Sectoral Composition of Government Spending, Distortionary Income Taxation, and Macroeconomic (In)stability*

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Abstract

This paper quantitatively examines the interrelations between sectoral composition of government spending and macroeconomic (in)stability in a two-sector real business cycle model with positive productive externalities in investment and distortionary income taxation through a stylized balanced-budget fiscal policy rule. We find that under endogenous public expenditures, the benchmark model always exhibits indeterminacy and sunspots provided the constant tax rate does not exceed a critical value. When the tax rate is raised to a higher level, a sufficiently high public-consumption share can destabilize the macroeconomy by generating belief-driven cyclical fluctuations. We also find that under the baseline parameterization with fixed government spending, the low-tax steady state is an indeterminate sink and the high-tax steady state is a saddle point, regardless of how public expenditures are divided between consumption and investment goods.

Keywords: Government Spending; Distortionary Income Taxation; Equilibrium (In)determinacy.

JEL Classification: E32; E62; O41.

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1 Introduction

Recently, there has been a growing literature examining the stabilization role of various fiscal policy rules within an otherwise standard real business cycle (RBC) model that exhibits multiple, indeterminate competitive equilibria under laissez faire.[1] As it turns out, many previous articles have focused on the macroeconomic (in)stability effects of changing the revenue or taxation side of the national budget,[2] hence left the impact of the government's spending policies mostly underexplored.[3] In our earlier work, Chang et al. (2015) study the quantitative inter-relations between sectoral composition of public expenditures and equilibrium (in)determinacy in a two-sector representative-agent macroeconomy with positive productive externalities in investment. However, when government purchases of consumption and investment goods are postulated as constant fractions of total public spending, these authors' setting with lump-sum taxation will not have enough equations to pin down the model's equilibrium allocations. It is left as a topic for future research to "investigate this formulation under [the more realistic] distortionary income tax policies" (Chang et al., 2015, p. 26). Accordingly, we will address this research question in the current paper.

Our analysis begins with embedding government purchases of goods and services into a discrete-time two-sector RBC model, as in Harrison (2001), with positive productive externalities present in the investment sector. Next, distortionary income taxation is incorporated through two stylized balanced-budget rules that are commonly adopted in the existing literature: Guo and Harrison (2004, henceforth GH) consider endogenous government spending financed by a fixed tax rate levied on the household's total income; whereas Schmitt-Grohé and Uribe (1997, henceforth SU) postulate that the government sets the proportional income tax rate to finance a pre-specified constant level of public expenditures. This analytical framework allows us to examine how the sectoral distribution of public spending affects each version of our model's local stability properties under parameter values that are consistent with post Korean-war U.S. time series data.

Generally speaking, our model's equilibrium dynamics are governed by the relative strength

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[1] See Benhabib and Farmer (1999) for an excellent survey on this RBC-based indeterminacy literature in which the terms "animal spirts", "sunspots" and "self-fulfilling prophecies" are used interchangeably.

[2] See, for example, Guo and Lansing (1998), Christiano and Harrison (1999), Guo and Harrison (2001, 2011, 2015), Greiner (2006), Giannitsarou (2007), Dromel and Pintus (2007, 2008), Chen and Guo (2013a, 2013b, 2014, 2016, 2017), Gokan (2013), Nourry, Seegmuller and Venditti (2013), and Abad, Seegmuller and Venditti (2017), among others.

[3] Previous macroeconomics research that has examined the (de)stablizing effects of public expenditures includes Schmitt-Grohé and Uribe (1997, section III), Raurich (2001), Gokan (2006), and Lloyd-Braga, Modesto and Seegmuller (2008), among others.

of two opposing effects on the household's intertemporal private-consumption Euler equation. Start the laissez-faire model from its non-stochastic steady state, and suppose that agents become optimistic about the economy's future. Acting upon this belief, the representative household will consume less and invest more today, thus raising next period's capital stock and output. On the one hand, *the MPK effect* refers to a lower real interest rate caused by an increase in today's private investment expenditures because of diminishing marginal product of capital. This channel in turn invalidates agents' initial optimism. On the other hand, due to the presence of positive externalities from producing investment goods, the social production possibility frontier that traces out the trade-off between private consumption and investment spending is convex to the origin. As a result, *the price effect* refers to a decrease in the relative price of investment as agents' rosy expectation reallocates more capital and labor inputs into the investment sector.

