Expecting the unexpected: Stressed scenarios for economic growth

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Abstract

We propose the construction of conditional growth densities under stressed factor scenarios to assess the level of exposure of an economy to small probability but potentially catastrophic economic and/or financial scenarios, which can be either domestic or international. The choice of severe yet plausible stress scenarios is based on the joint probability distribution of the underlying factors driving growth, which are extracted with a multi-level Dynamic Factor Model (DFM) from a wide set of domestic/worldwide and/or macroeconomic/financial variables. All together, we provide a risk management tool that allows for a complete visualization of the dynamics of the growth densities under average scenarios and extreme scenarios. We calculate Growth-in-Stress (GiS) measures, defined as the 5% quantile of the stressed growth densities, and show that GiS is a useful and complementary tool to Growth-at-Risk (GaR) when policymakers wish to carry out a multi-dimensional scenario analysis. The unprecedented economic shock brought by the COVID19 pandemic provides a natural environment to assess the vulnerability of US growth with the proposed methodology.

Keywords: Growth vulnerability, Multi-level factor model, Scenario analysis, Stressed factors

JEL codes: C32, C55, E32, E44, F44, F47, O41

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1 Introduction

In hindsight, the COVID-19 induced decline in GDP growth across the world economies had at least three common features. First, the decline was almost synchronous and world-wide; second, the magnitude of the decline was extraordinary by historical standards; and third, it was unpredictable. Given this historical experience and its lack of predictability, it seems natural to ask econometricians for the development of new tools to recreate extreme scenarios and provide warning signals of what to expect under the possibility of unexpected extreme economic and financial shocks. Properly speaking, we cannot characterize this exercise as forecasting but we can recreate a virtual future by canvassing extreme probabilistic scenarios that will teach us how resilient the present economic systems are.

The increasingly growing literature on macroeconomic risk measurement is now even more relevant precisely because of the havoc generated by the historically unprecedent decline in growth for most countries around the world observed during the COVID pandemic. It is desirable for policymakers to be prepared for extreme and unexpected shocks that could generate severe recessions as well as for the implementation of corrective measures to minimize their risks; see Kilian and Manganelli (2008) and Alessi et al. (2014) for the importance of having appropriate measures of risk for policymakers and central banks, respectively. The high costs of recessions underscore the need to strengthen the resilience of the economies, notably by assessing early on potential vulnerabilities that can lead to such costly events; see the discussion in Rohn et al. (2015), who describe more than 70 vulnerability indicators that could be monitored to assess country risks in OECD economies, and Ludvigson et al. (2021), who discuss the economic costs of the COVID19 pandemic in the US.

Recent popular macroeconomic risk indexes are based on estimates of the full probability distribution of growth; see, for example, Ravazzolo and Rothman (2016) for an early contribution proposing a recession index indicator for US GDP based on the growth density modelled as a function of oil prices; De Nicoló and Luccetta (2017), who propose using factor-augmented quantile regressions for industrial production growth and employment growth; and Chavleishvili and Manganelli (2019), who forecast euro area industrial production using quantile VAR models. More recently, Adrian et al. (2019) propose the

Growth at Risk (GaR) index, which is a lower quantile of the growth density modelled as a function of "domestic" underlying financial factors. GaR has become part of the toolbox of academics and policymakers to measure growth vulnerability. For example, it has been adopted by the International Monetary Fund (IMF) as the main quantitative criterion to gauge global financial stability risk; see Prasad et al. (2019) and Adrian et al. (2020) for descriptions of the use of GaR at the IMF.

Conceptually, GaR mimics the spirit of the popular Value-at-Risk (VaR) measure of financial risk and, consequently, it also shares its caveats. The VaR is an extreme quantile (often 1% or 5%) of the distribution of financial returns in normal times. However, a regulator should also be concerned about quantiles of the distribution of returns in adverse environments as, for example, catastrophic financial events; see, for example, the discussion by Acharya et al. (2012). Given that VaR is not designed to measure financial risk under these adverse environments or stressed conditions, there is a large number of works on financial stress testing and stress scenarios; see, for example, Borio et al. (2014), Flood and Korenko (2015) and Wang and Ziegel (2021).

Similarly, when dealing with economic growth, stress refers to the analysis of the conditional distribution of GDP growth when exposed to extreme shocks, which are rare and large in magnitude relative to the shocks expected during tranquil times. It is important to note that the growth densities from which the GaR is obtained are estimated under normal economic environments and not when the economy is stressed by an extremely large and unexpected shock. Using econometric tools designed to analyse the average effect of macroeconomic and financial variables on the quantiles of growth is bound to miss important effects, which arguably only arise when the system is affected by extreme shocks. Standard econometric tools may not be adequate to distinguish a potentially large growth decline in a normal economic environment from a potentially huge loss in a stressed economy working under adverse conditions. GaR is not designed to measure growth densities in stressed conditions and, consequently, it could not be adequate to measure the exposure of growth to potential extreme risks; see, for example, Plagborg-Möller et al. (2020), who show that GaR did not yield useful advanced warnings of tail risk during the COVID19 pandemic. Therefore, the need of developing new instruments to measure economic risk in adverse environments seems evident.

This paper contributes to the important literature on measuring growth vulnerability by proposing a methodology to construct stressful economic scenarios and to analyze the response of economic growth when the economy is under stress. We construct stressed growth densities as a complementary tool to the average growth densities. In doing so, policymakers will be able to evaluate the trade-off between building greater resilience in normal times and reduce downside risk in highly stressed periods; see Adrian and Liang (2018) for a discussion of this trade-off.

The proposed methodology to obtain stressed growth densities builds on the combination of three different procedures already available in the literature. First, we extract the latent factors driving growth by fitting a multi-level Dynamic Factor Model (DFM), proposed by Rodríguez-Caballero and Caporin (2019), to a vast array of worldwide/domestic and/or macroeconomic/financial variables, which are potential predictors of the distribution of growth for a particular country or area. The factor structure of the multi-level DFM allows for overlapping blocks of factors with factors common to all variables in the system and specific factors that can be particular to one or more blocks of variables.

Second, similarly to Adrian et al. (2019), we proceed to estimate factor-augmented quantile regressions using the estimated factors as regressors. Then, we use the estimated quantiles together with a smoothing approach to obtain one-step-ahead (and multi-step) forecasts of the conditional probability density of GDP growth. These forecasts deliver any quantile of interest under normal circumstances, that is, when the underlying factors driving growth are around their average values. Lower quantiles, like the 5% or 1% tails, provide an estimation of a potentially large but expected decline in growth (GaR).

Third, we obtain stressed scenarios (stressed factors) for the economy using the methodology behind the Growth in Stress (GiS) index proposed by González-Rivera et al. (2019). Under unexpected and rare circumstances, the factors underlying the distribution of growth are also under stress and thus, far from their average values. We quantify stress in the factors in a probabilistic way by considering the multivariate distribution of the factors and focusing on the values in the tails of their multivariate distribution. These values are the probabilistic stress scenarios. We estimate growth densities under these scenarios. Because stress is confined to the tails of the multivariate distribution of the factors, the policy maker will choose the tail quantile of this distribution depending on the desired level of resilience.

The proposed methodology provides the natural environment to perform stress testing exercises of growth. Therefore, in the empirical section of this paper, we build scenarios for US growth and analyse whether they could have been useful in the quarters preceding the COVID 19 pandemic. We first fit the multi-level DFM to extract the factors from a large set of variables that can be classified into four blocks, namely, domestic macroeconomic (DM), domestic financial (DF), worldwide macroeconomic (WM) and worldwide financial (WF) variables. We find a first pervasive factor common to all variables in the system, a second semi-pervasive factor common to the worldwide variables (regardless of whether they are macroeconomic or financial), and three additional non-pervasive factors, each of them common to a different subset of variables (worldwide financial, domestic macroeconomic, and worldwide macroeconomic variables). We compute the multivariate distribution of these factors and set the level of stress. Together with factor-quantile regression estimates, we are able to obtain stressed growth densities. We show that, for 2020Q2, US growth risk estimated by the 5%-quantile GaR was -15.29% (annualized quarter-over-quarter growth) and by the 5%-quantile GiS with 95% stress in the factors was -29.13%. The observed growth decline was -31.20% according to the IMF. The warning provided by GaR was rather conservative.

The rest of the paper is organized as follows. In Section 2, we describe the methodology to obtain growth densities in stressed scenarios. In particular, we describe how to specify and estimate a multi-level DFM to extract the relevant factors, how to obtain the distribution of the factors, and how to construct conditional densities of growth in "normal" as well as in "stressed" scenarios. In Section 3, we extract the factors from domestic/worldwide and/or financial/macroeconomic variables in the US and we compute the probability distribution of US GDP growth in "normal" times and under different stressed-factor scenarios. In Section 4, we conclude with some final considerations.

2 Stressed scenarios for economic growth: Methodology

In this section, we describe the methodology proposed to estimate the probability distribution of growth in stressed scenarios for the factors. We first describe how to obtain the distribution of the factors that will be used to obtain stressed scenarios in the context of the multi-level DFMs. Second, we describe the estimation of the distribution of growth in "normal" as well as in "stressed" scenarios, and the computation of the GiS.

2.1 Probability distribution of the factors: Scenarios under stress

Consider the following static DFM for the variables in X_t, the $N \times 1$ vector of observations at time t of the domestic/worldwide macroeconomic and financial variables used to extract the factors that explain the growth density in a given country or area

$$X_t = PF_t + \varepsilon_t, \tag{1}$$

where P is the $N \times r$ matrix of factor loadings, $F_t = (F_{1t}, ..., F_{rt})'$ is the $r \times 1$ vector of underlying unobserved factors at time t, and ε_t is the $N \times 1$ vector of idiosyncratic components, which are allowed to be weakly cross-sectionally correlated but uncorrelated with the factors, F_t. The factors, F_t, embed the information contained in the large number of potential predictors of the quantiles of growth, X_t. To uniquely identify the factors and loadings, we assume, as usual in this literature, that $\frac{F'F}{T} = I_r$, where $F = (F_1, ..., F_T)$ is an $r \times T$ matrix and $P'P$ is diagonal with its elements ordered from largest to smallest. After determining the number of factors, r, they are extracted by Principal Components (PC) from X_t.[1] Define $X = (X_1, ..., X_T)'$. The PC factors, $\widehat{F}_t$, are given by $\sqrt{T}$ times the eigenvectors corresponding to the r largest eigenvalues of XX' arranged in decreasing order while $\widehat{P}' = \frac{1}{T}\widehat{F}'Y$.[2]

[1] See also Giglio et al. (2016), who propose using Partial PC.

[2] In the context of PC factor extraction to explain the quantiles of growth, Adrian et al. (2019) consider $r = 1$ factor extracted from a set of domestic financial variables. In particular, they consider the Chicago Fed's National Conditions Index (NFCI), which provides a weekly update on US financial conditions in money markets, debt and equity markets and the traditional and "shadow" banking systems. In another application, González-Rivera et al. (2019) model the distribution of growth after extracting $r = 3$ factors from a set of international GDPs.

The multivariate probability density of the factors is needed to obtain probabilistic scenarios for the factors. From that density, it is possible to construct probability contours of the factors $g(F_t, \alpha) = 0$ at a desired probability or stress level α, say $\alpha = 95\%$, so that the contour is an ellipsoid that contains 95% of the values of F_t, with the most extreme 5% of events outside of the ellipsoid.[3] Under general conditions, Bai (2003) shows that, if $\frac{F'F}{T} = I_r$ and $\frac{\sqrt{N}}{T} \to 0$ when $N, T \to \infty$, at each moment of time, t, the asymptotic distribution of $\widehat{F}_t$ is given by

$$\sqrt{N}\left(\widehat{F}_t - F_t\right) \xrightarrow{d} N\left(0, \Sigma_P^{-1}\Gamma_t\Sigma_P^{-1}\right), \tag{2}$$

where $\Sigma_P = \lim_{N\to\infty} \frac{P'P}{N}$ and $\Gamma_t = \lim_{N\to\infty} \sum_{i=1}^{N}\sum_{j=1}^{N} p_i p_j' E(\varepsilon_{it}\varepsilon_{jt})$ with p_i' being the $1 \times r$ i'th row of P and ε_{it} being the idiosyncratic component corresponding to the i'th variable in X_t. The finite sample approximation of the asymptotic covariance matrix of $\widehat{F}_t$ can be estimated as follows

$$MSE_t = \left(\frac{\hat{P}'\hat{P}}{N}\right)^{-1} \frac{\hat{\Gamma}_t}{N} \left(\frac{\hat{P}'\hat{P}}{N}\right)^{-1}, \tag{3}$$

where $\hat{\Gamma}_t$ is an estimate of Γ_t; see Bai and Ng (2006) for estimators of Γ_t.

In this paper, the factors driving the quantiles of growth are extracted from a rich set of variables that are organized in blocks: domestic and worldwide variables and macroeconomic and financial variables. These blocks imply zeros in the loading matrix P as not all variables in X_t load on all r factors in the DFM. The factors could be extracted using PC from the full set of variables as explained above. However, PC does not take full advantage of the block structure and the estimated PC factors will not be optimal. Furthermore, it is important to note that the usual criteria for the determination of the number of factors are not very conclusive when the eigenvalues of the covariance matrix have not a clear break, as it is often the case when there are local factors that only load in subsets of variables. As a consequence, the corresponding estimated DFM could appear

[3]This proposal to obtain stressed factors is closely related to that of Haugh and Ruiz Lacedelli (2020), who carry out scenario analysis for derivative portfolios via DFMs expressed as state space models (SSMs) by computing and simulating from the distribution of unstressed risk factors conditional on a given scenario. It is also close to that of Wang and Ziegel (2021) in the context of scenarios for financial risk.

as either having weak common factors or with cross-sectionally correlated idiosyncratic errors; see, for example, the discussions by Moench et al. (2013) in the context of a hierarchical structure for the factors. Furthermore, the presence of zeros in the loadings may bias the estimates of the underlying factors; see Boivin and Ng (2006) and Breitung and Eickmeier (2016). To overcome these problems, instead of extracting PC factors from the DFM in (1), it is more appropriate to extract them from a multi-level DFM obtained after imposing the adequate zero restrictions on the matrix of loadings, P. Furthermore, the factors extracted from a multi-level DFM are more easily interpretable than those extracted using PC from the DFM in (1).

