Indeterminacy with Progressive Taxation and Sector-Specific Externalities[1]

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Abstract

This paper quantitatively examines the empirical plausibility of equilibrium indeterminacy and sunspot-driven cyclical fluctuations in a real business cycle model with two distinct production sectors that yield consumption and investment goods, together with separable or non-separable preferences. When calibrated to match the observed progressivity of the U.S. federal individual income tax schedule, each version of our model economy exhibits an indeterminate steady state under empirically realistic combinations of the household's labor supply elasticity and the degree of productive externalities in the investment goods sector. Therefore, macroeconomic instability due to agents' self-fulfilling expectations may in fact be a prevalent feature of the U.S. economy.

Keywords: Indeterminacy, Progressive Taxation, Sector-Specific Externalities.

JEL Classification: E30, E32, E62.

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1 Introduction

In the context of real business cycle (RBC) models, the work of Benhabib and Farmer (1994) and Farmer and Guo (1994) has started a large macroeconomics literature that explores the presence of an indeterminate steady state or balanced growth path under perfect foresight, and the neighboring stationary rational expectations equilibrium trajectories along which agents' animal spirits can be an independent source for endogenous cyclical fluctuations.[2] The original Benhabib-Farmer-Guo one-sector model economy exhibits a continuum of stationary sunspot equilibria under separable preferences and sufficiently strong increasing returns-to-scale in production. However, the degree of aggregate returns-to-scale needed for equilibrium indeterminacy is implausibly high within these authors' analytical framework. Considerable progress has been made since in asserting the empirical plausibility of self-fulfilling competitive equilibria. In particular, Benhabib and Farmer (1996), Weder (2000) and Harrison (2001) show that in a representative-agent model with two distinct production sectors that yield consumption and investment goods, the minimum degree of returns-to-scale in production for generating belief-driven business cycles is much less stringent, thus lies in the range of empirical plausibility. Nevertheless, all of these early studies postulate infinitely elastic labor supply, which is known to be inconsistent with the U.S. data at the micro-level. While subsequent research has considered lower labor supply elasticities,[3] these values are higher than that recommended by Chetty et al. (2011, 2012) for calibrating macroeconomic models to match an aggregate Frisch elasticity of 0.5 on the intensive margin.[4]

On the other hand, some recent research has examined the theoretical as well as quantitative interrelations between tax policies and equilibrium (in)determinacy within a representative-agent macroeconomy. For example, Guo and Harrison (2001, 2011) find that regressive income taxation may stabilize the economy against sunspot-driven business cycle fluctuations in a two-sector RBC model which possesses an indeterminate steady state under laissez faire; and that progressive taxes can operate like an "automatic destabilizer" in leading to indeterminacy and sunspots.[5] However, Guo and Harrison (2001, 2011) calibrate agents' labor supply elasticity to be two or above, and do not investigate the empirical plausibility on the requisite level of tax progressivity/regressivity for any of their results.

Motivated by the aforementioned gaps in this indeterminacy literature, we incorporate realistically plausible combinations of (i) the household's labor supply elasticity and (ii) the progressive tax schedule *a la* Guo and Lansing (1998) into a discrete-time two-sector RBC model, as in Harrison (2001), with positive productive externalities present in the investment

goods sector.[6] With regard to calibrating the level and slope parameters of our postulated fiscal policy rule, we follow Chen and Guo's (2013) empirical estimates from the U.S. federal individual income tax schedule for the $1966 - 2005$ period. For analytical completeness, our analysis considers two preference specifications that are commonly adopted in the real business cycle literature. Specifically, the household's utility function is assumed to be additively separable between consumption and hours worked in Model 1; whereas a non-separable preference formulation that does not exhibit income effect associated with agents' labor supply decision is studied in Model 2.[7]

We examine the local stability properties for each version of our model economy under parameter values that are consistent with post Korean-war U.S. time series data. It turns out that for a given value of the labor supply elasticity, equilibrium indeterminacy results with a lower threshold level of increasing returns-to-scale in investment when the tax schedule becomes more progressive, regardless of whether the household's utility function is separable or non-separable between consumption and labor hours. Intuitively, progressive income taxation generates a leftward shift of the convex social production possibility frontier, which in turn causes agents to reduce their optimism-driven consumption as well as investment expenditures. It follows that the investment effect that helps make for multiple equilibria is weakened, whereas the corresponding consumption and price effects are strengthened. Under the benchmark parameterization, the combined consumption and price effects are shown to outweigh the investment effect within each setting of our model. Therefore, indeterminacy and sunspots are *ceteris paribus* easier to occur in that lower investment externalities are needed.