We show that under GH's fiscal formulation, indeterminacy and sunspots always occur in our calibrated benchmark model with "relatively low" (constant) tax rates, regardless of how the government divides its purchases between consumption and investment goods. In this environment, positive income taxation shifts the convex social production possibility frontier downward, which will induce households to reduce their optimism-driven consumption as well as investment expenditures. It follows that the aforementioned price effect that helps make for multiple equilibria becomes weaker, whereas the corresponding after-tax MPK effect is strengthened because of a lower net real interest rate. As it turns out, the price effect continues to dominate provided the income tax rate does not exceed a certain threshold. We also show that when the tax rate is increased to a higher value, the model's steady state can either be a saddle point or a sink; and that the minimum level for the public-consumption share which yields equilibrium indeterminacy is monotonically increasing with respect to the income tax rate. Our numerical experiments find that in this case, the resulting after-tax MPK effect quantitatively outweighs the price effect as long as the fraction of government spending on consumption goods is zero or "relatively small", indicating that the model exhibits saddle-path stability and equilibrium uniqueness. Therefore, raising the public-consumption share to be above a corresponding critical value will lead to a dominating price effect. This in turn destabilizes the macroeconomy by generating expectations-driven business cycle fluctuations.

When the economy is subject to SU's balanced-budget rule, these exists a steady-state Laffer curve-type relationship between the tax rate and the resulting tax revenue. Specifically, the model possesses two interior steady states when the pre-specified (constant) level of public

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spending is lower than the revenue-maximizing counterpart. We find that under the baseline parameterization, the low-tax steady state is an indeterminate sink whereas the high-tax steady state is a saddle point, regardless of the sectoral composition on the government's purchases of goods and services. Intuitively, when agents become optimistic about the economy's future, the government is forced to decrease next period's income tax rate as total output rises. This implies that the net marginal product of capital is higher around the model's low-tax stationary equilibrium. As a result, the high-tax steady state is associated with a relatively stronger after-tax MPK effect, which in turn will stabilize the economy against sunspot-driven aggregate fluctuations. In sum, this paper shows that in the context of a two-sector RBC model with positive investment externalities, whether and how the government's sectoral spending policies affect macroeconomic (in)stability depends crucially on the exact formulation of the underlying fiscal policy rule.

The remainder of this paper is organized as follows. Section 2 describes the economy and analyzes its equilibrium conditions. Section 3 undertakes a quantitative investigation of local dynamics within calibrated versions of our model under the GH or SU fiscal policy rule. Section 4 concludes.

2   The Economy

We incorporate government purchases of goods and services into the discrete-time two-sector real business cycle (RBC) model *à la* Harrison (2001) under perfect foresight. Households live forever, and derive utility from consumption and leisure. The production side of the economy consists of two distinct sectors, consumption and investment. Based on the empirical findings of Harrison (2003), competitive firms in each sector use identical private technologies to produce their respective output, and positive productive externalities are present within the investment sector. The government balances its budget each period by levying distortionary income taxes to finance its expenditures. In particular, we consider two balanced-budget fiscal formulations: one in which the proportional income tax rate is a constant and public spending changes over time (GH); and the other in which the government sets a countercyclical income tax rate with fixed government expenditures (SU). Both specifications are nested in the following model.

2.1 Firms

In the consumption sector, output is produced by competitive firms with the following constant returns-to-scale Cobb-Douglas technology:

$$Y_{ct} = K_{ct}^{\alpha} L_{ct}^{1-\alpha}, \ 0 < \alpha < 1, \tag{1}$$

where K_{ct} and L_{ct} are the capital and labor inputs utilized in the production of consumption goods. Under the assumption that factor markets are perfectly competitive, the first-order conditions for these firms' profit maximization are

$$r_t = \alpha \frac{Y_{ct}}{K_{ct}}, \tag{2}$$

$$w_t = (1-\alpha) \frac{Y_{ct}}{L_{ct}}, \tag{3}$$

where r_t is the capital rental rate and w_t is the real wage rate.

Similarly, investment goods are produced by a unit measure of identical competitive firms using the private technology

$$Y_{It} = A_t K_{It}^{\alpha} L_{It}^{1-\alpha}, \tag{4}$$

where K_{It} and L_{It} are physical capital and labor hours in the investment sector, and A_t denotes productive externalities that each individual firm takes as given. Moreover, A_t is postulated to take the following specification:

$$A_t = (\bar{K}_{It}^{\alpha} \bar{L}_{It}^{1-\alpha})^{\theta}, \ \ \theta \geq 0, \tag{5}$$

where $\bar{K}_{It}$ and $\bar{L}_{It}$ represent the within-sector average levels of capital and labor services devoted to producing investment goods, and θ measures the degree of sector-specific externalities in the investment sector. In a symmetric equilibrium, all firms in the investment sector make the same decisions such that $K_{It} = \bar{K}_{It}$ and $L_{It} = \bar{L}_{It}$, for all t. As a result, (5) can be plugged into (4) to obtain the following aggregate production function for investment goods that may exhibit increasing returns-to-scale:

$$Y_{It} = K_{It}^{\alpha(1+\theta)} L_{It}^{(1-\alpha)(1+\theta)}, \tag{6}$$

where $\alpha(1+\theta) < 1$ to rule out the possibility of sustained economic growth. The first-order conditions that govern firms' demand for capital and labor in the investment sector are

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$$r_t = \alpha \frac{p_t Y_{It}}{K_{It}}, \tag{7}$$

$$w_t = (1 - \alpha) \frac{p_t Y_{It}}{L_{It}}, \tag{8}$$

where Y_{It} refers to the social technology for investment given by (6), and p_t denotes the relative price of investment to consumption goods at time t. Notice that firms in each sector will face the same equilibrium factor prices since capital and labor inputs are assumed to be perfectly mobile across the two production sectors.