Due to the particular structure of the data considered in this paper, with overlapping blocks of variables, we follow Rodríguez-Caballero and Caporin (2019) and extract the factors based on a multi-level DFM that decomposes the factor structure into different levels such that some factors are associated with the full cross-section of variables (pervasive factors) while some others either impact a specific subset of variables (non-pervasive factors) or several subsets of variables (semi-pervasive factors).[4] Consider the following example, with the variables in X_t divided in four blocks, $X_t = (X_{1t}, X_{2t}, X_{3t}, X_{4t})'$ and the multi-level DFM with $r = 8$ factors given by

$$X_t = \begin{bmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \\ X_{4t} \end{bmatrix} = \begin{bmatrix} p_{11} & 0 & p_{13} & p_{14} & p_{15} & 0 & 0 & 0 \\ p_{21} & p_{22} & p_{23} & 0 & 0 & p_{26} & 0 & 0 \\ p_{31} & 0 & 0 & p_{34} & 0 & 0 & p_{37} & 0 \\ p_{41} & p_{42} & 0 & 0 & 0 & 0 & 0 & p_{48} \end{bmatrix} \begin{bmatrix} F_{1t} \\ F_{2t} \\ F_{3t} \\ F_{4t} \\ F_{5t} \\ F_{6t} \\ F_{7t} \\ F_{8t} \end{bmatrix} + \varepsilon_t^*, \qquad (4)$$

where F_{1t} is a pervasive factor that loads in all the variables on the system, F_{2t}, F_{3t} and F_{4t} are semi-pervasive factors with loadings on X_{2t} and X_{4t}, X_{1t} and X_{2t}, and X_{1t} and X_{4t}, respectively. Finally, F_{5t}, F_{6t}, F_{7t} and F_{8t} are non-pervasive factors that load on X_{1t},

[4]This multi-level DFM is closely related to the three-level model proposed by Breitung and Eickmeier (2016).

X_{2t}, X_{3t}, and X_{4t}, respectively.

In order to specify the factor structure of the multilevel DFM, i.e. to determine the zeros in the loading matrix P, we follow Hallin and Liska (2011), who propose a statistical criteria based on analysing the pairwise correlations between the factors extracted by PC from each subset of variables separately. Due to the high variability in the number of factors detected by alternative statistical procedures, we determine the number of factors within each block by visual inspection of the scree plot; see Hindrayanto et al. (2016), who also use the scree plot.

Estimation of the multi-level DFM is challenging as the factor structure does not allow estimating one level after another. Consequently, estimation is based on the sequential procedure proposed by Breitung and Eickmeier (2016). First, initial estimates of the factors are obtained using canonical correlations and PC. Second, a sequential Least Squares procedure is implemented to estimate the loadings and factors; see Rodríguez-Caballero and Caporin (2019) for details about the estimation algorithm and for Monte Carlo results about its good finite sample performance.[5]

When the finite sample distribution of the factors, needed for the construction of scenarios, is estimated using the asymptotic approximation in (2), Poncela and Ruiz (2016) and Maldonado and Ruiz (2021) show that the associated regions for the factors will suffer from undercoverage due to the underestimation of the MSE when using (3). Consequently, González-Rivera et al. (2019) propose using the subsampling correction of the asymptotic distribution of the underlying factors of Maldonado and Ruiz (2021), which is designed to incorporate the uncertainty due to the estimation of the loadings. This correction is based on subsampling subsets of size N^* of series in the cross-sectional space, with each series containing all temporal observations. For each subsample, the loadings and factors are estimated by PC, obtaining $\hat{F}_t^{*(b)}$ and $\hat{P}^{*(b)}$, for $b = 1, ..., B$. The subsampling analogue of the MSE due to parameter uncertainty associated with the

[5]See Choi et al. (2018) for a similar estimation procedure and Aastveit et al. (2016) for an alternative estimation procedure for multi-level DFMs and a bootstrap procedure to construct confidence bounds for the factors.

estimation of the factor loadings, is estimated as follows

$$\frac{1}{B}\sum_{b=1}^{B}\left(\left(\hat{F}_t^{*(b)}-\hat{F}_t\right)\left(\hat{F}_t^{*(b)}-\hat{F}_t\right)'\right). \quad (5)$$

Finally, the finite sample MSE of $\hat{F}_t$ is estimated as

$$MSE_t^{*}=\frac{1}{N}\left(\frac{\hat{P}'\hat{P}}{N}\right)^{-1}\hat{\Gamma}_t\left(\frac{\hat{P}'\hat{P}}{N}\right)^{-1}+\frac{N^{*}}{NB}\sum_{b=1}^{B}\left(\left(\hat{F}_t^{*(b)}-\hat{F}_t\right)\left(\hat{F}_t^{*(b)}-\hat{F}_t\right)'\right); \quad (6)$$

see Maldonado and Ruiz (2021) for the good properties of this MSE when used to construct confidence ellipsoids for the underlying factors.[6]

2.2 The conditional distribution of growth in normal times: GaR

Let GDP_t be the Gross Domestic Product observed quarterly at time t, for $t = 1, ..., T$ and define the annualized quarter-over-quarter growth as $y_t = 400 \times \triangle \log(GDP_t)$. The h-step ahead τ^*-quantile of the conditional distribution of y_t is obtained by estimating the following factor-augmented quantile regression

$$q_{\tau^*}\left(y_{t+h}|y_t,F_t\right)=\mu(\tau^*,h)+\phi(\tau^*,h)y_t+\sum_{k=1}^{r}\beta_k(\tau^*,h)F_{kt}, \quad (7)$$

where $\mu(\tau^*, h), \phi(\tau^*, h)$ and $\beta_k(\tau^*, h)$, $k = 1, ..., r$, are parameters and F_t is the $r \times 1$ vector of underlying unobserved factors at time t, extracted as defined above from X_t, the set of N macroeconomic and/or financial potential predictors of growth.

The factor-augmented quantile regression model in (7) is appropriate for representing the potentially asymmetric and non-linear relationship between economic growth and the underlying factors; see, for instance, Plagborg-Möller et al. (2020) for evidence about

[6]Even though there is not yet a formal result on the asymptotic distribution of the factors extracted from multi-level models, we construct these regions based on the asymptotic distribution derived by Choi et al. (2018) for the pervasive factor, which is extracted in the first step and has the same asymptotic distribution derived by Bai (2003). For the rest of the factors, which are extracted based on the residuals from the previous step, we also assume asymptotic normality. Since they are based on residuals, their asymptotic MSE will be affected by parameter estimation uncertainty but this problem should be mitigated by extending the subsampling procedure of Maldonado and Ruiz (2021) to the multi-level DFM framework.

asymmetries in economic growth fluctuations. Factor-augmented quantile regressions are standard in modelling growth quantiles; see, Manzan (2005), Giglio et al. (2016), Adrian et al. (2019), González-Rivera et al. (2019), and Adrian et al. (2022), among others.[7] In practice, the underlying factors in (7) are replaced by estimated factors, $\hat{F}$, obtained from the multi-level DFM described above.

The parameters in equation (7) are estimated using the algorithm by Koenker and d'Orey (1987), which implements the estimator proposed by Koenker and Bassett (1978); see Ando and Tsay (2011) and Giglio et al. (2016) for its asymptotic properties. For a given quantile τ^*, and horizon h, the goodness of fit of the estimated factor-augmented quantile regressions is estimated by $R^1 = 1 - \frac{\sum_{t=2}^{T} \hat{\nu}_t[\tau^* I(\hat{\nu}_t \geq 0) + (\tau^* - 1) I(\hat{\nu}_t < 0)]}{\sum_{t=1}^{T} y_t[\tau^*(I(y_t \geq \bar{y}) + (\tau^* - 1) I(y_t < \bar{y})]}$, where $\hat{\nu}_t = y_t - \hat{\mu}(\tau^*, h) - \hat{\phi}(\tau^*, h) y_{t-h} - \sum_{k=1}^{r} \hat{\beta}_k(\tau^*, h) F_{kt-h}$, $\bar{y}$ is the sample mean of y_t and $I(\cdot)$ is an indicator function that takes value 1 if the argument is true and zero otherwise; see Koenker and Machado (1999). Note that R^1 is the natural analogue of the R^2 coefficient in a regression model.

After estimating (7) for different quantiles τ^*, we follow Adrian et al. (2019) and obtain the conditional distribution of growth by fitting the Skewed-t distribution of Azzalini and Capitanio (2003) to the estimated quantiles, $\hat{q}_{\tau^*}(y_{t+h}|y_t, F_t)$. At each moment of time t, the four parameters that define the Skewed-t distribution are estimated by minimizing the squared distance between the estimated quantiles and the corresponding quantiles of the Skewed-t distribution.[8] Denote this density by $\hat{k}_0(y_{t+h})$.

Adrian et al. (2019) propose measuring the h-step ahead growth risk at time t by GaR, which is defined as the τ quantile, most popular $\tau = 0.05$, of the estimated conditional distribution of growth, $\hat{k}_0(y_{t+h})$. Therefore, GaR is an extreme left quantile of the distribution of growth estimated as a function of the underlying estimated factors.

[7] De Nicoló and Luccetta (2017) also fit factor-augmented quantile regressions to measure the tail risk of industrial production and employment in the US. See also Carriero et al. (2022b) for alternative specifications of extreme quantiles of a distribution.

[8] Recently, Mitchell et al. (2023) propose an alternative non-parametric approach for constructing density forecasts from quantile regressions, according to which, the conditional quantile forecasts from the quantile regressions are mapped directly to a conditional density only assuming local uniformity between the quantile forecasts. The improvement of the nonparametric density when compared with the asymmetric Student density appears when the conditional distribution of growth is characterized by multimodalities instead of asymmetry. In an application to US GDP growth, they show that this nonparametric density matches or slightly improves upon the accuracy of the densities used by Adrian et al. (2019). Given that the improvement is only marginal, in this paper, we keep estimating the density by the more popular asymmetric Student density.

2.3 The conditional distribution of growth under stress: GiS

Given that GaR is computed under "non-stressed" conditions, i.e., when the underlying factors are fixed at their estimated averages, $\hat{F}_t$, it measures the vulnerability of the economy in the "normal" scenario. However, if an extreme event were to shock the economy, it would be of interest to analyse the probabilistic distribution of growth under unusual extreme circumstances. We consider that the extreme conditions will be reflected in the behaviour of the factors that drive growth, which could be themselves under stress. In this context, González-Rivera et al. (2019) propose GiS as an additional measure of vulnerability. Next, we describe GiS, the τ^*-quantile of economic growth densities under stressed factors.

Consider the factor-augmented quantile regression in (7) for a fixed quantile τ^*, and define the minimum value of $q_{\tau^*}(y_{t+h}|y_t, F_t)$ when the underlying factors are subject to α-probability stressed scenarios, as follows,

$$\min_{F_t} q_{\tau^*}(y_{t+h}|y_t, F_t) \tag{8}$$

$$s.t. \quad g(F_t, \alpha) = 0,$$

where $g(F_t, \alpha) = 0$ is a predetermined α-contour of the factors, i.e. an ellipsoid that contains the true factor vector, F_t, with probability α. The values of F_t on the boundary of the ellipsoid $g(F_t, \alpha) = 0$ are considered the extreme events of the factors.

In general, the constrained optimization problem in (8) requires the estimation of the iso-quantile surfaces, i.e., the combination of factors that generates the same value of the τ^*-quantile, as well as the search of the tangency point between these surfaces and the α-ellipsoid of the factors. When the number of underlying factors is larger than two, the constrained minimization is solved by using the simple binary mesh algorithm proposed by Flood and Korenko (2015).[9]

The optimization exercise in (8) is repeated for different τ^*-quantiles of growth (keep-

[9]Software is available in https://cran.r-project.org/web/packages/SyScSelection/index.html. In a spaced grid or mesh on the ellipsoid, the fineness parameter determines the number of points iterated along each dimension until the optimal combination of points is found. We choose a fineness parameter of 8. We have experimented with several values of the fineness parameter and our results are very robust to this choice.

ing the α-level of stress fixed). After fitting a Skewed-t density to the minimal growths corresponding to different estimated τ^*-quantiles, we obtain the conditional "stressed" density of growth. Denote this stressed density as $\hat{k}_\alpha(y_{t+h})$. Finally, for an α-level of stress of the factors, the h-step-ahead GiS is given by the τ-quantile of this stressed density as follows

$$GiS_{t+h} = \inf\left\{ y_{t+h} \mid \int_{-\infty}^{y_{t+h}} \hat{k}_\alpha(u)du \geq \tau \right\}. \tag{9}$$

We illustrate the construction of scenarios and the computation of the GiS with an example.[10] Consider that the growth quantile of interest is $\tau^* = 0.05$, which depends on two factors, F_{1t} and F_{2t}, as follows

$$q_{0.05}\left(y_{t+1}|F_t\right) = 1.07F_{1t} - F_{2t} - 3.35. \tag{10}$$

The factors are generated by a standardized bivariate normal distribution, with means 5 and 2, respectively, and covariance 0.5.

