The Cyclical Behavior of Unemployment and Vacancies with Loss of Skills during Unemployment

Victor Ortego-Marti*

University of California Riverside

November 11, 2013

Abstract

This paper studies the cyclical fluctuations in unemployment and vacancies in a search and matching model in which workers lose skills during periods of unemployment. Firms' profits fluctuate more because aggregate productivity affects the economy's human capital level. Moreover, wages for workers with lower levels of human capital are closer to the value of non-market time, leading to more rigid wages. Fluctuations in the vacancy-unemployment ratio are larger than is the case in the baseline search and matching model. For mid-range values of non-market time the improvement is substantial, and the model accounts for most observed labor market fluctuations.

*Department of Economics, University of California Riverside. Sproul Hall 3132, Riverside CA 92521. *Email:* victor.ortego-marti@ucr.edu. *Phone:* 951-827-1502. I am grateful to Alwyn Young for his comments and suggestions. I also thank Marcelle Chauvet, Jang-Ting Guo, Urmee Kahn, Dave Malueg and seminar participants at UC Riverside for helpful discussions.

1. Introduction

Unemployment has dramatic implications for workers' earnings and reemployment prospects. Unemployed workers often suffer important and permanent human capital losses, and workers with long unemployment spells become increasingly detached from the labor market. These negative impacts of unemployment on workers are absent from the baseline version of the Diamond-Mortensen-Pissarides model of equilibrium unemployment (henceforth DMP). At the same time, this workhorse model cannot generate fluctuations in unemployment and vacancies in line with those we observe in the data. Because these fluctuations are driven by firms' hiring decisions, the effect of unemployment on workers' human capital has important implications for the volatility of these fluctuations. Intuitively, firms are likely to hire fewer workers when the pool of unemployed workers worsens in recessions, and to hire more workers when the pool improves in booms. This leads to more volatile unemployment and vacancies. This paper examines whether introducing the loss of human capital during unemployment into an otherwise standard search model generates more sizable fluctuations in unemployment and vacancies.

The influential findings in Shimer (2005) show that, compared to empirical observations, the DMP model is unable to generate large enough fluctuations in its key variable, the ratio of vacancies to unemployment. As Shimer (2005) shows, the problem in the DMP framework is that wages respond too much to changes in productivity.[1] When labor productivity increases in a boom, the wage responds almost one to one, so the profits from posting a vacancy barely change. This dampens firms' incentive to hire more workers, resulting in only a mild increase in the number of posted vacancies. With vacancies unchanged, workers find jobs at the same rate and unemployment remains roughly constant. The same mechanism explains the mild response of vacancies and unemployment to lower labor productivity in recessions.

I present a search and matching model in which workers gradually lose human capital during unemployment. In this framework, labor market volatility increases because firms' hiring incentives fluctuate more. This happens for two reasons. First, aggregate productivity affects the human capital level of the pool of unemployed workers from which firms hire. When workers lose skills during unemployment, labor productivity depends both on aggregate productivity and the economy's human capital level. In the model this human capital level depends on workers' unemployment history—the cumulative duration of their

[1] See also Costain & Reiter (2008) and Hall (2005) for work with similar findings.

unemployment spells, since workers that have experienced more and longer unemployment spells have lower human capital levels. Because the economy's human capital improves when workers find jobs more quickly, and worsens when they take longer to find jobs, hiring becomes more profitable in booms and less so in recessions compared to a model without these human capital fluctuations. This leads firms to post more vacancies in booms and fewer vacancies in recessions.

The second mechanism increasing labor market fluctuations operates through wages. As workers accumulate more unemployment history, their productivity and wages decrease. Therefore, the wages of workers with longer unemployment history are closer to the value of non-market time—which includes unemployment benefits, home production and leisure. This makes wages more rigid for this group of workers, in the sense that their wages respond proportionally less to changes in aggregate productivity. As a result profits from hiring fluctuate more, leading to larger swings in the vacancy-unemployment rate ratio.

I study to what extent the model with loss of skills during unemployment is able to generate sizable fluctuations in the vacancy-unemployment ratio. I derive an expression for these fluctuations that depends only on a set of parameters standard in the literature. The model generates more fluctuations than the baseline DMP model, but the size of this improvement depends on the value of non-market time. Although the improvement is mild for low values of non-market time, the improvement in fluctuations is large for the mid-range values used in the literature. For this range of values the model generates fluctuations that closely match those observed in the data. In addition to generating larger fluctuations than the standard model, the model with human capital losses during unemployment produces more realistic labor market responses to changes in unemployment benefits.

Related literature.

This paper is motivated by the findings in Shimer (2005), Hall (2005) and Costain & Reiter (2008).[2] A number of papers address these findings. Hall & Milgrom (2008) consider alternating wage offers instead of the typical Nash bargaining assumption, which leads to some form of wage rigidity. Hagedorn & Manovskii (2008) use an alternative calibration to Shimer (2005) that leads to larger fluctuations in firms' profits. Pissarides (2009) adds a fixed component to vacancy costs, which makes the expected cost of posting a vacancy less volatile.[3] A number of papers generate larger fluctuations through some endogenous

[2]See also Mortensen & Nagypal (2007) and (2007b) for further discussion of the Shimer critique.

[3]See also the related work in Silva & Toledo (2009) and (2013).

wage rigidity. With wage rigidity, hiring profits become more volatile, leading to larger labor market fluctuations.[4] However, none of these papers look at the role of human capital depreciation during unemployment as a source of labor market fluctuations. The advantage of the approach in this paper is that the magnitude of the amplification in fluctuations depends on the rate at which workers lose skills during unemployment, which is directly observable from empirical micro evidence.

This paper is also related to a literature that combines search frictions with human capital depreciation during unemployment. In Pissarides (1992) unemployment becomes more persistent when unemployed workers lose skills during unemployment. Ljungqvist & Sargent (1998) offer an explanation for the high levels of unemployment in Europe compared to the US.[5] Coles & Masters (2000) show that job creation subsidies are a more efficient policy than training for the unemployed. Ortego-Marti (2012) shows that a model similar to that in this paper generates large amounts of frictional wage dispersion.[6] However, these papers do not investigate the effect on labor market fluctuations.

There is substantial empirical evidence on the effects of unemployment on workers' wages. Fallick (1996) and Kletzer (1998) review some of the findings in the early job displacement literature.[7] Although the size of the earnings losses depends on the data source and the period or location of the study, this literature finds large and very persistent earning losses among displaced workers.[8] This paper also draws from empirical evidence in Ortego-Marti (2012) about the effects of unemployment history on workers' wages.

The paper begins by describing the labor market in section 2. Section 3 derives the equilibrium wage and shows that it is closer to the value of non-market time for workers with longer histories of unemployment. Section 4 derives the distribution of unemployment history among workers. Using the equilibrium conditions in previous sections, section 5

[4] Menzio (2005) and Kennan (2010) achieve wage rigidity by adding some asymmetric information about match's productivity. Eyigungor (2010) combines specific capital and embodied technology. Rudanko (2011) studies the type of contracts offered by firms when workers are risk averse and unable to fully smooth consumption. In Gertler & Trigari (2009) wage rigidity is the result of staggered wages. Beauchemin & Tasci (2013) rely on shocks to job separations and matching efficiency instead of some form of endogenous wage rigidity, but find that this leads to counterfactual cyclicality of job separations.

[5] See also the related papers by den Haan, Haefke & Ramey (2005), and Ljungqvist & Sargent (2007) and (2008).

[6] See also Shimer & Werning (2006) and Pavoni (2011) for a study of the implications of the loss of skills during unemployment for unemployment insurance, and Fujita (2012) for the the effects on the secular decline in the job separation rate.

[7] See Couch & Placzek (2010) and von Wachter, Song & Manchester (2009) for more recent results.