2.2 Households

The economy is populated by a unit measure of identical infinitely-lived households. Each household maximizes its present discounted lifetime utility

$$\sum_{t=0}^{\infty} \beta^t \left[\log C_t - \frac{L_t^{1+\gamma}}{1+\gamma} \right], \quad 0 < \beta < 1, \text{ and } \gamma \geq 0, \tag{9}$$

where C_t and L_t are the household's consumption and hours worked, respectively; β is the discount factor, and γ denotes the inverse of the wage elasticity for labor supply. Notice that the period utility function in (9) is consistent with long-run balanced growth, a feature that is commonly adopted in the real business cycle literature. The budget constraint faced by the representative agent is given by

$$C_t + p_t I_t = (1 - \tau_t) \underbrace{(r_t K_t + w_t L_t)}_{=Y_t}, \tag{10}$$

where I_t is gross investment, $\tau_t \in [0, 1)$ represents the *distortionary* income tax rate, Y_t is national income or GDP,[4] and K_t is the household's capital stock that evolves according to the law of motion

$$K_{t+1} = (1 - \delta)K_t + I_t, \quad K_0 > 0 \text{ given}, \tag{11}$$

where $\delta \in (0, 1)$ is the capital depreciation rate. When $\tau_t = 0$, our model collapses to the original Harrison (2001) economy under laissez faire.

The first-order conditions for the household's dynamic optimization problem are

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[4]By contrast, Chang et al. (2015) examine the same model economy under non-distortionary lump-sum taxes (denoted as T_t) whereby the household's budget constraint is $C_t + p_t I_t + T_t = r_t K_t + w_t L_{t_t}$.

$$C_t L_t^\gamma = (1 - \tau_t)w_t, \tag{12}$$

$$\frac{1}{C_t} = \frac{\beta}{C_{t+1}} \left[\frac{(1 - \tau_{t+1})r_{t+1} + (1 - \delta)p_{t+1}}{p_t} \right], \tag{13}$$

$$\lim_{t \to \infty} \beta^t \frac{K_{t+1}}{C_t} = 0, \tag{14}$$

where (12) equates the slope of household's indifference curve to the after-tax real wage rate, (13) is the Euler equation for intertemporal choices of private consumption, and (14) is the transversality condition.

2.3 Government

The government collects its tax revenues $\tau_t Y_t$ to pay public spending on goods and services G_t produced by the consumption and investment sectors, and balances its budget each period. Hence, its period budget constraint is given by

$$\tau_t Y_t = G_t = G_{ct} + p_t G_{It}, \tag{15}$$

where G_{ct} and G_{It} are quantities of consumption and investment goods, respectively, purchased by the government. In our subsequent analyses, (15) becomes $G_t = \tau Y_t$ under the GH balanced-budget formulation, and $G = \tau_t Y_t$ per the SU specification. In addition, public expenditures on the consumption and investment goods are postulated to be constant fractions of the government's total spending:

$$\frac{G_{ct}}{G_t} = \phi \quad \text{and} \quad \frac{p_t G_{It}}{G_t} = 1 - \phi, \tag{16}$$

where $0 \leq \phi \leq 1$.[5] Finally, combining (10) and (15) yields the following aggregate resource constraint for the economy

$$C_t + p_t I_t + G_t = Y_t. \tag{17}$$

2.4 Competitive Equilibrium and Macroeconomic (In)stability

Since firms use identical private technologies and face the same factor prices across the two sectors, the fractions of capital and labor inputs utilized in the consumption sector are equal,

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[5]Under non-distortionary lump-sum taxes, Chang et al. (2015) postulate that government purchases of consumption and investment goods are constant fractions of (i) their respective sectoral output, and (ii) the economy's total output.

$$\frac{K_{ct}}{K_t} = \frac{L_{ct}}{L_t} \equiv \mu_t. \tag{18}$$

Using (1)-(2), (4)-(7) and (18), the equilibrium relative price of investment goods can be expressed as

$$p_t = \frac{1}{\left[(1 - \mu_t) K_t^\alpha L_t^{1-\alpha}\right]^\theta}. \tag{19}$$