The top panel of Figure 1 plots four iso-5%-quantile lines, i.e. four linear combinations of F_{1t} and F_{2t}, each of them implying the same value of the 5% quantile of growth. In particular, the green straight line represents $q_{0.05}\left(y_{t+1}|F_t\right) = -3.35$ while the black, blue and red straight lines represent $q_{0.05}\left(y_{t+1}|F_t\right) = -2.35$, $q_{0.05}\left(y_{t+1}|F_t\right) = -1.35$ and $q_{0.05}\left(y_{t+1}|F_t\right) = -0.5$, respectively. The top panel of Figure 1 also plots α-probability contours of the factors, for different probability levels α. Each contour can be thought as possible extreme realizations from the distribution of the factors. Note that the red iso-5%-quantile, which corresponds to a 5% quantile of growth of -0.5, crosses through the point of factor means. If, at time t, the realized factors are set to their mean values $F_{1t} = 5$ and $F_{2t} = 2$, the 5% quantile of growth roughly corresponds to the results that one would obtain from the GaR analysis, i.e. the GaR is -0.5. However, our framework allows us to also consider arbitrary stress scenarios for the factors and to assess their impact on the 5% quantile of growth. In the same figure, we illustrate the implications of the scenarios by highlighting three specific ones. The ellipse tangent to the green iso-5% quantile corresponds to the 99% contour. Therefore, in this case, we can think of the

[10] In this example, we are not smoothing the densities but considering the quantiles as directly obtained from the factor-augmented quantile predictive regression for τ^*.

factors stressed at $\alpha = 99\%$ level. The GiS associated to this level of stress of the factors is the value of the 5% quantile of growth corresponding to the green iso-5%-quantile line, which is -3.35. If the level of stress of the factors is smaller, for example, $\alpha = 93\%$, the GiS is given by the tangency point of the 93% contour with the black iso-5%-quantile line and the GiS is -2.35. Finally, if the level of stress of the factors is even smaller, $\alpha = 73\%$, the GiS is determined by the tangency point of the 73% contour with the blue line, which implies that the 5%-quantile of growth is -1.35. Note that there are big differences between the 5%-quantile of growth obtained under stressed factor scenarios and the GaR, which is obtained under "normal" circumstances, that is, when the factors are fixed at their averages, which correspond to the central point of the ellipse in Figure 1. Charts of this type can be used by policymakers to calibrate the severity of the stress, which can be arbitrarily set according to their own preferences.

The GiS measures the risk exposure of the economy to extreme movements in the underlying factors that drive growth. The policymaker could choose different α-levels of stress and generate the corresponding stressed densities of growth and GiS values.[11] By choosing different values of α in the constraint $g(F_t, \alpha) = 0$, i.e. different levels of stress in the factors, GiS provides an analysis of growth under different scenarios.[12] By working with the probability contours of the underlying factors, the policymaker can understand those scenarios in which severe but plausible factor values may substantially affect economic growth. For policymakers, knowledge of the growth density under stressed factors is a tool to assess whether the economy is too exposed to any of the factors and, if so, how to act to reduce exposure. In this sense, GiS underscores the arguments in Breuer et al. (2009), who argue that measures based on historical experience, as GaR, may risk to ignore plausible but harmful scenarios, as those we currently observe as a result of

[11] In this set up, the α-level of stress is chosen by the decision maker. It might be possible to choose α in an optimal way if the decision maker were to have a loss function that depends on GiS somehow. However, this is a different research question that may fit within the problem put forward by Manski (2021), who proposes the use of confidence sets for decision problems. The discussions by Granger and Machina (2006), Elliot and Timmermann (2016) and Watson and Holmes (2016) may also be relevant.

[12] Scenario analysis is rather popular in the context of financial markets; see Glasserman et al. (2015), who identify sensible combinations of stress to multiple factors to assess financial risk; Hagfors et al. (2016) for scenario analysis of electricity prices in the context of quantile regressions; European Central Bank (2006) for the importance of scenario analysis in the context of stress testing in the financial sector, Rebonato (2019) for financial stress testing based on Bayesian nets, and Haugh and Ruiz Lacedelli (2020) who carry out scenario analysis for derivative portfolios via DFMs. Finally, it is important to remark that the Basel Committee on Banking Supervision (2005) recommends choosing scenarios that are plausible and severe.

the COVID19 pandemic. The probability contours of the underlying factors provide a benchmark for plausibility and severity of the stressed factors. GiS captures plausibility by specifying how much stress to exercise into the tails of the factors' distribution, while severity is maximized by systematically searching for the worst growth case in the factor region determined by the chosen level of stress; see also Flood and Korenko (2015) and Breuer et al. (2009) for discussions on the trade-off between plausibility and severity of stress scenarios.

A final note on (the lack of) inference on GiS and GaR. Finding the uncertainty of the quantiles of $\hat{k}_0(y_{t+h})$ and $\hat{k}_\alpha(y_{t+h})$ is a challenging and interesting problem. With respect to $\hat{k}_0(y_{t+h})$, as far as we know, there are consistency results for the predicted quantiles in the factor-augmented regression models but not results are available for their asymptotic distribution; see Giglio et al. (2016). These results allow the evaluation of quantile forecasts. For example, Giglio et al. (2016) propose comparing the sequences of quantile forecast losses based on conditioning information, $\hat{\nu}_t\left[\tau^* I(\hat{\nu}_t \geq 0) + (\tau^* - 1) I(\hat{\nu}_t < 0)\right]$, to the quantile losses based on historical quantiles while, very recently, Corradi et al. (2023) also propose tests for the forecast accuracy of quantiles. However, obtaining asymptotic intervals for estimated quantiles poses some statistical challenges since it involves elements of nonparametric density estimation with resampling techniques to compute Mean Square Errors of the estimated quantiles. For example, Gregory et al. (2018) propose bootstrapping time series quantile regressions and illustrate its implementation in the context of VaR estimation. However, they do not consider the presence of estimated factors in the estimated quantile regressions. Alternatively, Gonçalves et al. (2017) propose using bootstrap to construct prediction intervals in the context of factor-augmented regressions but not for factor-augmented quantile regressions. Consequently, designing a proper bootstrap procedure that considers the presence of both estimated quantiles and estimated factors in quantile regressions is still needed. Furthermore, finding intervals for the estimated quantiles in factor-augmented quantile regressions does not solve the issue of finding intervals for GiS and GaR. It is important to note that, after estimating these regressions, GaR is calculated as the corresponding quantile of the smoothed distribution of growth, obtained by fitting a Skewed-t distribution. Consequently, even if one were able to obtain bootstrap replicates of the quantiles with good properties, it would be necessary to obtain a large number of bootstrap replicates of these smoothed densities (with and without stressed factors). With these bootstrap replicates of the smoothed densities, it would be possible to obtain the uncertainty surrounding GiS and GaR; see Chernozhukov et al. (2013) for the use of bootstrapping in the context of inference for counterfactual distributions. The computational burden involved in these simulations can be alleviated by using the fast bootstrap procedures proposed by Chernozhukov et al. (2022) in the context of quantile regressions. More importantly, even with this computational/numerical approach in place, it would be necessary to study the statistical properties of the newly proposed bootstrap procedure. This is beyond the scope of this paper.

Finally, note that, in the construction of scenarios for the τ^*-quantile of growth described above, α measures the level of stress to be chosen by the policymaker. Instead of looking at the conditional distribution of growth under stressed factors, one can use the methodology proposed in this paper to determine the maximum level of probability of the factors, γ, subject to a particular value of the τ^*-quantile of the distribution of growth of interest for the policymaker. The dual problem of (8) can be stated as follows

$$\max_{F_t} H(F_t) \tag{11}$$

$$s.t. \quad q_{\tau^*}(y_{t+h}|y_t, F_t) = \bar{q},$$

where $H(F_t)$ is the joint cumulative distribution function of the factors and $\gamma = \max H(F_t)$. Under the dual problem in (11), the policymaker chooses the value of the τ^*-quantile of growth that could be dangerous for the economy (say $\bar{q}$) and obtains the probability level of the factors leading to $\bar{q}$. Therefore, this dual problem is useful to find the probability of the factors such that the τ^*-quantile of growth does not exceed a predetermined level $\bar{q}$. If this probability is very small, then the chances for the economy going below $\bar{q}$ are scarce, while, if this probability is large, there is a large danger for the economy going below $\bar{q}$ and resilience measures can be implemented to avoid the negative implications.

The bottom panel of Figure 1 illustrates this dual problem for the same example described above. In this case, given that there are two factors with a joint normal distribution, and that the iso-quantile function is given by (10), the joint cumulative distribution function of the factors for which the τ^*-quantile of growth does not exceed $\bar{q}$ is given by

$$H(F_1, F_2) = \int_{-\infty}^{F_2} \int_{-\infty}^{\frac{\bar{q}+3.35+F_{2t}}{1.07}} w(F_{1t}, F_{2t}) dF_{1t} dF_{2t}, \tag{12}$$

where $w(F_{1t}, F_{2t}) = \frac{1}{2\pi\sqrt{0.75}} \exp\left\{-\frac{1}{1.5}\left[(F_{1t}-5)^2 + (F_{2t}-2)^2 - (F_{1t}-5)(F_{2t}-2)\right]\right\}$ is the joint density of the factors. The policymaker can calculate the probability of the combinations of F_1 and F_2 leading to the 5%-quantile of growth being below $\bar{q}$ by finding the maximum of $H(F_1, F_2)$. In particular, the probability of the factors for the 5%-quantile of growth being below -0.5 is 0.51, while the probabilities of the factors for the 5% quantile of growth being below -1.35, -2.35 and -3.35 are 0.17, 0.03 and 0.002, respectively.[13]

3 The distribution of growth in the US

In this section, we obtain the conditional probability distribution of US GDP growth based on factors extracted from a multi-level DFM, which considers a large system of macroeconomic and financial variables, some of which are domestic in the US and some are worldwide. The probability distribution is estimated in "normal" times and under different stressed-factor scenarios.

3.1 Underlying macroeconomic and financial factors

In order to estimate the factor-augmented quantile regression in (7), we consider annualized quarter-over-quarter real US GDP growth observed from 2005Q3 to 2021Q1. The in-sample period spans from 2005Q3 to 2020Q1 while the observations from 2020Q2 to 2021Q1 are reserved for out-of-sample exercises.

A strand of the literature analyses the conditional distribution of growth by focusing on factors extracted only from domestic financial variables. Adrian et al. (2019) estimate US growth densities as functions of a DF factor, in particular, the NFCI. Further works considering the DF factor are De Nicoló and Luccetta (2017), Adams et al. (2021), Catania et al. (2021), Ferrara et al. (2022) and Adrian et al. (2022), among many others.[14]

[13] We are very thankful to an anonymous referee for suggesting this alternative dual problem. In what follows, we focus on the construction of scenarios and the computation of the GiS.

[14] The ability of financial factors to predict future real economic activity has been discussed by Hatzius et al. (2010), Matheson (2012), Giglio et al. (2016), De Nicoló and Luccetta (2017), Menden and Proaño

The popularity of DF factors may be a consequence of the strong influence of domestic financial conditions in US during the 2008 Great Recession; see, for example, Dovern and van Roye (2014). The main argument for the link between financial factors and growth is based on the premise that financial prices incorporate market expectations of future price and output developments and, consequently, bear timely information on future economic conditions. However, other authors considering macroeconomic in addition to financial variables argue that the latter do not contribute much to distributional forecasts of growth; see, for example, Plagborg-Möller et al. (2020), Carriero et al. (2022a), Reichlin et al. (2020) and Çakmakli et al. (2021). Beyond the debate about whether financial and/or macroeconomic factors should be considered when modelling the conditional distribution of growth, other authors debate whether only domestic factors should be considered when assessing growth risk; see, for example, Mishkin (2011) and Breitung and Eickmeier (2016) for a discussion on the global character of some crisis, and Cerutti et al. (2019) on global financial factors. In general, they argue that forecasting growth risk based on only "domestic" factors could be misleading in the current globalized world. In this direction, Djogbenou (2020) propose a two-level DFM with two specific developed and emerging economy activity factors in addition to a world economic factor.[15]

In this paper, the factors underlying the conditional distribution of growth, which are used to estimate the factor-augmented quantile regression in (7), are extracted from a large set of financial and macroeconomic variables observed quarterly from 2005Q3 to 2020Q1 ($T = 59$ observations). These variables are classified into four different blocks. First, we consider the same domestic financial variables underlying the construction of the Chicago Fed's National Conditions Index (NFCI); see, Brave and Butters (2012) for a description of the NFCI. The cross-sectional dimension of this subset of variables, denoted as X_{1t}, is $N_1 = 105$ variables.[16] After standardization, we detect outliers using the procedure

(2017), Arrigoni et al. (2022) and Boyarchenko et al. (2020), among others. The link between economic and financial conditions has experienced a revival after the 2008 Great Recession; see, for example, Dovern and van Roye (2014). As pointed out by Ng and Wright (2013), using US data from 1960 to 2012, all the post-1982 recessions have originated in financial markets, and these recessions are different from recessions where financial markets play a passive role.

[15] There are other proposals with world and domestic financial factors. However, as far as we know, these factors have not been linked with economic growth; see Amiti et al. (2019) for a recent contribution.

[16] The NFCI is constructed on a weekly basis. We average weekly observations within each quarter to obtain observations with a quarterly frequency. For the attribution of weeks to overlapping quarters, we follow the same criteria as Adrian et al. (2019). Weeks that start in one quarter and end in the next one are fully assigned to the latter quarter.

in Kristensen (2014). We find one outlier in the variable ”T-note futures Euro/Dollar market depth” in 2008Q4.

Second, given the increasing globalization of the economy, we also consider the potential effect of worldwide financial factors on US growth; see, for example, Arregui et al. (2018), who show that, if deemed necessary, the rapid speed at which foreign shocks affect domestic financial conditions may make it difficult to react in a timely and effective manner. Daily observations of the variables within the worldwide financial block have been obtained from the ECB data base and aggregated by taking the quarterly average. They are denoted as X_{2t} and have cross-sectional dimension of $N_2 = 208$. Table 1 reports the variables within X_2 and the countries in which they have been observed, which represent 70% of the world's GDP at purchasing power parity; see Arrigoni et al. (2022), who also use these variables in their analysis. It is important to note that several variables corresponding to the US are among the variables included in X_2, namely, the term structure, the price earning ratio on national stock exchange (PER) and the historical volatility 30 days. As before, the worldwide financial variables are standardized and corrected for outliers. Two outliers are found in price earning ratio, one in Hungary in 2015Q2, which may be due to the brokerage scandals in this year, and another in Venezuela in 2018Q4, which may be attributed to large inflation and its repercussions in the stock market.

Third, an important strand of the literature claims that macroeconomic variables are better suited than financial variables to explain the growth distribution. Consequently, we also consider the effect of domestic macroeconomic factors on the conditional distribution of US growth. With this goal, we consider the popular database of McCracken and Ng (2016) with $N_3 = 248$ variables; De Nicoló and Luccetta (2017) and Plagborg-Möller et al. (2020) also use this dataset to extract factors to estimate factor-augmented quantile regressions. This subset of variables is denoted as X_{3t}.