For a given level of positive tax progressivity, we find that Model 1 under separable preferences is more susceptible to equilibrium indeterminacy when the household's labor supply elasticity rises. With more elastic labor supply, agents are more willing to move out of leisure into hours worked, which in turn reduces the requisite degree of investment externalities that fulfills their initial rosy anticipation about an expansion in future output. On the contrary, movements in total labor hours across time periods must be kept small in Model 2 in order to satisfy the intertemporal consumption Euler equation upon agents' optimistic expectation. Therefore, the smaller the labor supply elasticity, the easier it is to induce endogenous business cycles under no-income-effect preferences.

Of particular interest here is the empirical plausibility of the minimum level of productive externalities in the investment goods sector needed for indeterminacy and sunspots. Unlike Benhabib and Farmer (1996), Weder (2000) and Harrison (2001) with infinitely elastic labor supply, the required threshold investment externalities in our laissez-faire economy un-

der either preference formulation are too high *vis-à-vis* Harrison's (2003) empirical estimates. However, when calibrated to match with the observed tax progressivity in U.S., each version of our model exhibits an indeterminate steady state under empirically realistic combinations of the household's labor supply elasticity and returns-to-scale in the production of investment goods. Since equilibrium indeterminacy and belief-driven business cycles take place in the most empirically-relevant parameterizations of our model, we conclude that aggregate instability due to self-fulfilling expectations may in fact be a prevailing feature of the U.S. economy.

The remainder of this paper is organized as follows. Section 2 describes the model and analyzes its equilibrium conditions. Section 3 undertakes a quantitative investigation of macroeconomic (in)stability in a calibrated version of our model economy. Section 4 concludes.

2   The Model

Our model economy consists of households, firms and the government. In particular, we consider two preference formulations in a discrete-time two-sector real business cycle (RBC) model with the progressive tax policy *a la* Guo and Lansing (1998). Households live forever, and derive utility from consumption and leisure. In Model 1, the household utility is postulated to be additively separable between consumption and hours worked, as in Benhabib and Farmer (1994, 1996), Harrison (2001), and Guo and Harrison (2001, 2011). Model 2 examines a nonseparable preference specification that does not exhibit an income effect in labor supply, as in Meng and Yip (2008), Jaimovich (2008) and Guo and Harrison (2010). The economy also includes two production sectors that yield consumption and investment goods, respectively. For expositional simplicity, firms in each sector produce output using identical technologies, but positive productive externalities are limited to the investment goods sector. We further assume that there are no fundamental uncertainties present in the economy.

2.1   The Firms' Problems

In the consumption goods sector, output is produced by competitive firms using the following constant returns-to-scale Cobb-Douglas technology:

$$Y_{ct} = K_{ct}^{\alpha} L_{ct}^{1-\alpha}, \quad 0 < \alpha < 1, \tag{1}$$

where K_{ct} and L_{ct} are the capital and labor inputs used in the production of consumption goods. Under the assumption that factor markets are perfectly competitive, the first-order conditions for these firms' profit maximization are

$$r_t = \frac{\alpha Y_{ct}}{K_{ct}} \quad \text{and} \quad w_t = \frac{(1-\alpha)\,Y_{ct}}{L_{ct}}, \tag{2}$$

where r_t is the capital rental rate and w_t is the real wage.

Similarly, investment goods are produced by a unit measure of identical competitive firms with the production technology

$$Y_{It} = A_t K_{It}^{\alpha} L_{It}^{1-\alpha}. \tag{3}$$

Here, K_{It} and L_{It} are capital and hours worked utilized in the investment goods sector, and A_t represents productive externalities that each individual firm takes as given. In addition, A_t is specified as

$$A_t = (\bar{K}_{It}^{\alpha} \bar{L}_{It}^{1-\alpha})^{\theta}, \quad \theta \geq 0, \tag{4}$$