[8] Couch & Placzek (2010), Jacobson, LaLonde & Sullivan (1993), Schoeni & Dardia (2003), von Wachter, Song & Manchester (2009) use administrative data; Ruhm (1991) and Stevens (1997) use the PSID; and Carrington (1993), Farber (1997), Neal (1995), Topel (1990) use the Displaced Worker Survey (DWS) supplement of the Current Population Survey (CPS).

provides the equilibrium labor market tightness and its elasticity with respect to labor productivity, which measures labor market fluctuations. Finally, section 6 shows that for a mid-range value of non-market time labor market fluctuations are significantly higher in the model with unemployment history and similar to those observed in the data.

2. Search and Matching Model with Unemployment History

The labor market is subject to search and matching frictions.[9] Workers search for jobs and firms for workers. The number of matches formed is given by a matching function $m(N^U, N^V)$, where N^U is the number of unemployed workers and N^V is the number of vacancies. I assume the usual conditions for the matching function, that it is increasing in both its arguments and concave, and that it displays constant returns. With these assumptions, workers find jobs at a rate $f(\theta) = m(1, \theta)$, and firms fill their vacancies at a rate $q(\theta) = m(\theta^{-1}, 1)$, where labor market tightness θ is the vacancy-unemployment ratio, so $\theta \equiv N^V/N^U$. Separations occur at an exogenous rate s.

Within this standard DMP framework, I further assume that workers gradually lose human capital during unemployment at a constant rate δ. Because longer unemployment spells lead to larger human capital losses, a worker's human capital level depends on her complete history of unemployment spells. I use the term unemployment history to refer to a worker's cumulative duration of unemployment spells, and denote it by γ. Given unemployment history γ, the worker's human capital level is given by $h(\gamma)$. This human capital is net of other characteristics.[10] Normalizing $h(0) = 1$, the human capital of a worker with unemployment history γ is given by $h(\gamma)$.

There is some aggregate productivity in the economy that is common to all matches formed, which I denote by p. Once the firm and the worker meet, the productivity of the match is given by the product of aggregate productivity p and the worker's human capital level $h(\gamma)$.

Although workers are identical when they join the labor force, they find and lose jobs

[9]See Pissarides (2000) for an exposition of the search and matching approach to the labor market.

[10]This paper investigates fluctuations in unemployment-vacancy ratio generated by the behavior of unemployment history, hence the focus on human capital net of other observables such as education, occupation, etc. How these other aspects of human capital affect labor market fluctuations is out of the scope of this paper.

at random because of search frictions. This generates endogenous distributions $G^U(\gamma)$ and $G^E(\gamma)$ of unemployment history among unemployed and employed workers. To allow for stationary distributions $G^U(\gamma)$ and $G^E(\gamma)$, I assume that at a rate μ workers leave the labor force. Finally, during unemployment, workers receive payment flows b, which includes unemployment benefits, leisure and home production, and firms post vacancies at a flow cost k.

I denote by $U(\gamma)$ and $W(\gamma, p)$ the value functions of unemployment and employment given unemployment history γ. Given the previous assumptions, $U(\gamma)$ and $W(\gamma, p)$ satisfy the following Bellman equations

$$(r+\mu)U(\gamma) = b + f(\theta)(\max\{W(\gamma, p), U(\gamma)\} - U(\gamma)) + \frac{dU(\gamma)}{d\gamma}, \tag{1}$$

$$(r+\mu)W(\gamma, p) = w(\gamma, p) - s(W(\gamma, p) - U(\gamma)), \tag{2}$$

where r is the interest rate. Intuitively, (1) satisfies that the return to $U(\gamma)$ using the effective discount rate $r+\mu$—the left-hand side—, must equal the flow payments and changes in the capital value of $U(\gamma)$—the right-hand side. The right-hand side of (1) captures that unemployed workers receive payment flows b, they find a job at a rate $f(\theta)$, which carries a net gain of $W(\gamma, p) - U(\gamma)$ if the match is productive enough, and that the value of unemployment $U(\gamma)$ depreciates while the worker stays unemployed. Similarly, the right-hand side of (2) captures that employed workers receive wages $w(\gamma, p)$ and lose their job at a rate s, which carries a net loss of $W(\gamma, p) - U(\gamma)$.

Similarly, using $J(\gamma, p)$ to denote the value functions of a filled position, the following Bellman equation hold

$$(r+\mu)J(\gamma, p) = h(\gamma)p - w(\gamma, p) - sJ(\gamma, p). \tag{3}$$

Equation (3) captures that when the firm employs a worker with unemployment history γ, the match produces $h(\gamma)p$, the firm must pay wages $w(\gamma, p)$, and the job is destroyed at a rate s, which carries a net loss $J(\gamma, p)$.

I assume free entry in the market for vacancies, so firm post vacancies until the value of a vacancy is zero, i.e. $V = 0$. Because of search frictions, there are some rents from forming a match that must be split between the worker and the firm. I assume that this surplus from matching is split according to Nash Bargaining. Therefore, wages satisfy that workers get a share β of the surplus, and firms a share $1 - \beta$, where β is workers' bargaining power. As a result, Nash Bargaining implies

$$\beta J(\gamma, p) = (1 - \beta)(W(\gamma, p) - U(\gamma)). \tag{4}$$

In the model, workers that are too unproductive become unhireable. Their productivity is so low that the match yields a negative surplus. At that point neither the firm nor the worker find it profitable to form a match. This is formally captured in the following result.

Proposition 1. *There exists a unique $\bar{\gamma}$ such that matches are not formed if $\gamma > \bar{\gamma}$. In particular, the following results hold*

$$(r + \mu)U(\bar{\gamma}) = b \tag{5}$$

$$h(\bar{\gamma})p = w(\bar{\gamma}, p) \tag{6}$$

The formal proof is included in the appendix, but I provide some intuition here. Under the Nash Bargaining assumption, the firm must compensate the worker for her outside option, in this case $U(\gamma)$. This outside option includes the constant value of non-market time b. Because the value of output declines with unemployment history, output will be unable to cover for payments b to the worker if unemployment history is too large. At some point $\bar{\gamma}$ the value of the surplus is zero, and from (3) workers absorb all the output in the form of wages, i.e. $w(\bar{\gamma}, p) = h(\gamma)p$. Therefore, workers with unemployment history larger than $\bar{\gamma}$ do not lead to profitable matches and thus are not hired. They simply collect unemployment benefits b.[11]

Given this result, the Bellman equation (1) simplifies to

$$(r + \mu)U(\gamma) = b + f(\theta)(W(\gamma, p) - U(\gamma)) + \frac{dU(\gamma)}{d\gamma}\ , \ \forall \gamma \leq \bar{\gamma}. \tag{7}$$

Further, a posted vacancy with value V satisfies the following Bellman equation

$$rV = -k + q(\theta) \int_0^{\bar{\gamma}} J(\gamma, p) \frac{dG^U(\gamma)}{G^U(\bar{\gamma})}. \tag{8}$$

Equation (8) captures that the firm must pay a vacancy cost k while the vacancy remains

[11]Coles & Masters (2000) find a similar result, in their model long-term unemployment is the result of constant benefits with loss of skills during unemployment.

posted, and at a rate $q(\theta)$ the firm draws a job candidate from the pool of unemployed workers, with workers' unemployment history distributed according to $G^U(\gamma)$.

3. Wages

I begin by expressing wages as a function of productivity and $U(\gamma)$. I then proceed to solve for $U(\gamma)$ to derive the final expression for wages. Using (2) yields

$$(r+\mu+s)(W(\gamma,p)-U(\gamma)) = w(\gamma,p)-(r+\mu)U(\gamma). \tag{9}$$

Combing the above expression with (3) and Nash Bargaining gives wages as a function of $U(\gamma)$

$$w(\gamma,p) = \beta h(\gamma)p + (1-\beta)(r+\mu)U(\gamma). \tag{10}$$

Consider now equation (7). Solving for $U(\gamma)$ in (7) as a differential equation in γ gives

$$U(\gamma) = \int_{\gamma}^{\bar{\gamma}} (e^{-\alpha_1(\Gamma-\gamma)}b)d\Gamma + f(\theta)\int_{\gamma}^{\bar{\gamma}} \left(e^{-\alpha_1(\Gamma-\gamma)}\frac{\beta h(\gamma)p}{r+\mu+s}\right)d\Gamma + e^{-\alpha_1(\bar{\gamma}-\gamma)}U(\bar{\gamma}), \tag{11}$$

where $\alpha_1 \equiv r+\mu+\beta f(\theta)(r+\mu)/(r+\mu+s)$. The above expression provides some useful intuition. The value of unemployment captures all the future flows from unemployment, using α_1 as the discount factor. While unemployment history is lower than $\bar{\gamma}$ workers receive at least flow payments b—first term on the right-hand side of (11). In addition to this flow, at a rate $f(\theta)$ workers find a job and get a share β of output—second term on the right-hand side. Finally, when workers accumulate unemployment history $\bar{\gamma}$, they drop out of the labor force and no longer find jobs. At that point, the value of unemployment simply becomes $U(\bar{\gamma}) = b/(r+\mu)$.

