MORAL HAZARD WITH NON-ADDITIVE UNCERTAINTY: WHEN ARE ACTIONS IMPLEMENTABLE?

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Abstract. We provide sufficient conditions on the information structure for implementing actions in a moral hazard setting when Agent has non-probabilistic uncertainty. We show that under three different formulations of Agent's ambiguity attitude, contracts that partition the outcome space in two parts, and are piecewise constant on each part, are enough to implement an action.

Keywords: Moral hazard, non-probabilistic uncertainty, ambiguity aversion, implementability.

JEL Classification: D81, D82, D86

1. Introduction

This paper aims to characterize conditions for a two-part, piecewise-constant contract ('flat payment plus bonus', FPB henceforth) to be used to implement actions in moral hazard problems with non-additive uncertainty. Lopomo et al. (2011) provide sufficient conditions for FPB contracts to be uniquely optimal under Bewley-type preferences on a finite outcome space, when Principal has more precise information than Agent. We consider three different formulations of ambiguity-sensitive preferences on a continuum of outcomes, and identify sufficient conditions for FPB contracts to solve the implementation problem for any action other than the least costly one, regardless of whether Agent knows more or less than Principal.

Section 2 lays out the preliminaries, Section 3 presents the implementation results and we conclude with a discussion of our sufficient conditions in relation to the literature.

Date: March 4, 2018. The first author gratefully acknowledges the financial support from the Ministerio Economia y Competitividad (Spain) through grants ECO2014-55953-P and MDM 2014-0431.

2. Preliminaries

Let $\mathcal{Y} = [0, \overline{Y}]$ be the set of outcomes. Let $\mathcal{B}(\mathcal{Y})$ denote the Borel σ-algebra on $\mathcal{Y}$, $\Delta(\mathcal{Y})$ denote the set of Borel distributions on $\mathcal{Y}$ with the weak topology and $\mathbb{K}_{\Delta(\mathcal{Y})}$ denote the class of non-empty, compact, convex subsets of $\Delta(\mathcal{Y})$. Agent chooses an unobservable action a from a finite set $\mathcal{A}$. Let $g : \mathcal{A} \mapsto \mathbb{R}_+$ be a bounded, nonnegative function that describes the cost of effort to Agent, and denote $g(a_k) = g_k$. Principal's information about technology is characterized as a set-valued mapping from set of actions $\mathcal{A}$ to $\mathbb{K}_{\Delta(\mathcal{Y})}$ given by $Q^P(\cdot) : \mathcal{A} \mapsto \mathbb{K}_{\Delta(\mathcal{Y})}$. Similarly, Agent's information is characterized by another set-valued mapping, $Q^A(\cdot) : \mathcal{A} \mapsto \mathbb{K}_{\Delta(\mathcal{Y})}$. Furthermore, for all actions the associated set of probability distributions have a common support: for all $\{a, a'\} \in \mathcal{A}$, $\mathsf{supp}\, Q^A(a) = \mathsf{supp}\, Q^P(a) = \mathsf{supp}\, Q^A(a') = \mathsf{supp}\, Q^P(a') = \mathcal{Y}$.

Remark 1. Our formal set up can accommodate asymmetry of uncertainty, in particular both of the following cases: (I) Agent has (weakly) more precise knowledge about technology than Principal: $Q^A(a) \subset Q^P(a)$ for each $a \in \mathcal{A}$, and (II) Principal has (weakly) more precise knowledge about technology: $Q^P(a) \subset Q^A(a)$ for each $a \in \mathcal{A}$. However, since we focus on implementation alone, and not Principal's profit maximization, Agent's information sets are all that matter. Our results are interesting when Agent's information sets are sufficiently rich, in that they are not singletons (i.e. when Agent is not a standard expected utility maximizer). We will maintain that assumption. Both cases I and II can then be thought of as generalizations of Ghirardato (1994) and case II is also a generalization of the setting in Lopomo et al. (2011). At the contracting stage, both parties have common knowledge of Q^P and Q^A.[1]