We focus on competitive symmetric equilibria that consist of a set of prices $\{p_t, r_t, w_t\}_{t=0}^\infty$ and allocations $\{C_t, L_t, K_{t+1}\}_{t=0}^\infty$ which satisfies the household's and firms' first-order conditions. The equalities of demand by households and supply by firms in the consumption and investment sectors are given by $C_t + G_{ct} = Y_{ct}$ and $I_t + G_{It} = Y_{It}$. Moreover, both the capital and labor markets will clear whereby

$$K_{ct} + K_{It} = K_t, \tag{20}$$

$$L_{ct} + L_{It} = L_t. \tag{21}$$

Next, it is straightforward to derive our model' interior steady state(s) under either the GH or SU fiscal policy rule, respectively. We then take log-linear approximations to the economy's equilibrium conditions in a neighborhood of each steady state to obtain the following dynamical system:

$$\begin{bmatrix} \hat{K}_t \\ \hat{C}_t \end{bmatrix} = \mathbf{J} \begin{bmatrix} \hat{K}_{t+1} \\ \hat{C}_{t+1} \end{bmatrix}, \quad \hat{K}_0 \text{ given}, \tag{22}$$

where hat variables represent percentage deviations from their respective steady-state values, and $\mathbf{J}$ is the Jacobian matrix of partial derivatives of the transformed dynamical system. The economy exhibits saddle-path stability and equilibrium uniqueness when one eigenvalue of J lies inside and the other outside the unit circle. When both eigenvalues are outside the unit circle, the steady state becomes an indeterminate sink around which there are a continuum of stationary equilibrium trajectories that display cyclical fluctuations driven by agents' animal spirits or sunspots. When both eigenvalues are inside the unit circle, the steady state becomes a totally unstable source.

3   Quantitative Results

This section quantitatively examines the interrelations between the government's tax/spending policy and equilibrium (in)determinacy within a calibrated version of our two-sector RBC

model under the GH or SU balanced-budget formulation. Each period in the model is taken to be one quarter. As in Benhabib and Farmer (1996), Harrison (2001), Chang et al. (2015) and many other studies in the real business cycle literature, the labor share of national income, $1 - \alpha$, is chosen to be 0.7; the discount factor, β, is set to be $\frac{1}{1.01}$; the labor supply elasticity, γ, is equal to 0 (*i.e.* indivisible labor, *à la* Hansen [1985] and Rogerson [1988], that is infinitely elastic); and the capital depreciation rate, δ, is fixed at 0.025. Based on Harrison's (2003) empirical findings on the U.S. economy, we set the degree of productive externalities for investment θ to be 0.108.

3.1 GH Formulation with Constant Tax Rates

In this case with a time-invariant income tax rate τ, the economy possesses a unique interior steady state at which the fraction of factor inputs allocated to producing consumption goods is given by

$$\mu = 1 - \tau\left(1 - \phi\right) - \frac{\alpha\delta}{1/\beta - 1 + \delta}, \tag{23}$$

and the corresponding expressions of all remaining endogenous variables can be easily derived. Given the aforementioned parameterization, Figure 1 illustrates our model's local stability properties with Guo and Harrison's (2004) fiscal formulation in terms of the fraction of government spending from the consumption sector ϕ versus the fixed income tax rate τ. In particular, the $\phi - \tau$ space can be divided into regions of "Saddle" and "Sink". Below are our findings:

Result 1. When $0 \leq \tau \leq 0.098$, the economy always exhibits an indeterminate steady state, regardless of how public expenditures are divided between consumption and investment goods.

To understand the intuition behind this result, we note that the private-consumption Euler equation (13) under the GH fiscal policy rule is modified to

$$\frac{C_{t+1}}{C_t} = \beta\left[\frac{(1-\tau)r_{t+1} + (1-\delta)p_{t+1}}{p_t}\right]. \tag{24}$$

Start the laissez-faire ($\tau = 0$) model from its steady state at period t, and suppose that agents become optimistic about the economy's future. Acting upon this non-fundamental belief change, the representative household will consume less and invest more today, which in turn raises next period's capital stock, hours worked, output, and consumption. As a result, the

left-hand side of (24) will rise. For this alternative dynamic path to be justified as a self-fulfilling equilibrium, the (price-weighted) rate of return on K_{t+1} net of depreciation, *i.e.* the right-hand side of (24), needs to increase as well.

It turns out that our model's local dynamics are governed by the relative strength of two opposite effects. On the one hand, an increase in today's private investment that raises K_{t+1} will lead to a lower real interest rate r_{t+1} because of diminishing marginal product of capital as $\alpha(1+\theta) < 1$. Therefore, this *MPK effect* causes the right-hand side of (24) to fall. On the other hand, due to the presence of positive productive externalities in the investment sector, the economy's social production possibility frontier which traces out the trade-off between private consumption and investment spending is convex to the origin. As a result, the relative price of investment p_t will decrease upon agents' optimism that shifts more capital and labor inputs into producing investment goods ($\frac{\partial p_t}{\partial(1-\mu_t)} < 0$; see equation 19). Consequently, this *price effect* causes right-hand side of (24) to rise.