where $\bar{K}_{It}$ and $\bar{L}_{It}$ denote the economy-wide average levels of capital and labor devoted to producing investment goods, and θ measures the degree of sector-specific externalities in the investment goods sector. In a symmetric equilibrium, all firms in the investment goods sector make the same decisions such that $K_{It} = \bar{K}_{It}$ and $L_{It} = \bar{L}_{It}$, for all t. As a result, (4) can be substituted into (3) to obtain the following aggregate production function for investment that may display increasing returns-to-scale:

$$Y_{It} = K_{It}^{\alpha(1+\theta)} L_{It}^{(1-\alpha)(1+\theta)}, \tag{5}$$

where $\alpha(1+\theta) < 1$ to rule out the possibility of sustained economic growth. The first-order conditions that govern the firms' demand for capital and labor in the investment goods sector are

$$r_t = p_t \frac{\alpha Y_{It}}{K_{It}} \quad \text{and} \quad w_t = p_t \frac{(1-\alpha)\,Y_{It}}{L_{It}}, \tag{6}$$

where p_t denotes the relative price of investment to consumption goods at time t. Notice that firms in each sector face the same equilibrium factor prices since capital and labor inputs are assumed to be perfectly mobile across the two sectors.

2.2   The Household's Problem

The economy is populated by a unit measure of identical infinitely-lived households. Each household is endowed with one unit of time and maximizes its present discounted lifetime utility

$$\sum_{t=0}^{\infty} \beta^t U(C_t, L_t), \quad 0 < \beta < 1, \tag{7}$$

where β is the discount factor, and C_t and L_t are the representative household's consumption and hours worked, respectively. In this paper, we consider the following two specifications of the period utility function $U(\cdot)$ that are commonly adopted in the real business cycle literature:

$$U_1 = \log C_t - A\frac{L_t^{1+\chi}}{1+\chi}, \quad A > 0, \tag{8}$$

and

$$U_2 = \log(C_t - \Lambda\frac{L_t^{1+\chi}}{1+\chi}), \quad \Lambda > 0, \tag{9}$$

where $\chi \geqslant 0$ denotes the inverse of the wage elasticity for labor supply. The "indivisible labor" formulation of Hansen (1985) and Rogerson (1988) corresponds to the case of $\chi = 0$ whereby aggregate fluctuations in labor hours are caused by the household's extensive-margin responses (entering or out of employment). When $\chi > 0$, agents are able to adjust continuously along the intensive margin on the number of their hours worked.

The budget constraint faced by the representative household is

$$C_t + p_t I_t \leq (1 - \tau_t)(r_t K_t + w_t L_t) + TR_t, \tag{10}$$

where I_t is gross investment, K_t is the household's capital stock, TR_t denotes lump-sum transfer payments, and τ_t represents the income tax rate. The law of motion for the capital stock is given by

$$K_{t+1} = (1 - \delta)K_t + I_t, \quad K_0 > 0 \quad \text{given}, \tag{11}$$

where $\delta \in (0, 1)$ is the capital depreciation rate.

As in Guo and Lansing (1998), we postulate that τ_t takes the form

$$\tau_t = 1 - \eta\left(\frac{Y}{Y_t}\right)^\phi, \quad \eta \in (0, 1] \quad \text{and} \quad \phi \in [0, 1), \tag{12}$$

where $Y_t = r_t K_t + w_t L_t$ is the household's taxable income, and Y denotes the steady-state level of per capita income that is taken as given by each household. The parameters η and ϕ govern the level and slope of the tax schedule, respectively. Using (12), we obtain the expression for the marginal tax rate of income τ_t^m, which is defined as the change in taxes paid by the household divided by the change in its taxable income, as follows:

$$\tau_t^m \equiv \frac{\partial(\tau_t Y_t)}{\partial Y_t} = 1 - \eta(1 - \phi)\left(\frac{Y}{Y_t}\right)^\phi. \tag{13}$$

Households are postulated to take into account the way in which the tax schedule affects their earnings when they decide how much to work, consume and invest over their lifetimes.

Consequently, it is the marginal tax rate of income that governs the household's economic decisions.

In this paper, our analyses are restricted to environments in which $0 < \tau_t < 1$ and $0 < \tau_t^m < 1$ such that (i) the government does not have access to lump-sum taxes, (ii) the government can not confiscate all productive resources, and (iii) households have an incentive to provide labor and capital services to firms. Moreover, in order to guarantee the existence of an interior steady state, the economy's equilibrium after-tax interest rate, $(1 - \tau_t^m) r_t$, must be a monotonically decreasing function of K_t, which in turn imposes a lower bound on τ_t^m. In the steady state, the above considerations imply that $\eta \in (0,1)$ and $\phi \in \left(\frac{\alpha(1+\theta)-1}{\alpha(1+\theta)}, 1 \right)$, where $\frac{\alpha(1+\theta)-1}{\alpha(1+\theta)} < 0$.