2.1. Convex Capacities.

We assume that for each action a, the set of induced probability distributions $Q^A(a)$ is the core of a regular convex capacity C_a[2]:

Assumption 1. For each $a \in \mathcal{Y}$, $\exists C_a$, a convex capacity, such that $Q(a) = \mathsf{core}\,(C_a) = \{q \in \Delta(\mathcal{Y}) : q(E) \geq C_a(E), \quad \forall E \in \mathcal{B}(\mathcal{Y})\}$

Following Dyckerhoff and Mosler (1993), we define stochastic dominance of capacities.

[1] Dumav and Khan (2017) show that linear contracts provide a solution when Principal does not know Agent's information sets.

[2] A capacity on a measurable space $(\Omega, \mathcal{B})$ is a mapping $C : \mathcal{B} \to [0,1]$ such that $C(\emptyset) = 0$, $C(\Omega) = 1$ and $A \subset B \Rightarrow C(A) \leq C(B)$. Capacity C is coherent (or a *lower probability*) if for some set of probability measures $\mathcal{P}$, $C(A) = \inf_{P \in \mathcal{P}} P(A)$ for every $A \in \mathcal{B}$. A capacity is convex if for any $A, B \in \mathcal{B}$, $C(A) + C(B) \leq C(A \cap B) + C(A \cup B)$. A capacity is regular if $C(A) = \inf\{C(B) : A \subset B,\ B \in \mathcal{B},\ B\,\text{compact}\}$ ((Molchanov, 2005, Ch. 1)). Cores of convex capacities are well-defined (Schmeidler (1986)).

Definition 1. C_1 dominates C_2 with respect to a family of Borel measurable functions $\mathcal{F}$, denoted $C_1 \succsim_{\mathcal{F}} C_2$, if

$$\int^{Ch} f dC_1 \geq \int^{Ch} f dC_2 \qquad \forall f \in \mathcal{F} \tag{1}$$

where $\int^{Ch}$ is the Choquet integral (Choquet (1954)). Let $\mathcal{F} = \{f : \mathcal{Y} \to \mathbb{R},\ f \text{ increasing}, \mathcal{B}-\text{measurable}\}$. Then we have the following characterization of stochastic dominance (Proposition 1 in Dyckerhoff and Mosler (1993), proof omitted):

Proposition 1. $C_1 \succsim_{\mathcal{F}} C_2$ if and only if $C_1[t, \infty[\geq C_2[t, \infty[\quad \forall t \in \mathbb{R}$.

We also use the following result (Proposition 3 in Schmeidler (1986), proof omitted).

Proposition 2. For a convex capacity C, the Choquet integral is given by $\int^{Ch} f dC = \min_{q \in \mathsf{core}\,(C)} \int f dq$.

Let $\underline{q}_a$ denote the distribution in $Q(a) = \mathsf{core}\,(C_a)$ that attains the minimum (for a regular capacity this is well defined (Huber and Strassen (1973))).

2.2. **Payoffs and Timing.** We consider three alternative representations of ambiguity-sensitive preferences that evaluate a contract according to :

1. worst-case expected payoff, i.e., 'max-min' or MEU criterion (Gilboa and Schmeidler (1989));
2. 'α-max-min' or Hurwicz criterion (Hurwicz (1951));
3. probability-set-dominance criteria (Bewley (2002)).

We assume that Agent is risk-neutral over monetary payoffs.

A contract is a bounded, non-negative, $\mathcal{B}$ -measurable function $w : \mathcal{Y} \to \mathbb{R}_+$ that specifies output contingent payments and protects Agent with limited liability (i.e. $w(y) \geq 0$).