As in the no-government economy of Harrison (2001, p. 756) with identical values on α, β, γ and δ, the origin of Figure 1 with $\tau = G_t = 0$ indicates that since price effect is quantitatively stronger, the model's steady state is an indeterminate sink. Intuitively, the equilibrium rate of return on capital will rise to fulfill the household's initial rosy expectations in that the calibrated degree of investment sector-specific externalities ($\theta_{US} = 0.108$) is higher than the critical level of 0.0773. In this environment, positive income taxation ($\tau > 0$) generates a downward shift of the convex social production possibility frontier, which in turn induces agents to reduce their optimism-driven consumption as well as investment expenditures. It follows that the aforementioned price effect that helps make for multiple equilibria becomes weaker, whereas the corresponding after-tax MPK effect is strengthened because of a lower net real interest rate $(1-\tau)r_{t+1}$. Figure 1 shows that for all values of $\phi \in [0, 1]$ under the benchmark parameterization, the price effect continues to dominate when $0 < \tau \leq 0.098$. That is, regardless of how the government separates its purchases between consumption and investment goods, indeterminacy and sunspots always occur in our baseline model with "relatively low" (constant) income tax rates.

Result 2. When $\tau > 0.098$, the model's steady state can either be a saddle point or a sink. Moreover, the threshold level for the public-consumption share that yields equilibrium indeterminacy, denoted as $\phi_{\min}$, is monotonically increasing with respect to the income tax rate, *i.e.* $\frac{\partial \phi_{\min}}{\partial \tau} > 0$.

We first observe that for a given level of $\tau > 0.098$, the resulting after-tax MPK effect

outweighs the price effect as long as the fraction of government spending on consumption goods ϕ is zero, as shown in the vertical axis of Figure 1, or "relatively small". In this case, our model exhibits saddle-path stability and equilibrium uniqueness since the fall in the right-hand side of (24) is sufficiently high to invalidate agent's initial anticipation of a higher rate of return on capital. It follows that raising the public-consumption share to the corresponding critical level $\phi_{\min}$ ($= 0.5665$ when $\tau = 0.2$, which is also the steady-state ratio of government purchases to the economy's total output) or above leads to a dominating price effect. This in turn destabilizes the macroeconomy by generating belief-driven business cycle fluctuations.

Per the same reasoning for Result 1 under positive income taxation, an increase in the tax rate will enhance the after-tax MPK effect with a larger decrease in $(1-\tau)r_{t+1}$ and weaken the price effect with a smaller reduction in p_t. In this case, $\phi_{\min}$ will rise ($\frac{\partial \phi_{\min}}{\partial \tau} > 0$) to overturn the relative magnitude of these two opposing forces, which in turn change the model's steady state from being a saddle point to a sink. In sum, Results 1 and 2 together indicate that endogenous business cycles driven by agents' animal spirits do not arise within the benchmark parameterization of our two-sector RBC model under the GH balanced-budget formulation when (i) the income tax rate $\tau \in (0, 0.098]$ and the public-consumption share $\phi \in [0, 1]$; or (ii) $\tau > 0.098$ and $\phi < \phi_{\min}$.

Although infinitely elastic labor hours ($\gamma = 0$) are commonly postulated in many early RBC-based indeterminacy studies such as Benhabib and Farmer (1994) and Farmer and Guo (1994), recent empirical research by Chetty et al. (2011, 2012) reports that modern macroeconomic calibrations have taken on a larger labor supply elasticity than that observed in the micro-level evidence. Accordingly, the next result explores our model economy with lower labor supply elasticities.

Result 3. Given the same baseline calibrations on α, β, δ and θ, the economy always exhibits saddle-path stability and equilibrium uniqueness when $\gamma \geq 0.208$, regardless of the income tax rate and the sectoral composition of government spending on goods and services.

With less elastic labor supply (or when γ is higher than zero), agents are less willing to move out of leisure into hours worked at period $t+1$ upon an expected increase in the rate of return on today's investment. This will lead to smaller increases in L_{t+1} and r_{t+1} via firms' capital demand, which in turn dampens the representative household's initial optimism. Our numerical experiments find that independent of the government's tax and spending policies (*i.e.* for all combinations of $\tau \in [0, 1)$ and $\phi \in [0, 1]$), the model's steady state is a locally determinate saddle point provided the labor supply elasticity parameter is "sufficiently high"

to generate a relatively stronger after-tax MPK effect. Since there are no shocks to economic fundamentals within our model economy, the resulting unique equilibrium with $\gamma \geq 0.208$ will not display any cyclical fluctuations.