Given these restrictions on η and ϕ, it is straightforward to show that when ϕ is positive, the marginal tax rate (13) is higher than the average tax rate given by (12). In this case, the tax schedule is said to be "progressive". When ϕ is equal to zero, the average and marginal tax rates coincide at the level of $1 - \eta$, thus the tax schedule is "flat". When ϕ is negative, the tax schedule is said to be "regressive". Since the U.S. federal individual income tax schedule is progressive as it is characterized by several tax "brackets" (branches of income) that are taxed at progressively higher rates, the specification of $\phi > 0$ will be the focus of our model calibrations. To provide a useful benchmark for the subsequent quantitative analyses, we also examine the economy under laissez faire without income taxation ($\eta = 1$ and $\phi = 0$). As a result, the parametric constraints of $0 < \eta \leq 1$ and $0 \leq \phi < 1$ are imposed in (12).

The first-order conditions for the household's dynamic optimization problem are given by

$$AC_t L_t^\chi = (1 - \tau_t^m) w_t, \tag{14}$$

$$\Lambda L_t^\chi = (1 - \tau_t^m) w_t, \tag{15}$$

$$\frac{1}{C_t} = \frac{\beta}{C_{t+1}} \left[\frac{(1 - \tau_{t+1}^m) r_{t+1} + (1 - \delta)p_{t+1}}{p_t} \right], \tag{16}$$

$$\frac{1}{C_t - \Lambda \frac{L_t^{1+\chi}}{1+\chi}} = \frac{\beta}{C_{t+1} - \Lambda \frac{L_{t+1}^{1+\chi}}{1+\chi}} \left[\frac{(1 - \tau_{t+1}^m) r_{t+1} + (1 - \delta)p_{t+1}}{p_t} \right], \tag{17}$$

$$\lim_{t \to \infty} \beta^t \frac{K_{t+1}}{C_t} = 0, \tag{18}$$

$$\lim_{t \to \infty} \beta^t \frac{K_{t+1}}{C_t - \Lambda \frac{L_t^{1+\chi}}{1+\chi}} = 0, \tag{19}$$

where (14) and (15) equate the slope of the household's indifference curve to the after-tax real wage under U_1 and U_2, respectively. Since consumption C_t is missing from condition (15),

there is no income effect associated with the household's labor supply decision in Model 2. Furthermore, (16) and (17) are the standard Euler equations for intertemporal consumption choices; and equations (18) and (17) are the transversality conditions.

2.3 The Government

The government chooses the tax policy τ_t, and returns all its tax revenue to households as a lump-sum transfer TR_t. Hence, its period budget constraint is given by

$$TR_t = \tau_t Y_t. \tag{20}$$

Finally, combining (10) and (20) leads to the following aggregate resource constraint for the economy:

$$C_t + p_t I_t = Y_t. \tag{21}$$

2.4 Equilibrium and Local Dynamics

Since firms use identical production technologies and face the same factor prices across the two sectors, the fractions of capital and labor inputs used in the consumption goods sector are equal,

$$\frac{K_{ct}}{K_t} = \frac{L_{ct}}{L_t} \equiv \mu_t. \tag{22}$$

We focus on symmetric perfect-foresight equilibria that consist of a set of prices $\{p_t, r_t, w_t\}_{t=0}^{\infty}$ and quantities $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ that satisfies the household's and firms' first-order conditions. The equalities of demand by households and supply by firms in the consumption and investment goods sectors are given by $C_t = Y_{ct}$ and $I_t = Y_{It}$. In addition, both the capital and labor markets will clear whereby

$$K_{ct} + K_{It} = K_t, \tag{23}$$

$$L_{ct} + L_{It} = L_t. \tag{24}$$

It is straightforward to show that our model possesses a unique interior steady state. Specifically, the steady-state transfer payments to output ratio, fraction of factor inputs allocated to the consumption goods sector, and capital rental rate are given by