The timing of the contracting game is as follows:

(i) Principal offers a contract w;
(ii) Agent, knowing Q^A, chooses action $a \in \mathcal{A}$;
(iii) output y is realized;
(iv) payoffs are received: $y - w(y)$ to Principal and $w(y) - g(a)$ to Agent.

3. Implementation

Let $a \in \mathcal{A}$ be the action that Principal wants to implement. If a is the least cost action, then a flat payment of $g(a)$ would implement it for any of the three kinds of objective functions under consideration. For any other action, we have three sets of individual rationality (IR) and incentive compatibility (IC) conditions, one set for each case. These implementation conditions, (2) - (3), (7) - (8), (12) - (13), depend only on the payment scheme $w(y)$ and Agent's perception Q^A but neither on Principal's perceived ambiguity nor her ambiguity attitude.[3]

3.1. **Implementation with MEU Preferences.**

Action $a \in \mathcal{A}$ is implemented if it satisfies Agent's IR and IC, respectively:

$$\min_{q \in Q^A(a)} \int w(y)dq - g(a) \geq 0 \tag{2}$$

and

$$\min_{q \in Q^A(a)} \int w(y)dq - g(a) \geq \min_{p \in Q^A(a')} \int w(y)dp - g(a') \qquad \forall a' \in \mathcal{A}. \tag{3}$$

Proposition 3. With MEU preferences, if $a_k \in \mathcal{A}$ is implementable, then $\forall a_j \in \mathcal{A}$, $j \neq k$ and $g_j < g_k$, we have $Q^A(a_j) \setminus Q^A(a_k) \neq \emptyset$.

Proof. Suppose not. Let $w(y)$ implement a_k and let $Q^A(a_j) \subset Q^A(a_k)$ for some $a_j \in \mathcal{A}$ such that $g_j < g_k$. Combining the IR and IC conditions together with the fact that $Q^A(a_j) \subset Q^A(a_k)$ yields the following chain of inequalities that shows that Agent chooses a_j rather than a_k:

$$\min_{q \in Q^A(a_j)} \int w(y)dq - g_j \geq \min_{p \in Q^A(a_k)} \int w(y)dp - g_j > \min_{p \in Q^A(a_k)} \int w(y)dp - g_k \geq 0.$$

Here the last inequality shows that a_j is rational for Agent, while the strict inequality, which follows from $g_k > g_j$, together with the first inequality, which follows from the fact $Q^A(a_j) \subset Q^A(a_k)$ and that minimum of a non-negative-valued function does not get smaller over a larger set, establishes that Agent would prefer a_j rather than a_k. $\square$

[3] In Q^A, the 'best-case' and 'worst-case' distributions are endogenously determined for a given contract; Principal and Agent can disagree on these cases. Furthermore, since all these implementability conditions depend on Q^A, but not on Q^P, whether Principal has more or less precise knowledge of technology than Agent does not bear on implementability of an action.

The next Proposition shows that strengthening the necessary condition to stochastic dominance, and imposing bounds on the rate at which costlier actions improve outcomes becomes sufficient for implementation.

Assumption 2. $\forall a_j, a_k \in \mathcal{A}$, $j \neq k$ and $g_j < g_k$, $C_k \succsim_{\mathcal{F}} C_j$ and $C_k \neq C_j$.

Let $\mathbb{M}$ be the collection of all upper tail events in $\mathcal{B}$: $\mathbb{M} = \{\mathcal{M} \in \mathcal{B},\, \mathcal{M} = [y, \overline{Y}],\, y \in \mathcal{Y}\}$. The next two assumptions impose bounds on the rates of increase in upper tail capacities relative to the increase in costs.