Combing the result in proposition 1 with the wage in (10) gives the following condition for human capital at the terminal level of unemployment history $\bar{\gamma}$

$$h(\bar{\gamma})p = b. \tag{12}$$

Solving equation (11) and using (12), $(r+\mu)U(\gamma)$ simplifies to

$$(r+\mu)U(\gamma) = \left(\frac{(r+\mu+s+\delta\frac{r+\mu+s}{r+\mu})e^{-\alpha_1(\bar{\gamma}-\gamma)}}{r+\mu+s+\beta f(\theta)+\delta\frac{r+\mu+s}{r+\mu}} + \frac{(r+\mu+s)(1-e^{-\alpha_1(\bar{\gamma}-\gamma)})}{r+\mu+s+\beta f(\theta)} \right) b$$

$$+ \left(\frac{\beta f(\theta)}{r+\mu+s+\beta f(\theta)+\delta\frac{r+\mu+s}{r+\mu}} \right) h(\gamma)p. \qquad (13)$$

I now derive an expression for wages $w(\gamma,p)$ that provides some helpful intuition and allows for comparison with the baseline DMP model. Define the surplus $S(\gamma,p)$ of a match as

$$S(\gamma,p) = J(\gamma,p) + W(\gamma,p) - U(\gamma). \qquad (14)$$

Clearly, Nash bargaining implies that $W(\gamma,p) - U(\gamma) = \beta S(\gamma,p)$. Solving $U(\gamma)$ as a differential equation, and using that $(r+\mu)U(\bar{\gamma}) = b$, gives

$$(r+\mu)U(\gamma) = b + f(\theta)\int_{\gamma}^{\bar{\gamma}} (r+\mu)e^{-(r+\mu)(\Gamma-\gamma)}\beta S(\Gamma,p)d\Gamma. \qquad (15)$$

Compared to the standard DMP model, (15) differs in that workers must now integrate the match surplus over future values of unemployment history to account for human capital depreciation. This is the only difference between the expression for the value of unemployment in the two models.[12] The integral in (15) captures the option value of searching—the fact that unemployed workers find jobs at a rate $f(\theta)$, taking into account future depreciation in human capital if the worker takes longer to find a job. Applying Nash bargaining to (9) and (3), and substituting for $U(\gamma)$ using (15), gives the following expression for wages

$$w(\gamma,p) = \beta h(\gamma)p + (1-\beta)b + (1-\beta)f(\theta)\int_{\gamma}^{\bar{\gamma}} (r+\mu)e^{-(r+\mu)(\Gamma-\gamma)}\beta S(\Gamma,p)d\Gamma. \qquad (16)$$

Again, wages in the above expression differ from wages in the standard model only in that they take into account the depreciation process during unemployment.[13] However, the above expression offers some intuition for one of the channels that generates larger fluctuations

[12] More specifically, in the standard DMP model the value of unemployment U is given by $(r+\mu)U = b + f(\theta)\beta S(p)$. When $\delta = 0$, $\bar{\gamma} = \infty$ and $S(\gamma,p)$ does not depend on γ, i.e. $S(\gamma,p) = S(p)$. So when δ equals 0, (15) yields the Bellman equation for the DMP model, i.e. $(r+\mu)U = b + f(\theta)\beta S(p)$.

[13] In the standard model one can use the result that surplus $S(p)$ satisfies $(1-\beta)S(p) = k/q(\theta)$, thus giving the textbook expression $w(p) = \beta p + (1-\beta)b + \beta\theta k$. The last term, $\beta\theta k$, captures the option value of search, and corresponds to the integral term in (16) when $\delta = 0$.

in the model. Both the surplus from the match $S(\gamma, p)$ and the match productivity $h(\gamma)p$ decrease with unemployment history. Therefore, wages are closer to the value of non-market time b for workers with longer histories of unemployment. This leads to somewhat more rigid wages, in the sense that their elasticity with respect to labor productivity is smaller. That higher values for b lead to more rigid wages and larger fluctuations in labor market tightness is a known result in the literature.[14] However, in this paper some wages are closer to the value of non-market time b endogenously, not by assuming larger values for b.

Finally, substituting (13) into (10) gives the final expression for wages

$$w(\gamma, p) = (1-\beta)\left(\frac{(r+\mu+s+\delta\frac{r+\mu+s}{r+\mu})e^{-\alpha_1(\bar{\gamma}-\gamma)}}{r+\mu+s+\beta f(\theta)+\delta\frac{r+\mu+s}{r+\mu}} + \frac{(r+\mu+s)(1-e^{-\alpha_1(\bar{\gamma}-\gamma)})}{r+\mu+s+\beta f(\theta)}\right)b$$

$$+\beta\left(\frac{r+\mu+s+f(\theta)+\delta\frac{r+\mu+s}{r+\mu}}{r+\mu+s+\beta f(\theta)+\delta\frac{r+\mu+s}{r+\mu}}\right)h(\gamma)p. \tag{17}$$

Note that when δ equals 0, $\bar{\gamma}$ tends to infinity because (12) implies $\bar{\gamma} = -log(b/p)/\delta$. So the wage expression in (17) collapses to that in the standard DMP model for $\delta = 0$.[15]

4. Endogenous Unemployment History Distributions

When deciding how many vacancies to post, firms look at the expected profits from a match. Since the productivity of a match depends on aggregate productivity p and the human capital level $h(\gamma)$, computing expected profits requires knowledge of the endogenous distributions $G^U(\gamma)$ and $G^E(\gamma)$. To derive these distributions, I look at flows among different groups of workers. Consider the group of workers with unemployment history lower than a given γ, with $\gamma \leq \bar{\gamma}$. In steady-state the flows in and out of this group of workers must be equal. In particular, for a stationary distribution to exist the following flow equation must hold

$$g^U(\gamma)N + (f(\theta)+\mu)G^U(\gamma)N = sG^E(\gamma)E + \mu(E+N), \tag{18}$$

[14] See, among others, Mortensen & Nagypal (2007), Hagedorn & Manovskii (2008) and Hall & Milgrom (2008).

[15] That is, with $\delta = 0$ the wage collapses to

$$w = \frac{(1-\beta)(r+\mu+s)b + \beta(r+\mu+s+f(\theta))p}{r+\mu+s+\beta f(\theta)}.$$

where N and E are the number of unemployed and employed workers, and $g^U(\gamma) = dG^U(\gamma)/d\gamma$ is the probability distribution function. The left-hand side of (18) represents the flows out of the group of workers with unemployment history lower than γ. The first term accounts for workers who have unemployment history exactly equal to γ, and the second term accounts for workers who either find a job or leave the labor force at a rate μ. The right-hand side captures the flows into the group of workers with unemployment history lower than γ. The first term accounts for workers who lose their job and the last term accounts for new entrants to the labor market.