Finally, in order to incorporate the effect of the worldwide macroeconomy on economic growth, we also consider a set of annualized quarterly GDP growths of $N_4 = 63$ countries. Table 2 reports the countries used to extract the worldwide macroeconomic factors. The GDPs have been obtained from the IMF with the sample of countries chosen to maximize the amount of common data among them. The GDPs of these countries represent 91.62% of total GDP. Table 2 also reports the GDP and percentage over world GDP (in parenthesis) of each country, both according to World Bank. Note that the factors considered by González-Rivera et al. (2019) are extracted from a panel of annual growths corresponding to 83 countries obtained from the World Bank database. We also look for outliers using the procedure described by Kristensen (2014) and find two outliers in Thailand growth in 2011Q4 and 2012Q1. These outliers may be due to the severe flooding occurred during the 2011 monsoon season, which caused the fourth costliest economic disaster according to the World Bank; see Tanonue et al. (2020). China 2020Q1 and Ireland 2015Q1 are also outliers. We think that the main reason for the outlier in China is that the COVID19 affected China one quarter earlier than the rest of the world. With respect to the large Irish GDP growth, it could be due to the relocation of intellectual property of a number of large multinational corporations, which was triggered by the Irish low corporate tax rates. Given the size of these companies, the boost to GDP growth was correspondingly large. The subset of worldwide growths is denoted as X_{4t}.

We denote $X_t^* = (X_{1t}, X_{2t}, X_{3t}, X_{4t})'$ the entire set of domestic/worldwide and/or financial/macroeconomic variables with cross-sectional dimension $N = 624$ variables. It is important to note that to construct quarterly predictive distributions of real GDP growth, we use the conditioning information available at the moment the prediction is made. The US real GDP as well as all the variables in X_t^* used to extract the factors are final records at the time of writing. However, in most countries, national accounts are recorded quarterly and published late (often more than one month after the close of the quarter), and are subsequently revised. On the other hand, the variables published at a higher frequency than growth (monthly or even weekly), are known in advance.[17]

Our proposal is to consider the factors extracted from X_t^* and analyse their joint effect on the quantiles of the conditional distribution of US economic growth.[18] Given the block structure of the variables in X_t^*, we extract the factors by considering the multi-level DFM proposed by Rodríguez-Caballero and Caporin (2019). We start by extracting the PC factors separately from each of the four blocks of variables, X_1, X_2, X_3, and X_4. As

[17]The accuracy and timeliness of the estimated growth densities can be improved by augmenting the quarterly information with the available high frequency information. This is the proposal of Ferrara, Mogliani and Sahuc (2021). An interesting issue to investigate is the possibility of implementing the GiS methodology to construct a "nowcasting" measure of growth vulnerability in different scenarios.

[18]Busetti et al. (2021) also consider domestic and worldwide financial and real variables when modelling the distribution of Italian GDP. However, they do not pursue factor extraction as they focus on some individual variables.

proposed by Hallin and Liska (2011), we determine the factor structure by analysing the pairwise correlations among the factors separately extracted from each block of variables; see the Online Appendix A for details on the factors extracted from each block of variables and their correlations. After this analysis, we obtain the following specification of the multi-level DFM

$$X_t^* = \begin{bmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \\ X_{4t} \end{bmatrix} = \begin{bmatrix} p_{11} & 0 & p_{13} & p_{14} & 0 & 0 & 0 \\ p_{21} & p_{22} & p_{23} & 0 & p_{25} & 0 & 0 \\ p_{31} & 0 & 0 & p_{34} & 0 & p_{36} & 0 \\ p_{41} & p_{42} & 0 & 0 & 0 & 0 & p_{47} \end{bmatrix} \begin{bmatrix} F_{1t}^* \\ F_{2t}^* \\ F_{3t}^* \\ F_{4t}^* \\ F_{5t}^* \\ F_{6t}^* \\ F_{7t}^* \end{bmatrix} + \varepsilon_t^*, \tag{13}$$

where F_{1t}^* is a pervasive factor that loads in all the variables in X_t^*, F_{2t}^*, F_{3t}^* and F_{4t}^* are semi-pervasive factors with loadings in the worldwide (financial and macroeconomic), financial (domestic and worldwide), and domestic (financial and macroeconomic) variables, respectively. Finally, F_{5t}^*, F_{6t}^* and F_{7t}^* are non-pervasive factors that load on the worldwide financial, domestic macroeconomic, and worldwide macroeconomic variables, respectively. This factor structure explains the relation between the financial cycle and the business cycle, though both cycles have different characteristics; see Claessens et al. (2012), who, in a different context, has already pointed out that macroeconomic and financial dynamics could be driven by the same global and regional factors, and Breitung and Eickmeier (2016), who, in an application to a large macro-financial quarterly data set for 24 countries, conclude that financial variables strongly comove internationally, to a similar extent as macroeconomic variables.

Examining the structure of the multi-level DFM in (13), we note that the domestic financial variables, X_{1t}, load on the factor F_{3t}^*, which corresponds to the financial variables, and on the factor F_{4t}^*, which corresponds to the domestic variables. However, there is not a separate non-pervasive factor for the domestic financial variables alone. Once worldwide financial and domestic macroeconomic factors are taken into account, domestic financial factors do not appear explicitly in model (13). According to model (13), the information contained in the underlying domestic financial factors is already contained in the worldwide financial and domestic macroeconomic variables. This result is closely related to the question regarding the influence and extent of domestic financial conditions in a given country in the context of a globally integrated financial system, which has been attracting increased interest recently and continues to be hotly debated in policy and academic circles alike; see Breitung and Eickmeier (2016), who conclude that domestic factors are loosing weight as compared to international factors in an analysis of a large set of variables related to the US economy. Looking at the drivers of economic growth, Arregui et al. (2018) also conclude that common global components underlying financial conditions only account for about 20% to 40% of the variations in countries domestic financial conditions indexes. In the same vein, Brownlees and Souza (2019) conclude that it is unclear whether financial conditions are a relevant downside growth risk predictor during the COVID19 pandemic of 2020 and Chavleishvili and Manganelli (2019) find that severe financial shocks are transmitted to the real economy when the economy is simultaneously hit by a real negative shock. This result is in agreement with Reichlin et al. (2020), who conclude that the NFCI contains little advanced information on growth beyond what is already contained in the real economic indicators. Plagborg-Möller et al. (2020), estimating US growth risk, also conclude that the performance of a model with both a macroeconomic factor and a financial factor is indistinguishable from a model with only a macroeconomic factor.[19] They show that financial variables contribute little to distributional forecasts of growth, beyond the information contained in real indicators. In the same vein, Carriero et al. (2022a) find limited improvements in accuracy when using financial indicators in addition to macroeconomic indicators.

Given the arguments above about the lack of additional information in X_{1t} once X_{2t} and X_{3t} are taken into account, we simplify the model by considering only the variables in $X_t = (X_{2t}, X_{3t}, X_{4t})'$.[20] Following the same methodological steps described above, we

[19]Indeed, Plagborg-Möller et al. (2020) conclude that no predictors provide robust and precise advanced warnings about any features of GDP growth distribution other than the mean.

[20]The computational burden involved in the estimation of the distribution of the factors and in finding the tangency point between the corresponding countours and the iso-quantiles increases with the number of factors and can be very heavy if it is large. This computational complexity makes the problem instable when the number of factors is very large, increasing the noise involved in the computations. Consequently, by removing X_1, we have a more parsimonious model with all the information in it but avoiding superflous variables that not add additional information.

select the following final multi-level DFM[21]

$$X_t = \begin{bmatrix} X_{2t} \\ X_{3t} \\ X_{4t} \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 \\ p_{21} & 0 & 0 & p_{24} & 0 \\ p_{31} & p_{32} & 0 & 0 & p_{35} \end{bmatrix} \begin{bmatrix} F_{1t} \\ F_{2t} \\ F_{3t} \\ F_{4t} \\ F_{5t} \end{bmatrix} + \varepsilon_t, \tag{14}$$

where F_{1t} and F_{2t} are the pervasive and semi-pervasive factors that load in all variables and in the worldwide (financial and macroeconomic) variables of the system, respectively. The other three factors in model (14) correspond to non-pervasive factors that only load in the worldwide financial (F_{3t}), domestic macroeconomic (F_{4t}), and worldwide macroeconomic (F_{5t}) variables.

As mentioned above, estimation of model (14) is based on the sequential procedure described by Rodríguez-Caballero and Caporin (2019). Figure 2 plots the five factors extracted from the multi-level DFM in (14) together with their 95% confidence intervals obtained by the subsampling procedure explained above. Note that each factor is estimated conditional on the factors extracted in the previous level. We can observe that the worldwide financial factor, F_{3t}, increases during the crisis periods. Positive values of this factor indicate tighter financial conditions than average, while negative values indicate looser financial conditions than average. Neither the pervasive F_{1t} factor nor the non-pervasive F_{3t} factor warn about the plausibility of a forthcoming big decline in growth due to the COVID19 pandemic. However, the warnings coming from the semi-pervasive world factor, F_{2t}, and from the non-pervasive F_{4t} factor were strong, and that coming from the non-pervasive world macroeconomic F_{5t} factor was indeed very strong. It is this last factor that truly captures a sharp decline in the world macroeconomy.

3.2 The US conditional distribution of growth in normal times

After extracting the underlying factors from the multi-level DFM in (14), we estimate the corresponding factor-augmented quantile regression models in (7) for horizons $h = 1, 2, 3$

[21] Note that, in this case, we select 3 factors instead of two within the WM block.

and 4 and for quantiles of growth τ^* from 0.05 to 0.95 at intervals of 0.01. The estimated parameters are plotted in Figure 3 together with their corresponding 95% confidence intervals for $h = 1$ and 4.[22] Table 3 reports the estimated parameters for $h = 1, 2, 3$ and 4 and $\tau^* = 0.05$, 0.5 and 0.95 together with their corresponding p-values and the analogue coefficient of determination R^1. Several interesting insights on the conditional density of growth are obtained from Table 3 and Figure 3.[23]

In Table 3, we observe that the fit of the factor-augmented quantile regressions is rather large in the extreme quantiles with R^1 ranging, depending on h, from 39 to 49% for the 5% quantile and from 32 to 36% for the 95% quantile. For the median quantile, the fit is much lower, between 11 and 16%. The larger fit is the result of the significant effect of the five factors in the extreme 5% and 95% quantiles, which are more vulnerable than quantiles in the center of the distribution to economic and financial conditions. For the median quantile, the factors do not seem to be significant variables, with only a very small effect of F_3 in the short run ($h = 1$). Figure 3 confirms that the overall five factors are the most significant variables either in the extreme left tails or in the extreme right tails of the distribution of growth but their significance fades to zero in the median and neighbouring quantiles. The most remarkable feature of Figure 3 is the strong effect of F_2, F_3, and F_5 on the extreme 5% and neighbouring quantiles indicating that growth in recessions is mainly driven by worldwide macro and financial variables but in expansions (95% and neighbouring quantiles), it is mainly the worldwide financial factor F_3 that drives growth.[24] The joint effects of different factors with their different magnitude in the extreme left and right tails of the growth distribution generate the asymmetry of this distribution, which is in agreement with the findings in several current works; see, for instance, Adams et al. (2021), Baker et al. (2023), Bloom (2014), Jurado et al. (2015),

[22]The covariance matrix of the estimators has been obtained as proposed by Koenker and Bassett (1978) assuming i.i.d. errors.

[23]As a robustness check, the Online Appendix B reports the results of the estimated factor-augmented quantile predictive regressions when the factors are extracted either separatelly from each of the four blocks of variables (9 factors) or from the multi-level DFM in (13) with the domestic financial variables, X_{1t}, included (7 factors). We can observe that, in the first case, severe problems of multicollinearity may appear, while, in the second case, the increase in the R^1 coefficients is relatively small when considering the number of additional parameters that should be estimated.

[24]Recall that the subset of variables used to extract the worldwide financial factor also includes US financial variables. Therefore, this result does not contradicts the former literature about the impact of financial variables on macroeconomic activity; see, for example, Estrella and Trubin (2006) about the yield curve as a leading indicator of recessions and Stock and Watson (2003) about the role of asset prices as predictors of output and inflation.

Ludvigson et al. (2021), and Plagborg-Möller et al. (2020).[25]

At each moment of time t, smooth estimates of the growth distribution under average factor scenarios are obtained by fitting the Skewed-t distribution to the estimated quantiles of growth for $\tau^* = 0.05, 0.25, 0.5, 0.75$ and 0.95 from the factor-augmented predictive quantile regressions. The estimated densities from 2005Q4 to 2020Q1 are plotted in the top panel of Figure 4. The GaR_t measure is the 5%-quantile of the growth smoothed density at time t. Figure 5 plots three selected growth densities corresponding to 2008Q4 (just after the 2008 Great Recession), 2017Q1 (in a quarter of low uncertainty) and 2020Q2 (during the beginning of the COVID-19 crisis). Note that the location, scale and shape of the conditional growth densities change over time. In each of these densities, we mark its 5% quantile corresponding GaRs, namely, -7%, -0,5% and -16%, respectively. It is obvious that, according to the GaR, the vulnerability of the US economy was smaller in 2017 and much larger in 2020 than in 2008. However, these measures of vulnerability are obtained with the factors estimated at their average values.

3.3 The US conditional growth densities: a scenario analysis

In this subsection, we construct conditional one-step-ahead densities for US growth under stressed scenarios for the factors and calculate the associated GiS risk measures.