Assumption 3. $\forall a_k, a_j \in \mathcal{A}$, $j \neq k$ and $g_j < g_k$, we have

$$\frac{g_k - g_j}{g_k} \geq \frac{C_k(\mathcal{M}) - C_j(\mathcal{M})}{C_k(\mathcal{M})}$$

Assumption 4. $\forall a_l, a_k, a_j \in \mathcal{A}$, $l \neq k \neq j$ and $g_k < g_l < g_j$ we have, for all upper tail events $\mathcal{M}$,

$$\frac{g_l - g_k}{C_l(\mathcal{M}) - C_k(\mathcal{M})} \geq \frac{g_k - g_j}{C_k(\mathcal{M}) - C_j(\mathcal{M})} \tag{4}$$

Proposition 4. With MEU preferences, $a^k \in \mathcal{A}$ is implementable if Assumptions 2-4 hold.

Proof. Let $\underline{y}_k = \min_{p \in Q^A(a_k)} \int y dp$ and let $\mathcal{M}$ be the particular event $\{y \in \mathcal{Y} : y \in [\underline{y_k}, \overline{Y}]\}$. Consider contracts that reward Agent with a constant non-zero payment only above a certain performance level. For instance, consider a contract of the form:

$$w(y) = \begin{cases} b & \text{if } y \in \mathcal{M} \\ 0 & \text{if } y \in \mathcal{Y} \setminus \mathcal{M} \end{cases}$$

Such a contract implements a_k against a lower cost action a_j if

$$b \min_{q \in Q^A(a_k)} q(\mathcal{M}) - g_k \geq 0, \tag{5}$$

and

$$b \min_{q \in Q^A(a_k)} q(\mathcal{M}) - g_k \geq b \min_{q \in Q^A(a_j)} q(\mathcal{M}) - g_j \tag{6}$$

The IR condition (5) holds if

$$b \geq g_k / \underline{q}_k(\mathcal{M})$$

The iIC condition (6) holds if

$$b \geq g_k - g_j / (\underline{q}_k(\mathcal{M}) - \underline{q}_j(\mathcal{M}))$$

where $\underline{q}_k(\mathcal{M}) = \min_{q\in Q(a_k)} q(\mathcal{M})$;

Taken together, deviation to a lower cost action a_j can be prevented if we can find b for $\mathcal{M} = [\underline{y_k}, \overline{Y}]$ such that

$$b \geq \max\{g_k - g_j/(\underline{q}_k(\mathcal{M}) - \underline{q}_j(\mathcal{M})), g_k/\underline{q}_k(\mathcal{M})\} = g_k - g_j/(\underline{q}_k(\mathcal{M}) - \underline{q}_j(\mathcal{M})) \quad (*1)$$

The second equality follows from Assumption 3.

From Proposition 1 we have a characterization of stochastic dominance for convex capacities so that $C_k \succsim_{\mathcal{F}} C_j$ if and only if $C_k([y, \overline{Y}]) \geq C_j([y, \overline{Y}])$ for all $y \in \mathcal{Y}$ and the inequality holds as strict for $C_k \neq C_j$.

From Proposition 2, $C_a([y, \overline{Y}]) = \int^{Ch} 1_{[y,\overline{Y}]} dC_a = \min_{q\in Q(a)} \int 1_{[y,\overline{Y}]} dq$, and from Assumption 2, $C_k([y, \overline{Y}]) > C_j([y, \overline{y}])$. Letting $y = \underline{y}_k$ and $\mathcal{M} = [\underline{y}_k, \overline{Y}]$, we have $(\underline{q}_k(\mathcal{M}) - \underline{q}_j(\mathcal{M})) > 0$.

To prevent a deviation to a costlier action a_l that (by Assumption 2) stochastically dominates a_k, IC (6) holds if

$$b \leq g_l - g_k/(\underline{q}_l(\mathcal{M}) - \underline{q}_k(\mathcal{M})) \quad (*2)$$

It is straightforward to see that Assumption 4 ensures that ICs only need to be checked *locally*, against only the actions adjacent to a_k on either side, ranked in terms of cost.