$$\frac{TR}{Y} = 1 - \eta, \quad \mu = 1 - \frac{\alpha\delta\eta(1-\phi)}{\frac{1}{\beta} - 1 + \delta} \quad \text{and} \quad r = \frac{\mu^\theta\left(\frac{1}{\beta} - 1 + \delta\right)}{\eta\left(1 - \phi\right)\left(1 - \mu\right)^\theta}, \tag{25}$$

where time subscripts are left out to denote steady-state values. Given (25), the steady-state expressions of all other endogenous variables can be easily derived. We then take log-linear approximations to the model's equilibrium conditions in the neighborhood of this steady state to obtain the following dynamic system:

$$\left[\begin{array}{c} \hat{K}_t \\ \hat{C}_t \end{array} \right] = J \left[\begin{array}{c} \hat{K}_{t+1} \\ \hat{C}_{t+1} \end{array} \right], \quad \hat{K}_0 \quad \text{given,} \tag{26}$$

where hat variables denote percentage deviations from their steady-state values, and J is the Jacobian matrix of partial derivatives of the transformed dynamical system. The economy exhibits saddle-path stability and equilibrium uniqueness when one eigenvalue of J lies inside and the other outside the unit circle. When both eigenvalues are outside the unit circle, the steady state becomes an indeterminate sink around which there are a continuum of stationary equilibrium trajectories that display cyclical fluctuations driven by agents' animal spirits or sunspots. When both eigenvalues are inside the unit circle, the steady state becomes a totally unstable source.

3   Quantitative Analysis

This section examines the local stability properties for each version of our model economy under parameter values that are consistent with post Korean-war U.S. time series data. Each period in the model is taken to be one quarter. As is common in the real business cycle literature, the capital share of national income, α, is chosen to be 0.3; the discount factor, β, is set equal to 0.99; and the capital depreciation rate, δ, is fixed at 0.025. With regard to calibrating the tax-schedule parameters according to (12), we note that Chen and Guo (2013) follow the nonlinear least squares estimation methodology *a la* Cassou and Lansing (2004) and obtain year-by-year empirical estimates of η and ϕ from the U.S. federal individual income tax schedule for the $1966 - 2005$ period, with a resulting average $R^2 = 0.867$. Based on the mean values of Chen and Guo's (2013) point estimates, $\eta = 0.8$ and $\phi = 0.12$ are adopted in our benchmark formulation. Given the above parameterization, we then analyze the model's equilibrium dynamics for different combinations of χ and θ. In each parametric configuration, the preference parameters – A in (8) and Λ in (9) – are selected to ensure that the steady-state level of labor hours is equal to 1/3.

[Figure 1 About Here]

[Figure 2 About Here]

Under the baseline parameterization, Figures 1 and 2 depict the quantitative interrelations between the minimum degree of investment externalities $\theta_{\min}$, above which the economy

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exhibits an indeterminate steady state, and the parameter χ that governs the inverse of the household's labor supply elasticity for Models 1 and 2, respectively. Each figure also plots the local dynamics of our no-government economy with $\eta = 1$ and $\phi = 0$, as in Harrison (2001) and Guo and Harrison (2010), among others. It turns out that for a given value of χ, the dividing curve for "$\phi = 0.12$" under progressive taxes lies entirely below that for "no tax" under laissez faire in both figures. This implies that in our two-sector RBC model, equilibrium indeterminacy results with a lower threshold level of increasing returns-to-scale in investment when a progressive tax policy rule is present, regardless of whether the household utility is separable or non-separable between consumption and hours worked.

To intuitively understand the above result, start the laissez-faire ($\eta = 1$ and $\phi = 0$) model from its steady state, and suppose that agents become optimistic about the economy's future. Acting upon this belief, the representative household will consume less (the consumption effect) and invest more (the investment effect) today, thus raising the next period's capital stock and output. If the external effects in the firms' production processes are sufficiently strong, the rate of return on capital will rise because of a fall in the relative price of investment goods (the price effect). As a result, agents' initial rosy expectation can be justified as a self-fulfilling equilibrium. In this environment, progressive income taxation ($\phi > 0$) leads to a leftward shift of the convex social production possibility frontier, which in turn induces agents to reduce their optimism-driven consumption as well as investment expenditures. It follows that the aforementioned investment effect that helps make for multiple equilibria becomes weaker, whereas the corresponding consumption and price effects are strengthened. Figures 1 and 2 show that under the benchmark parameterization, the consumption and price effects together outweigh the investment effect within each setting of our model. Therefore, endogenous business cycles are *ceteris paribus* easier to occur, in the sense that lower investment externalities are needed, as the tax progressivity increases, *i.e.* $\frac{\partial \theta_{\min}}{\partial \phi} < 0$.