If we now consider the pool of unemployed workers, again flows out and into this group must be equal for the distribution to be stationary. This gives the following flow equation

$$f(\theta)G^U(\bar{\gamma})N + \mu N = sE + \mu(E + N), \tag{19}$$

where the left-hand side captures the flows out and the right-hand side the flows into the pool of unemployed. The above equation gives an expression for the ratio E/N

$$\frac{E}{N} = \frac{f(\theta)G^U(\bar{\gamma})}{s + \mu}. \tag{20}$$

Finally, consider the group of employed workers with unemployment history lower than a given γ. Because flows into this group equal flows out of it, the following flow equation holds

$$f(\theta)G^U(\gamma) = (s + \mu)G^E(\gamma)\frac{E}{N}. \tag{21}$$

Substituting (20) into the above flow equation gives the relationship between $G^E(\gamma)$ and $G^E(\gamma)$

$$G^U(\gamma) = G^E(\gamma)G^U(\bar{\gamma}), \quad \text{for all } \gamma \leq \bar{\gamma}. \tag{22}$$

Substituting (22) and (20) into (18) gives the following differential equation in $G^U(\gamma)$

$$g^U(\gamma) + \frac{\mu(f(\theta) + s + \mu)}{s + \mu}G^U(\gamma) = \frac{\mu(f(\theta)G^U(\bar{\gamma}) + s + \mu)}{s + \mu}. \tag{23}$$

Solving the differential equation yields

$$G^U(\gamma) = \frac{f(\theta)G^U(\bar{\gamma}) + s + \mu}{f(\theta) + s + \mu} \left(1 - e^{-\left(\frac{\mu(f(\theta)+s+\mu)}{s+\mu}\right)\gamma}\right). \tag{24}$$

Solving for the distribution $G^U(\bar{\gamma})$ only requires finding an expression for $G^U(\bar{\gamma})$. Evaluating (24) at $\gamma = \bar{\gamma}$ and solving for $G^U(\bar{\gamma})$ gives

$$G^U(\bar{\gamma}) = \frac{(s+\mu)\left(1 - e^{-\left(\frac{\mu(f(\theta)+s+\mu)}{s+\mu}\right)\bar{\gamma}}\right)}{s + \mu + f(\theta)e^{-\left(\frac{\mu(f(\theta)+s+\mu)}{s+\mu}\right)\bar{\gamma}}}. \tag{25}$$

Note that taking the derivative of (24) and using (25) shows that the conditional distribution $G^U(\gamma)/G^U(\bar{\gamma})$ is a truncated exponential with parameter $\alpha_3 \equiv \mu(f(\theta) + s + \mu)/(s + \mu)$, i.e.

$$\frac{g^U(\gamma)}{G^U(\bar{\gamma})} = \frac{\alpha_3 e^{-\alpha_3\gamma}}{1 - e^{-\alpha_3\bar{\gamma}}}. \tag{26}$$

One can also derive the distribution $G^U(\gamma)$ for $\gamma \geq \bar{\gamma}$ in a similar way. The only difference is that instead of (18) we have

$$g^U(\gamma)N + f(\theta)G^U(\bar{\gamma})N + \mu G^U(\gamma)N = sE + \mu(E + N). \tag{27}$$

The above expression uses that $G^E(\bar{\gamma}) = 1$, because only workers with $\gamma \leq \bar{\gamma}$ are employed by firms. The second term on the left-hand side of (27) also captures that only workers with unemployment history lower than $\bar{\gamma}$ find jobs. Since (21) is still satisfied for $\gamma = \bar{\gamma}$, substituting this into (27) gives

$$g^U(\gamma) + \mu G^U(\gamma) = \mu, \tag{28}$$

which implies

$$G^U(\gamma) = e^{\mu(\bar{\gamma}-\gamma)}G^U(\bar{\gamma}) + 1 - e^{\mu(\bar{\gamma}-\gamma)}, \text{ for all } \gamma \geq \bar{\gamma}. \tag{29}$$

5. Fluctuations in labor market tightness

I follow the standard approach in the literature and measure fluctuations in labor market tightness with comparative statics results. That is, I look at the response of the equilibrium labor market tightness to changes in labor productivity. Shimer (2005) and Mortensen & Nagypal (2007) show that, for the standard DMP model, the comparative statics results are a good approximation of the response of the full dynamic model.[16] Intuitively, with very persistent productivity and a high job finding rate, unemployment adjusts very quickly to its steady-state value. This result is more formally discussed in Mortensen & Nagypal (2007).[17]

In this paper, comparative statics also seem to be a good approximation of the full dynamic model. The other endogenous variables, labor market tightness θ and the terminal unemployment history level $\bar{\gamma}$, are jump variables that adjust immediately to their steady-state values. The only element in the model that may have a slow adjustment is the distribution of unemployment history $G^U(\gamma)$.[18] Because labor productivity depends on this distribution, comparative statics may be a weak approximation of the dynamic model if the adjustment is slow. However, one can simulate the response of the distribution of unemployment history to changes in the job finding rate. The simulations suggest that the distribution of unemployment history converges very quickly to its stationary distribution. Intuitively, the job finding rate is so large compared to the other parameters that determine the distribution $G^U(\gamma)$, that the response of $f(\theta)$ dominates the other flows. As a result, $G^U(\gamma)$ converges very quickly to its stationary distribution.

5.1. Equilibrium labor market tightness

Equilibrium labor market tightness θ is determined by the usual job creation condition. Combining (8) with the free entry condition for vacancies $V = 0$, and substituting (3) yields

$$\frac{k}{q(\theta)} = \int_0^{\bar{\gamma}} \frac{h(\gamma)p - w(\gamma, p)}{r + \mu + s} \cdot \frac{dG^U(\gamma)}{G^U(\bar{\gamma})}. \tag{30}$$

[16] See also Mortensen & Nagypal (2007b), Pissarides (2009) and Silva & Toledo (2013) for other work that uses the same approach.

[17] See also Pissarides (2009).

[18] A similar issue arises in the random matching model with endogenous separations, as discussed in Mortensen & Nagypal (2007b). There, the distribution of workers across productivities levels determines labor productivity. When aggregate productivity changes, so does this distribution. Whether or not this adjustment is slow is not addressed in their paper.

It will be convenient to denote the expected profit from filling a vacancy by the function $\Phi(\theta, \bar{\gamma}, p)$, i.e. define $\Phi(\theta, \bar{\gamma}, p)$ as

$$\Phi(\theta, \bar{\gamma}, p) \equiv \int_0^{\bar{\gamma}} (h(\gamma)p - w(\gamma, p)) \frac{dG^U(\gamma)}{G^U(\bar{\gamma})}. \tag{31}$$

The expected profit from filling a vacancy $\Phi(\theta, \bar{\gamma}, p)$ depends on $f(\theta)$ and p through the usual mechanism in the standard search and matching model. Higher $f(\theta)$ improves workers outside option, thus raising their wage, and higher p increases both match productivity and wages. However, in this model the expected value $\Phi(\theta, \bar{\gamma}, p)$ further depends on $f(\theta)$ and $\bar{\gamma}$ through the effect on the average human capital level $\int_0^{\bar{\gamma}} h(\gamma) dG^U(\gamma)/G^U(\bar{\gamma})$.

The derivation of $\Phi(\theta, \bar{\gamma}, p)$ requires taking many integrals and yields a somewhat cumbersome expression. Substituting the distribution $G^U(\gamma)$ from (24) into (31) and solving for the integrals eventually gives

$$\begin{aligned}
\Phi(\theta, \bar{\gamma}, p) = &\left\{ \frac{\alpha_3}{\alpha_3 + \delta} \cdot \frac{1 - e^{-(\delta + \alpha_3)\bar{\gamma}}}{1 - e^{-\alpha_3 \bar{\gamma}}} \cdot \frac{(1-\beta)(r + \mu + s + \delta \frac{r+\mu+s}{r+\mu})}{r + \mu + s + \beta f(\theta) + \delta \frac{r+\mu+s}{r+\mu}} \right\} p \\
&+ \left\{ \beta \left(\frac{r + \mu + s + f(\theta) + \delta \frac{r+\mu+s}{r+\mu}}{r + \mu + s + \beta f(\theta) + \delta \frac{r+\mu+s}{r+\mu}} - \frac{r + \mu + s + f(\theta)}{r + \mu + s + \beta f(\theta)} \right) \cdot \frac{\alpha_3}{\alpha_1 - \alpha_3} \cdot \frac{e^{-\alpha_3 \bar{\gamma}} - e^{-\alpha_1 \bar{\gamma}}}{1 - e^{-\alpha_3 \bar{\gamma}}} \right\} b \\
&- \left\{ (1-\beta) \frac{r + \mu + s}{r + \mu + s + \beta f(\theta)} \right\} b,
\end{aligned} \tag{32}$$

where $\alpha_3 \equiv \mu(f(\theta) + s + \mu)/(s + \mu)$ denotes the coefficient of the distribution in (24), and as before $\alpha_1 \equiv r + \mu + \beta f(\theta)(r + \mu)/(r + \mu + s)$. The appendix includes some further details on how to derive (32). A nice feature of (32) is that it depends only on known parameters that are standard in the literature.