With $(*1)$ and $(*2)$ the IC for action a_k then reduces to finding b such that

$$\frac{g_k - g_j}{\underline{q}_k(\mathcal{M}) - \underline{q}_j(\mathcal{M})} \leq b \leq \frac{g_l - g_k}{\underline{q}_l(\mathcal{M}) - \underline{q}_k(\mathcal{M})} \quad (*)$$

Assumption 4 then ensures that such b in $(*)$ is well-defined. □

3.2. **Implementation under $\alpha-$MEU Preferences.** Agent evaluates a contract according to a convex combination of worst-case and best-case scenarios, the former getting weight α and the latter $1-\alpha$. Action $a \in \mathcal{A}$ is implementable by $w(y)$ if it satisfies Agent's IR and IC, respectively:

$$\alpha \max_{q\in Q^A(a)} \int w(y)dq + (1-\alpha) \min_{q\in Q^A(a)} \int w(y)dq - g(a) \geq 0 \quad (7)$$

and

$$\alpha \max_{q\in Q^A(a)} \int w(y)dq + (1-\alpha) \min_{q\in Q^A(a)} \int w(y)dq - g(a) \geq$$
$$\alpha \max_{p\in Q^A(a')} \int w(y)dp + (1-\alpha) \min_{p\in Q^A(a')} \int w(y)dq - g(a') \qquad \forall a' \in \mathcal{A} \quad (8)$$

Take the conjugate capacity, $\overline{C}(E) = 1 - C(E^c)$. We have $\overline{C}(E) = \max_{q\in \mathsf{core}\,(C)} \int 1_E dq := \overline{q}(E)$.[4] For a convex capacity C_k its conjugate $\overline{C}_k$ is concave, and $C_k \succsim_{\mathcal{F}} C_j$ implies $\overline{C}_k \succsim_{\mathcal{F}} \overline{C}_j$. So we have $\overline{q}_k(E) > \overline{q}_j(E)$ for sets of the form $E = [y, \overline{Y}]$.

With the above observations, the same stochastic dominance condition as the one we used for MEU is also sufficient for α-MEU preferences.

For any $\alpha \in [0,1]$ let $C^\alpha(\mathcal{M}) = \alpha C(\mathcal{M}) + (1-\alpha)\overline{C}(\mathcal{M})$.

Assumption 5. $\forall a_k, a_j \in \mathcal{A}$, $j \neq k$ and $g_j < g_k$, and for all $\alpha \in [0,1]$, we have

$$\frac{g_k - g_j}{g_k} \geq \frac{C_k^\alpha(\mathcal{M}) - C_j^\alpha(\mathcal{M})}{C_k^\alpha(\mathcal{M})}$$

Assumption 6. $\forall a_l, a_k, a_j \in \mathcal{A}$, $l \neq k \neq j$ and $g_k < g_l < g_j$ we have, for all upper tail events $\mathcal{M} = [y, \overline{Y}]$, $y \in \mathcal{Y}$ and for all $\alpha \in [0,1]$

$$\frac{g(a_l) - g_k}{C_l^\alpha(\mathcal{M}) - C_k^\alpha(\mathcal{M})} \geq \frac{g(a_k) - g_j}{C_k^\alpha(\mathcal{M}) - C_j^\alpha(\mathcal{M})} \quad (9)$$

Proposition 5. *With α-MEU preferences, $a_k \in \mathcal{A}$ is implementable if Assumptions 2, 5 and 6 hold.*

Proof. Consider an FPB contract of the form given in Proposition 4. Such a contract prevents a downward deviation from a_k to a_j if

$$b\left(\alpha \min_{q\in Q(a_k)} q(\mathcal{M}) + (1-\alpha) \max_{q\in Q(a_k)} q(\mathcal{M})\right) - g_k \geq 0. \quad (10)$$

and

$$b\left(\alpha \min_{q\in Q^A(a_k)} q(\mathcal{M}) + (1-\alpha) \max_{q\in Q^A(a_k)} q(\mathcal{M})\right) - g_k \geq$$
$$b\left(\alpha \min_{q\in Q^A(a_j)} q(\mathcal{M}) + (1-\alpha) \max_{q\in Q^A(a_j)} q(\mathcal{M})\right) - g_j \quad (11)$$

[4] $C(E^c) = \int^{Ch} 1_{E^c} dC = \min_{q\in \mathsf{core}\,(C)} \int 1_{E^c} dq = \min_{q\in \mathsf{core}\,(C)} \int (1 - 1_E) dq = 1 - \max_{q\in \mathsf{core}\,(C)} \int (1_E) dq$.