On the other hand, Figure 1 demonstrates that for a given level of positive tax progressivity, $\theta_{\min}$ and χ are positively related $\left(\frac{\partial \theta_{\min}}{\partial \chi} > 0 \right)$ in our Model 1. The intuition for this result is the same as in Benhabib and Farmer's (1996) no-government economy under an additively separable utility formulation (8). With more elastic labor supply (or when χ falls), agents are more willing to move out of leisure into hours worked. Consequently, the investment effect becomes stronger, which in turn reduces the requisite degree of investment externalities that fulfills the household's anticipation of an expansion in future output. By contrast, Figure 2 shows that for a given level of positive tax progressivity, $\theta_{\min}$ and χ are negatively related $\left(\frac{\partial \theta_{\min}}{\partial \chi} < 0 \right)$ in our Model 2. The intuition for this finding is the same as in Guo and Harrison's

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(2010) laissez-faire economy under non-separable preferences (9). Upon agents' optimistic expectation, movements in total labor hours across time periods must be kept small in order to satisfy the associated intertemporal Euler equation (17). Therefore, the smaller the labor supply elasticity (or when χ rises), the easier indeterminacy and sunspots are to obtain, in that lower returns-to-scale in the production of investment goods are needed.[8]

For a quantitative illustration of the preceding results, Table 1 presents the values of $\theta_{\min}$ under several empirically realistic combinations of ϕ and χ. We note that while indivisible labor ($\chi = 0$) is postulated in many early indeterminacy studies such as Benhabib and Farmer (1994) and Farmer and Guo (1994), subsequent work has explored lower labor supply elasticities. For example, Guo and Harrison (2001) adopt 4 in their baseline parameterization, whereas Harrison and Weder (2013) undertake 2.2. However, recent research by Chetty et al. (2011, 2012) find that modern macroeconomic calibrations imply a much larger labor supply elasticity than that observed in the micro-level evidence, and recommend an aggregate Frisch elasticity of 0.5 on the intensive margin. Based on this suggestion, we examine the case with $\chi = 2$ in Table 1. In addition, Altonji (1986) reports that the estimated intertemporal labor supply elasticity is 0.067 in the U.S. economy, hence we also consider $\chi = 15$.

Table 1: Threshold Investment Externalities $\theta_{\min}$

	Model 1		Model 2	
	no tax	$\phi = 0.12$	no tax	$\phi = 0.12$
$\chi = 2$	0.207	0.140	0.404	0.232
$\chi = 15$	0.269	0.179	0.300	0.193

In addition to numerically verifying that (i) $\frac{\partial \theta_{\min}}{\partial \phi} < 0$ in both versions of our model economy, (ii) $\frac{\partial \theta_{\min}}{\partial \chi} > 0$ in Model 1, and (iii) $\frac{\partial \theta_{\min}}{\partial \chi} < 0$ in Model 2 (all discussed earlier), Table 1 shows that under the same parameterization of ϕ and χ, the required level of investment externalities for equilibrium multiplicity is lower in an economy with a separable utility function than that with no-income-effect preferences. As for the one-sector counterpart analyzed by Meng and Yip (2008) and Jaimovich (2008), this result illustrates the quantitative importance of the income effect associated with agents' labor supply decision in generating belief-driven cyclical fluctuations within two-sector RBC models.

Of particular interest here is the empirical plausibility of $\theta_{\min}$ for indeterminacy and sunspots within both versions of our model economy. Using durables as a proxy for the investment goods, Harrison (2003) reports that the point estimate on the degree of productive externalities in investment from a sample of U.S. two-digit manufacturing industry data is 0.15, with the upper bound of its 95% confidence interval equal be $\theta = 0.196$. Table 1 shows

that irrespective of the household's preference formulation, the threshold level of investment externalities in our "no tax" economy ($\eta = 1$ and $\phi = 0$) is too high *vis-à-vis* Harrison's (2003) empirical estimates. Nevertheless, since (as discussed above) equilibrium indeterminacy is more easily reached under progressive taxation, we find that the requisite value of $\theta_{\min}$ is empirically plausible in Model 1 under $\phi = 0.12$ together with $\chi = 2$ or $\chi = 15$; and that the same result holds in our Model 2 under $\phi = 0.12$ together with $\chi = 15$.