5.2. Response to aggregate productivity

Deriving the response of market tightness to aggregate productivity requires taking a number of integrals, which makes the solution somewhat cumbersome, but it is otherwise straightforward. Most importantly, one can find a closed form solution for this response that depends only on a set of known parameters.

Taking logs and differentiating (30) with respect to p yields

$$-\frac{q'(\theta)}{q(\theta)}\frac{d\theta}{dp} = \frac{\Phi_\theta(\theta,\bar{\gamma},p)\frac{d\theta}{dp} + \Phi_{\bar{\gamma}}(\theta,\bar{\gamma},p)\frac{d\bar{\gamma}}{dp} + \Phi_p(\theta,\bar{\gamma},p)}{\Phi(\theta,\bar{\gamma},p)}, \tag{33}$$

where the subscripts denote partial derivates, i.e. $\Phi_x(\theta,\bar{\gamma},p) \equiv \partial\Phi(\theta,\bar{\gamma},p)/\partial x$. The above equation provides some useful intuition. In response to changes in aggregate productivity p, the profitability of a job $\Phi(\theta,\bar{\gamma},p)$ changes through three channels. First, higher aggregate productivity p leads to higher job finding rates, which improves workers' average human capital level. This is captured by the first term on the right-hand side of (33). Second, with higher aggregate productivity some matches that were not profitable become now profitable, as is captured by the change in the terminal unemployment history $\bar{\gamma}$. This corresponds to the second term on the right-hand side of (33). Finally, an increase in p increases the profitability of the job as in the standard model. Note that the profitability of a job changes relatively more for matches with higher levels of unemployment history, because wages for those matches are closer to the value of non-market time b, as (16) shows.

Let η denote the elasticity of $q(\theta)$ with respect to θ, i.e. $\eta \equiv -q'(\theta)\theta/q(\theta)$. Further, let ε_θ denote the elasticity of θ with respect to p, i.e. $\varepsilon_\theta \equiv (d\theta/dp)\cdot(p/\theta)$. Because $f'(\theta)\theta/f(\theta) = 1-\eta$ and $\Phi(\theta,\bar{\gamma},p)$ depends on θ only through $f(\theta)$, we can express $\Phi_\theta(\theta,\bar{\gamma},p)\theta$ as

$$\Phi_\theta(\theta,\bar{\gamma},p)\cdot\theta = \Phi_{f(\theta)}(\theta,\bar{\gamma},p)(1-\eta)f(\theta). \tag{34}$$

Rearranging (33) and using the above result gives

$$\varepsilon_\theta = p\cdot\frac{\Phi_{\bar{\gamma}}(\theta,\bar{\gamma},p)\frac{d\bar{\gamma}}{dp} + \Phi_p(\theta,\bar{\gamma},p)}{\eta\Phi(\theta,\bar{\gamma},p) - \Phi_{f(\theta)}(\theta,\bar{\gamma},p)(1-\eta)f(\theta)} \tag{35}$$

A nice feature of (35) is that it is also uniquely determined by a set of standard parameters in the literature. This becomes clear by looking at $\Phi(\theta,\bar{\gamma},p)$ in (32). Because $\Phi(\theta,\bar{\gamma},p)$ in (32) depends on θ only through its effect on $f(\theta)$, its partial derivates Φ_i is also uniquely determined by this set of known parameters. The closed form expressions for the partials $\Phi_x(\theta,\bar{\gamma},p) \equiv \partial\Phi(\theta,\bar{\gamma},p)/\partial x$ in (35) are included in the appendix.

6. Quantifying labor market fluctuations

I now calibrate the model and quantify the amount of labor market fluctuations, as captured by the elasticity ε_θ of market tightness with respect to aggregate productivity p in (35). Labor market fluctuations are uniquely determined by $\{b, f(\theta), s, \mu, r, \beta, \eta, \delta\}$. For most of these values the calibration is standard in the literature.

6.1. Calibration

Table 1 summarizes the calibration values. Rates are measured monthly. While $f(\theta)$ is an endogenous variable, one can simply use the value for the job finding rate in Shimer (2005), which gives a monthly rate for $f(\theta)$ of 0.45. Shimer (2005) also finds that the separation rate s is 0.035 monthly. The rate at which workers leave the labor force μ is calibrated so that workers stay in the labor force for 40 years on average, which gives a value for μ of 0.0021. The interest rate r is consistent with a 5 percent annual interest rate. I use $\eta = 0.5$, which is within the range of values for the matching function in Petrongolo & Pissarides (2001).[19] Further, I set $\beta = \eta$, which satisfies the Hosios-Pissarides condition and corresponds to the efficient equilibrium. In section 6.2, I discuss in more detail the role of non-market time b, and show how different values of b affect the elasticity of market tightness ε_θ.

Finally, I build on evidence in Ortego-Marti (2012) to calibrate δ, the rate at which workers lose human capital during unemployment. This study uses the Panel Study of Income Dynamics (PSID) from 1968 to 1997 to construct workers' unemployment history γ_{it}. To find the effect of unemployment history on wages, Ortego-Marti (2012) regresses log wages on unemployment history and other covariates and worker observables, i.e.

$$\log W_{it} = \alpha_i - \delta\gamma_{it} + X_{it}\beta + \epsilon_{it}, \tag{36}$$

where X_{it} is a set of covariates commonly used in Mincerian regressions, such as experience, occupation, region and so on.[20] This empirical strategy finds that one added month of unemployment history is associated with a 1.22 % wage drop.[21] I use this estimate to calibrate

[19] Shimer (2005) uses a value of 0.28 for η, but as Mortensen & Nagypal (2007) argues, this value is on the lower bound of the parameter range and leads to lower labor market fluctuations.

[20] Ortego-Marti (2012) contains more details on the empirical work, and shows that the results are robust to different specifications.

[21] This value is smaller than the one found in the displaced workers literature. These studies focus on a set of workers that are very attached to their sector or employer—for example, they usually focus on workers with a minimum tenure on the job. Not surprisingly these workers have accumulated more specific human

the rate at which workers lose human capital during unemployment δ. The counterpart to this empirical value in the model is the average semi-elasticity of wages with respect to unemployment history γ, i.e. $E(\partial \log(w)/\partial\gamma)$. Using the expression for wages in (17) and differentiating with respect to unemployment history γ gives

$$\frac{dw}{d\gamma} = \left\{ \frac{\beta(1-\beta)f(\theta)\alpha_1 e^{-\alpha_1(\bar{\gamma}-\gamma)}\delta(\frac{r+\mu+s}{r+\mu})}{(r+\mu+s+\beta f(\theta)+\delta(\frac{r+\mu+s}{r+\mu}))(r+\mu+s+\beta f(\theta))} \right\} b$$

$$- \left\{ \beta \frac{r+\mu+s+f(\theta)+\delta(\frac{r+\mu+s}{r+\mu})}{r+\mu+s+\beta f(\theta)+\delta(\frac{r+\mu+s}{r+\mu})} \right\} \delta h(\gamma) p. \tag{37}$$

To find the average semi-elasticity in the model, I combine (17) and (37) and use numerical integration to calculate

$$E\left(\frac{dw/d\gamma}{w}\right) = \int_0^{\bar{\gamma}} \left(\frac{dw/d\gamma}{w}\right) \frac{dG^U(\gamma)}{G^U(\bar{\gamma})}. \tag{38}$$

The value for the depreciation rate δ is calibrated so that the average semi-elasticity in the model equals its empirical counterpart.