Rewriting and rearranging, condition (11) holds if

$$b \geq g_k - g_j / \left(\alpha(\underline{q}_k(\mathcal{M}) - \underline{q}_j(\mathcal{M})) + (1-\alpha)(\overline{q}_k(\mathcal{M}) - \overline{q}_j(\mathcal{M})) \right)$$

where $\underline{q}_k(\mathcal{M}) = \min_{q \in Q(a_k)} q(\mathcal{M})$; $\overline{q}_k(\mathcal{M}) = \max_{q \in Q(a_k)} q(\mathcal{M})$;

and condition (10) holds if

$$b \geq g_k / \left(\alpha \underline{q}_k(\mathcal{M}) + (1-\alpha)\overline{q}_k(\mathcal{M}) \right)$$

Incentive compatibility for downward deviation to a_j reduces to finding b for $\mathcal{M} = [\underline{y}_k, \overline{Y}]$ such that

$$\begin{aligned} b \geq \max \Big\{ & g_k - g_j / \left(\alpha(\underline{q}_k(\mathcal{M}) - \underline{q}_j(\mathcal{M})) + (1-\alpha)(\overline{q}_k(\mathcal{M}) - \overline{q}_j(\mathcal{M})) \right), \\ & g_k / \left(\alpha \underline{q}_k(\mathcal{M}) + (1-\alpha)\overline{q}_k(\mathcal{M}) \right) \Big\} \\ & = g_k - g_j / \left(\alpha(\underline{q}_k(\mathcal{M}) - \underline{q}_j(\mathcal{M})) + (1-\alpha)(\overline{q}_k(\mathcal{M}) - \overline{q}_j(\mathcal{M})) \right) \qquad (**1) \end{aligned}$$

Assumption 2 ensures $(\underline{q}_k(\mathcal{M}) - \underline{q}_j(\mathcal{M})) > 0$ and $(\overline{q}_k(\mathcal{M}) - \overline{q}_j(\mathcal{M})) > 0$, for $\mathcal{M} = [\underline{y}_k, \overline{Y}]$.

The IC to deter an upward deviation to a costlier action a_l is

$$b \leq g_l - g_k / \left(\alpha(\underline{q}_l(\mathcal{M}) - \underline{q}_k(\mathcal{M})) + (1-\alpha)(\overline{q}_l(\mathcal{M}) - \overline{q}_k(\mathcal{M})) \right) \qquad (**2)$$

Again, it is enough to check the ICs locally due to Assumption 6. With $(**1)$ and $(**2)$ the incentive compatibility for action a_k reduces to finding b such that

$$\begin{aligned} \frac{g_k - g_j}{\alpha(\underline{q}_k(\mathcal{M}) - \underline{q}_j(\mathcal{M})) + (1-\alpha)(\overline{q}_k(\mathcal{M}) - \overline{q}_j(\mathcal{M}))} & \leq b \\ & \leq \frac{g_l - g_k}{\alpha(\underline{q}_l(\mathcal{M}) - \underline{q}_k(\mathcal{M})) + (1-\alpha)(\overline{q}_l(\mathcal{M}) - \overline{q}_k(\mathcal{M}))} \qquad (**) \end{aligned}$$