To obtain plausible stress scenarios for the factors, first we need to construct the joint α%-confidence regions for the five factors extracted from the multi-level DFM. Next, we minimize the τ^*-quantile growth subject to a fixed ellipsoid with α-coverage as in (8). The minimization exercise takes place for different $\tau^* = 0.05, 0.25, 0.75$ and 0.95. The α-stressed conditional distributions of growth are obtained by fitting the Skewed-t distribution to the optimal estimated τ^*-quantiles. The bottom panel of Figure 4 plots the US one-step-ahead growth densities when the factors are stressed at the 95% level. As expected, we can observe that the stressed densities are located to the left of the non-stressed densities. It is interesting to see that by stressing the factors, the stressed densities tend to show increased uncertainty and asymmetry. In Figure 5, we offer a close-up of these densities in three specific quarters, 2008Q4, 2017Q1 and 2020Q2. In 2017Q1, the stressed

[25]The marginal effects of the factors have been analysed in the Online Appendix C.

and non-stressed densities are closer to each other and are approximately symmetric with low dispersion. However, in crisis periods like 2008Q1 and 2020Q2, both densities tend to move to the left showing increased uncertainty and pronounced asymmetry with a long left tail. The distance between the stressed and non-stressed densities is larger, mainly in the left tail. Both features are more acute in the COVID period than in the 2008 Great Recession.

The GiS_t measure is the 5%-quantile of the smoothed stressed density of growth at time t. Figure 5 also plots the GiS_t corresponding to the three selected quarters mentioned above. The distance between the GaR_t and the GiS_t depends on the quarter. In the most tranquil quarter, with higher average growth and less uncertainty, 2007Q1, the GiS is -6% while the GaR is -0.5%. However, in 2008Q1, when the average growth was smaller and the uncertainty larger, the GiS is -20% while the GaR is -8%. Finally, during the COVID pandemic in 2020Q2, the distance between the GiS (-29.13%) and the GaR (-15.19%) is 14%. The large GiS under the stressed factor scenario reveals the presence of a fat left tail in the distribution of US growth, which would go unnoticed by simply estimating the GaR, which assumes that the factors evolve according to an average scenario. Furthermore, it could be worth investigating whether the distance between the GaR and the GiS, could be signalling a crisis.

In Figure 6, we provide a different way to visualize the different implications of non-stressed and stressed growth densities. We plot the US actual quarterly growth over the sample period 2005Q4 to 2021Q1. The dashed lines are the estimated one-step ahead 5% (GaR) and 95% quantiles, which for the most part of the sample envelop the actual growth. We also plot the 5% and 95% quantiles of growth (light red) and the 25% and 75% quantiles (grey), when the factors are stressed at the 95% level. As before, the stressed density falls below the non-stress density and provides a complete assessment of the vulnerability of the economy in very different scenarios.

The densities plotted in Figures 4, 5, and 6 summarize our proposed tool for risk assessment. The policymaker has a complete visualization of growth dynamics under average and α-stressed scenarios of her choice, with warning signals coming from the quantiles in the left tail of the stressed densities of growth. An additional piece of information that the GiS methodology provides are the values of the factors in the α-stressed scenario that gives rise to the GiS warning. As an example, in 2020Q1, the values of the stressed factors in the 95% scenario were -1.26 (F_1), -5.74 (F_2), 1.94 (F_3), -0.19 (F_4) and -7.52 (F_5). We observe that the main factors contributing to the vulnerability of US growth at the time of the COVID pandemic were coming from the worldwide factor, F_2, and from the worldwide macroeconomic factor, F_5. Neither domestic information nor financial information *per se* were so influential.

Finally, in Table 4, we report numerical information regarding GaR and GiS for four quarters ahead ($h = 1, 2, 3, 4$), for three quantiles (τ = 5, 50, 95%), and for three different levels of stress (α = 70, 95, 99%). With information up to 2020Q1, the GaR warning for the following quarter 2020Q2 (beginning of the pandemic) was -15.29% decline in growth, the GiS (95%) warning was -29.13%, and the observed decline was -31.20%. GaR was rather conservative compared to GiS. Note that the 95% level of stress for the factors reflects that the COVID19 has been a truly exceptional event. Finally, note that, in the following quarters, the economy substantially improved due to all the fiscal and monetary stimuli pumped up into it. Since GiS and GaR are warnings with fixed information up to 2020Q1, they could not realistically capture the positive growth in the following quarters. From a policymaker point of view, the reading of GaR and GiS warnings several quarters into the future should inform about where the economy would have gone if no remedial measures were imposed at the outset. They show the path of no action. The GaR warning pointed out to a potential recovery in the four quarters ahead (τ = 5%, h = 4, GaR = 2.55%) and GiS (70%) pointed out to a mild improvement but still negative growth if the factors were kept at the chosen 70% stress level (τ = 5%, h = 4, GiS = -5.24%).

4 Final considerations

We propose a set of statistical tools to dynamically monitor the vulnerability of the economy. We are directly answering to the sentiment expressed by policymakers such as the former Chairman of the Federal Reserve Alan Greenspan: "Policymakers often have to act [...] even though [they] may not fully understand the full range of possible outcomes, [...]. As a result, [...] policymakers have needed to reach to broader, though less mathematically precise, hypotheses about how the world works ..." (quoted in Frydman and Goldberg

(2007) and Kwiatkowski and Rebonato (2011)), and Governor Brainard: "Policymakers tend to distinguish the most likely path, which I will refer to as the "modal" outlook, from risks around that path –events that are not the most likely to happen, but that have some probability of happening and that, if they do materialize, would have a one-sided effect" (Speech March 7, 2019, https://www.federalreserve.gov/newsevents/speech/brainard20190307a.htm). Aickman et al. (2021) also argue about the relevance for policy makers of "what if" exercises as those carried out in this paper.

Using the methodology described in this paper, it is possible to measure the effect of different factor scenarios on the density of growth. These comments refer to the rare or extreme event that even with a small probability of occurrence could bring catastrophic losses to the economy. We show how to select rare events in a probabilistic sense with the construction of plausible but stressful scenarios and we summarize their potential effect on the economy with GiS, the 5% quantile of the stressed conditional growth density, as a measure of risk or vulnerability index. To achieve this end, first, we have assumed that any quantile of the growth distribution is a function of a set of factors, extracted with a multi-level DFM from a wide set of macroeconomic and financial variables collected at the domestic and worldwide levels. Secondly, we have chosen severe and yet plausible stress scenarios based on the joint probability distribution of the underlying factors. This methodology allows the policymaker to choose the desired severity of the stress on the factors and to construct the density of growth under different scenarios. The macro-financial scenarios considered by the policymaker should be severe if she wants to be prepared for a large decline in growth as that observed during the COVID19 pandemic. In summary, we provide a risk management tool for the policymaker that allows for a complete visualization of growth dynamics under average and α-stressed scenarios of her choice with warning signals coming from the quantiles in the left tail of the stressed growth densities. We see GiS as a complementary measure to GaR. Applied systematically, GiS is an useful tool for policymakers wishing to carry out a multi-dimensional scenario analysis.

Finally, the proposed methodology to measure growth vulnerability could be implemented to measure risk in other variables of interest such as inflation or unemployment and/or other countries or regions; see Adams et al. (2021) for a very recent contribution considering risk in these two variables in addition to growth. These analysis are left for further research. From a methodological point of view, it is also of interest to design and investigate the properties of procedures to carry out inference on GaR and GiS; see Hendry and Petrix (2023) for the difficulties involved in testing the differences between scenarios. This investigation is also in our research agenda.

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Table 1: Variables included in the worldwide financial block: Term spread; Price earning ratio on national stock exchange (PER); Nominal effective exchange rate (NEER); Bilateral national exchange rate against USD (ER); Historical volatility 30 days (VOL); and Sovereign spread. Countries in which each of these variables is observed.

	Term spread	PER	NEER	ER	VOL	Sovereign spread
Argentina		x			x	x
Australia	x	x	x	x	x	
Austria	x	x			x	
Bahrain		x				
Belgium	x	x			x	
Brazil		x	x	x	x	x
Bulgaria		x			x	x
Canada	x	x	x	x	x	
Chile		x	x	x	x	x
China	x	x	x	x	x	x
Colombia		x	x	x	x	x
Cyprus		x				
Czech Republic		x	x	x	x	
Denmark	x	x			x	
Egypt		x				
Finland	x	x			x	
France	x	x			x	
Germany	x	x			x	
Greece		x			x	
Hong Kong		x				
Hungary	x	x	x	x	x	x
India	x	x	x	x	x	
Indonesia		x	x	x	x	x
Ireland		x			x	
Israel	x	x	x	x	x	
Italy	x	x			x	
Japan	x	x	x	x	x	
Korea	x	x	x	x	x	
Kuwait		x				
Luxembourg		x				
Malaysia	x	x	x	x	x	x
Malta		x				
Mexico		x	x	x	x	x
Morocco		x				
Netherlands		x			x	
New Zealand	x	x	x	x	x	
Norway	x	x	x	x	x	
Pakistan		x				
Peru		x	x	x	x	x
Philippines	x	x	x	x	x	x
Poland		x	x	x	x	x
Portugal					x	
Qatar		x				
Romania		x	x	x		
Russia	x	x	x	x	x	x
Singapore		x	x	x		
Slovenia		x				
South Africa	x	x	x	x	x	x
Spain		x			x	
Sri Lanka		x				
Sweden	x	x	x	x	x	
Switzerland	x	x	x	x	x	
Taiwan		x	x	x		
Thailand	x	x	x	x	x	
Turkey		x	x	x	x	x
United Arab Emirates		x				
United Kingdom	x	x	x	x	x	
United States	x	x			x	
Venezuela		x				
Vietnam					x	

Table 2: Countries included in the worldwide macroeconomic block: GDP in millions of US Dollars and, in parenthesis, percentage over total world GDP.

	Country	GDP (Percentage)		Country	GDP (Percentage)
1	South Africa	351,431.65 (0.40%)	33	Denmark	348,078.02 (0.40%)
2	Argentina	449,663.45 (0.51%)	34	Estonia	31,386.95 (0.04%)
3	Brazil	1,839,759.04 (2.10%)	35	Finland	268,761.20 (0.31%)
4	Canada	1,736,425.63 (1.98%)	36	France	2,715,518.27 (3.10%)
5	Chile	282,318.16 (0.31%)	37	Germany	3,845,630.03 (4.40%)
6	Colombia	323,802.81 (0.37%)	38	Greece	209,852.76 (0.24%)
7	Mexico	1,258,286.72 (1.43%)	39	Hungary	160,967.16 (0.18%)
8	Peru	226,848.05 (0.26%)	40	Iceland	24,188.04 (0.03%)
9	United States	21,374,418.88 (24.37%)	41	Ireland	388,698.71 (0.44%)
10	Venezuela	482,359.32 (0.55%)	42	Italy	2,001,244.39 (2.28%)
11	China	14,342,902.84 (16.35%)	43	Latvia	34,117.20 (0.04%)
12	Hong Kong	366,029.56 (0.42%)	44	Lithuania	54,219.32 (0.06%)
13	India	2,875,142.31 (3,28%)	45	Luxembourg	71,104.92 (0.08%)
14	Indonesia	1,119,190.78 (1.28%)	46	Malta	14,786.16 (0.02%)
15	Israel	395,098.67 (0.45%)	47	Moldova	11,955.44 (0.01%)
16	Japan	5,081,769.54 (5.79%)	48	Netherlands	909,070.40 (1.04%)
17	Jordan	43,743.66 (0.05%)	49	Norway	403,336.36 (0.46%)
18	Kazakhstan	180,161.74 (0.21%)	50	Poland	592,164.40 (0.68%)
19	Korea, Rep.	1,642,383.22 (1.87%)	51	Portugal	237,686.08 (0.27%)
20	Malaysia	364,701.52 (0.42%)	52	Romania	250,077.44 (0.29%)
21	Philippines	376,795.51 (0.43%)	53	Russia	1,699,876.58 (1.94%)
22	Singapore	372,062.53 (0.42%)	54	Serbia	51,409.17 (0.06%)
23	Thailand	543,649.98 (0.62%)	55	Slovenia	53,742.16 (0.06%)
24	Taiwan	586,000 (0.67%)	56	Slovak Republic	105,422.30 (0.12%)
25	Vietnam	261,921.24 (0.30%)	57	Spain	1,394,116.31 (1.59%)
26	Turkey	754,411.71 (0.86%)	58	Sweden	530,832.91 (0.61%)
27	Austria	446.314,74 (0.51%)	59	Switzerland	703,082.44 (0.80%)
28	Belarus	63,080.46 (0.07%)	60	Ukraine	153,781.07 (0.18%)
29	Belgium	529,606.71 (0.60%)	61	United Kingdom	2,827,113.18 (3.22%)
30	Czech Republic	246,489.25 (0.28%)	62	Australia	1,392,680 (1.59%)
31	Croatia	60,415.55 (0.07%)	63	New Zealand	206,928.77 (0.24%)
32	Cyprus	24,564.65 (0.03%)			

Table 3: Estimates of the parameters of h-step-ahead factor-augmented quantile regression models for the $\tau = 5\%$, 50% and 95% quantiles of the US growth distribution. Estimation sample from 2005Q3 up to 2020Q1. p-values in parenthesis and in bold parameters significant at the 10% significance level.