6.2. Results

Shimer (2005) reports that in the data labor market tightness—the vacancy-unemployment ratio—is 20 times more volatile than labor market productivity. However, many factors other than labor productivity affect fluctuations in market tightness. To address this issue, Mortensen & Nagypal (2007) regress market tightness on labor productivity, which yields a coefficient of 7.56. This regression coefficient corresponds to the elasticity of market tightness with respect to aggregate productivity, and is thus the empirical counterpart to (35). Intuitively, the search and matching framework with aggregate productivity as its only source of fluctuations should only aim to account for this level of fluctuations. Any fluctuations in θ that are not correlated with aggregate productivity p cannot be accounted for without including further sources of fluctuations. Therefore, I investigate whether the model in this paper is able to generate a value of 7.56 for the elasticity ε_θ.

The model in general generates larger fluctuations in market tightness θ compared to

capital on average, so displacement leads to larger wage losses. Using their values would improve the results, so the chosen calibration for δ is not controversial.

the baseline DMP model.[22] However, the magnitude of the improvement depends on the level of non-market time b. For the mechanism in this paper to generate a significantly higher level of fluctuations, the value of non-market time must be in the mid-range of values commonly used in the literature. Table 2 shows the elasticity of market tightness ε_θ for different values of b. This elasticity is then compared to that in the baseline DMP model—which corresponds to $\delta = 0$. However, in the model with unemployment history the average productivity in the economy is $\bar{p} = E(h(\gamma)p|\gamma \leq \bar{\gamma})$, whereas in the baseline model it is simply p. So both models are comparable only if the replacement ratio $b/\bar{p}$ is the same. To allow for comparison between the two models, table 2 provides the replacement ratio implied in the model with unemployment history. It further reports the elasticity of market tightness in the baseline model using the implied replacement ratio as the value for non-market time b. This gives the same replacement ratio in both models.[23]

Table 2 includes the results. For low values of non-market time b the improvement is modest. When non-market time equals 0.30, the elasticity of market tightness increases from 1.55 to 1.86 in the model with unemployment history, a 20% improvement compared to the baseline. However, this value still falls short of the empirical elasticity of 7.56. With this value of non-market time the replacement ratio is 0.42, which is similar to the replacement ratio in Shimer (2005).[24] However, when b is sufficiently high, the model with unemployment history is a significant improvement over the baseline and generates labor market fluctuations similar to those observed in the data. With a non-market time value of 0.65 the elasticity of non-market time increases to 7.36, a 49% improvement compared to the elasticity of 4.93 in the baseline model and close to the empirical value of 7.56. In this case the replacement ratio is 0.78, slightly above the value in Hall & Milgrom (2008).[25] In this case the model with unemployment history accounts for most of fluctuations in market tightness. For slightly larger values of non-market time the model has no trouble generating sizable fluctuations. When b is 0.70—which implies a replacement ratio of 0.82—the elasticity in the model with unemployment history is 12.80, compared to 5.96 in the baseline. For the higher value for b of 0.75—which implies a replacement ratio of 0.85—the elasticity goes as high as 40.69,

[22] As a robustness check, the elasticity ε_θ using the calibration in Shimer (2005) with $\delta = 0$ and $\mu = 0$ delivers an elasticity of 1.72, the same value that Mortensen & Nagypal (2007) find for the baseline DMP model with the Shimer calibration, as one would expect.

[23] If instead ones uses the same value for b in both models, in table 2 fluctuations in the baseline model would be lower, but would leave those in the model with unemployment history unchanged.

[24] Shimer (2005) uses a replacement ratio of 0.40 for his calibration, based on unemployment insurance replacement ratios.

[25] Hall & Milgrom (2008) use a value of 0.73 based on empirical Frisch elasticities.

whereas for the baseline model the elasticity is 7.23.

Further, the model with unemployment history has the advantage of generating larger fluctuations in market tightness without worsening the model's response to changes in unemployment benefits. As Costain & Reiter (2008) point out, when the value of non-market time b is large, labor market tightness responds too much to changes in labor market policies, such as an increase in unemployment benefits. I calculate the elasticity $\varepsilon_{\theta,b}$ of θ with respect to changes in b. Taking logs of (30) and differentiating with respect to b yields

$$\varepsilon_{\theta,b} = b \cdot \frac{\Phi_{\bar{\gamma}}(\theta,\bar{\gamma},p)\frac{d\bar{\gamma}}{db} + \Phi_b(\theta,\bar{\gamma},p)}{\eta\Phi(\theta,\bar{\gamma},p) - \Phi_{f(\theta)}(\theta,\bar{\gamma},p)(1-\eta)f(\theta)}. \tag{39}$$

As with the elasticity with respect to labor productivity p, the above elasticity depends only on the parameters $\{b, f(\theta), s, \mu, r, \beta, \eta, \delta\}$. Table 3 shows the ratio of ε_θ to $\varepsilon_{\theta,b}$, and compares it to the same ratio in the baseline DMP model.[26] As Costain & Reiter (2008) point out, with higher values of non-market time b the response of θ to changes in b—say, because of a change in unemployment insurance—is dampened. This is true both in the standard model and in the model of this paper, as the low ratios of the elasticities ε_θ to $\varepsilon_{\theta,b}$ show. However, the ratio of these two elasticities is still higher for the model with unemployment history. The ratio is twice as large when b is 0.4, and is still around 20 % larger for the higher value of 0.7.

7. Conclusion

The DMP search and matching framework has become the workhorse model for the study of labor markets. Motivated by the findings in Shimer (2005), this paper studies labor market fluctuations in a search and matching model in which workers lose some skills during unemployment. Firms' hiring incentives fluctuate more in this model. First, because changes in aggregate productivity affect the human capital level of the pool of unemployed workers. Their human capital worsens in recessions and improves in booms. Second, workers' human capital decreases with unemployment history, and as a result so do their wages. This leads to more rigid wages for workers with longer unemployment history because their wages are closer to the value of non-market time. I show in this paper that these two mechanisms deliver larger fluctuations in the vacancy-unemployment ratio. The improvement in the size

[26] Again, using the same strategy for the choice of b in the baseline model, so that both models are comparable.

of fluctuations is mild for low values of non-market time, but it is sizable for a mid-range of values standard in the literature. In this latter case the model can generate labor market fluctuations that are in line with those in the data. Finally, the model improves the response of labor markets to changes in unemployment insurance benefits. Unemployment affects workers' labor market fortunes. This paper suggests that the effect of unemployment on workers human capital has important implications for firms' hiring decisions, and leads to larger fluctuations in labor markets.

References

Beauchemin, K. & Tasci, M. (2013), 'Diagnosing labor market search models: a multiple-shock approach', *Macroeconomic Dynamics* **FirstView**, 1–25.

Carrington, W. J. (1993), 'Wage losses for displaced workers: Is it really the firm that matters?', *The Journal of Human Resources* **28**(3), pp. 435–462.

Coles, M. & Masters, A. (2000), 'Retraining and long-term unemployment in a model of unlearning by not doing', *European Economic Review* **44**(9), 1801 – 1822.

Costain, J. S. & Reiter, M. (2008), 'Business cycles, unemployment insurance, and the calibration of matching models', *Journal of Economic Dynamics and Control* **32**(4), 1120 – 1155.

Couch, K. A. & Placzek, D. W. (2010), 'Earnings losses of displaced workers revisited', *American Economic Review* **100**(1), 572–89.

den Haan, W. J., Haefke, C. & Ramey, G. (2005), 'Turbulence and unemployment in a job matching model', *Journal of the European Economic Association* **3**(6), 1360–1385.

Eyigungor, B. (2010), 'Specific capital and vintage effects on the dynamics of unemployment and vacancies', *American Economic Review* **100**(3), 1214–1237.

Fallick, B. C. (1996), 'A review of the recent empirical literature on displaced workers', *Industrial and Labor Relations Review* **50**(1), pp. 5–16.

Farber, H. S. (1997), 'The changing face of job loss in the united states, 19811995', *Brookings Papers on Economic Activity: Microeconomics* pp. 55–128.

Fujita, S. (2012), 'Declining labor turnover and turbulence'. mimeo, Federal Reserve Bank of Philadelphia.