Condition (9) then ensures that such b in $(**)$ is well-defined. $\square$

3.3. **Implementation under Bewley-type Preferences.** With Bewley-type preferences, the IR and IC conditions, respectively, become

$$\int w(y)dq - g(a) \geq 0 \qquad \forall q \in Q^A(a) \tag{12}$$

and

$$\int w(y)dq - g(a) \geq \int w(y)dp - g(a') \qquad \forall q \in Q^A(a), \qquad \forall p \in Q^A(a'), \qquad \forall a' \in \mathcal{A}$$

$$(13)$$

Proposition 6. *With Bewley-type preferences, $a^k \in \mathcal{A}$ is implementable only if, $\forall a_j \in \mathcal{A}$, $j \neq k$ and $g_j < g_k$, we have $Q^A(a_j) \cap Q^A(a_k) = \emptyset$*

Proof. Suppose not, i.e. suppose that a_k is implementable with $w(y)$, but there is a cheaper action a_j such that $Q^A(a_j) \cap Q^A(a_k) \neq \emptyset$. Consider $q \in Q^A(a_k) \cap Q^A(a_j)$. For all such q, $\int w(y)dq - g_j > \int w(y)dq - g_k$ and hence the IC condition (13) for implementation a_k is violated. □

Remark 2. This is a sharper version of Proposition 1 in Lopomo et al. (2011).

Assumption 7. $\forall a_k, a_j \in \mathcal{A}$, $j \neq k$ and $g_j < g_k$, for all upper tail events $\mathcal{M} = [y, \overline{Y}]$, $y \in \mathcal{Y}$, we have

$$\frac{g_k - g_j}{g_k} \geq \frac{C_k(\mathcal{M}) - \overline{C}_j(\mathcal{M})}{C_k(\mathcal{M})}$$

and

Assumption 8. $\forall a_l, a_k, a_j \in \mathcal{A}$, $l \neq k \neq j$ and $g_j < g_k < g_l$ we have, for all upper tail events $\mathcal{M} = [y, \overline{Y}]$, $y \in \mathcal{Y}$,

$$\frac{g(a_l) - g_k}{\overline{C}_l(\mathcal{M}) - C_k(\mathcal{M})} > \frac{g(a_k) - g_j}{C_k(\mathcal{M}) - \overline{C}_j(\mathcal{M})} \tag{14}$$

Proposition 7. With Bewley-type preferences, $a_k \in \mathcal{A}$ is implementable if Assumptions 2, 8 and 7 hold.

Proof. Consider the same kind of FPB contract as in the MEU case. Note that IC boils down to

$$\min_{q \in Q_k} \int w(y)dq - g_k \geq \max_{p \in Q_j} \int w(y)dp - g_j \qquad \forall a_j \in \mathcal{A} \tag{15}$$

The rest of the proof is analogous to the MEU case, if we replace C_j with $\overline{C_j}$. □

4. Conclusion

We provide conditions for two-part, piecewise-constant contracts to implement actions in a moral hazard setting with non-additive uncertainty, under three different formulations of ambiguity attitude for Agent. Our necessary conditions for implementation are comparable to the one in Hermalin and Katz (1991) for the standard Bayesian model, and those in

Ghirardato (1994) and Lopomo et al. (2011) for non-additive models. All these conditions stipulate a necessary amount of 'disjointedness' between the sets of probabilities generated by different actions.

Our sufficiency conditions turn out to be very similar in all three cases, and also shares some substantive similarities to those in Lopomo et al. (2011). Our stochastic dominance assumption is comparable to the MLRP condition on extreme points in Lopomo et al. (2011), and helps make downward deviations unattractive enough. Our other substantive assumption entails that any upward deviation does not increase the probability of upper sets by more than the proportionate increase in effort costs. Again, the role played by this assumption is very similar to the 'concavity of distribution' assumption in Grossman and Hart (1983) and Lopomo et al. (2011). Given that these contracts provide just the minimum level of variability needed for implementation, our results suggest they could be robustly optimal across different formulations of ambiguity attitude.

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