	μ	ϕ	Global	Worldwide	WF	DM	WM	R^1
$\tau = 0.05$								
$h = 1$	**-2.62**	0.15	0.68	**2.19**	**-1.20**	**-1.21**	**3.44**	0.49
	(0.00)	(0.54)	(0.26)	(0.00)	(0.01)	(0.03)	(0.00)	
$h = 2$	**-2.03**	**-0.39**	**2.27**	**1.83**	**-1.65**	**-0.59**	**3.61**	0.46
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	
$h = 3$	**-4.55**	**0.91**	**2.61**	**1.35**	**-1.24**	**-2.00**	**-1.09**	0.40
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	
$h = 4$	-0.73	**-1.08**	0.15	-1.03	**-3.12**	-0.83	-0.73	0.39
	(0.39)	(0.01)	(0.87)	(0.22)	(0.00)	(0.31)	(0.36)	
$\tau = 0.5$								
$h = 1$	**2.04**	-0.19	0.45	-0.01	**-0.87**	0.48	0.58	0.16
	(0.00)	(0.37)	(0.38)	(0.99)	(0.02)	(0.28)	(0.19)	
$h = 2$	**2.39**	-0.15	0.09	-0.30	-0.58	0.34	0.03	0.11
	(0.00)	(0.40)	(0.83)	(0.40)	(0.07)	(0.36)	(0.94)	
$h = 3$	**1.95**	0.13	-0.14	-0.21	-0.56	-0.22	-0.34	0.11
	(0.00)	(0.50)	(0.76)	(0.60)	(0.11)	(0.59)	(0.39)	
$h = 4$	**2.57**	**-0.31**	0.19	-0.18	-0.31	0.26	0.28	0.16
	(0.00)	(0.02)	(0.54)	(0.52)	(0.19)	(0.34)	(0.30)	
$\tau = 0.95$								
$h = 1$	**4.33**	**-0.24**	**1.30**	**0.62**	**-1.06**	0.23	**-0.59**	0.36
	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)	(0.21)	(0.00)	
$h = 2$	**4.52**	**-0.34**	**0.77**	0.23	-0.28	**0.86**	-0.32	0.22
	(0.00)	(0.00)	(0.01)	(0.33)	(0.17)	(0.00)	(0.18)	
$h = 3$	**3.45**	**0.18**	-0.26	0.14	**-0.82**	-0.31	**-0.67**	0.35
	(0.00)	(0.05)	(0.25)	(0.49)	(0.00)	(0.12)	(0.00)	
$h = 4$	**4.73**	**-0.56**	0.31	**0.53**	**-1.08**	-0.15	-0.14	0.32
	(0.00)	(0.00)	(0.23)	(0.02)	(0.00)	(0.51)	(0.51)	

Table 4: US growth risk (in annualized percentage over previous quarter). The table reports h-step-ahead forecasts of the 5% quantile of growth with information up to 2020Q1 and computed by GaR (without stressing the underlying factors) and by GiS (with factors stressed at 70%, 95% and 99%).

	$h = 1$ 2020Q2	$h = 2$ 2020Q3	$h = 3$ 2020Q4	$h = 4$ 2021Q1
Observed	-31.20	33.89	4.50	6.30
$\tau = 0.05$				
GaR	-15.29	-18.07	-1.04	2.55
GiS(70%)	-25.49	-29.01	-9.70	-5.24
GiS(95%)	-29.13	-32.84	-12.53	-7.99
GiS(99%)	-31.48	-35.39	-14.58	-9.74
$\tau = 0.50$				
GaR	-4.94	-3.94	2.36	3.10
GiS(70%)	-12.29	-10.63	-3.06	-1.78
GiS(95%)	-14.83	-12.96	-4.96	-3.51
GiS(99%)	-16.49	-14.51	-6.18	-4.62
$\tau = 0.95$				
GaR	0.98	3.00	4.34	3.40
GiS(70%)	-6.04	-3.22	0.24	0.02
GiS(95%)	-8.42	-5.25	-1.23	-1.20
GiS(99%)	-9.96	-6.56	-2.17	-2.00

Figure 1: Illustration of factor scenario construction and GiS and GaR. **Upper panel**: The straight lines represent the iso-5% quantile growth for $q_{0.05}(y_{t+1}|F_t) = -3.35$ (green), $q_{0.05}(y_{t+1}|F_t) = -2.35$ (black), $q_{0.05}(y_{t+1}|F_t) = -1.35$ (blue), and $q_{0.05}(y_{t+1}|F_t) = -0.5$ (red). The ellipses represent the contours of the bivariate standard Normal probability density of the factors with means 5 and 2, respectively. The outer ellipse contains the factors with a 99% probability, while each of the inner ellipses contains the factors with the same probability of the previous one minus 1%. **Lower panel**: Bivariate standard Normal probability density of factors with means 5 and 2, together with the cuts of the density obtained for the same values of the iso-5% quantiles of growth as those in the top panel.

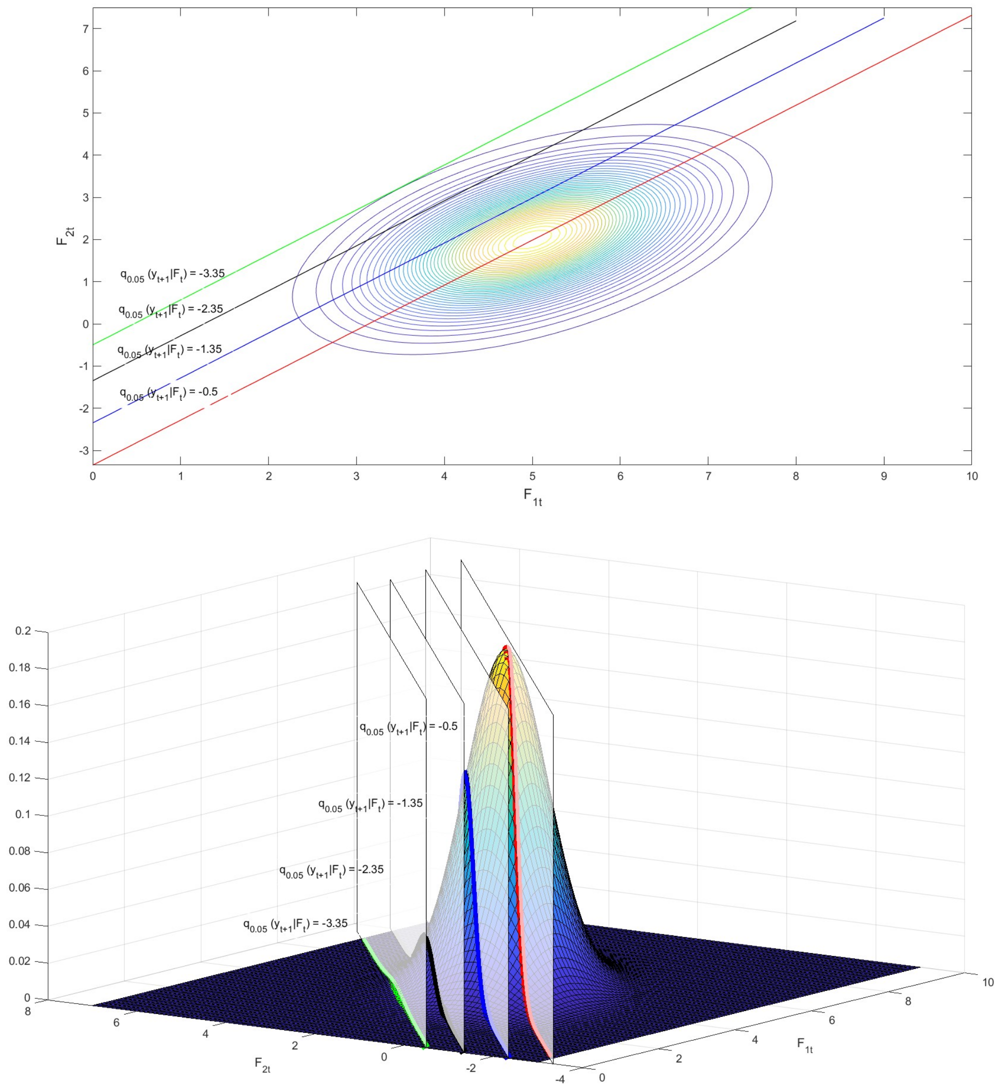
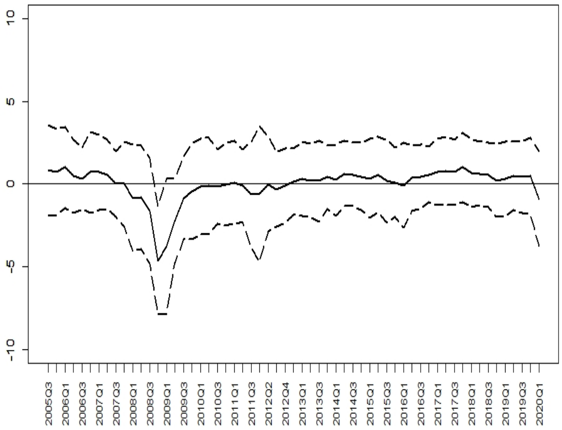
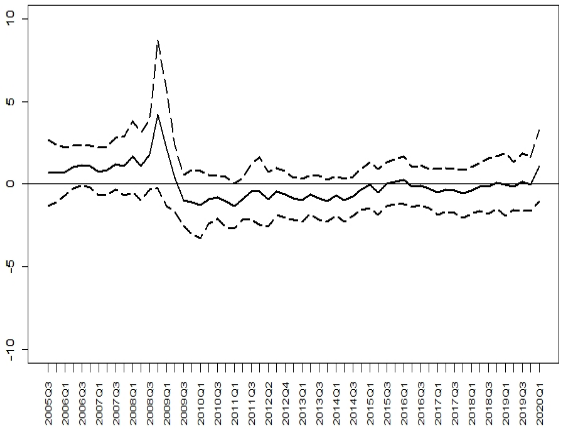


Figure 2: Estimated factors from multi-level DFM with their pointwise 95% confidence bounds. Total factor F_1, worldwide factor F_2, worldwide financial factor F_3, domestic macroeconomic factor F_4, and worldwide macroeconomic factor F_5. Estimation sample from 2005Q3 up to 2020Q1.



(a) F_1

(b) F_2



(c) F_3

(d) F_4

(e) F_5

Figure 3: Estimated parameters of the factor-augmented predictive quantile regressions for each quantile of the growth distribution ranging from $\tau^* = 0.5$ to $\tau^* = 0.95$ and for horizons $h = 1$ (black) and 4 (red) lines. The shade areas represent the 95% confidence pointwise intervals for the parameters (blue for $h = 1$ and light red for $h = 4$).

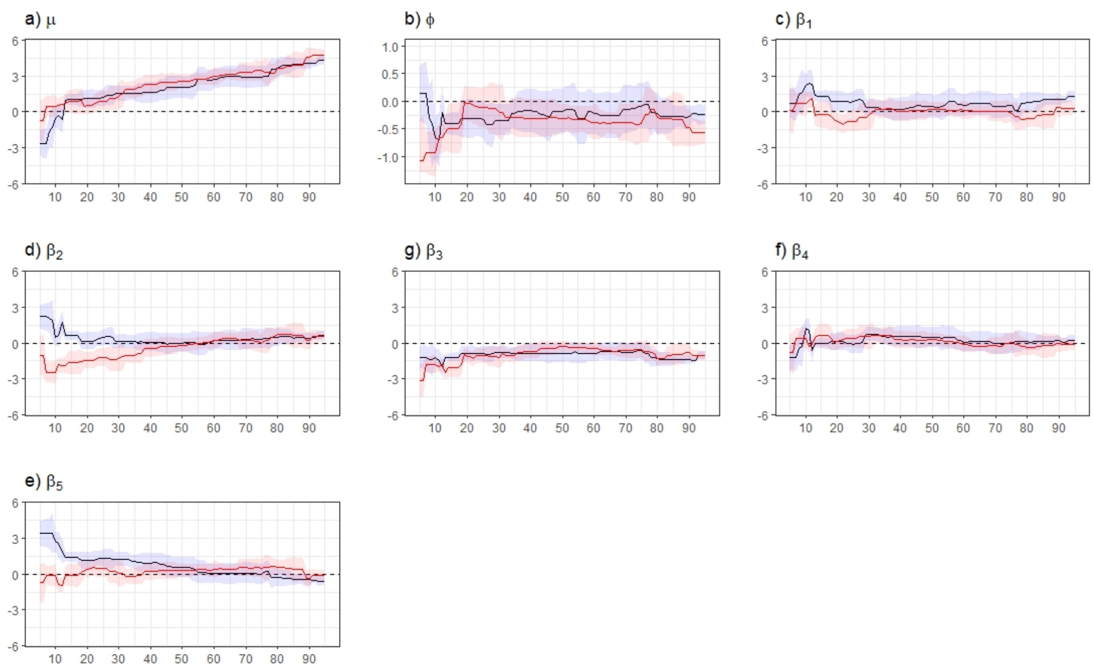


Figure 4: One-step-ahead US growth densities estimated from the factor-augmented quantile regression model with multi-level factors. The densities are calculated when factors are centered at their means (upper panel), and when they are stressed at the 95% level (lower panel).

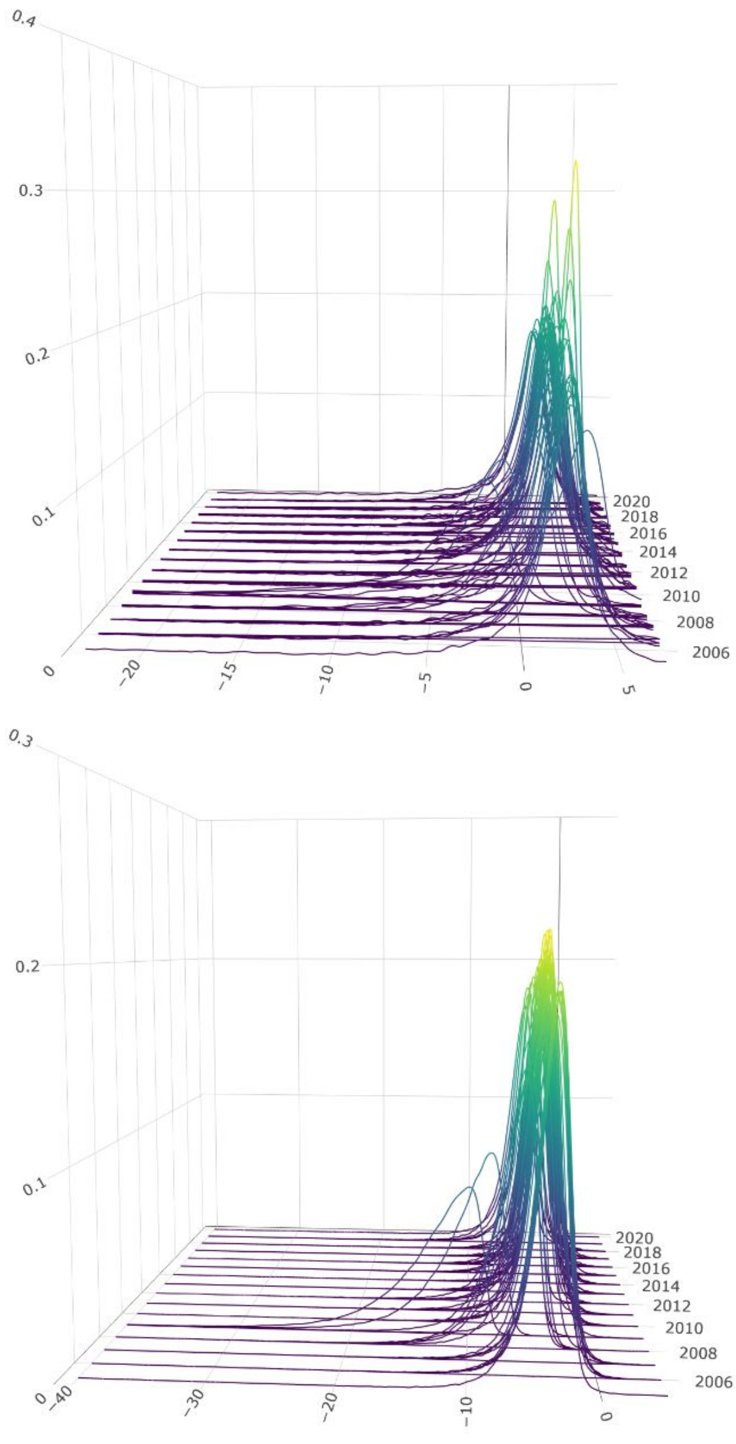


Figure 5: One-step-ahead US conditional growth densities in 2008Q4 (after the 2008 Great Recession), 2017Q1 (low uncertainty), and 2020Q2 (COVID-crisis). Densities calculated when factors are centered at their means (black) and when they are stressed at the 95% level (blue). The vertical dashed lines in black (GaR) and blue (GiS) correspond to the values of the 5% quantile of their respective densities.