Gertler, M. & Trigari, A. (2009), 'Unemployment fluctuations with staggered nash wage bargaining', *Journal of Political Economy* **117**(1), pp. 38–86.

Hagedorn, M. & Manovskii, I. (2008), 'The cyclical behavior of equilibrium unemployment and vacancies revisited', *American Economic Review* **98**(4), pp. 1692–1706.

Hall, R. E. (2005), 'Employment fluctuations with equilibrium wage stickiness', *American Economic Review* **95**(1), pp. 50–65.

Hall, R. E. & Milgrom, P. R. (2008), 'The limited influence of unemployment on the wage bargain', *American Economic Review* **98**(4), pp. 1653–1674.

Jacobson, L. S., LaLonde, R. J. & Sullivan, D. G. (1993), 'Earnings losses of displaced workers', *American Economic Review* **83**(4), pp. 685–709.

Kennan, J. (2010), 'Private information, wage bargaining and employment fluctuations.', *Review of Economic Studies* **77**(2), 633 – 664.

Kletzer, L. G. (1998), 'Job displacement', *The Journal of Economic Perspectives* **12**(1), pp. 115–136.

Ljungqvist, L. & Sargent, T. J. (1998), 'The european unemployment dilemma', *Journal of Political Economy* **106**(3), 514–550.

Ljungqvist, L. & Sargent, T. J. (2007), 'Understanding european unemployment with matching and search-island models', *Journal of Monetary Economics* **54**(8), 2139 – 2179.

Ljungqvist, L. & Sargent, T. J. (2008), 'Two questions about european unemployment', *Econometrica* **76**(1), 1–29.

Menzio, G. (2005), 'High frequency wage rigidity'. mimeo, University of Pennsylvania.

Mortensen, D. T. & Nagypal, E. (2007), 'More on unemployment and vacancy fluctuations', *Review of Economic Dynamics* **10**(3), 327 – 347.

Mortensen, D. T. & Nagypal, E. (2007b), 'Labor-market volatility in matching models with endogenous separations', *Scandinavian Journal of Economics* **109**(4), 645–665.

Neal, D. (1995), 'Industry-specific human capital: Evidence from displaced workers', *Journal of Labor Economics* **13**(4), pp. 653–677.

Ortego-Marti, V. (2012), 'Unemployment history and frictional wage dispersion'. mimeo.

Pavoni, N. (2011), 'Optimal unemployment insurance, with human capital depreciation, and duration dependence', *International Economic Review* .

Petrongolo, B. & Pissarides, C. A. (2001), 'Looking into the black box: A survey of the matching function', *Journal of Economic Literature* **39**(2), pp. 390–431.

Pissarides, C. A. (1992), 'Loss of skill during unemployment and the persistence of employment shocks', *Quarterly Journal of Economics* **107**(4), pp. 1371–1391.

Pissarides, C. A. (2000), *Equilibrium Unemployment Theory*, MIT Press, Cambridge.

Pissarides, C. A. (2009), 'The unemployment volatility puzzle: Is wage stickiness the answer?', *Econometrica* **77**(5), 1339–1369.

Rudanko, L. (2011), 'Aggregate and idiosyncratic risk in a frictional labor market', *American Economic Review* **101**(6), 2823–43.

Ruhm, C. J. (1991), 'Are workers permanently scarred by job displacements?', *American Economic Review* **81**(1), pp. 319–324.

Schoeni, R. & Dardia, M. (2003), 'Estimates of earnings losses of displaced workers using california administrative data'. University of Michigan Population Studies Center Report 03543.

Shimer, R. (2005), 'The cyclical behavior of equilibrium unemployment and vacancies', *American Economic Review* **95**(1), 24–49.

Shimer, R. & Werning, I. (2006), 'On the optimal timing of benefits with heterogeneous workers and human capital depreciation'. mimeo, University of Chicago.

Silva, J. I. & Toledo, M. (2009), 'Labor turnover costs and the cyclical behavior of vacancies and unemployment', *Macroeconomic Dynamics* **13**, 76–96.

Silva, J. I. & Toledo, M. (2013), 'The unemployment volatility puzzle: The role of matching costs revisited', *Economic Inquiry* **51**(1), 836–843.

Stevens, A. H. (1997), 'Persistent effects of job displacement: The importance of multiple job losses', *Journal of Labor Economics* **15**(1), pp. 165–188.

Topel, R. (1990), 'Specific capital and unemployment: Measuring the costs and consequences of job loss', *Carnegie-Rochester Conference Series on Public Policy* **33**(0), 181 – 214.

von Wachter, T., Song, J. & Manchester, J. (2009), 'Long-term earnings losses due to mass layoffs during the 1982 recession: An analysis using U.S. administrative data from 1974 to 2004'. mimeo, Columbia University.

Technical Appendix

Proof of Proposition 1

Define the match surplus as $S(\gamma, p) \equiv J(\gamma, p) + W(\gamma, p) - U(\gamma)$. Nash Bargaining implies that

$$W(\gamma, p) - U(\gamma) = \beta S(\gamma, p), \tag{A1}$$

$$J(\gamma, p) = (1 - \beta) S(\gamma, p). \tag{A2}$$

Using (2) and (3) gives

$$(r + \mu + s)(W(\gamma, p) - U(\gamma)) = w(\gamma, p) - (r + \mu)U(\gamma), \tag{A3}$$

$$(r + \mu + s)J(\gamma, p) = h(\gamma)p - w(\gamma, p). \tag{A4}$$

Combining the above equations with Nash Bargaining and solving for wages gives

$$S(\gamma) = \frac{h(\gamma)p - (r + \mu)U}{r + \mu + s}. \tag{A5}$$

Given that an unemployed worker can always decide to just keep the value of non-market time b, clearly $(r + \mu)U(\gamma) \geq b$. Therefore

$$S(\gamma) \leq \frac{h(\gamma)p - b}{r + \mu + s}. \tag{A6}$$

As γ tends to infinity, productivity $h(\gamma)p$ tends to zero, so there exists a $\bar{\gamma}$ such that $S(\bar{\gamma}, p) = 0$. As a result, $J(\bar{\gamma}, p) = 0$ and $W(\bar{\gamma}, p) = U(\bar{\gamma})$. In particular, using (1) and (3) it follows that

$$h(\bar{\gamma})p = w(\bar{\gamma}, p), \tag{A7}$$

$$(r + \mu)U(\bar{\gamma}) = b. \tag{A8}$$

Derivation of the expected value of a filled job $\Phi(\theta, \bar{\gamma}, p)$

This part gives further details on how to derive the expression for $\Phi(\theta, \bar{\gamma}, p)$ in (32). Integrating the different terms in the above expression yields the following expression for

$\Phi(\theta, \bar{\gamma}, p)$

$$\Phi(\theta, \bar{\gamma}, p) = \{\phi_1 \cdot \phi_2 \cdot \phi_3 + \phi_4 \cdot \phi_5 \cdot \phi_6\} \, p - \{\phi_7 \cdot \phi_5 \cdot \phi_8 + \phi_9\} \, b, \tag{A9}$$

where, using $\alpha_3 \equiv \mu(f(\theta) + s + \mu)/(s + \mu)$ to denote the coefficient in the distribution (24), the terms ϕ_i are given by

$$\begin{aligned}
\phi_1 &= \frac{\alpha_3}{\delta + \alpha_3} \\
\phi_2 &= \frac{1 - e^{-(\delta+\alpha_3)\bar{\gamma}}}{1 - e^{-\alpha_3 \bar{\gamma}}} \\
\phi_3 &= (1-\beta) \left(\frac{r + \mu + s + \delta \frac{r+\mu+s}{r+\mu}}{r + \mu + s + \beta f(\theta) + \delta \frac{r+\mu+s}{r+\mu}} \right) \\
\phi_4 &= \beta \left(\frac{r + \mu + s + f(\theta) + \delta \frac{r+\mu+s}{r+\mu}}{r + \mu + s + \beta f(\theta) + \delta \frac{r+\mu+s}{r+\mu}} \right) \\
\phi_5 &= \frac{\alpha_3}{\alpha_1 - \alpha_3} \\
\phi_6 &= \frac{e^{-(\alpha_3+\delta)\bar{\gamma}} - e^{-(\alpha_1+\delta)\bar{\gamma}}}{1 - e^{-\alpha_3 \bar{\gamma}}} \\
\phi_7 &= \beta \left(\frac{r + \mu + s + f(\theta)}{r + \mu + s + \beta f(\theta)} \right) \\
\phi_8 &= \frac{e^{-\alpha_3 \bar{\gamma}} - e^{-\alpha_1 \bar{\gamma}}}{1 - e^{-\alpha_3 \bar{\gamma}}} \\
\phi_9 &= (1-\beta) \left(\frac{r + \mu + s}{r + \mu + s + \beta f(\theta)} \right)
\end{aligned} \tag{A10}$$

