authorities may have, and to analyze this game exhaustively. But we learn a few simple

lessons already. First, if the game is non-cooperative, clearly its outcome will hinge on which one of the two authorities has (or is perceived to have) greater commitment power. It stands to reason that the fiscal authority prefers low borrowing costs over high ones, hence it has a strong incentive to grow the bond supply in the long run at approximately the rate of inflation that the monetary authority prefers. However, if the fiscal authority is able to commit to a high rate of bond issuance, then a benevolent monetary authority also has an incentive to give in and accept the long-run inflation rate that the fiscal authority prefers.

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