For a sensitivity analysis, we allow the tax-slope parameter ϕ to take on the values of 0.175 and 0.23 – these are respectively one and two standard deviations above Chen and Guo's (2013) average point estimate; and then report the resulting threshold level of investment externalities needed for multiple equilibria in Table 2. As illustrated in Table 1 under $\phi = 0.12$, our Model 1 and Model 2 with $\chi = 15$ continue to exhibit empirical plausibility for indeterminacy and sunspots as the tax schedule becomes more progressive because of $\frac{\partial \theta_{\min}}{\partial \phi} < 0$. Table 2 also shows that when the household's labor supply elasticity is equal to 0.5 ($\chi = 2$), the requisite $\theta_{\min}$ for equilibrium indeterminacy becomes empirically plausible within Model 2 under the highest possible level of tax progressivity $\phi = 0.23$ which can be regarded as realistic.

Table 2: Sensitivity Analysis on $\theta_{\min}$				
	Model 1		Model 2	
	$\phi = 0.175$	$\phi = 0.23$	$\phi = 0.175$	$\phi = 0.23$
$\chi = 2$	0.130	0.122	0.211	0.192
$\chi = 15$	0.165	0.153	0.179	0.165

In sum, the local stability properties of a two-sector real business cycle model depends on (i) the slope of the tax schedule ϕ, (ii) the wage elasticity of agents' labor supply decision χ, and (iii) the level of positive productive externalities in the investment goods sector θ. This section shows that our model economy with separable or no-income-effect preferences, when calibrated to match with the observed tax progressivity in U.S., exhibits an indeterminate steady state under realistically plausible combinations of the labor supply elasticity and returns-to-scale in producing investment goods. Since equilibrium indeterminacy and endogenous business cycles take place in the most empirically-relevant parameterizations of our model, macroeconomic instability due to self-fulfilling expectations may in fact be a prevalent feature of the U.S. economy.

4   Conclusion

This paper has examined how the quantitative interrelations between (i) the tax progressivity of the fiscal policy rule, (ii) the household's labor supply elasticity and (iii) the degree of

increasing returns-to-scale in producing investment goods affect the equilibrium dynamics of a two-sector real business cycle model with separable or non-separable preferences. Under a progressive tax schedule calibrated to match that observed in the U.S. and a non-perfectly elastic labor supply consistent with the micro-level evidence, we find that the threshold level of investment externalities needed for indeterminacy and sunspots is empirically plausible within both versions of our model economy. This result implies that aggregate instability due to agents' self-fulfilling expectations may in fact be a prevailing feature of the U.S. economy.

5 Footnotes

1. We thank Yong Wang (Editor), an anonymous referee, Been-Lon Chen, Shu-Hua Chen, Alain Venditti, Yan Zhang and seminar participants at Society for the Advancement of Economic Theory Conference for helpful comments and suggestions. Part of this research was conducted while Guo was a visiting research fellow of economics at Academia Sinica, Taipei, Taiwan, whose hospitality is greatly appreciated. Of course, all remaining errors are our own.

2. See Benhabib and Farmer (1999) for an excellent survey on this indeterminacy literature in which the terms animal spirits, sunspots, and self-fulfilling prophecies are used interchangeably.

3. For example, Guo and Harrison (2001) adopt 4 in their baseline parameterization, whereas Harrison and Weder (2013) undertake 2.2.

4. Interestingly, Huang and Meng (2012) show that equilibrium indeterminacy can emerge in a one-sector RBC model with sticky wages, regardless of the magnitude of the household's labor supply elasticity.

5. Guo and Harrison (2011) point out an error in Guo and Harrison's (2001) description of the household's and government's budget constraints, and then show that all of the authors' earlier results remain qualitatively unchanged.

6. In the continuous-time version of a two-sector RBC model under laissez faire, Garnier, Nishimura and Venditti (2007, 2013) and Nishimura and Venditti (2010) also examine the requisite conditions for indeterminacy and sunspots when the household's labor supply elasticity takes on an empirically plausible value.

7. Meng and Yip (2008) and Jaimovich (2008) show that with this no-income-effect preference formation, a one-sector RBC economy always exhibits saddle-path stability and equilibrium uniqueness under laissez faire. This result is overturned by Guo and Harrison (2010) in a two-sector RBC model with sufficiently strong investment externalities.