3.2 SU Formulation with Constant Government Spending

This subsection quantitatively investigates our model's equilibrium dynamics with Schmitt-Grohé and Uribe's (1997) balanced-budget rule whereby the government endogenously sets the proportional income tax rate to finance a pre-specified constant level of public expenditures. In this case, the number of the model's interior steady state(s) can be zero, one or two. Specifically, it is straightforward to show that the government's tax revenue $(= G)$ is equal to zero when the steady-state tax rate $\tau_{ss} = 0$ or 1^{6}; and that the analytical relationship between $G > 0$ and $\tau_{ss} \in (0, 1)$ is given by

$$\frac{\partial G}{\partial \tau_{ss}} = -\frac{G}{\tau_{ss}} \left\{ \frac{\lambda \tau_{ss}^2 - [\lambda(1-\alpha) - \alpha\delta(1-\alpha\theta)]\tau_{ss} - \alpha\delta[1-\alpha(1+\theta)]}{(1-\tau_{ss})(\alpha\delta + \lambda\tau_{ss})[1-\alpha(1+\theta)]} \right\}, \tag{25}$$

where $\lambda \equiv (1-\phi)(1/\beta - 1 + \delta) - \alpha\delta$. Next, under our benchmark calibrations of α, β, γ $(= 0)$, δ and θ, Figure 2 depicts the Laffer curve-type relationship between G and τ_{ss} when the public-consumption share ϕ takes on the extreme value of 0 or 1. Notice that there exists a unique steady-state tax rate in the interval $(0, 1)$ that maximizes the level of public spending denoted as G^{*}.[7] This in turn implies that for a given level of $\phi \in [0, 1]$, our model possesses zero (two) interior steady states(s) provided $G > (<) G^{*}$, as shown in Figure 3. Therefore, any small deviation from the revenue-maximizing steady state with G^{*} and $\tau_{ss}(G^{*})$ will lead to its disappearance, or the emergence of dual stationary equilibria. This result implies that the economy undergoes a saddle-node bifurcation which may cause the hard loss of equilibrium stability, *i.e.* the occurrence of a radical qualitative change in the behavior of the dynamical system (22), as the government purchases of goods and services pass through the critical value G^{*}. Figure 3 also shows that when $G \in (0, G^{*})$, the resulting steady states in our model are characterized by τ_{ss}^1 and τ_{ss}^2, where $\tau_{ss}^1 < \tau_{ss}(G^{*}) < \tau_{ss}^2$; and their associated local stability properties will be discussed below.

Result 4. Under our benchmark parameterization on α, β, γ, δ and θ, together with a

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[6]Since we focus on the model's equilibrium conditions in a neighborhood of its steady state, the specification with $G = \tau_{ss} = 0$ displays local indeterminate dynamics that are identical to those under laissez faire.

[7]Setting $\frac{\partial G}{\partial \tau_{ss}} = 0$ yields a quadratic equation in τ_{ss}, as in the denominator for the curly brackets of (25). Solving this quadratic equation numerically under the baseline calibrated parameter values, we obtain one root for $\tau_{ss} \in (0, 1)$ and the other for $\tau_{ss} > 1$.

positive level of $G < G^*$, the low-tax steady state is a sink whereas the high-tax steady state is a saddle point, regardless of the sectoral composition of government spending on goods and services.

To understand the intuition for this finding, we note that the private-consumption Euler equation with the SU tax formulation is given by (13). When households become optimistic about the economy's future and decide to invest more today, the government is forced to decrease next-period's tax rate τ_{t+1} as total output Y_{t+1} rises. Since $\tau_{ss}^1 < \tau_{ss}^2$, the corresponding steady-state GDP levels will exhibit $Y_{ss}^1 > Y_{ss}^2$, which in turn implies that $r_{ss}^1 > r_{ss}^2$, where r is the capital rental rate. It follows that in a neighborhood of the steady state with τ_{ss}^1 and Y_{ss}^1, the after-tax real interest rate $(1 - \tau_{t+1})r_{t+1}$ is higher than that around τ_{ss}^2 and Y_{ss}^2. As a result, the after-tax MPK effect discussed earlier is weaken around the low-tax steady state, which helps fulfill agents' initial rosy anticipation with belief-driven cyclical fluctuations. On the other hand, in the presence of SU's countercyclical (or regressive) balanced-budget rule, a higher tax rate τ_{ss}^2 together with a lower output Y_{ss}^2 will enhance the after-tax MPK effect, invalidating the household's original optimism and thus saddle-path stability emerges. In sharp contrast to Result 2 under Guo and Harrison's (2004) flat-tax schedule, these (in)determinacy results with the SU fiscal policy are obtained irrespective of the sectoral composition of public expenditures among consumption and investment goods.

As in the previous subsection, our next result examines the model's equilibrium dynamics with lower labor supply elasticities.

Result 5. When $\gamma \leq 0.068$, while keeping other parameter values unchanged, the economy displays the same local stability properties as those described in Result 4.