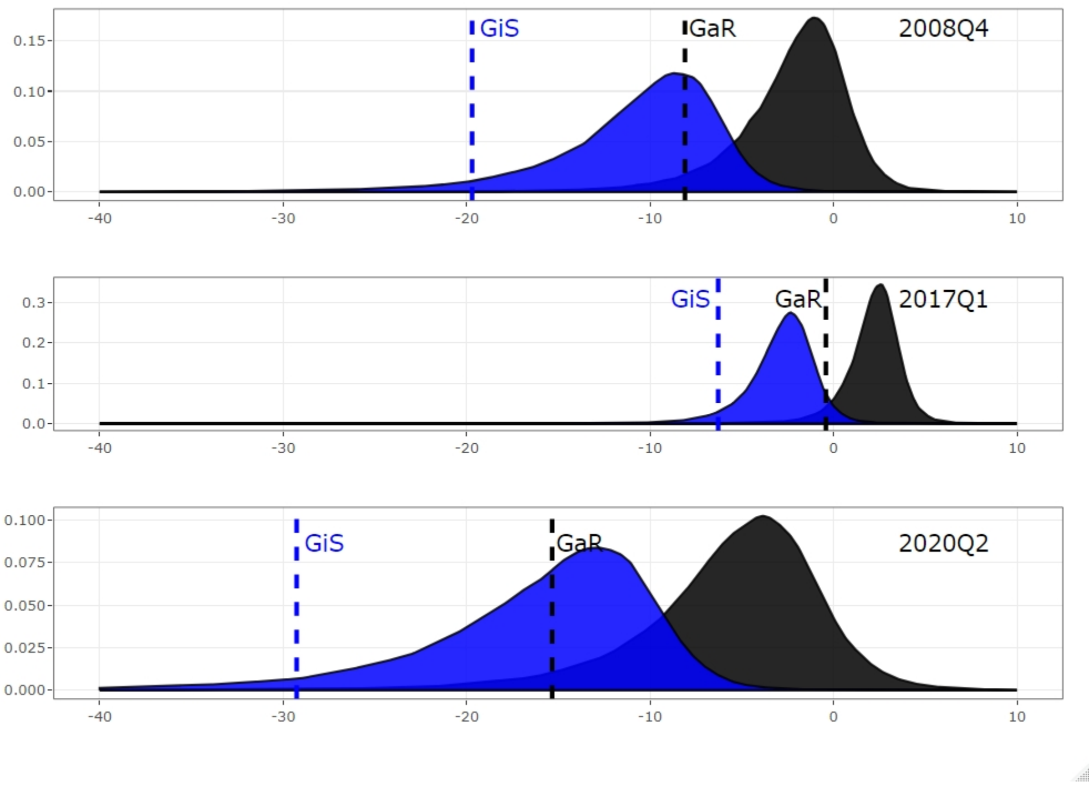
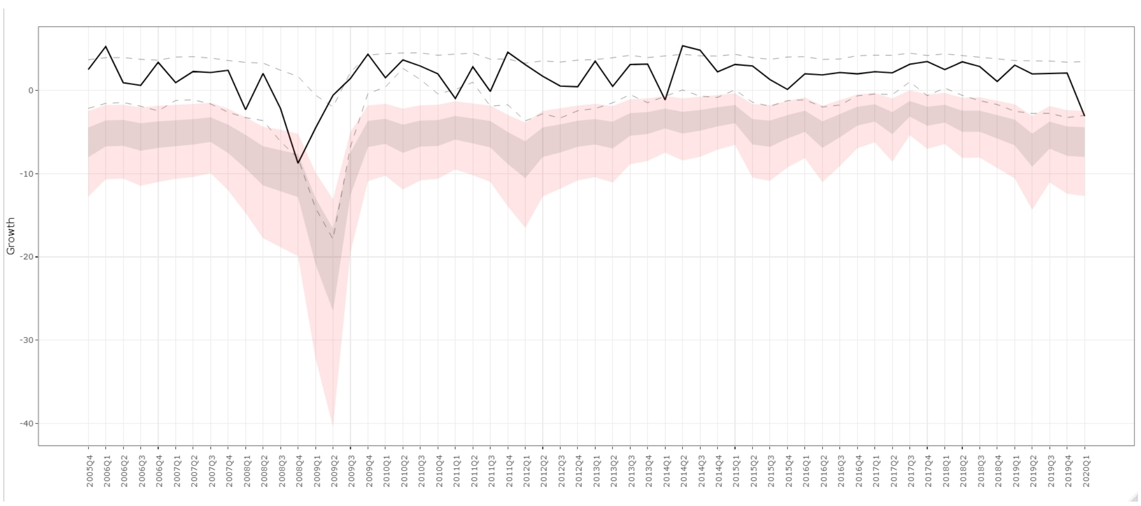


Figure 6: US quarterly growth (annualized rates, black line). 5% (GaR) and 95% quantiles (dashed lines) of the conditional one-step-ahead distribution of growth. In shades of light red (grey) the 5% (GiS) and 95% (25% and 75%) quantiles of the conditional α-stressed density with $\alpha = 95\%$.



Online Appendix
Expecting the unexpected: Stressed scenarios for economic growth

A Factors obtained in each subset of variables

Consider $X_t^* = (X_{1t}, X_{2t}, X_{3t}, X_{4t})'$, the entire set of domestic/worldwide and/or financial/macroeconomic variables, where X_{1t} is the block of $N_1 = 105$ domestic financial (DF) variables, X_{2t} contains the $N_2 = 208$ worldwide financial (WF) variables, X_{3t} is the block of $N_3 = 248$ domestic macroeconomic (DM) variables, and X_{4t} contains the $N_4 = 63$ worldwide macroeconomic (WM) variables. In order to determine the number of factors within each individual block, $X_{it}, i = 1, ..., 4$, we analyse visualy the scree plot in Figure A.1, which also reports the percentage of total variability explained by each factor. The number of factors chosen within X_1 and X_3 is 2, while X_2 has 3 factors and X_4 one. Each of the factors chosen within each block explain at least 10% of the total variability. Note that the procedure implemented to specify the factor structure of the multi-level DFM requires that there are at least two factors within each block of variables. Consequently, we also consider two factors in X_4. Therefore, there are 9 common factors altogether.[26]

The specification of the factor structure of the multi-level DFM proposed by Hallin and Liska (2011) rely on the analysis of the pairwise correlations of the factors estimated separately from each block of variables. Consequently, we compute the pairwise correlations among the 9 factors. To avoid the effect of influential points on the estimated correlations, we delete from their computation, the factors corresponding to 2008Q4, 2009Q1 and 2020Q1. The estimated pair-wise cross-correlations are reported in Figure A.2 together with scatter plots of the estimated factors. We can observe that there are two important correlations to be considered when building the multi-level factor model structure. First, there is a strong positive correlation between the second WM factor and the first WF factor while the first WM factor and the second WF factors are negatively correlated. On the other hand, worldwide and domestic factors are only correlated when we look at the financial factors with the second DF factor having a mild positive correlation with the first WF factor and a stronger negative correlation with the third WF factor. The pairwise correlations plotted in Figure A.2 suggest the following block structure in the factors, with some of them being common to all variables (pervasive), other being common to one

[26]Using the criteria proposed by Alessi et al. (2010), the number of common factors in the block of DF variables is 2, while the number of factors in the block of WF variables is 6. On the other hand, the number of common factors in the blocks of DM and WM variables is 3.

(non-pervasive) or several (semipervasive) blocks of variables,

$$X_t^* = \begin{bmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \\ X_{4t} \end{bmatrix} = \begin{bmatrix} p_{11} & 0 & p_{13} & p_{14} & 0 & 0 & 0 \\ p_{21} & p_{22} & p_{23} & 0 & p_{25} & 0 & 0 \\ p_{31} & 0 & 0 & p_{34} & 0 & p_{36} & 0 \\ p_{41} & p_{42} & 0 & 0 & 0 & 0 & p_{47} \end{bmatrix} \begin{bmatrix} F_{1t}^* \\ F_{2t}^* \\ F_{3t}^* \\ F_{4t}^* \\ F_{5t}^* \\ F_{6t}^* \\ F_{7t}^* \end{bmatrix} + \varepsilon_t^*, \qquad \text{(A.1)}$$

Figure A.1: Scree plots of each block of variables: Domestic Financial (first row, first column), Domestic Macroeconomic (first row, second column), Worldwide Financial (second row, first column), and Worldwide Macroeconomic (second row, second column). For each factor, we report the percentage of explained total variability.

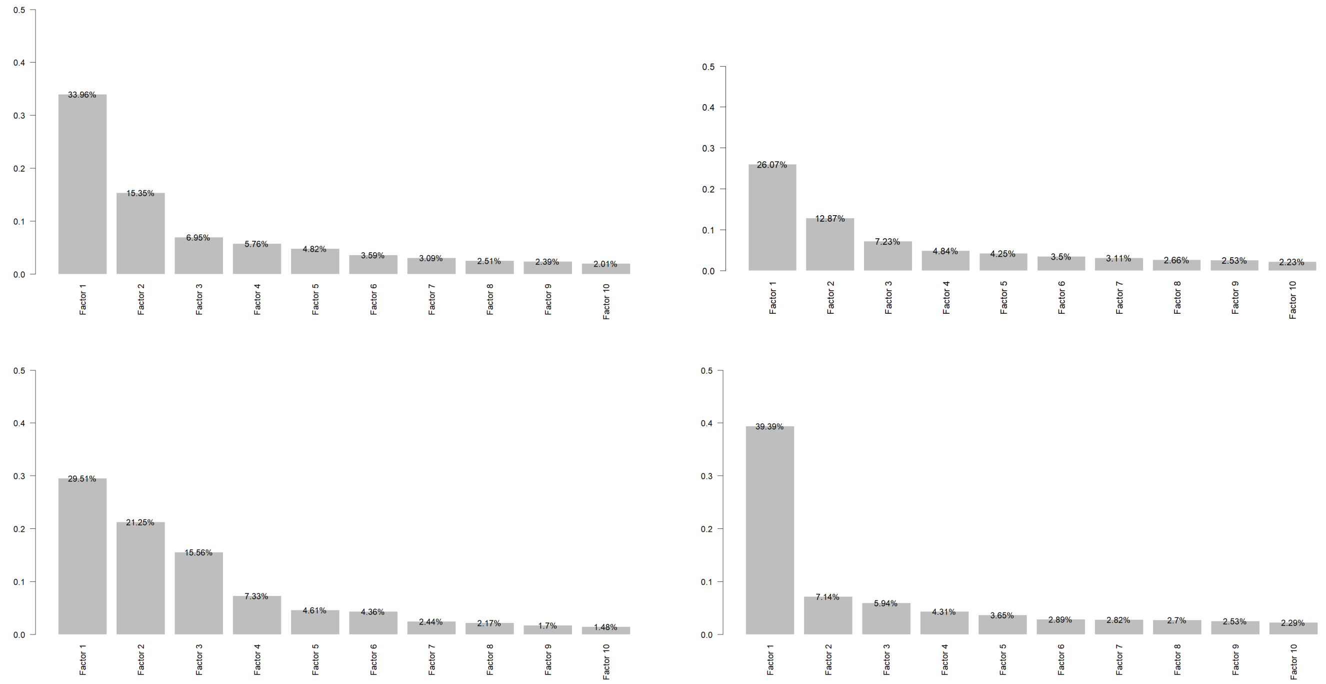
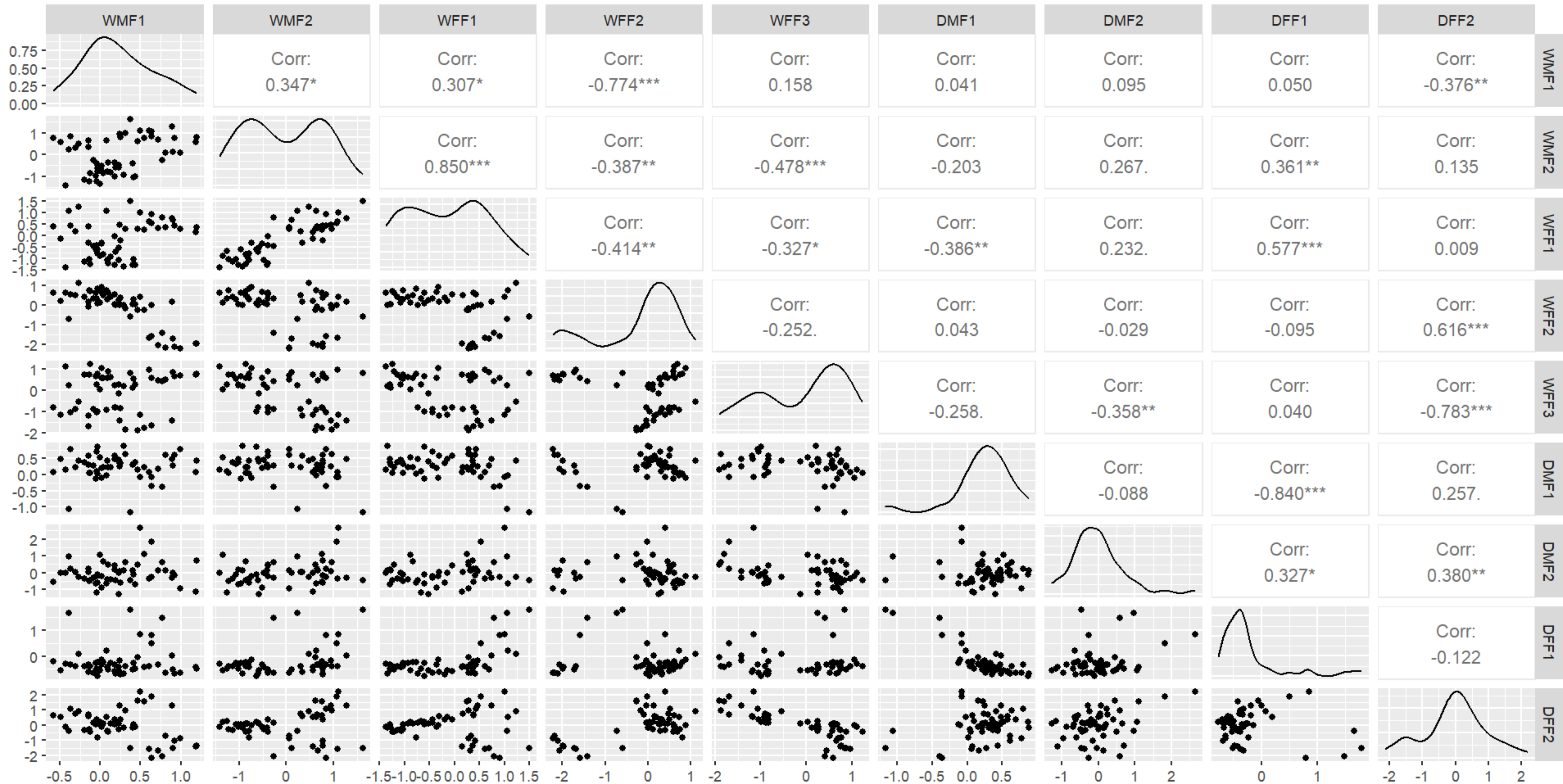