Rearranging, after using that $e^{-\delta\bar{\gamma}} p = h(\bar{\gamma}) p = b$ implies that $\phi_6 \cdot p = \phi_8 \cdot b$, gives

$$\Phi(\theta, \bar{\gamma}, p) = \{\phi_1 \cdot \phi_2 \cdot \phi_3\} \, p + \{(\phi_4 - \phi_7) \cdot \phi_5 \cdot \phi_8 - \phi_9\} \, b. \tag{A11}$$

Replacing the values for ϕ_i in the above expression and rearranging gives (32) in the text.

Derivation of the partial effects $\Phi_x(\theta, \bar{\gamma}, p)$

This section includes the derivation of the partial effects $\Phi_f(\theta, \bar{\gamma}, p)$, $\Phi_{\bar{\gamma}}(\theta, \bar{\gamma}, p)$ and $\Phi_p(\theta, \bar{\gamma}, p)$. Substituting these values into (35) gives the expression of the elasticity of labor market tightness θ with respect to p.

To calculate $\Phi_f(\theta, \bar{\gamma}, p)$, it is sufficient to derive the expressions for the partials of ϕ_i with respect to f using the expressions in (A10). Such derivatives are given by

$$
\begin{aligned}
\frac{\partial \phi_1}{\partial f} &= \frac{\delta\mu}{(s+\mu)(\delta+\alpha_3)^2} \\
\frac{\partial \phi_2}{\partial f} &= \frac{\bar{\gamma}(e^{-(\delta+\alpha_3)\bar{\gamma}} - e^{-\alpha_3\bar{\gamma}})}{(1-e^{-\alpha_3\bar{\gamma}})^2} \cdot \frac{\mu}{s+\mu} \\
\frac{\partial \phi_3}{\partial f} &= -\frac{\beta(1-\beta)(r+\mu+s+\delta\frac{r+\mu+s}{r+\mu})}{(r+\mu+s+\beta f(\theta)+\delta\frac{r+\mu+s}{r+\mu})^2} \\
\frac{\partial \phi_4}{\partial f} &= -\frac{\partial \phi_3}{\partial f} \\
\frac{\partial \phi_5}{\partial f} &= \frac{\alpha_1\frac{\mu}{s+\mu} - \alpha_3\beta\frac{r+\mu}{r+\mu+s}}{(\alpha_1-\alpha_3)^2} \\
\frac{\partial \phi_6}{\partial f} &= \frac{\bar{\gamma}e^{-\delta\bar{\gamma}}\left\{\frac{\partial\alpha_3}{\partial f}e^{-\alpha_3\bar{\gamma}}(e^{-\alpha_1\bar{\gamma}}-1) + \frac{\partial\alpha_1}{\partial f}e^{-\alpha_1\bar{\gamma}}(1-e^{-\alpha_3\bar{\gamma}})\right\}}{(1-e^{-\alpha_3\bar{\gamma}})^2} \\
\frac{\partial \phi_7}{\partial f} &= \frac{\beta(1-\beta)(r+\mu+s)}{(r+\mu+s+\beta f(\theta))^2} \\
\frac{\partial \phi_8}{\partial f} &= e^{\delta\bar{\gamma}} \cdot \frac{\partial \phi_6}{\partial f} \\
\frac{\partial \phi_9}{\partial f} &= -\frac{\partial \phi_7}{\partial f},
\end{aligned}
\tag{A12}
$$

where $\partial\alpha_3/\partial f = \mu/(s+\mu)$ and $\partial\alpha_1/\partial f = \beta(r+\mu)/(r+\mu+s)$.

Since only ϕ_2, ϕ_6 and ϕ_8 depend on $\bar{\gamma}$, to calculate $\Phi_{\bar{\gamma}}(\theta, \bar{\gamma}, p)$ we only need the following

$$
\begin{aligned}
\frac{\partial \phi_2}{\partial \bar{\gamma}} &= \frac{(\delta+\alpha_3)e^{-(\delta+\alpha_3)\bar{\gamma}}(1-e^{-\alpha_3\bar{\gamma}}) - \alpha_3 e^{-\alpha_3\bar{\gamma}}(1-e^{-(\delta+\alpha_3)\bar{\gamma}})}{(1-e^{-\alpha_3\bar{\gamma}})^2} \\
\frac{\partial \phi_6}{\partial \bar{\gamma}} &= e^{-\delta\bar{\gamma}}\left\{\frac{((\alpha_1+\delta)e^{-\alpha_1\bar{\gamma}} - (\alpha_3+\delta)e^{-\alpha_3\bar{\gamma}})(1-e^{-\alpha_3\bar{\gamma}}) - (e^{-\alpha_3\bar{\gamma}} - e^{-\alpha_1\bar{\gamma}})\alpha_3 e^{-\alpha_3\bar{\gamma}}}{(1-e^{-\alpha_3\bar{\gamma}})^2}\right\} \\
\frac{\partial \phi_8}{\partial \bar{\gamma}} &= \frac{(\alpha_1 e^{-\alpha_1\bar{\gamma}} - \alpha_3 e^{-\alpha_3\bar{\gamma}})(1-e^{-\alpha_3\bar{\gamma}}) - (e^{-\alpha_3\bar{\gamma}} - e^{-\alpha_1\bar{\gamma}})\alpha_3 e^{-\alpha_3\bar{\gamma}}}{(1-e^{-\alpha_3\bar{\gamma}})^2}
\end{aligned}
\tag{A13}
$$

Finally, $\Phi_p(\theta, \bar{\gamma}, p)$ follows immediately from (A10)

$$
\Phi_p(\theta, \bar{\gamma}, p) = \phi_1 \cdot \phi_2 \cdot \phi_3 + \phi_4 \cdot \phi_5 \cdot \phi_6. \tag{A14}
$$

Tables

Table 1: Calibration values

Variable	Description	Target/Source	Value
$f(\theta)$	Job finding rate	Shimer (2005)	0.45
s	Separation rate	Shimer (2005)	0.035
μ	Rate labor force exit	40 years working life	0.0021
r	Interest rate	Annual interest rate 5%	0.0041
η	Elasticity of $q(\theta)$	Petrongolo & Pissarides (2001)	0.5
r	Interest rate	Annual interest rate 5%	0.0041

Note.- Rates are expressed monthly. See section 6 for details.

Table 2: ELASTICITY OF LABOR MARKET TIGHTNESS ε_θ

Non-market time b	Replacement ratio $b/\bar{p}$	Baseline	Unemployment history
0.30	0.42	1.55	1.86
0.40	0.53	2.31	2.25
0.50	0.65	2.17	3.08
0.60	0.74	4.17	5.06
0.65	0.78	4.93	7.36
0.70	0.82	5.96	12.80
0.75	0.85	7.23	40.69

Note.- To allow for comparison, the elasticity for the baseline model is computed using the implied replacement ratio $b/\bar{p}$ as the value of non-market time. See text for more details.

Table 3: RATIO OF ELASTICITIES ε_θ TO $\varepsilon_{\theta,b}$

Non-market time b	Baseline	Unemployment history
0.4	1.89	3.85
0.7	1.23	1.56

Note.- To allow for comparison, the elasticity for the baseline model is computed using the implied replacement ratio $b/\bar{p}$ as the value of non-market time. See text for more details.