Per the same reasoning for Result 3, less elastic hours worked ($\gamma > 0$) strengths the after-tax MPK effect since it will reduces the net rate of return on investment $(1 - \tau_{t+1})r_{t+1}$ upon agents' optimistic expectations, thus causing the right-hand side of (13) to fall. Our numerical experiments find that for all values of $\phi \in [0, 1]$, the price effect continues to dominate around the low-tax steady state provided the labor supply elasticity parameter is "sufficiently low" with $\gamma \leq 0.068$. In addition, the high-tax steady state is always a saddle point, regardless of how elastic the household's labor supply decision is and how public expenditures are divided between consumption and investment goods. In sum, Results 4 and 5 together imply that the occurrence of endogenous business cycles driven by agents' animal spirits within our two-sector RBC model under the SU balanced-budget formulation does not depend on the government's sectoral spending policy governed by ϕ.

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4   Conclusion

In this paper, we have examined the quantitative interrelations between government purchases, distortionary income taxation and equilibrium (in)determinacy in Harrison's (2001) representative-agent macroeconomy with two production sectors. Under Guo and Harrison's (2004) balanced-budget formulation, we find that regardless of how public expenditures are divided between consumption and investment goods, our benchmark model always exhibits indeterminacy and sunspots provided the (fixed) income tax rate does not exceed a critical value. When the tax rate is raised to a higher level, the model's steady state can either be a saddle point or a sink; and the threshold for the public-consumption share above which aggregate instability occurs is shown to be monotonically increasing in the income tax rate. Under Schmitt-Grohé and Uribe's (1997) fiscal formulation with a pre-specified (constant) level of government spending lower than the revenue-maximizing counterpart, we find that independent of the sectoral composition of public expenditures, the low-tax steady state is an indeterminate sink and the high-tax steady state is a saddle point in our baseline parameterization. To summarize, this paper shows that in the context of a two-sector real business cycle model with positive productive externalities in investment, whether and how the government's sectoral spending policies generate macroeconomic (in)stability depends crucially on the exact formulation of the underlying fiscal policy rule.

This paper can be extended in several directions. For example, it would be worthwhile to explore our model economy under a non-separable preference formulation *à la* Abad et al. (2017); a non-balanced budget with national debt *à la* Schmitt-Grohé and Uribe's (1997, pp. 990-991); a progressive tax policy *à la* Guo and Lansing (1998); or productive/utility-generating government spending *à la* Guo and Harrison (2008). These possible extensions will allow us to examine the robustness of this paper's theoretical results and policy implications, as well as further enhance our understanding of the (de)stabilization effects of public expenditures within a multi-sector representative-agent macroeconomy. We plan to pursue these research projects in the near future.