Figure A.2: Pairwise correlations between PC factors estimated separately from each block of variables (top triangle), marginal densities of the estimated factors (main diagonal) and scatter plots of estimated factors (lower triangle).



B Factor-augmented quantile predictive regressions based on alternative number of factors: A robustness check

After extracting the common factors separatelly from each block of variables,[27] we compute the factor-augmented quantile regressions by considering as regressors each subset of factors separately and by introducing all 9 factors together. The estimated parameters are reported in Table B.1 together with the corresponding R^1 coefficients. Three main conclusions emerge from Table B.1. Let us look first to the estimates reported when each set of factors is introduced separately. We can observe that different quantiles of the distribution of growth may depend on different factors. Note that, when $\tau^* = 0.05$, the largest R^1 is obtained when the DM factors are used as regressors, while the largest R^1 is obtained for DF when $\tau^* = 0.5$. Finally, when $\tau = 0.95$ the maximum R^1 is 0.25 regardless of whether DM or WF factors are used as regressors. In any case, the differences between the fit of the four factor-augmented quantile regressions are not large. The second conclusion from the results of the four separate regressions reported in Table B.1 is that the signs of the estimated parameters are as expected. The effect of financial factors on the $\tau^* = 0.05$ quantile is negative while macroeconomic factors have a positive effect.

When looking at the results of the factor-augmented quantile regression estimated with the 9 factors, we can observe that the fit coefficient, R^1, clearly increases up to 0.63. However, note that the signs of the factors are not as expected, a clear signal of the presence of strong multicolineality among the regressors (factors).

Comparing Tables B.1 and 3, we can observe that the fit of the factor-augmented quantile regressions based on the multi-level factors is much higher at the extreme quantiles than the fit of the quantile regressions with factors extracted from separate DFM based on subsets of variables and slightly smaller than when the factor-augmented quantile regressions are estimated with all 9 original factors extracted separately from each block. However, it is important to point out that the R^1 coefficient is not corrected by the number of parameters to be estimated. Furthermore, a large number of factors may be computationally problematic when estimating the joint densities of the factors and the tangency points between the iso-quantile lines and the cuntours of the factors.

Finally, the robustness check on the factors used to estimate the quantiles of the distribution of growth is carried out by estimating the factor-augmented predictive quantile regressions when the factors are extracted from the multi-level DFM in equation (13) with 7 factors and X_t^* including the DF variables. The results are reported in Table B.2. When looking at the results for $\tau = 0.05$, we can observe the problems pointed out above about the signs of the financial factors being positive while they are expected to be negative. Once more, this effect could be due to the presence of multicolinearity when the factors are extracted from the multi-level DFM in (13). In any case, the coefficients R^1 obtained hen using the 7 factors from this latter model are only slightly larger than those reported in Table 3 for the multi-level DFM with only 5 factors obtained without including the DF

[27]Note that the correlation between the first DF factor and the NFCI is 97%.

Table B.1: Estimates of the parameters of 1-step-ahead factor-augmented regression models for the 5%, 50% and 95% quantiles of the US growth distribution. The factors are extracted separately from the domestic financial (DF), worldwide financial (WF), domestic macroeconomic (DM) and worldwide macroeconomic (WM) blocks of variables. Estimation sample from 2005Q3 up to 2020Q1. p-values in parenthesis.

	All factors			DF factors			WF factors			DM factors			WM factors		
	$\tau = 0.05$	$\tau = 0.5$	$\tau = 0.95$	$\tau = 0.05$	$\tau = 0.5$	$\tau = 0.95$	$\tau = 0.05$	$\tau = 0.5$	$\tau = 0.95$	$\tau = 0.05$	$\tau = 0.5$	$\tau = 0.95$	$\tau = 0.05$	$\tau = 0.5$	$\tau = 0.95$
μ	**-1.78**	**1.93**	**4.15**	**-1.69**	**1.95**	**4.59**	**-4.94**	**2.07**	**4.53**	-0.34	**2.15**	**4.69**	**-4.36**	**1.91**	**4.77**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
ϕ	**0.33**	-0.12	**-0.22**	0.04	-0.13	-0.13	**1.17**	-0.10	**-0.14**	**-0.61**	-0.11	-0.24	**0.94**	0.04	-0.02
	(0.07)	(0.61)	(0.01)	(0.79)	(0.37)	(0.77)	(0.00)	(0.59)	(0.00)	(0.00)	(0.39)	(0.49)	(0.00)	(0.64)	(0.94)
β_1	**-2.17**	-0.30	**-1.05**	**-3.18**	**-1.64**	-1.17									
	(0.03)	(0.81)	(0.03)	(0.00)	(0.00)	(0.26)									
β_2	**-1.50**	-0.34	-0.20	**-0.44**	**0.51**	0.44									
	(0.02)	(0.67)	(0.52)	(0.10)	(0.02)	(0.54)									
β_3	0.74	-0.70	**0.58**				-0.43	**-0.87**	**-0.20**						
	(0.17)	(0.32)	(0.04)				(0.53)	(0.02)	(0.00)						
β_4	**3.07**	0.97	-0.17				-0.20	-0.40	**-0.88**						
	(0.00)	(0.21)	(0.57)				(0.73)	(0.20)	(0.00)						
β_5	**1.11**	0.10	**-0.50**				-0.94	**-0.51**	**-1.00**						
	(0.03)	(0.88)	(0.04)				(0.11)	(0.10)	(0.00)						
β_6	0.98	0.48	**1.25**							**4.47**	**1.22**	**1.70**			
	(0.40)	(0.75)	(0.04)							(0.00)	(0.00)	(0.04)			
β_7	0.54	-0.02	**0.35**							**-0.36**	-0.03	0.11			
	(0.11)	(0.96)	(0.04)							(0.06)	(0.86)	(0.81)			
β_8	**2.12**	1.00	-0.30										0.25	**0.67**	0.94
	(0.00)	(0.22)	(0.33)										(0.71)	(0.00)	(0.28)
β_9	**2.96**	**0.98**	0.11										**1.54**	0.00	-0.38
	(0.00)	(0.05)	(0.58)										(0.00)	(0.98)	(0.52)
R^1	0.63	0.23	0.41	0.36	0.14	0.17	0.34	0.10	0.25	0.39	0.12	0.25	0.31	0.09	0.07

variables.

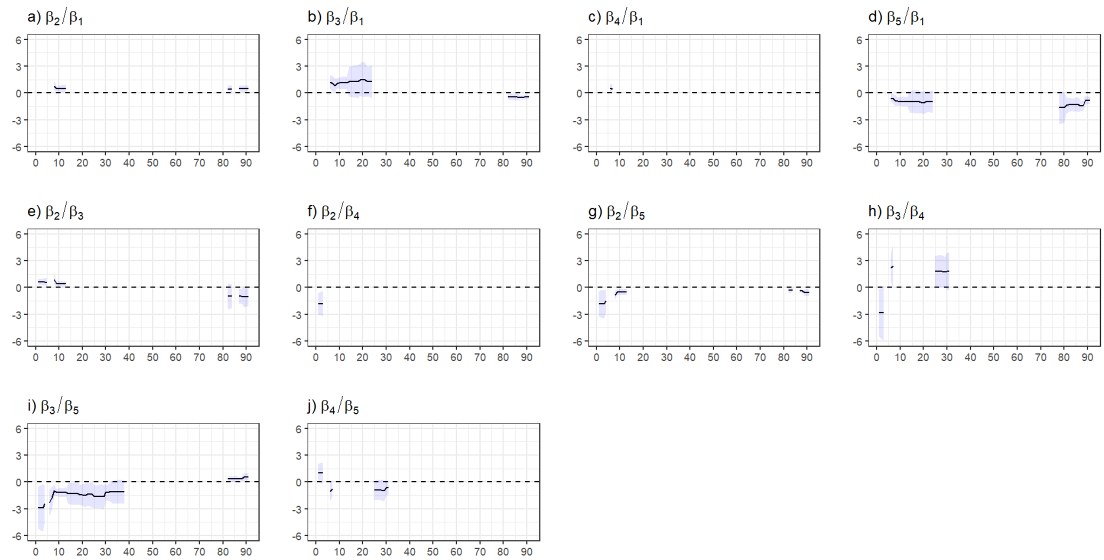
Table B.2: Estimates of the parameters of 1-step-ahead factor-augmented predictive quantile regression models for the 5%, 50% and 95% quantiles of the US growth distribution. The factors are extracted from the multi-level DFM with $X^* = (X_1, X_2, X_3, X_4)$. Estimation sample from 2005Q3 up to 2020Q1. p-values in parenthesis and in bold parameters significant at 10% significance level.

	$\tau = 0.05$	$\tau = 0.5$	$\tau = 0.95$
μ	**-0.88**	**2.03**	**4.33**
	(0.06)	(0.00)	(0.00)
ϕ	**-0.64**	**-0.35**	**-0.27**
	(0.00)	(0.05)	(0.00)
Global factor	**0.26**	**0.14**	**0.13**
	(0.00)	(0.00)	(0.00)
Worldwide factor	0.09	-0.02	**0.05**
	(0.19)	(0.78)	(0.01)
Domestic factor	**2.57**	**0.90**	**1.30**
	(0.00)	(0.03)	(0.00)
Financial factor	0.36	**-0.75**	**-0.29**
	(0.40)	(0.05)	(0.03)
WF factor	**11.63**	**4.48**	**-1.95**
	(0.00)	(0.01)	(0.00)
DM factor	**5.29**	**1.81**	**0.99**
	(0.00)	(0.00)	(0.00)
WM factor	**1.35**	**1.61**	**0.65**
	(0.01)	(0.00)	(0.00)
R^1	0.54	0.19	0.39

C Marginal effects

The ratios between the estimated parameters of the one-step-ahead factor-augmented predictive quantile regressions represent the pairwise marginal rate of substitution between factor i and factor j, denoted as $\text{MRS}(i,j) = \beta_i/\beta_j$ i.e. the variation in factor j needed to maintain constant the quantile of growth when factor i varies in one unit, given all other factors in the model being constant. Note that $\text{MRS}(i,j)$ gives a notion about the relative price of the risk of the factors.[28] These MRS are plotted in Figure C.1 together with their 95% confidence bounds, obtained only when both estimated regression parameters are significant.[29] Among the most important conclusions from the MRS plotted in Figure C.1, we can observe that, for the lower quantiles of the distribution of growth, the MRS between the worldwide financial factors and the global factor is larger than 1 in absolute value, while the MRS between the worldwide macroeconomic factor and the global factor is approximately equal to one. Finally, the MRS between the worldwide financial factors and the worldwide macroeconomic factor is also larger than one in absolute value.

Figure C.1: Ratios of estimated parameters of the factor-augmented predictive quantile regressions for each quantile of the growth distribution ranging from $\tau^* = 0.5$ to $\tau^* = 0.95$ and for horizon $h = 1$. The shade areas represent the 95% confidence pointwise intervals for the ratios.



References

Alessi, L., Barigozzi, M., and Capasso, M. (2010). Improved penalization for determining the number of factors in approximate factor models. *Statistics and Probability Letters*, 80(1):1806–1813.

[28]We are very grateful to a referee for her/his suggestion of estimating the MRSs.

[29]The 95% confidence bounds are constructed using the delta method; see Lye and Hirschberg (2018) for a discussion on the construction of confidence bounds for ratios.

Hallin, M., and Liska, R. (2011). Dynamic factor models in the presence of blocks. *Journal of Econometrics*, 163:29-41.

Lye, J. and Hirschberg, J. (2018). Ratios of parameters: Some econometric examples. *Australian Economic Review*, 51(4):578602.