8. In a laissez-faire two-sector RBC model with CES production functions and sector-specific externalities, Nishimura and Venditti (2006) obtain the same result under separable preferences that exhibit a sufficiently high intertemporal elasticity of substitution in consumption.

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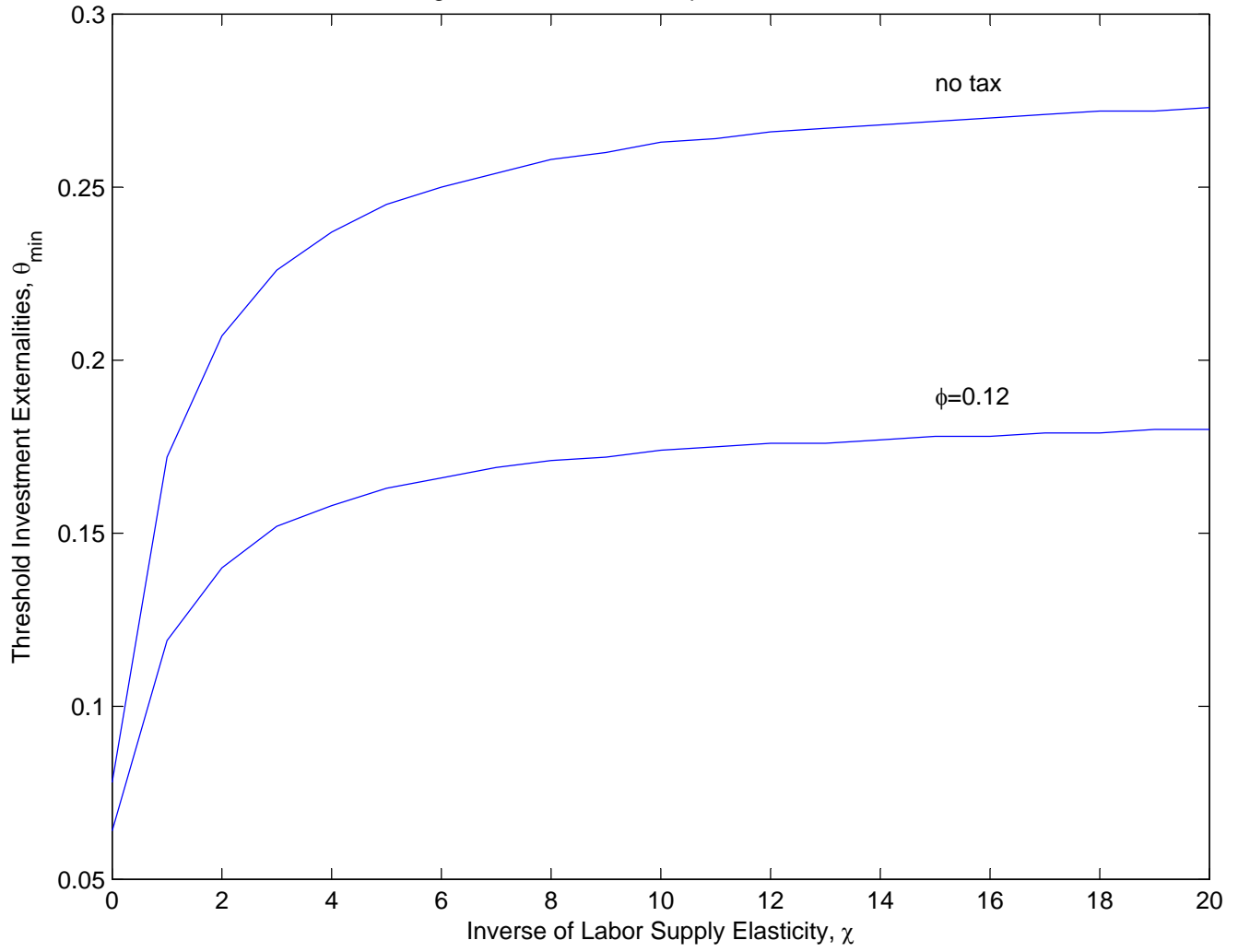
Figure 1: Model 1 with Separable Preferences

Figure 2: Model 2 with No-Income-Effect Non-Separable Preferences