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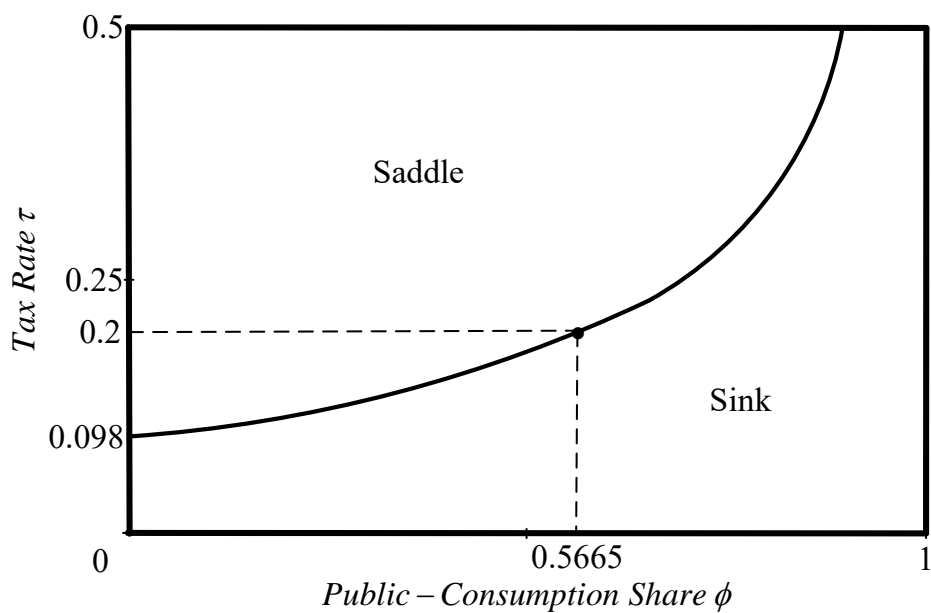
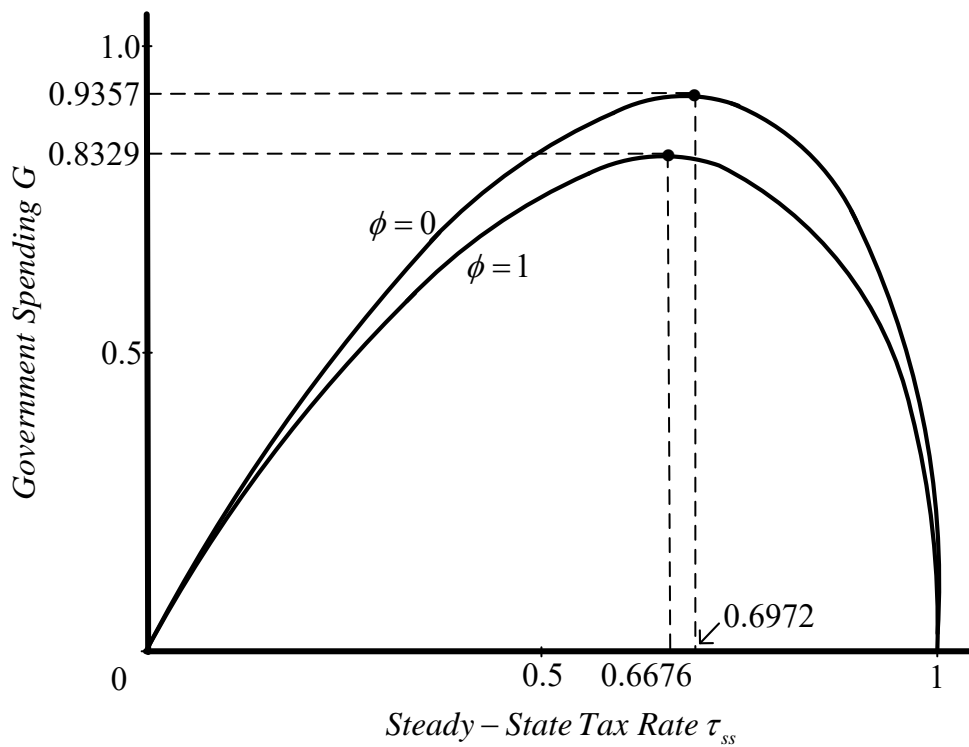Figure 1. Benchmark Model under the GH Balanced-Budget Rule
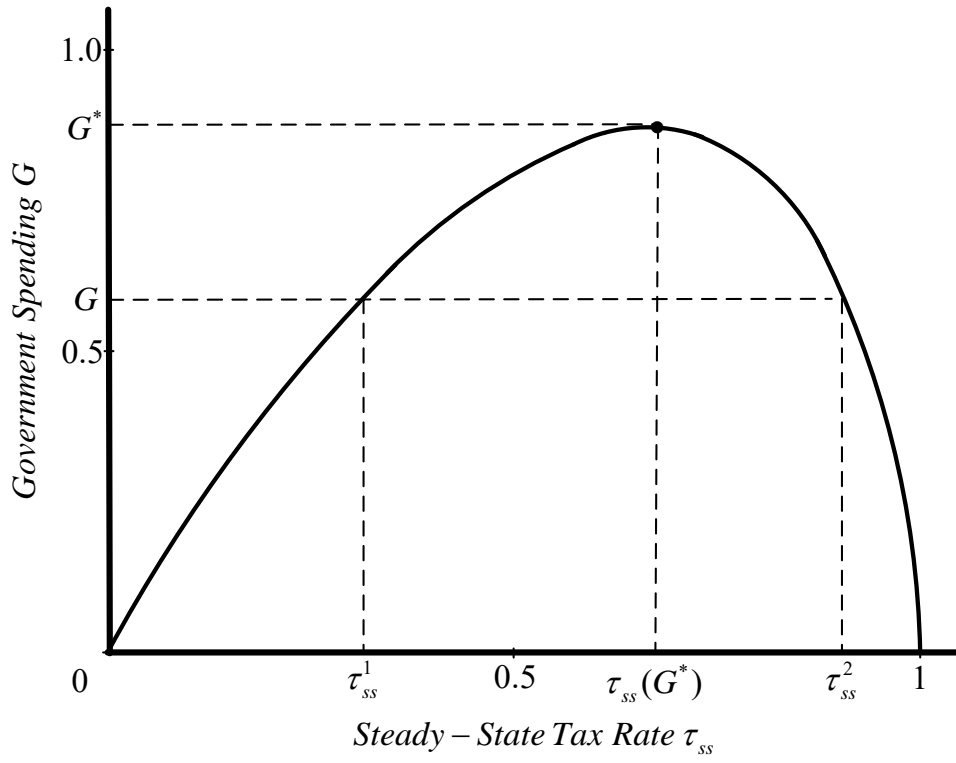

Figure 2. Benchmark Model under the SU Balanced-Budget Rule

Figure 3. Steady-State Laffer Curve